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# **Discussion** paper





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#### TESTING FOR MEAN-VARIANCE SPANNING WITH SHORT SALES CONSTRAINTS AND TRANSACTION COSTS: THE CASE OF EMERGING MARKETS

By Frans A. de Roon, Theo E. Nijman and Bas J.M. Werker

January 1998

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## Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets

#### January 1998

#### Abstract

In this paper we propose tests for mean-variance spanning and intersection in case investors face market frictions such as short sales constraints and transaction costs. We show how regression techniques can be used to test for mean-variance spanning and intersection in case there are such frictions. The tests are applied to address the question whether US investors can extend their efficient set by investing in emerging markets in the presence of short sales constraints and transaction costs. The results show that although in the absence of market frictions mean-variance spanning of the emerging markets by three mature market indices is strongly rejected, the evidence that emerging markets provide significant diversification benefits is much weaker when short sales constraints, transaction costs and ownership restrictions are taken into account.

#### 1 Introduction

The question whether or not an investor can extend his efficient set by including additional assets in his portfolio has recently received considerable attention in the literature. If extension of the efficient set is not possible for one specific mean-variance utility function, the mean-variance frontier of the benchmark assets and of the benchmark assets plus the additional assets intersect, i.e., they have one point in common. If extension of the efficient set is not possible for any mean-variance utility function, the mean-variance frontier of the initial assets spans the frontier of the larger set of the initial assets plus the additional assets. These concepts are discussed by Huberman & Kandel (1987), who propose regression-based tests of the hypotheses of spanning and intersection for mean-variance investors. It is well known by now that testing whether there is a significant shift in the mean-variance frontier from adding assets to the investment opportunity set is tantamount to testing whether there is a significant shift in the volatility bounds of the kernels that price the assets under consideration (e.g., DeSantis, 1994, Bekaert & Urias, 1996) and that the issue is also very closely related to performance evaluation (see, e.g., Jobson & Korkie, 1988, Chen & Knez, 1996). DeRoon, Nijman & Werker (1997) show how tests for spanning can be extended to allow for other utility functions, and to allow for zero investment securities such as futures contracts and for the presence of nonmarketable risks.

Tests for intersection and spanning have been applied to numerous problems in the finance literature. For instance, DeSantis (1994), Harvey (1995), and Bekaert & Urias (1996) perform tests whether investors can realize a significant shift in the mean-variance frontier if they invest part of their wealth in emerging markets besides their investments in well-developed western markets. Similarly, Glen & Jorion (1993) investigate whether or not there are significant benefits from currency hedging for a mean-variance investor and Chen & Knez (1996) and Cumby & Glen (1990) discuss applications to performance measurement of mutual funds.

A crucial assumption in almost all tests for extension of the efficient set that have been proposed in the literature, is the absence of market frictions such as short sales restrictions and transaction costs. For many investors, however, such frictions are important facts of life. The aim of this paper is to extend the tests for mean-variance spanning and intersection in order to take these market frictions into account. The paper is therefore related to Hansen, Heaton & Luttmer (1995) who derive the asymptotic distribution

of specification error bounds allowing for market frictions, and to Luttmer (1996) who analyzes the impact of market frictions on volatility bounds. Especially the region subset test considered by Hansen, Heaton & Luttmer is closely related to some results presented in this paper. They do not consider testing for spanning however. Glen & Jorion (1993) have proposed an alternative way to test for spanning in case of short sales constraints on the additional assets, but their test is more restrictive than ours in a number of ways. A detailed comparison of these test procedures and the one proposed in this paper will be presented in Section 3.

Transaction costs and short sales constraints are important in many investment problems, but perhaps their presence is most predominant in the case of emerging markets. Using the Emerging Market Data Base (EMDB) of the International Finance Corporation (IFC) both DeSantis (1994) and Harvey (1995) show that the mean-variance frontier that is based on welldeveloped western markets only, significantly shifts outward when the emerging markets are included. However, these results presuppose that there are no transaction costs or any other market frictions for both the developed and the emerging markets. Using returns on closed-end country funds Bekaert & Urias (1996) try to overcome this problem, since the returns on these funds are attainable to investors. Based on emerging market country funds Bekaert & Urias find only mixed evidence for the diversification benefits of emerging markets. Although the use of country funds adjusts for the effect of transaction costs and short sales constraints that investors face in emerging markets. it does not account for short sales constraints and transaction costs on the country funds themselves or on the benchmark assets.

We provide direct evidence on the effect of transaction costs and short selling constraints on the diversification benefits of emerging markets, by using the same IFC Indices as in DeSantis and Harvey, but incorporating these market frictions in our testing methodology. Our results show that the test statistics are affected in a nontrivial way by the presence of short sales constraints and transaction costs and that it is important to account for these effects in both the emerging markets as well as the benchmark assets. Although the evidence against mean-variance spanning is weaker when short sales constraints on both the emerging markets and the benchmark assets are taken into account, the hypothesis of spanning can still be rejected for many emerging markets. However, when incorporating transaction costs it is much harder to reject the hypothesis of spanning, at least when investors trade their portfolio on a monthly basis. For investors that trade their portfolio

less frequently their is still evidence in favor of the diversification benefits of emerging markets. For investors that have an investment horizon of one month, the critical level of transaction costs above which the hypothesis of spanning can not be rejected are usually smaller than the estimates of the size of these transaction costs that have been reported in the literature. Even though the hypothesis of spanning is still rejected for a number of emerging markets when there are short sales constraints and transaction costs, suggesting diversification benefits are still possible, these results must be interpreted with caution since foreign ownership restrictions may prevent investors from realizing these benefits. Indeed, when performing some of the spanning tests for the IFC Investable Indices, which take foreign ownership restrictions into account but which are available for a shorter sample period only, there is hardly any evidence left against the hypothesis of spanning.

The plan of this paper is as follows. In Section 2 we first of all formulate the hypotheses of mean-variance spanning and intersection in case of short sales restrictions. Regression-based tests for these hypotheses are proposed in Section 3. In Section 4 the analysis is extended to the case of transaction costs. The empirical results on investing in emerging markets are presented in Section 5 and in the final section we will offer some concluding remarks.

#### 2 Mean-variance spanning with short sales constraints

Consider a set of K assets, whose gross returns are given by the vector  $R_{t+1}$ . Investors can hold portfolios  $w \in C \subset \mathbb{R}^K$  such that  $w'\iota_K = 1$ , where  $\iota_K$  is a K-vector containing only ones. The set of returns available to investors is therefore given by:

$$X = \{R^p_{t+1} : R^p_{t+1} = w'R_{t+1}, \ w \in C, \text{ and } w'\iota_K = 1\}.$$

Let us first of all reconsider the case where there are no market frictions, i.e.,  $C = \mathbb{R}^K$ . If, in addition, the Law of One Price holds, there exists a stochastic discount factor  $M_{t+1}$  such that:

$$E[M_{t+1}R_{t+1} | I_t] = \iota_K,$$
 (1)

where  $I_t$  denotes the information set that is available to investors at time t (see, e.g., Duffie, 1996). In this paper we will restrict ourselves to uncondi-

tional versions of (1) and to unconditional mean-variance spanning. Extensions of our results to the conditional case are straightforward however (see, e.g., DeSantis, 1994, DeRoon, Nijman & Werker, 1997).

In case of mean-variance optimizing behaviour, the stochastic discount factor is a linear function of the asset returns. If  $m(v)_{t+1}$  is a mean-variance stochastic discount factor that has expectation v, then  $m(v)_{t+1}$  is given by:

$$m(v)_{t+1} = v + \alpha'(R_{t+1} - E[R_{t+1}]),$$
 (2a)

$$\alpha = Var[R_{t+1}]^{-1}(\iota_K - vE[R_{t+1}]),$$
 (2b)

From Hansen & Jagannathan (1991) we know that the discount factor given in (2) has the lowest variance of all stochastic discount factors with expectation v, that price  $R_{t+1}$  correctly. It is also well known by now that  $w = \alpha/(\alpha' \iota_K)$  is a mean-variance efficient portfolio that has a zero-beta return equal to 1/v.

Now consider the presence of market frictions such as short sales constraints and transaction costs. These can be dealt with by letting C be a particular subset of  $\mathbb{R}^K$  and/or by adjusting the vector of returns  $R_{t+1}$  to reflect the frictions. In case of short sales constraints for instance,  $C = \mathbb{R}_+^K$ , the nonnegative part of  $\mathbb{R}^K$ . When there are short sales constraints on the portfolio holdings, the condition in (1) must be replaced by:

$$E[m(v)_{t+1}R_{t+1}] \le \iota_K. \tag{3}$$

Here the inequality sign applies componentwise. The mean-variance efficient frontier without short sales can be found by solving the problem:

$$\max_{\{w\}} w' E[R_{t+1}] - \frac{1}{2} \gamma w' Var[R_{t+1}] w,$$
s.t.  $w' \iota_K = 1$  and  $w_i \ge 0, \forall i,$  (4)

where  $\gamma$  is the coefficient of risk aversion. From the Kuhn-Tucker conditions, mean-variance efficient portfolios  $w^*$  satisfy:

$$E[R_{t+1}] - \eta \iota_K + \delta = \gamma Var[R_{t+1}]w^*,$$

$$w_i^* \geq 0,$$

$$\delta_i \geq 0,$$

$$\delta_i w_i^* = 0, \forall i.$$

$$(5)$$

The vector  $\delta$  contains the Kuhn-Tucker multipliers for the restrictions that the portfolio weights are nonnegative. The Lagrange multiplier for the restriction that  $w'\iota_K=1$ , is equal to  $\eta$ , the intercept of the line that is tangent to the mean-variance frontier in mean-standard deviation space.

Now take the mean-variance efficient portfolio for which  $\eta=1/v$ , with v the expectation of a stochastic discount factor that prices  $R_{t+1}$  correctly subject to short sales constraints. Denote by  $R_{t+1}^{(v)}$  the L-dimensional subvector of  $R_{t+1}$  that only contains the returns of the assets for which the short sales constraints in (5) are not binding and let superscripts v refer to this subset. It is straightforward to show that the mean-variance efficient portfolio in (5) is equal to the mean-variance efficient portfolio without short sales constraints of the assets in  $R_{t+1}^{(v)}$  only:

$$E[R_{t+1}^{(v)}] - \frac{1}{v}\iota_L = \gamma^{(v)}Var[R_{t+1}^{(v)}]w^{(v)}$$
 and (6)  
 $E[R_{t+1}] - \frac{1}{v}\iota_K + \delta^{(v)} = \gamma^{(v)}Cov[R_{t+1}, R_{t+1}^{(v)}]w^{(v)},$ 

where  $Cov[R_{t+1}, R_{t+1}^{(v)}]$  is the  $K \times L$ -dimensional covariance matrix of  $R_{t+1}$  and its subvector  $R_{t+1}^{(v)}$ . Thus the mean-variance efficient portfolio for a set of assets with return vector  $R_{t+1}$  subject to short sales constraints, is simply the mean-variance efficient portfolio for the subset of assets for which the restrictions are not binding (see, e.g., Markowitz, 1991). Observe that for the assets that are in  $w^{(v)}$  the Kuhn-Tucker multipliers  $\delta_i^{(v)}$  in (6) are zero.

Since the mean-variance stochastic discount factor is a linear function of the mean-variance efficient portfolio, in case of short sales restrictions the mean-variance stochastic discount factor that prices  $R_{t+1}$ ,  $m_R(v)_{t+1}$ , is equal to

$$m_R(v)_{t+1} = v + \alpha^{(v)'}(R_{t+1}^{(v)} - E[R_{t+1}^{(v)}]),$$
 (7)  
 $\alpha^{(v)} = Var[R_{t+1}^{(v)}]^{-1}(\iota_L - vE[R_{t+1}^{(v)}]).$ 

The *L*-dimensional vector of projection coefficients  $\alpha^{(v)}$  is of course proportional to the vector of mean-variance efficient portfolio weights  $w^{(v)}$ :  $w^{(v)}$ 

 $<sup>^1</sup>$ It can easily be seen from (6) that if we take the portfolio  $w^{(v)}$  as the benchmark portfolio, the vector of Kuhn-Tucker multipliers  $\delta^{(v)}$  is proportional to the vector  $\alpha_J(v)$  of Jensen's alphas of the returns  $R_{t+1}$  with respect to this benchmark portfolio. The vector of Jensen's alphas can be obtained as the intercept in a regression of the excess returns  $R_{t+1} - \frac{1}{v}\iota_K$  on the excess returns  $R_{t+1} - \frac{1}{v}\iota_L$  and a constant.

 $\alpha^{(v)}/t_L'\alpha^{(v)} = -\alpha^{(v)}/\gamma^{(v)}v$ . It is shown in Appendix A that the stochastic discount factor as defined in (7) has the lowest variance of all stochastic discount factors that have expectation v and that price  $R_{t+1}$  correctly subject to short sales constraints, as long as v > 0. Therefore, in case of short sales constraints the duality between mean-variance frontiers and volatility bounds still holds.

Next consider a set of N additional assets with return vector  $r_{t+1}$  besides the set of K benchmark assets with return vector  $R_{t+1}$ . Mean-variance spanning of the assets  $r_{t+1}$  by the benchmark assets  $R_{t+1}$  occurs if the mean-variance stochastic discount factors that price  $R_{t+1}$  correctly, also price  $r_{t+1}$ , i.e., if:

$$E[m_R(v)_{t+1}r_{t+1}] \le \iota_N, \tag{8}$$

holds for all values of v. Substituting (7) into (8), this is equivalent to:

$$vE[r_{t+1}] + Cov[r_{t+1}, R_{t+1}^{(v)}]Var[R_{t+1}^{(v)}]^{-1}(\iota_L - vE[R_{t+1}^{(v)}]) \le \iota_N.$$
 (9)

The inequality sign in (8) reflects the fact that there are short sales constraints on  $r_{t+1}$ . In the absence of short sales constraints on  $r_{t+1}$ , the inequality becomes an equality. If there is only one value of v for which (8) holds, then there is intersection. If  $m_R(v)_{t+1}$  prices  $r_{t+1}$ , agents whose marginal utility corresponds with  $m_R(v)_{t+1}$ , can not increase their utility by including the assets  $r_{t+1}$  in their portfolio besides the benchmark assets  $r_{t+1}$ . Because of the short sales constraints agents can only increase their utility by including an asset with return  $r_{t,t+1}$  if  $E[m_R(v)_{t+1}r_{t,t+1}] > 1$ .

#### 3 Testing for intersection and spanning

#### 3.1 Testing for intersection

Absent short sales constraints and any other market frictions, the hypotheses of mean-variance intersection and spanning are equivalent to the condition that

$$E[m_R(v)_{t+1}r_{t+1}] = \iota_N, \tag{10}$$

for one value of v (intersection) or for all values of v (spanning), where

$$m_R(v)_{t+1} = v + (\iota_K - vE[R_{t+1}])'Var[R_{t+1}]^{-1}(R_{t+1} - E[R_{t+1}]).$$

It is well known that in this case tests for intersection and spanning can be based on the regression

$$r_{t+1} = a + BR_{t+1} + \varepsilon_{t+1},$$
 (11)

with  $E[\varepsilon_{t+1}] = 0$  and  $E[\varepsilon_{t+1}R_{t+1}] = 0$ . Intersection for a given value of v implies that  $av + (B\iota_K - \iota_N) = 0$ , while spanning implies that a = 0 and  $B\iota_K - \iota_N = 0$  (Huberman & Kandel, 1987, DeSantis, 1994, Bekaert & Urias, 1996). Alternatively, GMM-tests can be used to test for intersection and spanning (DeSantis, 1994, Hansen, Heaton & Luttmer, 1995, Chen & Knez, 1996).

As shown in the previous section, if there are short sales restrictions on the benchmark assets  $R_{t+1}$ , the stochastic discount factor  $m_R(v)_{t+1}$  is a linear function of  $R_{t+1}^{(v)}$  only, and if there are short sales restrictions on the additional assets  $r_{t+1}$ , then the equality in (10) becomes an inequality. For a given value of v, the restrictions implied by intersection can be derived by substituting (7) into (8), which results in (9). These restrictions are equivalent to the restrictions that in the regression

$$r_{t+1} = a^{(v)} + B^{(v)}R_{t+1}^{(v)} + \varepsilon_{t+1}^{(v)}, \tag{12}$$

it holds true that

$$va^{(v)} + (B^{(v)}\iota_L - \iota_N) \le 0.$$
 (13)

Intuitively, since in case of short sales constraints the mean-variance efficient portfolio of  $R_{t+1}$  for a given value of v consists of positions in only those assets for which the constraints are not binding, intersection requires that there is intersection at the unrestricted frontier of  $R_{t+1}^{(v)}$  rather than at the unrestricted frontier of  $R_{t+1}$ . The inequality in (13) reflects the short sales constraints on  $r_{t+1}$ . If some elements on the left hand side of (13) are negative this would imply that a more efficient portfolio could be reached by taking short positions in the corresponding elements of  $r_{t+1}$ . Since such portfolios are unattainable with short sales constraints however, the inequality sign reflects that this situation would not violate the hypothesis that there is intersection.

A Wald test can be used to test the inequality constraints in (13) (see, e.g., Kodde & Palm, 1986). Denote the left hand side of (13) as  $v\alpha_J(v)$ , where  $\alpha_J(v)$  is the N-dimensional vector of Jensen's alphas of the assets  $r_{t+1}$  relative to the mean-variance efficient portfolio of  $R_{t+1}^{(v)}$  with zero-beta return

1/v. The sample equivalent of  $\alpha_J(v)$  is  $\widehat{\alpha}_J(v)$ , and the estimated  $N\times N$  covariance matrix of  $\widehat{\alpha}_J(v)$ ,  $Var[\widehat{\alpha}_J(v)]$ , can be obtained from the restricted covariance matrix of the OLS-estimates of (12), where the restrictions are given by  $va^{(v)} + (B^{(v)}\iota_L - \iota_N) = 0$ . Following Kodde & Palm, under the null hypothesis and standard regularity conditions, the test statistic

$$\xi(v) = \min_{\{\alpha_J(v) \le 0\}} (\widehat{\alpha}_J(v) - \alpha_J(v))' Var[\widehat{\alpha}_J(v)]^{-1} (\widehat{\alpha}_J(v) - \alpha_J(v)), \tag{14}$$

is asymptotically distributed as a mixture of  $\chi^2$  distributions. For the case considered here, where we test whether there is intersection for the N assets  $r_{t+1}$ , the probability of  $\xi(v)$  exceeding a given value c is, under the null-hypothesis, given by (see, e.g., Kodde & Palm, 1986)

$$\Pr\{\xi(v) \ge c\} = \sum_{i=0}^{N} \Pr\{\chi_i^2 \ge c\} w(N, i, Var[\hat{\alpha}_J(v)]),$$
 (15)

where  $w(N, i, Var[v\hat{\alpha}_J])$  are probability weights<sup>2</sup>. Given the estimated covariance matrix  $Var[\hat{\alpha}_J(v)]$ , the probabilities can be determined using numerical simulation, as proposed by Gouriéroux et al. (1982). Alternatively, without calculating the weights, Kodde & Palm (1986) show that an upper and a lower bound on the p-values of  $\xi(v)$  are given by

$$p_{up}[\xi(v)] = \frac{1}{2} \Pr\{\chi_{N-1}^2 \ge \xi(v)\} + \frac{1}{2} \Pr\{\chi_N^2 \ge \xi(v)\}$$

$$p_{low}[\xi(v)] = \frac{1}{2} \Pr\{\chi_1^2 \ge \xi(v)\}.$$
(16)

Of course, when implementing the intersection test in empirical applications, it is usually the case that for a particular value of v we do not observe which assets are in  $R_{t+1}^{(v)}$ , but have to derive this information from the asset returns in our sample. It is shown in Appendix B that this does not affect the limit distribution of the Wald test statistic for the restrictions in (13) however, if v corresponds to an efficient portfolio where none of the weights in  $w^{(v)}$  is exactly zero (i.e.,  $w_i^* = 0$  and  $\delta_i > 0$ ). If this latter situation occurs, then it is easily verified that the size of the test (conditional on  $R_{t+1}^{(v)}$ ) does not depend on  $R_{t+1}^{(v)}$ , and hence the unconditional size equals the one chosen,

<sup>&</sup>lt;sup>2</sup>The weights  $w(N, i, Var[v\widehat{\alpha}_J])$  are the probabilities that (N-i) of the N elements of a vector with a  $N(0, Var[v\widehat{\alpha}_J])$  distribution are strictly negative.

which shows the validity of our test. Further discussion of this point will follow at the end of this section.

As in Gibbons, Ross & Shanken (1989) it can be shown that the test statistic in (14) also has an interpretation in terms of Sharpe ratios:

$$\xi(v) = T \frac{\tilde{\theta}(v)^2 - \theta(v)^2}{1 + \theta(v)^2},\tag{17}$$

where  $\theta(v)^2$  is the maximum Sharpe ratio that can be obtained from the excess returns  $R_{t+1}^{(v)} - 1/v$ , and  $\tilde{\theta}(v)^2$  is the maximum Sharpe ratio that can be obtained from the excess returns  $R_{t+1}^{(v)} - 1/v$  and  $r_{t+1} - 1/v$ , with short sales constraints on  $r_{t+1}$  only. Therefore, the familiar interpretation of intersection tests in terms of performance measures as can be found in Jobson & Korkie (1989) for instance, also holds when there are short sales constraints.

At this point it is useful to compare our test procedure with the one proposed by Glen & Jorion (1993). Glen & Jorion calculate the mean-variance efficient portfolio of the benchmark assets subject to short sales constraints, with 1/v equal to the observed risk free rate. Because of the existence of a risk free asset the hypotheses of intersection and spanning coincide in this case. The mean returns of all assets, i.e., both  $R_{t+1}$  and  $r_{t+1}$ , are then adjusted such that the calculated portfolio is mean-variance efficient without short sales constraints. Thus, the mean returns are adjusted such that the calculated portfolios would yield Jensen's alpha's equal to zero. Using these adjusted returns and assuming normality, a new set of T returns is simulated and a test statistic based on Sharpe ratios is calculated, but with short sales constraints on all the available assets rather than on  $r_{t+1}$  only. By repeating this process many times an empirical distribution of the test statistic can be obtained. Our procedure has the advantage that it yields a known distribution for the test-statistic in (14). Apart from this, our procedure has the advantage that we avoid the assumption that one of the assets is riskless and that the test can also be used to test for spanning.

#### 3.2 Testing for spanning

Up to now we considered tests for intersection. Spanning implies that the restrictions in (13) hold for all relevant values of v. Notice that for a given set of K asset returns  $R_{t+1}$ , there is only a finite number of subsets with  $L^{(v)}$ 

elements,  $L^{(v)} \in \{1, 2, ..., K\}$ , with  $R_{t+1}^{(v)}$  the  $L^{(v)}$ -dimensional vector containing the returns on the subset of the assets . Let  $V^{[j]}$  be the set of those values of v for which the subset of assets for which the short sales constraints in the mean-variance efficient portfolios are not binding is the same, and denote the  $L^{[j]}$ -dimensional vector of returns for these assets as  $R_{t+1}^{[j]}$ , i.e.,  $R_{t+1}^{[j]} = R_{t+1}^{(v)}$  if and only if  $v \in V^{[j]}$ . Similarly, each variable or parameter that refers to the set  $R^{[j]}$  will be denoted with a superscript [j]. Since for  $v \in V^{[j]}$  the meanvariance efficient frontier of  $R_{t+1}$  coincides with the mean-variance frontier of  $R_{t+1}^{[j]}$ , the mean-variance frontier of  $R_{t+1}$  with short sales constraints consists of a finite number of parts of the unrestricted mean-variance frontiers of the subsets  $R_{t+1}^{[j]}$ . It follows that the return on the additional assets  $r_{t+1}$ are spanned by the returns on the benchmark assets  $R_{t+1}$  if

$$E[m_R^{[j]}(v)_{t+1}r_{t+1}] \le \iota_N, \ \forall j,$$
 (18)

where  $m_R^{[j]}(v)_{t+1}$  is the mean-variance pricing kernel that is linear in  $R_{t+1}^{[j]}$ . If there are only short sales constraints on the benchmark assets  $R_{t+1}$  and not on the additional assets  $r_{t+1}$ , the inequality in (18) becomes an equality. If there are only short sales constraints on  $r_{t+1}$  and not on  $R_{t+1}$ ,  $R_{t+1}^{[j]} = R_{t+1}$ .

Intuitively, since if there are short sales constraints the mean-variance frontier of  $R_{t+1}$  consists of parts of the unrestricted mean-variance frontiers of the subsets of returns  $R_{t+1}^{[j]}$ , j = 1, 2, ...M,  $r_{t+1}$  can only be spanned by the returns  $R_{t+1}$  if it is spanned by the M subsets of  $R_{t+1}$ . It follows then that if there are no short sales constraints on the assets  $r_{t+1}$ , there is mean-variance spanning if and only if in the M regressions

$$r_{t+1} = a^{[j]} + B^{[j]} R^{[j]}_{t+1} + \varepsilon^{[j]}_{t+1},$$
 (19a)  
 $a^{[j]} = 0 \text{ and } B^{[j]} \iota^{[j]} = \iota_N.$  (19b)

$$a^{[j]} = 0 \text{ and } B^{[j]}\iota^{[j]} = \iota_N, \tag{19b}$$

where  $\iota^{[j]}$  is an  $L^{[j]}$ -dimensional vector consisting of ones. The hypothesis that there is spanning can therefore easily be tested by using a multivariate regression of  $r_{t+1}$  on all  $R_{t+1}^{[j]}$  and using a Wald test for the Huberman-Kandel restrictions in each of these regressions. If there are also short sales restrictions on  $r_{t+1}$ , then the conditions in (18) imply that we should again use the multivariate regression in (19a), but now the restrictions imposed are that

$$a^{[j]}v + B^{[j]}\iota^{[j]} \le \iota_N$$
, for all  $v \in V^{[j]}$ . (20)

Denoting  $v_{\min}^{[j]} = \min_{v \in V^{[j]}} v$ , and  $v_{\max}^{[j]} = \max_{v \in V^{[j]}} v$ , the restrictions in (20) are satisfied if there is intersection for  $v_{\min}^{[j]}$  and for  $v_{\max}^{[j]}$ , since in that case there is also intersection for all the intermediate values of  $v^{[j]}$ . Therefore, testing for spanning comes down to jointly testing the restrictions:

$$a^{[j]}v_{\min}^{[j]} + B^{[j]}\iota^{[j]} \leq \iota_N,$$
 (21)  
 $a^{[j]}v_{\max}^{[j]} + B^{[j]}\iota^{[j]} \leq \iota_N,$ 

for j=1,...,M. Again, the test-statistic for the inequality restrictions in (21) is standard and is now based on testing simultaneously for intersection for  $v_{\min}^{[j]}$  and  $v_{\max}^{[j]}$ , j=1,2,...,M, analogous to (14). The *p*-values can be obtained from (15) by replacing  $\chi_N^2$  with  $\chi_{2MN}^2$ . Similarly, without calculating the weights in (15), upper and lower bounds on the *p*-values can be obtained from (16) by replacing  $\chi_N^2$  and  $\chi_{N-1}^2$  by  $\chi_{2MN}^2$  with  $\chi_{2MN-1}^2$  respectively.

In order to have an indication of the power of the spanning test with short sales constraints, Figure 1 presents the power as a function of the intercept  $a^{[j]}$  in the spanning regression (19a). The figure contains the power function for the Wald spanning test-statistics in case there are no frictions, in case there are short sales constraints for the new asset  $r_{t+1}$  only, and in case there are short sales constraints for both the new assets as well as for the benchmark assets. Because in case of short sales constraints on the benchmark assets the test depends on the benchmark assets that are included in the segments, the power function for the case of short sales constraints on all assets is shown when the elements in  $R^{[j]}$  are known as well as when they have to be estimated. The slope coefficients  $B^{[j]}$  are chosen such that there is spanning under all null hypotheses (with and without short sales constraints) if the intercept  $a^{[j]}$  is equal to zero.

For each value of  $a^{[j]}$  the power is derived from a series of 1000 simulations. For each simulation 10 years of monthly data are generated and the empirical power for a 5%-rejection rate is determined. The benchmark indices are assumed to be the three stock indices used in the empirical application in Section 5, where the data generating process for these indices is based on the summary statistics in Table 1, assuming multivariate normality. For the new asset the monthly standard deviation of the residual  $\varepsilon_{t+1}$  in (19a) is 10%, which is representative for the emerging markets that are analyzed in Section 5. The order of magnitude of the intercept  $a^{[j]}$  that is used in the simulations is also representative for the emerging markets. Notice that  $a^{[j]} < 0$ , which is tantamount to a negative Jensen measure  $\alpha_J(v)$  in our simulations, implies that there is spanning if there are short sales constraints. Therefore, we can expect low power for negative values of  $a^{[j]}$ , which is confirmed in Figure 1.

Because the test statistics with short sales constraints are calculated in such a way that they have a maximum size of 5% over all of the null hypothesis, it should come as no surprise that the power of these tests is smaller than 5% for  $a^{[j]} = 0$ .

As the figure shows, the different tests are very similar with respect to power. Notice that for the spanning test with short sales constraints on all assets, the power functions for the case where the elements in  $R^{[j]}$  have to be estimated and for the case where these elements are known, are almost identical. This confirms the result in Appendix B that shows that the limit distribution of the spanning test is not affected by the fact that the elements in  $R^{[j]}$  have to be estimated. Also notice that according to Figure 1 the power of the tests with short sales constraints is very close to the power of the optimal Wald spanning test when there are no constraints. Therefore, Figure 1 suggests that the intersection and spanning tests with short sales constraints proposed here have desirable power properties.

The intersection and spanning tests presented here are closely related to the region subset tests in Hansen, Heaton & Luttmer (HHL) (1995). In the region subset tests of HHL the interest is in testing whether, given the initial asset returns  $R_{t+1}$ , including the additional returns  $r_{t+1}$  causes a significant shift in the volatility bound. For a given mean v of the stochastic discount factor this simply amounts to a test for intersection. The region subset test of HHL is based on the minimum variance stochastic discount factor  $m(v)_{t+1}$  that prices the assets in  $R_{t+1}$  and  $r_{t+1}$  subject to short sales constraints:

$$m(v)_{t+1} = v + \alpha_R^{(v)'}(R_{t+1}^{(v)} - E[R_{t+1}^{(v)}]) + \alpha_r^{(v)'}(r_{t+1}^{(v)} - E[r_{t+1}^{(v)}]).$$

This is similar to the minimum variance stochastic discount factor in (7), but now based on all the assets rather than the benchmark assets  $R_{t+1}$  only. The intersection hypothesis is now equivalent to the hypothesis that the coefficients associated with  $r_{t+1}$ ,  $\alpha_{\tau}^{(v)}$ , are zero. As pointed out by HHL (1995), the asymptotic distribution of the estimate of  $\alpha_{i}^{(v)}$  is nonstandard if  $\alpha_{i}^{(v)}$  equals zero, because in that case it is impossible to distinguish between assets that have a zero coefficient and assets whose short sales constraints are binding. Since in the region subset tests the null hypothesis is that the coefficients  $\alpha_{\tau}^{(v)}$  are zero, it is under the hypothesis of interest that the limit distribution of  $\alpha_{\tau}^{(v)}$  is nonstandard (see HHL (1995) for further details).

The interest in this paper is in the hypothesis of spanning rather than intersection. As shown above, testing for spanning with short sales constraints amounts to simultaneously testing for intersection at those values of v for which one of the weights in the mean-variance efficient portfolio of the benchmark assets  $R_{t+1}^{[j]}$  is zero, i.e., in  $v_{\min}^{[j]}$  and  $v_{\max}^{[j]}$ , which suggests that the limit distribution for the spanning test may be nonstandard as well. Recall however, that the spanning test is based on testing for intersection in  $v_{\min}^{[j]}$  and  $v_{\min}^{[j]}$  because these are the two extreme values of  $v^{[j]}$  for which the short sales constraints on  $R_{t+1}^{[j]}$  are not binding. Intersection at  $v_{\min}^{[j]}$  and  $v_{\max}^{[j]}$  implies intersection at all the intermediate values of  $v^{[j]}$  and therefore spanning. Thus, since the spanning test described above is essentially based on testing intersection for all values of  $v^{[j]}$  for which the short sales constraints on  $R_{t+1}^{[j]}$  are not binding, the problem encountered in the region subset test of HHL does not occur.

Another way to look at this is the following. HHL estimate the minimum variance stochastic discount factor  $m(v)_{t+1}$  under non-negativity constraints (which essentially induces the nonstandard limit distribution) and end up with testing equality restrictions. On the other hand, our regression-based estimator is unrestricted with a standard asymptotic distribution, but we end up with - more difficult - inequality restrictions that have to be tested. This latter problem is well-studied in the literature however (see, e.g., Gouriéroux et al., 1982, and Kodde & Palm, 1986).

## 4 Mean-variance spanning with transaction costs

When taking transaction costs into account it is useful to differentiate between the return on a long position in asset i,  $\tau_i^{\ell}R_{i,t+1}$ , and the return on a short position in asset i,  $\tau_i^{s}R_{i,t+1}$  (see, e.g., Luttmer, 1996). Let  $\tilde{R}_{t+1}$  be a 2K-dimensional vector, the first K elements of which are the returns on the long positions in the assets  $i=1,\ldots,K$ , and the last K elements of which are the returns on the short positions in these same assets. Thus,  $\tilde{R}_{i,t+1} = \tau_i^{s}R_{i,t+1}$  and  $\tilde{R}_{i+K,t+1} = \tau_i^{s}R_{i,t+1}$ . One way to motivate this kind of transaction costs is to assume that investors have to pay a bid/ask spread when buying or (short) selling the asset at time t. Thus, letting  $a_i > 0$  and  $b_i > 0$  be the ask and the bid spread respectively, as a percentage of the price  $P_{i,t}$ ,  $\tau_i^{\ell}$  is defined by  $\tau_i^{\ell}R_{i,t+1} = P_{i,t+1}/((1+a_i)P_{i,t})$ , implying that  $\tau_i^{\ell} = 1/(1+a_i)$ , and  $\tau_i^{s}$  is

defined by  $\tau_i^s R_{i,t+1} = P_{i,t+1}/((1-b_i)P_{i,t})$ , implying that  $\tau_i^s = 1/(1-b_i)$ . Of course,  $\tau_i^t$  and  $\tau_i^s$  can be interpreted as any kind of proportional transaction costs associated with long and short positions in the assets.

Considering  $\widetilde{R}_{t+1}$  as the vector of returns on 2K different assets, transaction costs can now be handled by requiring that investors can not go short in the first K assets  $(C = \mathbb{R}_+^K)$  and can not go long in the last K assets  $(C = \mathbb{R}_-^K)$ . Analogously to the case of short sales constraints, mean-variance efficient portfolios follow from the Kuhn-Tucker conditions of the problem:

$$\max_{\{\widetilde{w}\}} \widetilde{w}' E[\widetilde{R}_{t+1}] - \frac{1}{2} \widetilde{w}' Var[\widetilde{R}_{t+1}] \widetilde{w}$$
 s.t.  $\widetilde{w}' \iota_{2K} = 1$  and  $\widetilde{w}_i \geq 0$ ,  $\widetilde{w}_{K+i} \leq 0$ ,  $i = 1, 2, ..., K$ ,

which are:

$$E[\tilde{R}_{t+1}] - \frac{1}{v}\iota_{2K} + \tilde{\delta} = \gamma Var[\tilde{R}_{t+1}]\tilde{w}^*, \qquad (22)$$

$$\tilde{w}_i^* \geq 0, \tilde{w}_{K+i}^* \leq 0, i = 1, 2, ..., K,$$

$$\tilde{\delta}_i \geq 0, \tilde{\delta}_{K+i} \leq 0, i = 1, 2, ..., K,$$

$$\tilde{\delta}_i \tilde{w}_i^* = 0, \forall i.$$

Let  $\widetilde{m}_R(v)_{t+1}$  be the mean-variance stochastic discount factor that prices  $\widetilde{R}_{t+1}$  correctly and let  $\widetilde{R}_{t+1}^{(v)}$  be the L-dimensional subvector of  $\widetilde{R}_{t+1}$  for which the constraints on the short and long positions are not binding. The notation is therefore analogous to the case of short sales constraints only. The mean-variance stochastic discount factor is now given by:

$$\widetilde{m}_{R}(v)_{t+1} = v + \widetilde{\alpha}^{(v)'}(\widetilde{R}_{t+1}^{(v)} - E[\widetilde{R}_{t+1}^{(v)}]),$$

$$\widetilde{\alpha}^{(v)} = Var[\widetilde{R}_{t+1}^{(v)}]^{-1}(\iota_{L} - vE[\widetilde{R}_{t+1}^{(v)}]),$$
(23)

where it is of course again the case that  $\tilde{w}^{(v)} = -\tilde{\alpha}^{(v)}/\gamma^{(v)}v$ , and where  $\tilde{w}^{(v)}$  is the mean-variance efficient portfolio of  $\tilde{R}_{t+1}^{(v)}$  with zero-beta return 1/v.

In a similar way, we consider long and short positions in the N additional asset as 2N different assets. The returns on long position in the additional assets are given by  $(\tilde{r}_{t+1}^{\ell})_k = \tau_k^{\ell} r_{k,t+1}, \ k = 1, 2, ..., N$ , while the returns on

<sup>&</sup>lt;sup>3</sup>Alternatively, we may also include a bid/ask spread at t+1, by letting  $\tau_i^{\ell} = (1-b_i)/(1+a_i)$  and  $\tau_i^{s} = (1+a_i)/(1-b_i)$ .

short positions are given by  $(\tilde{\tau}_{t+1}^s)_k = \tau_k^s r_{k,t+1}$ , k = 1, 2, ..., N. The returns on the additional assets are then spanned by the benchmark assets if

$$E[\widetilde{m}_{R}^{[j]}(v)_{t+1}\widetilde{r}_{t+1}^{\ell}] \leq \iota_{N}, \ \forall j,$$

$$E[\widetilde{m}_{R}^{[j]}(v)_{t+1}\widetilde{r}_{t+1}^{s}] \geq \iota_{N}, \ \forall j.$$

$$(24)$$

As in case of short sales constraints, let  $V^{[j]}$  be the set of those values of v for which the subsets of assets for which the constraints on the long and short positions are not binding are the same, with j=1,2,...,M. Therefore, we can test for mean-variance spanning of  $\tilde{r}_{t+1}$  by  $\tilde{R}_{t+1}$  by testing whether in the 2M regressions

$$\tilde{r}_{t+1}^{\ell} = a_{\ell}^{[j]} + B_{\ell}^{[j]} \tilde{R}_{t+1}^{[j]} + \varepsilon_{\ell,t+1}^{[j]},$$

$$\tilde{r}_{t+1}^{s} = a_{s}^{[j]} + B_{s}^{[j]} \tilde{R}_{t+1}^{[j]} + \varepsilon_{s,t+1}^{[j]},$$
(25)

the following restrictions hold:

$$a_{\ell}^{[j]}v_{\min}^{[j]} + B_{\ell}^{[j]}\iota^{[j]} \leq \iota_{N}, \qquad (26)$$

$$a_{\ell}^{[j]}v_{\max}^{[j]} + B_{\ell}^{[j]}\iota^{[j]} \leq \iota_{N},$$

$$a_{s}^{[j]}v_{\min}^{[j]} + B_{s}^{[j]}\iota^{[j]} \geq \iota_{N},$$

$$a_{s}^{[j]}v_{\max}^{[j]} + B_{s}^{[j]}\iota^{[j]} \geq \iota_{N}, \forall j.$$

#### 5 Empirical results for emerging markets

In this section we will test whether US investors that have a well-diversified international stock portfolio can improve upon their efficient set by investing in emerging markets. We use 17 indices from the Emerging Markets Data Base (EMDB) of the International Finance Corporation (IFC). According to the IFC, a country's stock market is an emerging market if that country is classified as either a low- or a middle-income economy by the World Bank, which means that in 1994 the country had to have a per capita GNP of \$8,955 or less. To obtain a sufficiently long data period, monthly observations on the Global Indices are used over the period of January 1985 until June 1996, for six Latin American Countries, seven Asian Countries, one European, one Mideast, and two African countries. Except for Indonesia, Portugal, and Turkey, which are left out of the sample because of many

missing observations, these are the same emerging markets as used by Harvey (1995). DeSantis (1994) also uses these emerging markets, except for Thailand. As noted by Bekaert & Urias (1996), apart form short sales constraints and transaction costs, the returns on the IFC Global Indices may be unattainable to investors because of foreign ownership restrictions, e.g. This problem does not occur with the IFC Investable Indices, which account for these restrictions. The problem with the Investable Indices however, is that there is only a limited sample available. An overview of the available data is given in Appendix C, from which we see that for 9 of the emerging markets in our dataset the Investable Indices are available from January 1989 onwards, while all other indices have a later starting date, which may be as late as November 1993 (Zimbabwe). For Nigeria the Investable Index is not available at all. Notwithstanding this limited availability, some spanning tests will be presented for both the Global and the Investable Indices to show the effect of ownership restrictions. Unless explicitly stated otherwise, the empirical results in this section are based on the IFC Global Indices. The Morgan Stanley Capital International (MSCI) Indices for the USA, Europe and Japan serve as the benchmark assets. Similar indices are also used as reference assets by DeSantis (1994) and Harvey (1995). For all these indices we use (unhedged) monthly holding returns in US dollars. The indices for both the emerging markets and for the developed markets are calculated with dividends reinvested. All data are obtained from Datastream.

Some basic summary statistics for net monthly holding returns are given in Table 1. Panel A of Table 1 provides summary data on the three benchmark indices. Since our test statistics for spanning involve tests for intersection for several values of v, the expectation of the stochastic discount factor, it is useful to restrict the possible range of v beforehand. An upper bound on v is obtained if we do not impose the requirement that investors should invest all their wealth in the available assets, but may choose to invest only part of their wealth, i.e.,  $0 \le w' \iota \le 1$  (see also Luttmer, 1996). In effect this allows for the possibility to take long positions in a risk free asset with zero net return. This implies that the upper bound for v is 1. If we move upward along the mean-variance frontier, v decreases until 1/v equals the intercept of the asymptote of the lines tangent to the mean-variance frontier. This intercept is equal to the expected return on the global minimum variance portfolio,  $E[R_{t+1}^{GMV}]$ , implying that the lower bound on v is given by  $v = 1/E[R_{t+1}^{GMV}]$ . Table 1 shows that if there are no short sales constraints or transaction costs on the benchmark assets, v is in the range between 0.986 and 1.000. The

maximum attainable Sharpe ratios at these boundaries for the benchmark assets are 0.06 and 0.37 respectively. Of course these boundaries have to be adjusted in case there are short sales constraints and/or transaction costs on the benchmark assets  $R_{t+1}$ . Table 1 also presents summary statistics in case there are short sales constraints on the benchmark indices and in case there exists a transaction cost of 0.125% or 0.5% per month when buying or selling the indices. Although a 0.5% transaction cost is a more realistic estimate of the round trip costs for these benchmark indices than 0.125%, we also consider a 0.125% transaction cost per month to allow for an investment horizon that is longer than one month, thereby decreasing the transaction cost on a monthly basis. It is easy to show that if the proportional transaction cost for a holding period of k months is  $\tau$  and returns are i.i.d., the mean-variance frontier (portfolios) for this holding period can be obtained from monthly returns with a transaction cost of  $\tau^{1/k}$ . Therefore, with a four-month holding period, the implied monthly transaction cost is  $(1.005)^{\frac{1}{4}} \approx 1.00125$ .

Panel B of Table 1 shows some summary statistics for the emerging markets. The codes that are used for the emerging markets are explained in Appendix C. A quick look at the data reveals that the emerging markets indices are usually much more variable than the benchmark indices, but also have higher average returns. For the monthly holding returns we use, the average standard deviation of the emerging markets indices is 11.66% and the average expected return is 2.41%, compared with 5.54% and 1.46% for the benchmark indices. Table 1 also provides some information on the diversification possibilities of each emerging market relative to the three benchmark indices if transaction costs are negligible and short selling is allowed. Since the Wald test-statistic for intersection for a given value of v is a quadratic function of v, we can solve for the range of values of v for which the test statistic is smaller than the  $\alpha$ %-critical value. The third and fourth column of Panel B give the (unrestricted) range of v for which the hypothesis that the mean-variance efficient frontier of the three benchmark indices plus the emerging market intersects the mean-variance efficient frontier of the three benchmark indices only, can not be rejected at the 5% statistical significance level. For instance, in case of Argentina, the hypothesis of intersection (neglecting market frictions) can not be rejected at the 5% level if  $0.999 \le v \le 1.089$ . Columns 5 and 6 of Panel B translate these values of v into expected portfolio returns for the three benchmark indices. Thus, for investors that initially hold minimum variance portfolios of the three benchmark indices with expected returns that are either below 1.42% per month

or above 1.45% per month, including Argentina yields a shift in the frontier that is statistically significant at the 5% level. If no bounds are reported in Panel B of Table 1, this means that the intersection-test never rejects at the 5% level for that emerging market.

Although these results indicate that most emerging markets can offer significant diversification possibilities, it is not clear whether such diversification benefits are actually attainable. For one thing, it may very well be the case that the diversification benefits offered by the emerging markets can only be realized if short positions are taken in emerging markets, the benchmark assets, or both. Whether or not the shifts in the mean-variance frontiers are statistically significant once short sales constraints are taken into account will be the subject of the next section.

Finally, Panel C of Table 1 shows the average returns and standard deviations for two subperiods. Bekaert et al. (1996) provide ample evidence that especially the behaviour of the emerging returns has been changing over time. One important reason for this are the many liberalizations that have taken place in the emerging markets (see, e.g., Bekaert, 1995), causing the emerging markets to become more integrated with the developed markets. As noted by Bekaert et al. (1996), most of the capital market liberalizations in the emerging markets took place before 1992. For this reason we split our sample in a pre-1992 and a post-1992 period. From the average returns and the standard deviations of the returns it is obvious that there are important differences between the pre-1992 and post-1992 period, both for the emerging and for the developed markets. For one thing, the average monthly returns and the standard deviations have decreased in the post-1992 period relative to the pre-1992 period, for both the emerging and the developed markets, although there are also a number of individual emerging markets for which the average returns and/or the standard deviations of the returns have increased in the post-1992 period. The average return for the benchmark indices has decreased from 1.81% per month in the pre-1992 period to 0.92% in the post-1992 period. The relatively high average return in the pre-1992 period is mainly due to the high returns in the first two years of our sample period. For the emerging markets the average returns in the two subperiods are 3.08% and 1.36% per month respectively. The standard deviations likewise decreased over the two subperiods. However, the stylized fact that both the average returns and the volatility in the emerging markets are higher than in the developed markets is present in both the pre-1992 and the post-1992 period. Also, whereas the average correlations between the benchmark returns were 0.46 and 0.34 in the two subperiods, the correlations between the emerging markets and between the emerging markets and the benchmark assets were usually rather low in both subperiods, despite the liberalizations of the emerging markets. Therefore, although the return characteristics for the emerging markets may have changed after the liberalizations, these stylized facts suggest that in the post-1992 period there may still be diversification benefits from including emerging markets in a portfolio of the benchmark assets considered here.

#### 5.1 Results for spanning tests with short sales constraints

The analysis in the previous section already suggested that, in the absence of market frictions, many emerging markets yield diversification benefits relative to the benchmark indices for the US, Europe, and the world. Table 2 shows Wald test-statistics for the hypothesis that the returns on these three indices span the returns for each emerging market. In this table and the following, the emerging markets are organized according to their geographical region: Latin America, Asia, and "Other". For each group, the first line shows the spanning test-statistic and the associated p-value in case there are no short sales restrictions on either the benchmark assets or the emerging markets. In this case, the hypothesis of mean-variance spanning is readily rejected at the 5% significance level for 9 out of the 17 emerging markets. A joint test for spanning of all the emerging markets in a geographical group always rejects the null hypothesis of spanning. These results merely confirm the findings of, e.g., DeSantis (1994) and Harvey (1995). As noted before however, these diversification benefits may not be attainable to investors, since they may require short selling of the emerging markets indices, the benchmark indices, or both.

If we do not allow investors to go short in the emerging markets, while still retaining the possibility to sell the benchmark indices short, the main conclusion does not change. The second line for each geographical group in Table 2 shows that there are now 10 out of 17 rejections at the 5% significance level. Notice that the rejections that are found do not always coincide with a rejection in the no-friction case. Taking into account short sales constraints on the emerging markets causes decreases in the Wald test-statistic that are often nontrivial. When performing a joint spanning test for all emerging

markets within a geographical group, the effect of short sales constraints is strongest for the group "Other". However, the hypothesis of spanning can always be rejected at the 5% significance level, reflecting the fact that the short sales constraints on the emerging markets are usually not binding. Because of the high average returns in the emerging markets, investors with low risk aversions can benefit from buying the emerging markets asset and selling (part of) their benchmark assets.

It may be the case though, that investing in the emerging markets only extends the efficient set when the portfolio of the benchmark assets already contains short positions. To account for short sales restrictions on the benchmark assets as well, Table 2 also presents spanning tests in case there are short sales restrictions on both the emerging markets and the benchmark assets. These results are presented in the third line for each geographical group in Table 2. The effect of short sales restrictions is more pronounced in this case. If investors are not allowed to short sell any of the assets, the hypothesis of spanning can be rejected at the 5% significance level for only 5 markets: Chile, Colombia, Pakistan, the Philippines, and Zimbabwe. Joint tests for each geographical group always reject the hypothesis of spanning however. The results in Table 2 therefore show that our benchmark investors can still benefit from investing in those markets, even though there may be short sales constraints.

Even for the five emerging markets for which the null hypothesis is rejected, the diversification benefits may not be attainable however, because of foreign ownership restrictions. Bekaert (1995) discusses several measures of the extent of foreign ownership restrictions in emerging markets. One such measure, for instance, is the ratio of the IFC Investable Index over the IFC Global Index, since the Investable Index takes into account foreign ownership restrictions on each stock traded in an emerging markets. Except for Colombia, Bekaert (1995) reports ratios that are rather low for these five countries (in particular for Zimbabwe). Thus, except possibly for Colombia, the diversification benefits suggested by Table 2 may be difficult or impossible to obtain.

To shed some further light on this issue, the last line for each geographical group in Table 2 gives the results for the spanning tests in case the IFC Investable Indices are used instead of the Global Indices. The null hypothesis is again whether the emerging market indices are spanned by the benchmark assets in case there are short sales constraints on both the emerging markets and the benchmark assets. For three of the five markets just mentioned,

Pakistan, the Philippines and Zimbabwe, the hypothesis of spanning can not be rejected for the Investable Indices, suggesting that the ownership restrictions are indeed binding for these countries. For Colombia the hypothesis of spanning can be rejected at the 10% level, but it is only in case of Chile that we can still reject spanning at the 5% level. Joint tests for all emerging markets within a geographical group reject the hypothesis of spanning only for Latin America.

Summarizing, it is clear that the hypothesis of mean-variance spanning is easily rejected in case there are no market frictions. In case there are short sales constraints there is still a lot of evidence against this hypothesis. Especially when there are only short sales constraints on the emerging markets but not on the benchmark indices, the shifts from the mean-variance frontiers of the benchmark indices to the frontiers of the benchmark indices plus the emerging markets are often statistically significant. The number of countries for which the hypothesis of spanning can be rejected is much smaller once there are also short sales constraints on the benchmark indices, although the joint tests for each geographical still reject the hypothesis of spanning in all cases. However, the countries for which the rejections remain significant even after allowing for short sales constraints, seem to be countries for which ownership restrictions are particularly severe. Taking into account ownership restrictions as well as short sales constraints, the hypothesis of spanning can only be rejected for Latin America.

#### 5.2 Results for spanning tests with transaction costs

In this section we consider the effects of transaction costs on the hypothesis that the mean-variance frontier of the benchmark indices spans the frontiers of the benchmark indices plus the individual emerging markets. We assume that investors have to pay a transaction cost of either 0.125% or 0.5% per transaction when buying or (short) selling the benchmark assets. Notice that the proportional transaction costs considered here can be interpreted as a round trip cost. Although 0.5% may be a more realistic estimate of the round trip transaction costs for the benchmark indices, observe that since we use monthly returns this implicitly assumes that trading takes place once a month. The effect of transaction costs as high as 0.5% may be particularly severe with this rather high trading frequency. Therefore, to mitigate this effect, we also allow for a 0.125% transaction cost, which may be a more realistic estimate for investors who have an investment horizon of, say, four

months. As already noted in the discussion of Table 1, with a 0.5% transaction cost per month on the benchmark assets, v is in the range between 1.000 and 0.993 as shown in Table 1, where investors want to take long positions in the MSCI Indices for the USA, Europe and Japan, or in the indices for the USA and Europe only. A transaction cost of either 0.125% or 0.5% per month precludes investors from taking any short position in the benchmark indices.

With a 0.125% transaction cost per month, v is in the range between 1.000and 0.990, and investors also want to take long positions in the USA, Europe and Japan index or in the USA and Europe index only. As noted by Bekaert & Urias (1996), the IFC Indices for the emerging markets are characterized by high transaction costs and other market frictions. Therefore, we consider the effect of increasing the transaction costs on those indices to a level as high as eight times the level for the benchmark indices. The results for the spanning tests with a 0.125% transaction cost on the benchmark assets and an increasing transaction cost on the emerging markets are presented in Table 3. Here the null hypothesis is that long positions in each emerging market are spanned by long positions in the three mature market indices. To put things into perspective, the results in Table 3 should be compared with the results in the third line of each geographic group in Table 2, where there are short sales restrictions on the emerging markets as well as on the benchmark assets. The first line of each geographic group in Table 3 shows the results if there is a 0.125% transaction cost on both the benchmark assets and the emerging markets. A quick look at Table 2 and 3 shows that the effect of a 0.125% transaction in itself is not very dramatic, since the values of the test statistics and the associated p-values are roughly of the same order of magnitude in the two tables. In case of a 0.125% transaction cost, the hypothesis of spanning can be rejected at the 5% level for 9 emerging markets. The joint tests also reject the hypothesis of spanning for each geographical region at the 5% level.

The most interesting result of Table 3 is perhaps the effect of an increase in the transaction cost on the emerging markets, while keeping the transaction cost for the benchmark assets at 0.125%. Doubling the transaction cost on the emerging markets to 0.250% has only a minor effect on the individual markets. However, an increase to 0.5% per month leaves us with only 5 rejections at the 5% level, while the joint tests only reject the null hypothesis for Latin America. The rejections at the 5% level are for Argentina, Chile, Colombia, the Philippines and Zimbabwe. As noted in the previous section, except for Colombia, these are markets in which foreign ownership

restrictions probably prevent investors to realize the diversification benefits that are potentially offered by these emerging markets. However, even with a transaction cost as high as 1.0% on the emerging markets, i.e., 8 times as high as for the benchmark assets, there is still some evidence against the hypothesis of spanning for both Chile and Colombia, as well as for all of the Latin American countries together. Nonetheless, for the bulk of the emerging markets, increasing the transaction costs leaves us with little evidence in favor of diversification benefits. Notice though, that these are the transaction costs that investors have to pay when they trade their portfolio every month. If the round trip cost for emerging markets is in the order of magnitude of, say, 0.5%, then the results in Table 3 suggest that investing in emerging markets is worthwhile if investors trade once every two months or less.

These results are confirmed by the results in Table 4, where the effect of transaction costs is shown for the Investable Indices. As in Table 3, in Table 4 it is assumed that there is a 0.125% transaction cost on the benchmark assets, and there are two levels of transaction costs for the emerging markets: 0.125% and 0.5% respectively. With a 0.125% transaction costs it is only for some Latin American countries that the hypothesis of spanning can be rejected. Joint tests for the three geographical groups also only reject for Latin America. In case of a 0.5% transaction costs there are still rejections at the 5% level for the Latin American countries. Therefore, as with the short sales constraints only, in case of transaction costs the results in Table 3 and 4 show that spanning can only be rejected for Latin America.

Finally, Table 5 gives some idea of the transaction costs that are needed to keep investors out of the emerging markets. Starting with a round trip cost of 0.5% for the benchmark assets and assuming monthly trading, Table 5 presents levels of transaction costs in the emerging markets above which the hypothesis of spanning can not be rejected at the 5% and 10% level respectively. For instance, in case of Argentina a round trip cost below 1.50% is needed to reject spanning by the benchmark assets at the 10% level and a round trip cost below 0.60% is needed to reject spanning at the 5% level. The estimates of 0.00% in case of Brazil for instance, imply that spanning can never be rejected at the 10% level, no matter how low the transaction costs are. The estimates in Table 5 suggest that with a 0.5% round trip costs on the benchmark assets, transaction costs for the emerging markets need not be particularly high to keep investors out of these markets. It is only in a few cases that a transaction cost of at least two times the level in the benchmark assets is needed to keep investors out of the market. (Admittedly this is a

rather aggressive interpretation of the results in Table 5, since the fact that we can not reject the hypothesis of spanning by no means implies that there is spanning.) Once more, if the hypothesis of spanning is not to be rejected, a transaction cost more than two times the one for the benchmark assets is needed for only three markets: Chile, Colombia, and the Philippines.

To get some further intuition about the importance of these transaction costs, the third line for each geographic group in Table 5 gives an estimate of the actual round trip costs in the emerging markets. These estimates are from Barings Securities and reported by Bekaert et al. (1996). The reported transaction costs are calculated from the percentage spread, which is the difference between the offer and bid price divided by the average of the offer and bid price for a security. To obtain a spread for each country, the percentage spreads of individual stocks are weighted by the capitalization of each stock within each country (see Bekaert et al., 1996).

Interestingly, except for Colombia and the Philippines, the actual transaction costs are always higher than the calculated 5%-bounds in Table 5. Even for Colombia and the Philippines the actual transaction is rather high compared with the other markets, and is close to the estimated 5%-bound. Also, both the 5%-bound and the actual transaction cost are highest for the same market: Chile. Taken together, the evidence presented in this section suggests that the individual emerging markets are spanned by the three benchmark indices when allowing for transaction costs. This conclusion is based on investors that trade their portfolio on a monthly basis however. For investors that trade their portfolio less frequently there is still evidence that there are diversification benefits from investing in emerging markets, even after transaction costs.

#### 5.3 Spanning tests for the post-liberalization periods

As already suggested by the summary statistics in Panel C of Table 1 and by previous studies (e.g., Bekaert, 1995), the liberalizations that have taken place in many emerging markets may have altered the return distributions in those markets in a nontrivial way. To see the effects of these liberalizations on some of our results we repeat the spanning tests for the no-frictions case and for the case where there are short sales constraints, for the periods after major liberalizations of the emerging markets. Appendix C provides the last major liberalization date for each emerging stock market as reported by Bekaert (1995). Starting from the month after this liberalization until the

end of our sample period, we repeat the analysis in Table 2 for each emerging market. The results for these subperiods are presented in Table 6. For each geographical group of emerging markets, the last column of Table 6 similarly presents joint spanning tests for the period from the last liberalization in the geographical group until the end of the sample period. Since there are no major liberalizations for the group "Other", we do no report results for this group in Table 6.

Spanning tests in case there are no market frictions are presented in the first row of each geographical group in Table 6. Except for Colombia and Thailand, the hypothesis of spanning can not be rejected for any of the emerging markets in Latin America and Asia at the 10% level. Joint tests for all emerging markets within each geographical group still reject the null hypothesis at for all geographical groups. Thus, for the post-liberalization period, there is much less evidence against the hypothesis that there is mean-variance spanning, even in case there are no frictions.

This is also the case for the remainder of Table 6, which shows the test statistics in case there are short sales constraints. In case there are short sales constraints on all assets the hypothesis of spanning can only be rejected at the 5% level for Colombia. The joint tests for the emerging markets in Latin America and in Asia never reject the null hypothesis. These results also hold true when the Investable Indices are used instead of the Global Indices. Therefore, whereas the hypothesis of spanning is strongly rejected when data for the whole sample period are used and there are no market frictions, there is hardly any evidence against spanning, with or without market frictions, for the subperiods after liberalization of the emerging markets.

#### 6 Concluding remarks

In this paper we showed how regression techniques can be used to test for mean-variance spanning and intersection in case there are short sales constraints and/or transaction costs. When there are short sales constraints on the benchmark assets, the mean-variance frontier consists of parts of the mean-variance frontiers of subsets of the set of benchmark assets. If the benchmark assets are to span a new set of assets, there has to be spanning for each subset of the benchmark assets. This can be incorporated in regression based test for spanning, by using a multivariate regression in which the returns on the new assets are regressed on the returns of the relevant

subsets of the benchmark assets. Short sales restrictions on the new assets require to test for inequality restrictions rather than equalities. Following the ideas presented for instance in Luttmer (1996), transaction costs can be handled by looking at short and long positions in an asset as two different securities. Transaction costs can then be dealt with in the same way as short sales constraints.

There is substantial evidence available in the literature that suggests that, in the absence of market frictions, US-investors can benefit from including emerging markets assets in their well-diversified international portfolio of developed market assets. We try to shed some further light on this issue by testing whether emerging market indices are spanned by developed market indices when investors face short sales constraints and/or transaction costs. The hypothesis of spanning can still be rejected for many emerging markets if there are short sales constraints on both the emerging markets and the benchmark assets. However, the markets for which spanning can still be rejected when there are short sales constraints, appear to be markets for which foreign ownership restrictions may be particularly severe. Taking into account transaction costs, the evidence against spanning is much weaker, although there is still evidence in favor of the diversification benefits of emerging markets for holding periods of two months or longer. When we estimate the minimum amount of transaction costs that are needed in order not to reject the hypothesis of spanning with monthly trading, this lower bound on the transaction costs is lower than estimates of the actual transactions in all but two markets. Therefore, the analysis in this paper suggests that in determining the potential diversification benefits of emerging markets for US-investors, it is important to take into account real-life market frictions such as short sales constraints and transaction costs. However, there is still evidence in favor of these diversification benefits, even after allowing for short sales constraints and transaction costs. Finally, if we limit the analysis to the subperiods after some major liberalizations in the emerging stock markets have taken place. there is little evidence against the hypothesis of spanning, even if there are no market frictions.

There are still some open issues that have not been considered in this paper and that are left for future research. For instance, except for splitting the sample according to market liberalizations, our results do not account for time variation in expected returns and volatilities. Previous studies suggest that there is such time variation, and that it is important to consider dynamic trading strategies. Future research plans to take these issues into account as

well.

#### A Duality between mean-variance frontiers and volatility bounds with short sales constraints

In this appendix we show that the duality between mean-variance frontiers and volatility bounds still holds when there are short sales constraints on the assets. In particular, we show that the stochastic discount factor with expectation v > 0, that has the lowest variance among all stochastic discount factors that have expectation v and that price the returns  $R_{t+1}$  correctly subject to short sales constraints, is a linear function of the return on a mean-variance optimal portfolio with zero-beta rate 1/v, subject to short sales constraints.

With short sales constraints on the K assets with return vector  $R_{t+1}$  the set of returns available to investors is given by:

$$X^p = \{R_{t+1}^p : R_{t+1}^p = w'R_{t+1}, w \ge 0 \text{ and } w'\iota_K = 1\}.$$

Valid stochastic discount factors  $M_{t+1}$  satisfy:

$$E[M_{t+1}R_{t+1}] \le \iota_K,\tag{27}$$

where there are strict equalities for the assets for which the short sales constraints are not binding (otherwise the agent with a utility function corresponding to  $M_{t+1}$  would sell part of his holding of  $R_{i,t+1}$  until an equality is obtained). Recall that  $M_{t+1}$  is proportional to the derivative of an agents derived utility of wealth function, given his optimal portfolio choice,  $w^*$ . Let  $u(w'R_{t+1})$  be a derived utility of wealth function (strictly increasing and concave). The problem that the agent has to solve is

$$\max_{\{w\}} E[u(w'R_{t+1})] - \eta(w'\iota_K - 1) + w'\delta,$$

where  $\delta$  is the K-dimensional vector of Kuhn-Tucker multipliers for the condition that  $w \geq 0$ . The first order conditions of the optimization problem imply

$$E[u'(w''R_{t+1})R_{t+1}] - \eta \iota_K + \delta = 0, \qquad (28)$$

$$w^{*'}\delta = 0,$$
  
 $\delta_i = 0 \text{ if } w_i > 0$   
 $\delta_i > 0, \forall i,$ 

implying that  $M_{t+1} = u'(w^{*t}R_{t+1})/\eta$  is a valid stochastic discount factor. Notice that the first order conditions imply that

$$E[M_{t+1}w^{*'}R_{t+1}] = 1. (29)$$

Let  $X^*$  be the set of returns on optimal portfolios subject to short sales constraints:

$$\begin{array}{rcl} X_{:}^{\bullet} & = & \{R_{t+1}^{p}: R_{t+1}^{p} = w^{\bullet\prime} R_{t+1}, w^{\bullet} \geq 0, w^{\bullet\prime} \iota_{K} = 1, \text{ and} \\ \exists M_{t+1} \text{ s.t. } E[M_{t+1} R_{t+1}] & \leq & \iota_{K} \text{ and } E[M_{t+1} w^{\bullet\prime} R_{t+1}] = 1\}, \end{array}$$

and observe that  $X^* \subset X$ .

For a stochastic discount factor with expectation v, define excess returns  $R_{t+1}^e \equiv R_{t+1}^p - 1/v$ . Using obvious notation it follows that for  $R_{t+1}^p \in X^*$ ,

$$0 = E[M(v)_{t+1}R_{t+1}^e] = vE[R_{t+1}^e] + \rho_{RM}\sigma_R\sigma_M \qquad (30)$$

$$\Rightarrow \left| \frac{E[R_{t+1}^e]}{\sigma_R} \right| \leq \frac{\sigma_M}{v}.$$

Thus, the maximum (absolute value of the) Sharpe ratio that can be obtained from the set of optimal portfolio returns,  $X^*$ , gives a lower bound on the volatility of admissable stochastic discount factors with expectation v (see, e.g., Hansen & Jagannathan, 1991).

First consider the returns that are in X, i.e., the set of all possible portfolio returns subject to short sales constraints. The set of mean-variance efficient portfolios is characterized by (5):

$$E[R_{t+1}] - \eta \iota_K + \delta = \gamma Var[R_{t+1}]w^*,$$

$$\delta_i = 0 \text{ if } w_i > 0,$$

$$\delta_i \geq 0 \forall i.$$
(31)

Now take the mean-variance efficient portfolio for which  $\eta=1/v$ . Denoting by  $R_{t+1}^{(v)}$  the *L*-dimensional subvector of  $R_{t+1}$  that only contains the returns of the assets for which the short sales constraints in (5) are not binding, it is

straightforward to show that the mean-variance efficient portfolio in (31) is equal to the mean-variance efficient portfolio without short sales constraints of the assets in  $R_{t+1}^{(v)}$  only:

$$E[R_{t+1}^{(v)}] - \frac{1}{v}\iota_L = \gamma^{(v)}Var[R_{t+1}^{(v)}]w^{(v)}$$
 and (32)  
 $E[R_{t+1}] - \frac{1}{v}\iota_K + \delta = \gamma^{(v)}Cov[R_{t+1}, R_{t+1}^{(v)}]w^{(v)},$ 

where  $Cov[R_{t+1}, R_{t+1}^{(v)}]$  is de  $K \times L$ -dimensional covariance matrix of  $R_{t+1}$  and its subvector  $R_{t+1}^{(v)}$ , and  $\gamma^{(v)} = (w^{(v)} E[R_{t+1}^{(v)}] - 1/v) / (w^{(v)} Var[R_{t+1}^{(v)}] w^{(v)})$ . The maximum Sharpe ratio is therefore equal to

$$\{(E[R_{t+1}^{(v)}] - \frac{1}{v}\iota_L)'Var[R_{t+1}^{(v)}]^{-1}(E[R_{t+1}^{(v)}] - \frac{1}{v}\iota_L)\}^{\frac{1}{2}}.$$
 (33)

Since this is the maximum Sharpe ratio that is attainable over all feasible portfolio returns, equation (33) gives a lower bound on the volatility of all admissible stochastic discount factors with expectation v. We can go one step further however, since this lower bound is actually attained by the stochastic discount factor that is linear in the asset returns  $R_{t+1}^{(v)}$ :

$$m_R(v)_{t+1} = v + \alpha^{(v)'}(R_{t+1}^{(v)} - E[R_{t+1}^{(v)}]),$$
 (34)  
 $\alpha^{(v)} = Var[R_{t+1}^{(v)}]^{-1}(\iota_L - vE[R_{t+1}^{(v)}]).$ 

Since the portfolio  $w^{(v)}$  in (32) with  $\eta = 1/v$  is given by

$$w^{(v)} = \frac{1}{\gamma} Var[R_{t+1}^{(v)}]^{-1} (E[R_{t+1}] - \frac{1}{v}\iota_L) = -\frac{1}{v\gamma}\alpha^{(v)},$$

we have, by using (6) that

$$E[m_R(v)_{t+1}R_{t+1}] = vE[R_{t+1}] - v\gamma Cov[R_{t+1}, R_{t+1}^{(v)}]w^{(v)} = \iota_K - v\delta \le \iota_K,$$
 (35) if  $v > 0$ .

Thus, the stochastic discount factor in (34) satisfies (1), implying that the portfolio return  $w^{(v)}$ ,  $R_{t+1}^{(v)}$  that maximizes the Sharpe ratio over all returns in X, is also in  $X^*$ . Therefore,  $m_R(v)_{t+1}$  is a stochastic discount factor that attains the volatility bound. This is straightforward, since, using (34),

$$\frac{\sigma[m_R(v)_{t+1}]}{v} = \{(\frac{1}{v}\iota_L - E[R_{t+1}^{(v)}])'Var[R_{t+1}^{(v)}]^{-1}(\frac{1}{v}\iota_L - E[R_{t+1}^{(v)}])\}^{\frac{1}{2}},$$

which is equal to the maximum Sharpe ratio in (33).

#### B Proof of validity of the test

In this appendix we prove a simple but useful lemma. This lemma shows that the fact that we possibly use the incorrect regressions in our spanning and intersection tests (due to sample variation) is asymptotically negligible. Short sales restrictions on the benchmark assets are handled by testing for spanning and intersection on subsets of the available assets, where there is only a finite number of such subsets. The probability of choosing the right subsets tends to one, and this turns out to be a sufficient condition for the validity of the tests.

Suppose that we are given a finite number of Wald test-statistics,  $\xi(v^{[1]})_T$ , ...,  $\xi(v^{[M]})_T$ , as defined in (14), where T is the sample size. Let the space of all possible values of v be partitioned in  $V^{[1]}$ , ...,  $V^{[M]}$ , with the interpretation that, depending on the value of the parameter v, one of the test statistics  $\xi(v^{[1]})_T$  has desirable properties. Let j indicate the set  $V^{[j]}$  to which  $v^{[j]}$  belongs. If  $v_0$  denotes the true value of v, one would like to use the test  $\xi(v_0)_T$  of course, but this is not possible, since  $v_0$  is unknown. Assume however, that we are given a parameter estimate  $\hat{v}_T$ , such that, under  $v_0$ 

$$\Pr{\{\hat{v}_T \in V^{[j]}\} \to 1, T \to \infty.}$$

Now we have the following result:

Lemma 1 For each  $c \in \mathbb{R}$ , we have

$$\lim_{T \to \infty} \Pr\{\xi(\hat{v}_T)_T \le c\} - \Pr\{\xi(v_0)_T \le c\} = 0.$$

Proof. The proof is very straightforward, using:

$$\begin{split} \Pr\{\xi(\widehat{v}_T)_T & \leq \ c\} = \sum_{j=1}^M \Pr\{\xi(v^{[j]})_T \leq c \text{ and } v^{[j]} = \widehat{v}_T\} = \\ \Pr\{\xi(v_0)_T & \leq \ c\} - \Pr\{\xi(v_0)_T \leq c \text{ and } v \neq v_0\} + \\ \sum_{v \neq v_0} \Pr\{\xi(v)_T & \leq \ c \text{ and } v = \widehat{v}_T\}, \end{split}$$

and that the latter two terms converge to zero.

Country	Code	Starting Date IFC Investable	Liberalization
Argentina	ARG	Jan 1989	Dec 1989
Brazil	BRA	Jan 1989	Jul 1991
Chile	CHI	Jan 1989	Apr 1990
Colombia	COL	Mar 1991	Feb 1991
Mexico	MEX	Jan 1989	May 1989
Venezuela	VEN	Feb 1990	Dec 1990
India	IND	Dec 1992	Nov 1992
Korea	KOR	Feb 1992	Jan 1992
Malaysia	MAL	Jan 1989	12.2
Pakistan	PAK	Apr 1991	Feb 1991
Philippines	PHI	Jan 1989	Nov 1991
Taiwan	TAI	Feb 1991	Jan 1991
Thailand	THA	Jan 1989	
Greece	GRE	Jan 1989	
Jordan	JOR	Jan 1989	
Nigeria	NIG	Not available	
Zimbabwe	ZIM	Nov 1993	

#### C Available data for the emerging markets

This appendix describes some characteristics of the data that are used. The table gives the first month that the IFC Investable Indices for the emerging markets used in this paper appear in the sample. The Global Indices are always available from January 1985 onwards. The sample period ends in June 1996. The last column of the table contains the last major liberalization date of the emerging stock market, based on Bekaert (1995).

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Table 1: Summary statistics:

Panel A provides summary statistics for monthly dollar returns on the MSCI Indices that serve as the benchmark assets. Panel B provides summary statistics for the IFC Emerging Markets Data Base. The sample period is January 1985 until June 1996, giving a total of 138 observations. GMV is the Global Minimum Variance Portfolio. v is the expectation of the stochastic discount factor. "Sh" is

the maximum attainable Sharpe ratio.

Panel A	: Benchm	ark indices		
	average	stand.dev.	skewness	kurtosis-3
USA	1.38%	4.16%	-1.14	6.09
Europe	1.58%	4.91%	-0.56	1.80
Japan	1.43%	7.55%	0.21	0.51
	Correl.			
USA	1.000	0.605	0.211	
Europe		1.000	0.493	
Japan			1.000	
	No fricti	ions		
<b>GMV</b>	1.42%	3.88%		
$v_{\mathrm{min}}$	0.986	$\mathrm{Sh}(v_{\mathrm{min}})$	0.055	
$v_{\mathrm{max}}$	1.000	$Sh(v_{max})$	0.370	
	No short	sales allowe	d	
$v_{\min}$	0.984	$Sh(v_{min})$	0.086	
$v_{\mathrm{max}}$	1.000	$Sh(v_{max})$	0.370	
	0.125% t	ransaction c	osts	
$v_{\mathrm{min}}$	0.990	$\mathrm{Sh}(v_{\mathrm{min}})$	0.085	
$v_{\rm max}$	1.000	$Sh(v_{max})$	0.338	
	0.50% tr	ansaction cos	sts	
$v_{\mathrm{min}}$	0.993	$Sh(v_{min})$	0.085	
$v_{\rm max}$	1.000	$Sh(v_{max})$	0.243	

Panel	B: Emergi	ng markets			7.1	
	Avg(%)	Std.dev.(%)	$v_1$	$v_2$	$m(v_1)(\%)$	$m(v_2)(\%)$
ARG	5.10	27.76	0.999	1.089	1.45	1.42
BRA	3.01	19.23				
CHI	3.38	7.95	0.974	0.880	1.38	1.42
COL	2.83	8.94	0.987	0.946	1.90	1.41
MEX	3.18	13.10				
VEN	2.01	13.81	1.000	0.962	1.45	1.40
IND	1.55	9.80	0.999	0.973	1.45	1.39
KOR	1.64	8.33	1.021	0.925	1.43	1.41
MAL	1.37	7.64				
PAK	1.43	7.09	0.998	0.975	1.46	1.38
PHI	3.41	10.37	0.975	1.059	1.38	1.43
TAI	2.72	14.34				
THA	2.39	8.69	1.005	1.213	1.44	1.42
GRE	2.11	11.81	1.034	0.674	1.43	1.42
JOR	0.64	4.86	1.005	0.986	1.44	2.56
NIG	1.69	15.27	1.061	0.871	1.43	1.42
ZIM	2.51	9.21	0.991	0.957	1.51	1.40

	jan-85 -	dec-91	jan-92 -	jun-96
	Avg(%)	Std.dev.(%)	Avg(%)	Std.dev.(%)
USA	1.51	5.03	1.17	2.23
EUR	1.96	5.70	1.00	3.28
JAP	1.96	7.96	0.61	6.87
ARG	7.74	34.44	0.98	10.26
BRA	3.01	22.44	3.02	12.96
CHI	4.38	8.08	1.83	7.55
COL	3.65	8.69	1.56	9.24
MEX	4.87	14.14	0.54	10.91
VEN	3.60	13.68	-0.45	13.77
IND	1.83	8.78	1.12	11.27
KOR	2.30	8.71	0.62	7.67
MAL	1.01	7.99	1.93	7.10
PAK	2.23	6.10	0.19	8.32
PHI	4.08	11.43	2.38	8.46
TAI	3.57	15.96	1.40	11.37
THA	2.63	8.68	2.03	8.78
GRE	3.28	14.13	0.30	6.55
JOR	0.59	5.31	0.72	4.09
NIG	0.59	9.99	3.39	21.01
ZIM	3.08	8.83	1.61	9.78

Table 2: Spanning tests with short sales constraints

The table presents test results for the hypothesis that there is mean-variance spanning of emerging markets by three benchmark assets, which are the MSCI indices for the USA, Europe and Japan. The numbers in the table are Wald test statistics. The numbers in parentheses are p-values associated with the Wald test statistics. The tests are based on monthly returns for the period January 1985 until June 1996. The results for the IFC Investable Indices are for January 1989 until June 1996, or on a shorter period if no data for the IFC Investable Index was available. NA = not available.

Latin	Americ							
	Arg	Bra	Chi	Col	Mex	Ven		All
	No rest	rictions						
Wald	4.43	1.30	17.54	21.11	3.31	18.40		61.07
(p)	(0.109)	(0.522)	(0.000)	(0.000)	(0.192)	(0.000)		(0.000)
	No shor	rt sales of	emerging	markets				
Wald	3.72		14.46			3.72		31.44
(p)	(0.032)	(0.157)	(0.001)	(0.003)	(0.053)	(0.035)		(0.000)
	No shor							
Wald	3.70	1.21	14.36	9.88	3.23	3.69		31.74
(p)	(0.075)	(0.313)	(0.000)	(0.003)	(0.109)	(0.070)		(0.000)
	Investal	ble indices	, no shor	t sales				
Wald	2.01	2.11	6.74	4.42	1.65	5.46		14.81
(p)	(0.181)	(0.175)	(0.012)	(0.057)	(0.219)	(0.024)		(0.027)
Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	
	No restr	rictions						
Wald	32.50	6.11	0.74	44.07	6.05	2.36	4.60	86.16
(p)	(0.000)	(0.047)	(0.689)	(0.000)	(0.048)	(0.307)	(0.100)	(0.000)
	No shor	t sales of	emerging	markets				
Wald	3.92	1.47	0.05	4.71	6.05	1.87	3.28	14.82
(p)	(0.029)	(0.148)	(0.481)	(0.017)	(0.011)	(0.098)	(0.044)	(0.004)
	No shor	t sales						
Wald	3.89	1.46	0.05	4.68	6.00	1.86	3.25	14.71
(p)	(0.069)	(0.256)	(0.680)	(0.042)	(0.017)	(0.194)	(0.082)	(0.029)
(80) (B)		le indices	200	117 777	S (1)	· 100 (f)	6.	
Wald	0.21	0.17	0.97	2.04	0.88	0.67	0.61	3.93
(p)	(0.614)	(0.627)	(0.329)	(0.196)	(0.366)	(0.398)	(0.429)	(0.578)

(Continued)

Other	r				
	Gre	Jor	Nig	Zim	All
	No restr	rictions			
Wald	4.28	76.37	4.29	24.29	105.07
(p)	(0.118)	(0.000)	(0.117)	(0.000)	(0.000)
200	No shor	t sales of	emerging	markets	
Wald	1.59	1.09	0.83	8.38	11.16
(p)	(0.125)	(0.186)	(0.227)	(0.001)	(0.013)
-0.4	No shor	t sales	<i>10</i> 16		
Wald	1.58	1.08	0.83	8.31	11.08
(p)	(0.227)	(0.312)	(0.387)	(0.006)	(0.036)
2.5	Investab	le indices	, no shor	t sales	
Wald	1.59	1.02	NA	3.21	3.25
(p)	(0.240)	(0.334)	NA	(0.105)	(0.391)

Table 3: Spanning tests with transaction costs

The table presents test results for the hypothesis that there is mean-variance spanning of emerging markets by three benchmark assets, which are the MSCI indices for the USA, Europe and Japan when there are transaction costs. The table assumes that there is a 0.125 percent transaction cost on the benchmark assets. The numbers in the table are Wald test statistics. The numbers in parentheses are the p-values associated with the Wald test statistics. The tests are based on monthly returns for January 1985 until June 1996. NA = not available.

Latin	Americ	a						
	Arg		Chi			Ven		All
	0.125 %	tr. cst.	on emerg	ing marke	ets			
Wald			13.97			3.28		29.74
(p)			(0.000)			(0.042)		(0.000)
	0.250 %	tr. cst.	on emerg	ing marke	ets			
Wald			12.67					26.99
(p)	(0.040)	(0.176)	(0.001)	(0.002)	(0.069)	(0.051)		(0.000)
			on emergi					
Wald	3.06	0.77	10.26	6.70	1.81	2.29	15	21.93
(p)	(0.043)	(0.226)	(0.001)	(0.009)	(0.112)	(0.080)		(0.000)
	1.000 %	tr. cst.	on emerg	ing marke	ts	***************************************		,
Wald	2.41	0.35	6.20	3.91	0.76	1.25		13.51
(p)	(0.068)	(0.323)	(0.011)	(0.036)	(0.226)	(0.142)		(0.012)
Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	All
			on emergi					
Wald			0.04			1.80	3.13	13.43
(p)	(0.044)	(0.154)	(0.456)	(0.029)	(0.012)	(0.105)	(0.049)	(0.011)
	0.250 %	tr. cst.	on emergi	ng marke	ts		,	
Wald	2.80		0.00			1.55	2.57	11.38
(p)	(0.065)	(0.194)	(0.944)	(0.052)	(0.015)	(0.128)	(0.065)	(0.020)
	0.500 %	tr. cst.	on emergi	ng marke	ts	,		,
Wald	1.92		0.00			1.11	1.62	7.90
(p)	(0.103)	(0.325)	(0.789)	(0.105)	(0.029)	(0.162)	(0.127)	(0.086)
	1.000 %	tr. cst. c	on emergi	ng marke	ts	(/	(/	(5.555)
Wald	0.65	0.00	0.00	0.36	2.19	0.44	0.37	3.23
(p)	(0.250)	(0.803)	(0.637)	(0.314)	(0.075)	(0.291)	(0.320)	(0.408)

(Continued)

Othe	r				
	Gre	Jor	Nig	Zim	All
	0.125 %	tr. cst.	on emerge	ing markets	
Wald	1.46	0.62	0.73	7.66	9.85
(p)	(0.119)	(0.269)	(0.242)	(0.006)	(0.022)
	0.250 %	tr. cst.	on emergi	ing markets	, ,
Wald	1.19	0.25	0.59	6.86	8.40
(p)	(0.158)	(0.344)	(0.263)	(0.005)	(0.027)
				ng markets	,
Wald	0.73	0.00		5.39	6.18
(p)	(0.215)	(0.547)	(0.314)	(0.013)	(0.080)
	1.000 %			ng markets	(/
Wald	0.15	0.00		2.98	3.10
(p)	(0.386)	(0.539)	(0.478)	(0.057)	(0.280)

Table 4: Testing for spanning with transaction costs: Investable indices The table presents test results for the hypothesis that there is mean-variance spanning of the IFC Investable Indices for the emerging markets by three benchmark assets, which are the MSCI indices for the USA, Europe and Japan when there are transaction costs. The table assumes that there is a 0.125 percent transaction cost on the benchmark assets. The numbers in the table are Wald test statistics. The numbers in parentheses are the p-values associated with the Wald test statistics. The tests are based on monthly returns for January 1989 until June 1996, or on a shorter period if no data for the IFC Investable Index was available. NA = not available.

Latin	Americ	a						
	Arg	Bra	Chi	Col	Mex	Ven		All
200000	0.125 %	tr. cst.	on emerg	ing marke	ets			
Wald	2.17	1.80	6.47	4.31	1.65	5.10		9.72
(p)	(0.099)	(0.129)	(0.012)	(0.023)	(0.137)	(0.022)		(0.040)
	0.500 %	tr. cst.	on emerg	ing marke	ets			
Wald	1.82			3.38				7.12
(p)	(0.114)	(0.146)	(0.023)	(0.040)	(0.235)	(0.030)		(0.102)
Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	All
	0.125 %	tr. cst.	on emergi	ing marke	ts			
Wald	0.15	0.10	0.96	1.88	0.43	0.62	0.74	2.73
(p)	(0.453)	(0.487)	(0.215)	(0.115)	(0.321)	(0.279)	(0.248)	(0.438)
				ing marke				
Wald	0.02	0.00	0.24	1.29	0.09	0.29	0.20	1.92
(p)	(0.566)	(0.652)	(0.369)	(0.178)	(0.429)	(0.388)	(0.371)	(0.584)
Other		191						, ,
	Gre	Jor	Nig	Zim				All -
	0.125 %	tr. cst.	on emergi	ng marke	ts			
Wald	1.49	0.79	NA	3.03				3.06
(p)	(0.141)	(0.233)	NA	(0.057)				(0.216)
	0.500 %	tr. cst. c	on emergi	ng marke	ts			, , , ,
Wald	0.93	0.08	NA	2.42				2.42
(p)	(0.221)	(0.447)	NA	(0.092)				(0.274)

Table 5: Transaction cost bounds

The table presents estimated transaction cost bounds for the emerging markets in order to reject spanning of each emerging market by three benchmark assets at the 5% and 10% significance level. The three benchmark assets are the MSCI indices for the USA, Europe and Japan. The table assumes that there is a 0.5 percent transaction cost on the benchmark assets. The estimated transaction costs are in percentages per month. All results are based on monthly returns for January 1985 until June 1996. The actual transaction costs are from Baring Securities as reported by Bekaert et al. (1996). NA = not available.

Latin Amer	ica						
	ARG	BRA	CHI	COL	MEX	VEN	
10%-bound	1.50	0.00	1.70	1.50	0.30	0.70	
5%-bound	0.60	0.00	1.50	1.20	0.00	0.30	
actual tr.cst.	1.55	0.85	3.93	1.00	0.93	NA	
Asia							
	IND	KOR	MAL	PAK	PHI	TAI	THA
10%-bound	0.40	0.20	0.00	0.50	1.40	0.30	0.75
5%-bound	0.10	0.00	0.00	0.30	1.10	0.00	0.40
actual tr.cst.	1.50	NA	0.69	0.38	0.94	0.47	0.70
Other				×			
	GRE	JOR	NIG	ZIM			
10%-bound	0.20	0.00	0.00	1.40			
5%-bound	0.00	0.00	0.00	0.10			
actual tr.cst.	0.48	0.58	NA	NA			

Table 6: Spanning test for the post-liberalization periods

The table presents test results for the hypothesis that there is mean-variance spanning of emerging markets by three benchmark assets, which are the MSCI indices for the USA, Europe and Japan, after liberalizations in the emerging markets have taken place. For each emerging market results are shown for the period after liberalization of the stock market has taken place, as reported in Appendix C. If there is no liberalization during the sample period, the whole sample period is used. The numbers in the table are Wald test statistics. The numbers in parentheses are p-values associated with the Wald test statistics. The tests are based on monthly returns for the month after liberalization (or from January 1985) until June 1996.

Latin	Americ	a					
	Arg	Bra	Chi	Col	Mex	Ven	All
	No restr	rictions					
Wald	2.12	2.28	4.38	5.14	1.02	1.85	18.12
(p)	(0.347)	(0.320)	(0.112)	(0.077)	(0.599)	(0.397)	(0.000)
	No shor	t sales of	emerging	markets			
Wald	0.76	0.85	3.52	4.01	0.93	0.01	5.60
(p)	(0.221)	(0.242)	(0.036)	(0.031)	(0.201)	(0.524)	(0.163)
***	No shor	t sales	100				
Wald	0.27	0.40	3.51	4.01	0.85	0.01	5.03
(p)	(0.606)	(0.298)	(0.061)	(0.026)	(0.194)	(0.480)	(0.148)
	Investat	le indices	, no shor	t sales			
Wald	0.27	0.38	3.94	4.43	0.95	3.78	5.57
(p)	(0.603)	(0.301)	(0.047)	(0.023)	(0.170)	(0.026)	(0.127)

Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	All
	No restr	rictions						
Wald	3.16	3.33	0.74	4.42	3.19	1.31	4.60	16.50
(p)	(0.206)	(0.189)	(0.689)	(0.110)	(0.203)	(0.520)	(0.100)	(0.000)
	No shor	t sales of	emerging	markets				3
Wald	0.12	0.12	0.05	1.95	2.72	0.70	3.28	1.33
(p)	(0.433)	(0.436)	(0.481)	(0.112)	(0.065)	(0.249)	(0.044)	(0.633)
7770775	No shor	t sales			A			
Wald	0.12	0.13	0.05	1.67	2.67	0.55	3.25	1.30
(p)	(0.451)	(0.404)	(0.680)	(0.094)	(0.068)	(0.226)	(0.082)	(0.655)
17.7	Investab	le indices	, no shor	t sales				
Wald	0.21	0.18	0.97	2.03	1.63	0.53	0.61	3.11
(p)	(0.403)	(0.369)	(0.329)	(0.088)	(0.114)	(0.236)	(0.429)	(0.362)

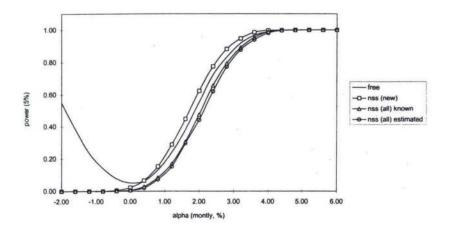


Figure 1: The figure presents the power function as a function of the intercept in (19a). For each value of the intercept (alpha) the power is derived from a series of 1000 simulations with 120 monthly observations each, when the rejection rate is 5%. The label "free" refers to the spanning test when there are no short sales constraints, "nss (new)" refers to the spanning test when there are only short sales constraints on the new assets, "nss (all) known" refers to the spanning test when there are short sales constraints on all assets and the relevant subsets of the benchmark assets are known, and the label "nss (all) estimated" refers to the spanning test with short sales constraints on all assets where the relevant subsets are estimated in each simulation run.

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