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Testing for Mobility Dominance

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Abstract:
This paper proposes tests for stochastic dominance in mobility based on the empirical likelihood ratio. Two views of mobility are considered, either based on measures of absolute mobility or on transition matrices. First-order and second-order dominance conditions in mobility are first derived, followed by the derivation of statistical inferences techniques to test a null hypothesis of non dominance against an alternative of mobility dominance. An empirical analysis, based on the US Panel Study of Income Dynamics (PSID), is performed by comparing four income mobility periods ranging from 1970 to 1990.

Keywords: Mobility, stochastic dominance, transition matrices, empirical likelihood ratio, bootstrap tests

JEL Classification: C10, C12, C13, D31, J60

1 Introduction

Two sources of income mobility are usually considered in the literature. The first arises from a temporal reallocation of incomes across individuals, and the second comes from changes in total income (Fields and Ok 1996). Ever since Prais (1955), a variety of mobility measures have been proposed to capture the effect of these on the income distribution. These include relatively elementary measures based on statistics such as the coefficient of correlation between individuals at different dates (see for instance Atkinson, Bourguignon, and Morrisson 1992, Peters 1992, Bjorklund and Jantti 1997, Chadwick and Solon 2002, or Corak 2006) as well as more elaborate measures based on transition matrices and other measures of dynamic processes.

Some of the studies have adopted a relative concept, treating mobility as a re-ranking in which individuals change position (Shorrocks 1978a, Bartholomew 1996). Others have treated mobility as an absolute concept in which any change in individuals' incomes from the initial situation is equivalent to greater mobility (Fields and Ok 1996; also see D'Agostino and Dardanoni 2005 for a discussion). A further group of studies are based on specific interpretations of mobility, notably reflecting its implications for social welfare. Shorrocks (1978b) and Maasoumi and Zandvakili (1986), for instance, measure mobility as the ratio of income inequality averaged over several years relative to mean inequality in each year; see also Chakravarty, Dutta, and Weymark (1985), Atkinson (1983), Markandya (1984), Conlisk (1989) and Dardanoni (1993).

Though a number of studies have addressed the measurement of mobility, relatively few have examined how and whether normatively robust comparisons of mobility can be made across countries or time periods — see Atkinson (1983), Conlisk (1989), Dardanoni (1993), Mitra and Ok (1998), Benabou and Ok (2001), and Formby, Smith, and Zheng (2003) for some of the more important exceptions. In addition, aside from Fields, Leary, and Ok (2002) who explicitly integrate the cumulative density function to derive conditions for stochastic dominance, and Formby, Smith, and Zheng (2004) who perform comparisons between two vectors of mobility measures (using composite hypotheses), most studies tend to focus on comparing estimates of mobility indices without implementing explicit statistical testing procedures.

This paper tries to fill some of this gap by demonstrating the use of empirical likelihood ratio procedures to test for the existence of stochastic dominance in mobility. Two types of mobility measures are retained: absolute measures and those based on transition matrices. These measures have the advantage of being relatively easy to reconcile with tools such as the cumulative density function, a feature which is convenient for the application of the empirical likelihood methods recently proposed by Davidson and Duclos (2006).

The remainder of this paper is organized as follows. Section 2 presents mobility measures and a theoretical framework for their partial ranking. Here we expand on several examples of the two types of measures retained as well as on conditions for stochastic dominance in mobility. Section 3 describes methods of statistical inference for ranking mobility based on empirical likelihood ratios and resampling procedures. Section 4 provides an illustration of the methods to robust comparisons of mobility across time in the United States. The final section concludes.

2 Mobility measures and stochastic dominance

The concept of income mobility captures the extent to which the income distribution changes over time across a given set of individuals. Let an initial distribution of income be denoted by $x = (x_1, x_2, \ldots, x_i, \ldots, x_n) \in \mathbb{R}^n_+$ where x_i is the income of the i-th individual in a population of size n. Let y_i be the income of individual i at a later date (or the income of a descendent of i if we were to think of intergenerational mobility), such that a new income distribution $y = (y_1, y_2, \ldots, y_{i,} \ldots, y_n) \in \mathbb{R}^n_+$ is generated by the transformation process $(x \to y)$.

A mobility measure can be defined as a continuous function $M : \mathbb{R}^n_+ \times \mathbb{R}^n_+ \to \mathbb{R}$. If two other distributions x' and y' are characterized by the transformation process $(x' \to y')$, we say that process $(x \to y)$ is more mobile than the process $(x' \to y')$ if $M(x, y) \geq M(x', y')$.

Several such measures of mobility exist. This papers considers two classes of them. They reflect two sorts of mobility movements, to wit, structural (or growth) mobility and exchange (or lateral) mobility. Structural mobility increases with a general fall or rise in incomes, while exchange mobility captures changes in positions between individuals.[1] Measures of absolute mobility fall into the first class, while transition matrices are part of the second class.

2.1 Measures of absolute mobility

Fields and Ok (1996) propose measuring absolute income mobility using individual-specific temporal distances between incomes. A generic measure they propose is defined as:

$$M_d\,(x, y) = \left(\sum_{i=1}^{n} |x_i - y_i|^{\alpha} \right)^{1/\alpha} \quad \forall \quad x, y \in \mathbb{R}^n_+, \quad \text{and} \quad \alpha > 0. \tag{1}$$

More generally, let $\Delta_i = |x_i - y_i|$, with $i = 1, \ldots, n$. Δ_i helps assess the contribution of individual i to total mobility. Consider a class of functions $M_d = M_d(\Delta) =$

[1] See Fields and Ok (1999) for a more detailed discussion.

$M_D(\Delta_1, \ldots, \Delta_i, \ldots, \Delta_n)$. We can draw on Willig (1981), Shorrocks (1983), and Foster and Shorrocks (1988) in the context of welfare measurement to establish conditions for dominance in such types of mobility measures. First, analogous to the property of monotonicity (let us refer to it as H_A) of welfare functions (conferred by the Pareto principle), we can postulate that a rise in Δ_i must, *ceteris paribus*, entail an increase in mobility. Second, we can postulate a symmetry assumption (H_B) that imposes that $M_d(\Delta) = M_d(\Gamma\Delta)$, where Γ is a permutation matrix. This says that a re-ranking of the mobility contributions of individuals leaves total mobility unchanged.

Let $\mathcal{M}_d^+$ represent the class of mobility functions that satisfy assumptions H_A and H_B. Let Δ and Δ' be two profiles of individual mobility contributions with cumulative distribution functions (cdf) given by $F(z)$ and $G(z)$, respectively. Let $Q_F(p)$ and $Q_G(p)$ be the corresponding quantile functions, given by the inverse of the cdf's. By analogy with the welfare dominance conditions shown *inter alia* in Duclos and Araar (2006), Theorem 1 shows conditions for first-order dominance in mobility.

Theorem 1 (First-order mobility dominance) *The following conditions are equivalent:*
(i) $M_d(\Delta) \geq M_d(\Delta') \quad \forall \quad M_d \in \mathcal{M}_d^+$,
(ii) $F(z) \leq G(z) \quad \forall \quad z \in [0, +\infty[$,
(iii) $Q_F(p) \geq Q_G(p) \quad \forall \quad p \in [0, 1]$.

Second-order dominance conditions are obtained when an assumption H_C of Schur-concavity of the function M_d is added. Let $\mathcal{M}_d^{++}$ then be the class of mobility functions that are consistent with assumptions H_A, H_B and H_C. Define $F^2(z)$ and $G^2(z)$ as

$$F^2(z) = \int_0^z F(t)dt \quad \text{and} \quad G^2(z) = \int_0^z G(t)dt. \tag{2}$$

Also define the generalized Lorenz curves $GL_F(p)$ and $GL_G(p)$ as

$$GL_F(p) = \int_0^p Q_F(q)dq \quad \text{and} \quad GL_G(p) = \int_0^p Q_G(q)dq, \tag{3}$$

with $p \in [0, 1]$. Theorem 2 shows conditions for second-order dominance in mobility.

Theorem 2 *The following conditions are equivalent:*
(i) $M_d(\Delta) \geq M_d(\Delta') \quad \forall \quad M_d \in \mathcal{M}_d^{++}$,
(ii) $F^2(Z) \leq G^2(Z) \quad \forall \quad Z \in [0, +\infty[$,
(iii) $GL_F(p) \geq GL_G(p) \quad \forall \quad p \in [0, 1]$.

For the class of mobility functions proposed by Fields and Ok (1996), the Schur-concavity assumption is verifiable for $\alpha \leq 1$. Mitra and Ok (1998), for their part, focus on a class of Schur-convex functions by setting $\alpha \geq 1$. They develop an approach to partial rankings in which $(x \to y)$ is more mobile than $(x' \to y')$ if:

$$\sum_{i=1}^{n}\Delta_i^{\alpha}(x,y) \geq \sum_{i=1}^{n}\Delta_i^{\alpha}(x',y') \quad \forall \quad \alpha \geq 1. \tag{4}$$

Mitra and Ok (1998) then derive another, more easily testable, condition for a partial ranking. Arranging Δ and Δ' in descending order j, $(x \to y)$ is more mobile than $(x' \to y')$ if and only if:

$$\sum_{j=1}^{s}\Delta_j(x,y) \geq \sum_{j=1}^{s}\Delta_j(x',y') \quad \forall \quad s = 1,\ldots,n. \tag{5}$$

This is an alternative approach to that of the generalized Lorenz dominance one in Theorem 2, where the Δ_i's are arranged in increasing order. Here, we weigh the most mobile individuals more heavily, which is compatible with the Schur-convexity assumption implied by the choice of $\alpha \geq 1$. If we let $\alpha \leq 1$, we have Schur-concavity and an "egalitarian mobility transfer" is desirable. In this case, the usual generalized Lorenz approach is preferred.

2.2 Measures based on transition matrices

2.2.1 The transition matrix

For simplicity, let income dynamics be described by a discrete Markov process with η states. Let $P = [P_{kl}]$ be a nonsingular $(\eta \times \eta)$ transition matrix (*i.e.* P^d is strictly positive for sufficiently large values of the integer d) such that there exists a vector of steady-state probabilities π that uniquely solves the equation $\pi^\tau = \pi^\tau P$ (τ designates the transpose). The element P_{kl} represents the probability that an individual initially in state k will end up in state l.

Let us return to the process $(x \to y)$, where $x = (x_1, x_2, \ldots, x_i, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_i, \ldots, y_n)$. Assume that x_i and y_i are bounded above by the strictly positive values $\overline{x}$ and $\overline{y}$, respectively. Let Ω_k be one of the η partitions of the interval $[0, \overline{x}]$, representing a class of rank k, and Γ_l be one of the η partitions of the interval $[0, \overline{y}]$, representing a class of rank l. Define p_i as the probability associated with each pair (x_i, y_i), such that $\sum\limits_{i=1}^{n} p_i = 1$. The expression for the conditional probability P_{kl} is given by:

$$P_{kl} = \frac{\Pr\left([x_i \in \Omega_k] \cap [y_i \in \Gamma_l]\right)}{\Pr\left(x_i \in \Omega_k\right)} = \frac{\sum\limits_{i=1}^{n} p_i I\left(x_i \in \Omega_k, y_i \in \Gamma_l\right)}{\sum\limits_{i=1}^{n} p_i I\left(x_i \in \Omega_k\right)} \tag{6}$$

where $\Pr$ stands for probability and $I(\cdot)$ is an indicator function taking the value one when the argument is true and zero otherwise. We also observe the following marginal probabilities:

$$\pi_k = \sum_{i=1}^{n} p_i I\left(x_i \in \Omega_k\right) \quad \text{and} \quad \pi_l = \sum_{i=1}^{n} p_i I\left(y_i \in \Gamma_l\right). \tag{7}$$

The vectors of these marginal probabilities are represented respectively by $\pi\left(k\right)$ and $\pi\left(l\right)$, with $\pi\left(k\right) = \pi\left(l\right) = \pi$ at the steady state.

2.2.2 Mobility and social welfare

The dominance conditions derived by Atkinson (1983) allow mobility to be ranked for a class of welfare functions satisfying a certain set of properties. Let us consider the case of two periods—specifically, a process $(x \to y)$. Let $F(x, y)$ be the bivariate cdf and $F(x)$ and $F(y)$ be the marginal cdf. A social welfare function defined over the two time periods can be defined as

$$W = \int_0^{\overline{x}} \int_0^{\overline{y}} U\left(x, y\right) dF\left(x, y\right) dxdy. \tag{8}$$

Let $G(x, y)$ represent the bivariate cdf of another process $(x' \to y')$. $G(x)$ and $G(y)$ are the marginal cdf, and assume for now that $F(x) = G(x)$ and $F(y) = G(y)$.

If a mobility process is described by the transition matrix P, we have:

$$\pi^{\tau}(l) = \pi^{\tau}(k)P \quad \text{or} \quad \pi_l = \sum_{k=1}^{\eta} P_{kl}\pi_k \quad \forall \quad l = 1, \ldots, \eta, \tag{9}$$

where $\pi(k)$ and $\pi(l)$ are two $(\eta \times 1)$ vectors. Let x^k and y^l be incomes of rank k and l respectively. Expected welfare in (8) becomes:

$$W = \sum_{k=1}^{\eta}\sum_{l=1}^{\eta} U\left(x^k, y^l\right) P_{kl}\pi_k. \tag{10}$$

A distribution F stochastically dominates a distribution G if the expected utility level F generates is at least as high as that for G for all utility functions in some class of U. A first-order class of U is made of the U functions for which $U_x > 0$, $U_y > 0$ and $U_{xy} \leq 0$. Atkinson (1983) shows that, for identical marginal distributions [*i.e.* $F(x) = G(x)$ and $F(y) = G(y)$], the process $(x \to y)$ first-order dominates the process $(x' \to y')$ if and only if:

$$F(x^k, y^l) \leq G(x^k, y^l) \quad \forall \quad k, l = 1, \ldots, \eta. \tag{11}$$

It is also possible to derive a second-order dominance condition that is weaker than the above first-order condition. A distribution F second-order dominates a distribution G if the expected utility level it generates is at least as high for all utility functions U such that $U_x > 0$, $U_y > 0$, $U_{xy} \leq 0$, $U_{xy} < 0, U_{xxy}, U_{xyy} \geq 0$ and $U_{xxyy} \leq 0$. This is equivalent to

$$F^2\left(x^k, y^l\right) \leq G^2\left(x^k, y^l\right) \quad \forall \quad k, l = 1, \ldots, \eta \tag{12}$$

where

$$F^2\left(x, y\right) = \int_0^x \int_0^y F\left(s, t\right) dtds \text{ and } G^2\left(x, y\right) = \int_0^x \int_0^y G\left(s, t\right) dtds. \tag{13}$$

A difference between $\pi\left(k\right)$ and $\pi\left(l\right)$ would allow incorporation of structural (or growth) mobility. Normalizing incomes such as to equalize the marginal distributions, *i.e.* $\pi\left(k\right) = \pi\left(l\right) = \pi$, we can isolate pure mobility. This is what is done in the illustration below — see also Dardanoni (1993) Formby, Smith, and Zheng (2003) for alternative approaches to this exercise.

3 Methods of statistical inference

The tests for dominance in mobility derived in this section extend the use of the empirical likelihood ratio statistics developed in Davidson and Duclos (2006). They are based on an intersection-union approach that makes it possible to test directly for strict dominance of one distribution over another. For measures of absolute mobility, the dominance condition pertains to a univariate distribution, and the Davidson and Duclos method is thus directly applicable. A comparison of measures based on transition matrices requires an extension to the two-dimensional case; this has been suggested and partly investigated by Batana and Duclos (2008).

3.1 Inference with measures of absolute mobility

Let two distributions of individual mobility Δ and Δ' derive from the process $(x \to y)$ and $(x' \to y')$. Also let the distributions of Δ and Δ' be denoted by $F(Z)$ and $G(Z)$, respectively, with sample analogues (with sample sizes equal to N_F and N_G respectively) given by

$$\widehat{F}(Z)=\frac{1}{N_F}\sum_{s=1}^{N_F} I\left(\Delta_s \leq Z\right) \quad \text{and} \quad \widehat{G}(Z)=\frac{1}{N_G}\sum_{h=1}^{N_G} I\left(\Delta_h' \leq Z\right). \tag{14}$$

The problem of maximizing the unconstrained empirical likelihood function (ELF) is as follows:

$$\max_{p_s^F,p_h^G}\sum_{s=1}^{N_F}\log p_s^F+\sum_{h=1}^{N_G}\log p_h^G \quad \text{subject to} \quad \sum_{s=1}^{N_F} p_s^F=1, \sum_{h=1}^{N_G} p_h^G=1, \tag{15}$$

where p_s^F and p_h^G represent respectively the probabilities of Δ_s and Δ_h' occurring. The solution to this problem yields $p_s^F=\frac{1}{N_F}$ and $p_h^G=\frac{1}{N_G}$, so that the maximized value of the unconstrained likelihood is:

$$\widehat{ELF}_{UC}=-N_F\log N_F-N_G\log N_G. \tag{16}$$

Strict dominance in mobility of G by F, *i.e.* stochastic dominance in mobility of $(x'\rightarrow y')$ by $(x\rightarrow y)$, implies that $G(Z)>F(Z)$ for all Z. This condition is violated if there exists a point Z for which $G(Z)\leq F(Z)$. To test the null hypothesis of non-dominance of G by F, the natural null to be tested is therefore

$$H_0: G(Z)\leq F(Z) \text{ for at least one } Z \tag{17}$$

against the alternative hypothesis

$$H_0: G(Z)>F(Z) \text{ for all } Z. \tag{18}$$

The condition of dominance of Δ' by Δ in the samples is met when $\widehat{F}(Z)\leq\widehat{G}(Z)\ \forall\ Z$. When that condition holds, the ELF maximization problem (15) can then be recast to impose a non-dominance constraint. This constraint is given by:

$$\sum_{h=1}^{N_G} p_h^G I\left(\Delta_h'\leq Z\right)\leq\sum_{s=1}^{N_F} p_s^F I\left(\Delta_s\leq Z\right). \tag{19}$$

for some Z. Maximization of (15) subject to (19) gives the constrained empirical likelihood $\widehat{ELF}_C(Z)$, and leads to the following expressions:

$$p_h^G = \frac{I(\Delta'_h \leq Z)}{\theta} + \frac{[1 - I(\Delta'_h \leq Z)]}{\phi}, \tag{20}$$

$$p_s^F = \frac{I(\Delta_s \leq Z)}{N - \theta} + \frac{[1 - I(\Delta_s \leq Z)]}{N - \phi}, \tag{21}$$

where:

$$\theta = \frac{N \times N_G(Z)}{N_G(Z) + N_F(Z)} \text{ and } \phi = \frac{N \times M_G(Z)}{M_G(Z) + M_F(Z)},$$

$$N_G(Z) = \sum_{h=1}^{N_G} I\left(\Delta'_h \leq Z\right) \text{ and } N_F(Z) = \sum_{s=1}^{N_F} I\left(\Delta_s \leq Z\right),$$

$$M_G(Z) = N_G - N_G(Z), \;\; M_F(Z) = N_F - N_F(Z), \;\text{ and }\; N = N_G + N_F.$$

The statistic $LR(Z)$ is given by the difference between the value of the unconstrained maximum likelihood ($\widehat{ELF}_{UC}$) and the value of the maximum likelihood ($\widehat{ELF}_C(Z)$) constrained at Z. For values of Z for which we have in the samples that $\widehat{G}(Z) \leq \widehat{F}(Z)$, the constraint is not binding and, consequently, $LR(Z)$ is nil. When $\widehat{G}(Z) > \widehat{F}(Z)$ for some Z, the constraint becomes binding and ($\widehat{ELF}_C$) is then less than ($\widehat{ELF}_{UC}$), so that $LR(Z)$ assumes a strictly positive real value:

$$LR\,(Z) = \left\{ \begin{array}{cc} 0 \quad \text{if} & \widehat{G}(Z) \;\leq\; \widehat{F}(Z) \\ > 0 \quad \text{otherwise} & \end{array} \right\}. \tag{22}$$

$LR(Z)$ is then given by:

$$LR\,(Z) = 2 \left\{ \begin{array}{c} N \log N - N_G \log N_G - N_F \log N_F \\ +N_G(Z) \log N_G(Z) + N_F(Z) \log N_F(Z) \\ +M_G(Z) \log M_G(Z) + M_F(Z) \log M_F(Z) \\ - [N_G(Z) + N_F(Z)] \log [N_G(Z) + N_F(Z)] \\ - [M_G(Z) + M_F(Z)] \log [M_G(Z) + M_F(Z)] \end{array} \right\} \tag{23}$$

The final test statistic LR for testing non-dominance is obtained by minimizing $LR(Z)$ over all values of Z:

$$LR = \min_Z LR(Z) \tag{24}$$

One could use the methods of Davidson and Duclos (2006) to show that LR follows asymptotically a χ^2 distribution under the null of non-dominance. Finite sample refinements can, however, be profitably obtained by bootstrapping the value of LR. This is done by computing the statistics $LR_b(Z)$ and LR_b for each of $b = 1, ..., B$ (B is set to 399 in the illustration below) bootstrap samples of both distributions simultaneously. These bootstrap samples are generated with the probabilities given by (29), where

$$Z = \arg\min_Z LR(Z). \tag{25}$$

The p-value of the bootstrap test is then given by the proportion of the statistics LR_b that are greater than LR.

To test for second-order dominance, we consider condition (iii) from Theorem 2. This is dominance in terms of generalized Lorenz curves. Unlike in the case of first-order dominance, we here reject dominance of G by F if $GL_F(p) \leq GL_G(p)$ for some p. The constraint (19) becomes:

$$\sum_{s=1}^{N_F} p_s^F \Delta_s I\left(\Delta_s \leq \hat{Z}_F(p)\right) = \sum_{h=1}^{N_G} p_h^G \Delta'_h I\left(\Delta'_h \leq \hat{Z}_G(p)\right), \tag{26}$$

where $\hat{Z}_F(p)$ and $\hat{Z}_G(p)$ correspond to sample quantiles at percentile p. The Lagrangian, $\mathcal{L}$, is given by

$$\begin{aligned}\mathcal{L} = \sum_s \log p_s^F + \sum_h \log p_h^G + \lambda_F\left(1 - \sum_s p_s^F\right) + \lambda_G\left(1 - \sum_h p_h^G\right) \\ - \mu\left[\sum_s p_s^F \Delta_s I_s\left(\hat{Z}_F(p)\right) - \sum_h p_h^G \Delta'_h I_h\left(\hat{Z}_G(p)\right)\right],\end{aligned} \tag{27}$$

where λ_F, λ_G and $\mu \in \mathbb{R}$ are the Lagrange multipliers. These equations cannot be solved analytically. However, starting from the first-order conditions, we can derive numerical procedures to find a solution. The solutions use

$$\lambda_F + \lambda_G = N_F + N_G = N \tag{28}$$

$$p_h^G = \frac{1}{N - \lambda_F - \mu\Delta'_h I_h\left(\hat{Z}_G(p)\right)} \quad \text{and} \quad p_s^F = \frac{1}{\lambda_F + \mu\Delta_s I_s\left(\hat{Z}_F(p)\right)}. \tag{29}$$

and solve for $\widehat{\lambda}_F$ and $\widehat{\mu}$:

$$(\widehat{\lambda}_F, \widehat{\mu}) = \underset{\lambda_F, \mu\in\mathbb{R}}{\arg\min} - \sum_s \log\left[\lambda_F + \mu\Delta_s I_s\left(\hat{Z}_F(p)\right)\right] - \sum_h \log\left[N - \lambda_F - \mu\Delta'_h I_h\left(\hat{Z}_G(p)\right)\right]. \tag{30}$$

Once estimated, $\widehat{\lambda}_F$ and $\widehat{\mu}$ allow calculation of the probabilities and the likelihood ratio for each pair $(\hat{Z}_F(p), \hat{Z}_G(p))$. However, $\hat{Z}_F(p)$ and $\hat{Z}_G(p)$ are endogenous since both depend respectively on p_s^F and p_h^G. Thus, from some initial pair $(\hat{Z}_F(p)^0, \hat{Z}_G(p)^0)$, the estimated probabilities are used to compute a new pair $(\hat{Z}_F(p)^1, \hat{Z}_G(p)^1)$. The solution to problem (30) is then iterated and new probabilities are re-estimated by using the expressions in (29). The process is stopped at the i-th iteration when the differences between $(\hat{Z}_F(p)^{i-1}, \hat{Z}_G(p)^{i-1})$ and $(\hat{Z}_F(p)^i, \hat{Z}_G(p)^i)$ become numerically insignificant. Both the generalized Lorenz and the Mitra and Ok (1998) approaches are considered for such a test in the illustration below.

3.2 Inference with transition matrices

Consider a transition matrix $P = [P_{kl}]$. Consider also the expression $\sum\limits_{k=1}^{K}\sum\limits_{l=1}^{L}\pi_k P_{kl}$. If we replace P_{kl} and π_k by their expressions from equations (6) and (7), we obtain:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\pi_k P_{kl} = \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=1}^{n} p_i I\left(x_i \in \Omega_k, y_i \in \Gamma_l\right) \quad \forall \quad K, L = 1, 2, \ldots, \eta. \tag{31}$$

Denote $\Omega_k = [X_{k-1}, X_k]$ and $\Gamma_l = [Y_{l-1}, Y_l]$, with $\Omega_1 = [0, X_1]$, $\Omega_\eta = [X_{\eta-1}, \overline{x}]$, $\Gamma_1 = [0, Y_1]$, and $\Gamma_\eta = [Y_{\eta-1}, \overline{y}]$. We set each Ω_k and Γ_l to contain the same sample proportions. X_k and Y_l $(k, l = 1, \ldots, \eta)$ are then the η quantiles retained for each dimension. Equation (31) can be rewritten as:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\pi_k P_{kl} = \sum_{i=1}^{n} p_i I\left(x_i \leq X_k, y_i \leq Y_l\right) \quad \forall \quad k, l = 1, 2, \ldots, \eta. \tag{32}$$

Let two cumulative bivariate densities, F and G, describe the two distributions of $(x \to y)$ and $(x' \to y')$ respectively, with $\Omega_k = [X_{k-1}, X_k]$ and $\Gamma_l = [Y_{l-1}, Y_l]$ being associated to F and $\Omega'_k = \left[X'_{k-1}, X'_k\right]$ and $\Gamma'_l = \left[Y'_{l-1}, Y'_l\right]$ being associated to G. We distinguish between 2 cases, depending on how the transition matrix is calculated.

The first case corresponds to the situation in which we are only concerned with pure, or exchange, mobility (see Atkinson 1983 and Dardanoni 1993). Here, we impose the steady-state and pure mobility assumptions that $\hat{F}(\hat{X}_k) = \hat{F}(\hat{Y}_k) = \hat{G}(\hat{X}'_k) = \hat{G}(\hat{Y}'_k) = \pi_k$ and $k = 1, ..., \eta$, where the $\hat{X}$'s and $\hat{Y}$'s are the sample quantiles and the $\hat{F}$ and $\hat{G}$ are the empirical (or sample) distributions of F and G. In this first case, those sample quantiles are determined from the joint distributions of x and y in both $\hat{F}$ and $\hat{G}$.

The second case integrates growth, as suggested by Formby, Smith, and Zheng (2003). Here, the growth that is considered is that observed in the dynamics of each population separately. The normalization imposed here is that the two distributions F and G have the same initial quantiles in the x dimension. The values of these quantiles are then fixed and used to assess the degree of mobility in the y dimension. Here therefore, the equality that is imposed is that $\hat{F}(\hat{X}_k) = \hat{G}(\hat{X}'_k)$, with $\hat{X}_k = \hat{Y}_k$ and $\hat{X}'_k = \hat{Y}'_k$ for $k = 1, ..., \eta$. For simplicity, we focus below on the first case; the extension to the second case will also be briefly mentioned.

According to the Atkinson (1983) and Dardanoni (1993) mobility dominance criteria, the constraint that F does not dominate G in the samples implies, for at least one pair (k, l), that

$$\sum_{j=1}^{N_G} p_j^G I\left(x'_j \leq \hat{X}'_k, y'_j \leq \hat{Y}'_l\right) \leq \sum_{i=1}^{N_F} p_i^F I\left(x_i \leq \hat{X}_k, y_i \leq \hat{Y}_l\right). \tag{33}$$

$\hat{X}'_k$ and $\hat{Y}'_l$ are the sample quantiles (generated by the empirical distribution $\hat{G}$) that correspond to the same percentiles as the sample quantiles $\hat{X}_k$ and $\hat{Y}_l$ for the empirical distribution $\hat{F}$. The testing strategy follows a similar procedure to that of the previous section. The maximization results yield:

$$p_j^G(k,l) = \frac{I\left(x'_j \leq \hat{X}'_k, y'_i \leq \hat{Y}'_l\right)}{\theta} + \frac{\left[1 - I\left(x'_j \leq \hat{X}'_k, y'_i \leq \hat{Y}'_l\right)\right]}{\phi}, \tag{34}$$

$$p_i^F(k,l) = \frac{I\left(x_i \leq \hat{X}_k, y_i \leq \hat{Y}_l\right)}{N - \theta} + \frac{\left[1 - I\left(x_i \leq \hat{X}_k, y_i \leq \hat{Y}_l\right)\right]}{N - \phi}, \tag{35}$$

with

$$\theta = \frac{N \times N_G(k,l)}{N_G(k,l) + N_F(k,l)}, \phi = \frac{N \times M_G(k,l)}{M_G(k,l) + M_F(k,l)},$$

$$N_G(k,l) = \sum_{j=1}^{N_G} I\left(x'_j \leq \hat{X}'_k, y'_j \leq \hat{Y}'_l\right), N_F(k,l) = \sum_{i=1}^{N_F} I\left(x_i \leq \hat{X}_k, y_i \leq \hat{Y}_l\right),$$

$$M_G(k,l) = N_G - N_G(k,l), \;\; M_F(k,l) = N_F - N_F(k,l) \quad \text{and} \quad N = N_G + N_F.$$

The likelihood ratio is again first obtained by considering the difference between the unconstrained and the constrained empirical likelihood function for some fixed (k, l). This equals

$$LR\,(k,l) = 2\left\{\begin{array}{c} N\log N - N_G\log N_G - N_F\log N_F \\ +N_G(k,l)\log N_G(k,l) + N_F(k,l)\log N_F(k,l) \\ +M_G(k,l)\log M_G(k,l) + M_F(k,l)\log M_F(k,l) \\ -\left[N_G(k,l) + N_F(k,l)\right]\log\left[N_G(k,l) + N_F(k,l)\right] \\ -\left[M_G(k,l) + M_F(k,l)\right]\log\left[M_G(k,l) + M_F(k,l)\right] \end{array}\right\}. \tag{36}$$

As above, the pairs $(\hat{X}_k, \hat{Y}_L)$ and $(\hat{X}'_k, \hat{Y}'_L)$ are endogenous, making it necessary to perform several iterations. For each iteration, the constraint that $\hat{F}(\hat{X}_k) = \hat{F}(\hat{Y}_k) = \hat{G}(\hat{X}'_k) = \hat{G}(\hat{Y}'_k) = \pi_k$ for $k = 1, ..., \eta$ is imposed.

The test statistic LR is given by minimizing $LR(k,l)$ over all possible values of (k,l). The bootstrap p-value is obtained as above, by computing bootstrap LR statistics on samples generated under the probabilities given by (34) and (35) at the value of (k,l) that minimizes $LR(k,l)$ in the initial samples. For the case of growth mobility, the above remains the same except that the constraints imposed on the empirical quantiles is now that $\hat{F}(\hat{X}_k) = \hat{G}(\hat{X}'_k)$, $\hat{X}_k = \hat{Y}_k$, and $\hat{X}'_k = \hat{Y}'_k$ for $k = 1, ..., \eta$.

To test for second-order mobility dominance, we use the following useful expression (see Atkinson and Bourguignon 1982) for $F^2\,(x,y)$:

$$F^2\,(x,y) = \int_0^x \int_0^y (x-s)(y-t)F\,(s,t)\,dtds. \tag{37}$$

The non-dominance constraint (33) then becomes:

$$\begin{aligned} &\sum_{j=1}^{N_G} p_j^G(\hat{X}'_k - x'_j)(\hat{Y}'_l - y'_j)I\left(x'_j \leq \hat{X}'_k, y'_j \leq \hat{Y}'_l\right) \\ &\qquad \leq \sum_{i=1}^{N_F} p_i^F(\hat{X}_k - x_i)(\hat{Y}_l - y_i)I\left(x_i \leq \hat{X}_k, y_i \leq \hat{Y}_l\right). \end{aligned} \tag{38}$$

The rest of the procedure is analogous to the first-order procedure described above. The solutions to this problem are computed numerically also using a similar procedure to that for testing second-order absolute mobility dominance.

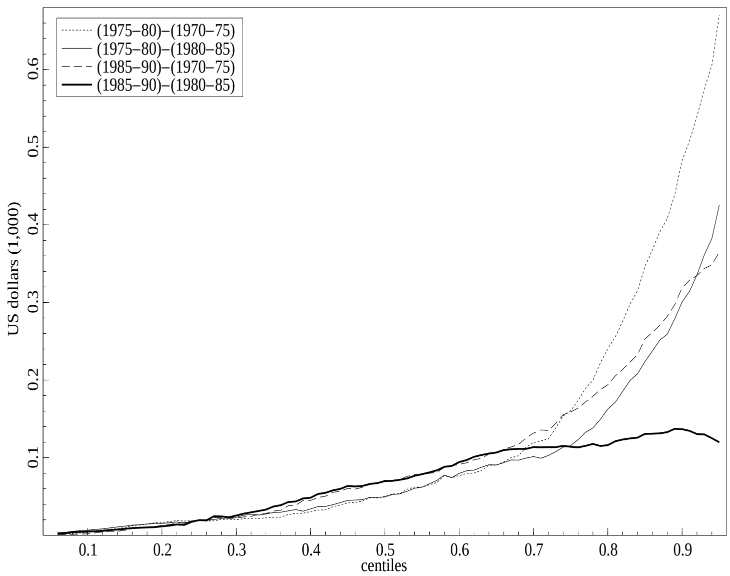
4 Empirical Illustration

Our data comes from the US PSID (Panel Study of Income Dynamics) surveys. The analysis is restricted to those individuals aged 24 to 49 that are not self-employed. Four periods are used to compare temporal mobility in the US: 1970–75, 1975–80, 1980–85 and 1985–90. For instance, mobility in the 1970–75 period is assessed by comparing the income status of our individuals in 1970 to their status in 1975. All incomes are evaluated at 1990 prices.

4.1 Absolute mobility

Tests for first-order mobility dominance show overall an absence of dominance in mobility across the four periods. For second-order dominance tests, both generalized Lorenz and Mitra and Ok (1998) approaches are considered. Figure 1 shows four differences between the generalized Lorenz curves of distances (recall (3)). These curves illustrate four possible dominance relationships, for dominance of the periods 1970-75 and 1980-85 by the periods 1975-80 and 1985-90. All of the differences are positive in the sample. The four curves in the figure intersect, a feature which thus excludes the presence of other dominance relations.

Figure 1: Differences in the generalized Lorenz curves of the income distances across time



As to the Mitra and Ok (1998) approach (Figure 2), which cumulates distances from the largest to the lowest, only the distribution for 1975–80 dominates in the sample all others — the differences between the 1975–1980 curve and the three other curves being positive. The three other curves intersect excluding as above the existence of additional dominance relations.

Statistical inference confirms most of these relations. In all of the following tables, statistical inference can be made by comparing the reported p-values to conventional critical levels, which we implicitly set here to 5% level. The p-values that are shown are the probability that the null hypothesis of non-dominance of G by F be wrongly rejected. The rejection of this null (when the p-value is low) implies that F statistically dominates G in mobility (F displays greater mobility).

Tables 1 and 2 show indeed that 5 relations out of the 7 previously identified are significant at the conventional 5% level. The two remaining dominance relations do not appear to be significant, as evidenced by their p-values that exceed 5%.

Figure 2: Differences between the curves implied by the Mitra and Ok (1998) approach

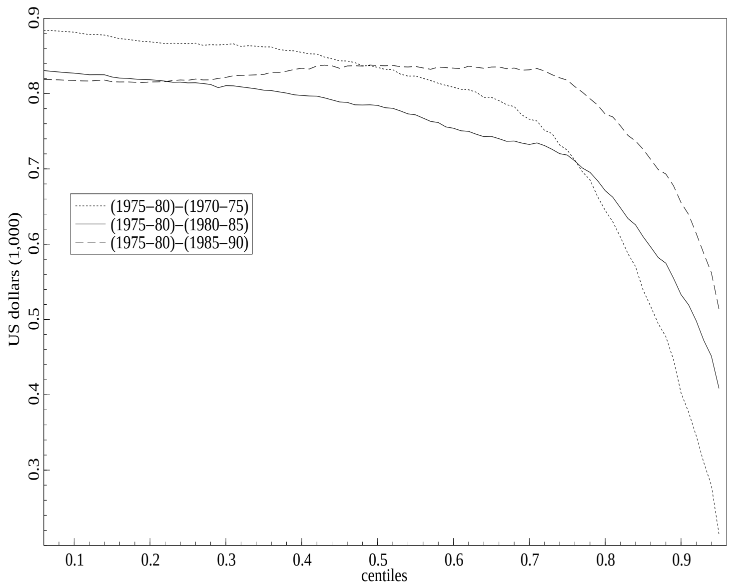


Table 1: Results of the statistical tests on differences in the generalized Lorenz curves of the income distances across time

G is not dominated by F	LR ratio	p-value
1970–75 vs 1975–80	4.653	0.000**
1970–75 vs 1985–90	2.898	0.010*
1980–85 vs 1975–80	3.362	0.025*
1980–85 vs 1985–90	1.108	0.208

(*) and (**) denote that the statistics are significant at 5% and 1% levels respectively.

Table 2: Results of the statistical tests on differences in the curves of the Mitra and Ok (1998) approach

G is not dominated by F	LR ratio	p-value
1970–75 vs 1975–80	1.089	0.163
1980–85 vs 1975–80	2.859	0.008**
1985–90 vs 1975–80	3.015	0.000**

(**) denotes that the statistics are significant at 1% level.

4.2 Transition matrices

The transition matrices were estimated by grouping the population into five income classes, as in Formby, Smith, and Zheng (2004). These income classes are defined by the vector of quintiles x and y, for the first-period and the second-period income distributions respectively.

For the first case (exchange mobility), we normalize the distributions by fixing $F(x) = F(y) = G(x) = G(y) = \pi = (0.2, 0.2, 0.2, 0.2, 0.2)$. The tests are then performed on the matrix $\left[\sum_{k=1}^{K}\sum_{l=1}^{L-1} 0.2(Q_{kl} - P_{kl})\right]$, excluding the element $k = K$ and $l = L$ which are equal to zero by definition. In this first case, it proves impossible to find dominance relations across periods.

For the second case, we only set $F(x) = G(x) = \pi = (0.2, 0.2, 0.2, 0.2, 0.2)$. The quantiles computed for the initial x period are used to define income classes for the second y period. In this second case, the test is performed on the matrix $\left[\sum_{k=1}^{K}\sum_{l=1}^{L-1} 0.2(Q_{kl} - P_{kl})\right]$ since only the elements $l = L$, are equal to zero by definition. The appendices provide further details for each of these two types of normalizations.

The estimated transition matrices within each of the three periods 1970-75, 1980-85 and 1985-90 are then estimated as:

$$\widehat{P}_{1970-75} = \begin{bmatrix} 0.480 & 0.297 & 0.118 & 0.075 & 0.032 \\ 0.190 & 0.369 & 0.254 & 0.125 & 0.061 \\ 0.079 & 0.190 & 0.308 & 0.301 & 0.118 \\ 0.021 & 0.057 & 0.186 & 0.401 & 0.423 \\ 0.014 & 0.021 & 0.075 & 0.165 & 0.598 \end{bmatrix}$$

$$\widehat{P}_{1980-85} = \begin{bmatrix} 0.554 & 0.271 & 0.100 & 0.061 & 0.014 \\ 0.210 & 0.415 & 0.248 & 0.092 & 0.043 \\ 0.145 & 0.216 & 0.314 & 0.238 & 0.120 \\ 0.049 & 0.090 & 0.183 & 0.316 & 0.330 \\ 0.049 & 0.061 & 0.067 & 0.189 & 0.619 \end{bmatrix}$$

$$\widehat{P}_{1985-90} = \begin{bmatrix} 0.449 & 0.278 & 0.138 & 0.078 & 0.057 \\ 0.169 & 0.363 & 0.322 & 0.138 & 0.077 \\ 0.081 & 0.120 & 0.265 & 0.321 & 0.140 \\ 0.041 & 0.062 & 0.148 & 0.389 & 0.418 \\ 0.034 & 0.029 & 0.054 & 0.163 & 0.663 \end{bmatrix}$$

Here two relations show dominance in the samples: both the periods 1975-80 and 1985-90 dominate in the samples the 1980-85 period. However, the inference results (see Table 3) show that these dominance relations are not statistically significant.

Table 3: First-order growth mobility dominance

G is not dominated by F	LR ratio	p-value
1980-85 vs 1975-80	0.409	0.376
1980-85 vs 1985-90	0.205	0.459

To test for second-order dominance, we maximize as above the empirical likelihood without and with constraint (38), and normalizations of the quantiles also being carried out in the same manner as above.

Table 4: Second-order mobility dominance

G is not dominated by F	LR ratio	p-value
Exchange mobility		
1975-80 vs 1970-75	0.135	0.650
1975-80 vs 1985-90	0.023	0.808
Growth mobility		
1970-75 vs 1985-90	0.499	0.501
1975-80 vs 1970-75	0.241	0.376
1975-80 vs 1985-90	1.364	0.330
1980-85 vs 1985-90	0.174	0.702

The sampling estimates suggest six potential dominance relations to be formally tested, including 2 relations in the case of exchange mobility and 4 in the case of growth mobility. As we see in Table 4, none of these dominance relations proves to be statistically significant since all of the p-values are too high to reject the null of non-dominance. Statistical significance is, however, obtained if the dominance relations are restricted by dropping the last columns of the transition matrices. Two dominance relations — one for each case — are then statistically significant at a 5% level (Table 5), with the 1985-90 period dominating the period 1975-80 in both cases.

5 Conclusion

A number of mobility measures have been proposed in the literature. This paper proposes procedures for testing for whether mobility is robustly greater in a distribution A than in a distribution B over different possible classes of mobility measures. For this, it draws on the frameworks for making partial orderings of mobility proposed by Atkinson

Table 5: Second-order dominance (restricted to the first four quintiles of income)

G is not dominated by F	Ratio LR	p-value
Exchange mobility		
1970-75 vs 1985-90	0.088	0.822
1975-80 vs 1970-75	0.135	0.120
1975-80 vs 1980-85	0.057	0.970
1975-80 vs 1985-90	1.900	$0.010*$
Growth mobility		
1970-75 vs 1985-90	0.744	0.506
1975-80 vs 1970-75	0.769	0.313
1975-80 vs 1985-90	2.080	$0.036*$
1980-85 vs 1985-90	0.591	0.719

(*) denotes that the statistics are significant at 5% level.

(1983), Conlisk (1989), Dardanoni (1993), Mitra and Ok (1998), Formby, Smith, and Zheng (2003) and others.

Following in the footsteps of Maasoumi and Trede (2001) and Formby, Smith, and Zheng (2004), this paper proposes statistical tests for comparing mobility, with the difference being that the procedure relies here on nulls of non-dominance. Two types of comparisons are made. The first is based on absolute mobility measures. The second uses transition matrices. The tests are performed on the null hypothesis of non-dominance versus the alternative hypothesis of dominance. The analysis covers both first- and second-order stochastic dominance.

Illustrations are performed from the US PSID data by comparing four five-year periods between 1970 and 1990. There are no first-order absolute mobility dominance relationships across these periods because the sample dominance curves intersect. Two approaches are taken to assess second-order dominance: the classical generalized Lorenz approach and the Mitra and Ok (1998) approach. Our analysis identifies several dominance relations in the samples, most of which end up being statistically significant. For measures based on transition matrices, two first-order dominance relations (both in the case of growth mobility) are observed in the sample, none of which being statistically significant. Several second-order dominance relations exist in the samples, only two of which being statistically significant. These results suggest that it may sometimes be difficult to obtain rankings of distributions that are robust over wide classes of mobility indices, and that it can also be important to perform statistical tests of such rankings since sample rankings of mobility may not always be strong enough to infer population ones.

6 Appendices

6.1 Appendix 1: Normalizing the data to consider exchange mobility

Let X_K and Y_K be the empirical quintiles from the distribution F, and X'_K, and Y'_K be the ones from the distribution G, with $K, L = 1, ..., 5$. We wish to normalize the observations (x', y') of G into $(\widetilde{x}, \widetilde{y})$ so that their respective quintiles be given by X_K and Y_K. To do this, we can apply the formula:

$$\widetilde{x}_j = X_{K-1} + (x'_j - X'_{K-1})\frac{X_K - X_{K-1}}{X'_K - X'_{K-1}}, \forall\ j = 1, ..., N_G \text{ with } X'_{K-1} < x'_j \leq X'_K. \quad (39)$$

We set $X_0 = X'_0 = 0$. The same procedure can be applied for estimating $\widetilde{y}_j$:

$$\widetilde{y}_j = Y_{L-1} + (y'_j - Y'_{L-1})\frac{Y_L - Y_{L-1}}{Y'_L - Y'_{L-1}}, \forall\ j = 1, ..., N_G \text{ with } Y'_{L-1} < y'_j \leq Y'_L. \quad (40)$$

Again, we set $Y_0 = Y'_0 = 0$. After these transformations, the new observations $(\widetilde{x}, \widetilde{y})$ drawn from G have the same quintiles as those obtained for F.

6.2 Appendix 2: Normalizing the data to consider growth mobility

Here, only the initial income quintiles are considered, namely X_K for F and X'_K for G, with $K = 1, ..., 5$. These quintiles are used for determining the final income classes. Let the proportions of individuals in the final income classes be respectively given by P_K and P'_K. We need to transform (x', y') into $(\widetilde{x}, \widetilde{y})$ so that the initial income quintiles $\widetilde{x}$ be equal to X_K. Moreover, this transformation should let the proportions P'_K unchanged. We first follow the procedure described in (39) for computing $\widetilde{x}_j$. The $\widetilde{y}_j$ are then determined as follows:

$$\widetilde{y}_j = X_{K-1} + (y'_j - X'_{K-1})\frac{X_K - X_{K-1}}{X'_K - X'_{K-1}}, \forall\ j = 1, ..., N_G \text{ with } X'_{K-1} < y'_j \leq X'_K. \quad (41)$$

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