Testing multivariate uniformity: the distance-to-boundary method

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Given a random sample taken on a compact domain $\mathcal{S} \subset \mathbb{R}^d$, we propose a new method for testing the hypothesis of uniformity of the underlying distribution. The test statistic is based on a geometric notion of depth, namely, the distance of each observation to the boundary of $\mathcal{S}$.

This test has a number of interesting properties: Unlike most available methods, it is computationally feasible even in higher dimensions, it is distribution-free for a wide range of choices of S, it can be adapted to the case when the support $\mathcal{S}$ is unknown and also allows for one-sided versions. Moreover, in many cases, the power of this test increases as the dimension d grows. This somewhat surprising feature can be empirically observed in another recent test (relying on a completely different idea, see Liang et al., 2000) for multivariate uniformity, but does not appear in other standard procedures (based on Kolmogorov-Smirnov or χ^2-type tests).

We study the properties of this test from both a theoretical and practical point of view. In particular, an extensive simulation study is given in order to compare the performance of our methods with two recent alternative procedures. The conclusions suggest that the proposed test provides a quite satisfactory balance between statistical power, computational simplicity, and flexibility of application for different dimensions and supports.

References

Liang, J.J., Fang, K.T., Hickernell, F.J. and Li, R. (2000). Testing multivariate uniformity and its applications. Mathematics of Computation 70 , 337–355.