

Testing naturalness at 100 TeV

Chuan-Ren Chen,^a Jan Hajer,^{b,c} Tao Liu,^c Ian Low^{d,e} and Hao Zhang^f

^a*Department of Physics, National Taiwan Normal University,
Taipei 116, Taiwan*

^b*Institute for Advanced Studies, The Hong Kong University of Science and Technology,
Clear Water Bay, Kowloon, Hong Kong S.A.R., China*

^c*Department of Physics, The Hong Kong University of Science and Technology,
Clear Water Bay, Kowloon, Hong Kong S.A.R., China*

^d*Department of Physics and Astronomy, Northwestern University,
Evanston, IL 60208, U.S.A.*

^e*High Energy Physics Division, Argonne National Laboratory,
Argonne, IL 60439, U.S.A.*

^f*Institute of High Energy Physics, Chinese Academy of Sciences,
Beijing 100049, China*

E-mail: crchen@ntnu.edu.tw, jan@hajer.com, taoliu@ust.hk,
ilow@northwestern.edu, zhanghao@ihep.ac.cn

ABSTRACT: Solutions to the electroweak hierarchy problem typically introduce a new symmetry to stabilize the quadratic ultraviolet sensitivity in the self-energy of the Higgs boson. The new symmetry is either broken softly or collectively, as for example in supersymmetric and little Higgs theories. At low energies such theories contain naturalness partners of the Standard Model fields which are responsible for canceling the quadratic divergence in the squared Higgs mass. Post the discovery of any partner-like particles, we propose to test the aforementioned cancellation by measuring relevant Higgs couplings. Using the fermionic top partners in little Higgs theories as an illustration, we construct a simplified model for naturalness and initiate a study on testing naturalness. After electroweak symmetry breaking, naturalness in the top sector requires $a_T = -\lambda_t^2$ at leading order, where λ_t and a_T are the Higgs couplings to a pair of top quarks and top partners, respectively. Using a multivariate method of Boosted Decision Tree to tag boosted particles in the Standard Model, we show that, with a luminosity of 30 ab^{-1} at a 100 TeV pp -collider, naturalness could be tested with a precision of 10 % for a top partner mass up to 2.5 TeV.

KEYWORDS: Beyond Standard Model, Higgs Physics, Technicolor and Composite Models

ARXIV EPRINT: [1705.07743](https://arxiv.org/abs/1705.07743)

Contents

1	Introduction	1
2	A simplified model for naturalness	2
3	Collider analysis	7
3.1	Simulation of signal and background	7
3.2	Event reconstruction	10
3.3	Results	11
4	Discussion and conclusion	14
A	Higher order corrections	16

1 Introduction

The naturalness problem [1], motivated by the smallness of the Standard Model (SM) particle masses in comparison to a UV cutoff, has been the driving force behind theoretical developments in particle physics for several decades. Especially after the discovery of the 125 GeV Higgs boson at the Large Hadron Collider (LHC) in 2012 [2, 3], there is a renewed urgency to understand why its mass is so much smaller than the next known scale in particle physics, the Planck scale, given the quadratic ultraviolet sensitivity in the self-energy of the Higgs. In the past decades, many theories of naturalness have been proposed. Typically a new symmetry which protects the squared mass of the SM-like Higgs boson from quadratic UV-sensitivity is introduced. Two eminent examples are supersymmetry [4] and spontaneously broken global symmetry [5, 6], which require the existence of naturalness partners coupling to the Higgs boson. The symmetry-protected relation among the Higgs couplings with the SM particles and their partners, called the naturalness sum rule, ensures cancellation of the quadratic divergence in the squared Higgs mass. Confirming or refuting theories of naturalness is one of the most important goals for the current and future high energy collider experiments.

Among the naturalness partners, the partner of the SM top quark typically plays the most prominent role. Currently, a tremendous amount of effort at the LHC has been going into searches for top partners [7–10], whose discovery potential at a next-generation hadron collider has also been studied [11]. However, the discovery of a partner-like particle is only the first step in confirming theories of naturalness. In order to ensure that the discovered partner-like particle is not a naturalness impostor, but indeed the one that yields the cancellation of the quadratic UV-sensitivity in the Higgs self-energy, one needs to further measure the naturalness sum rule. In this article, we propose a novel way to

probe the naturalness sum rule at hadron colliders, in order to facilitate the testability of theories of naturalness. For convenience, we rephrase the naturalness sum rule $\Delta m_H^2|_{\text{SM}} + \Delta m_H^2|_{\text{NP}} = 0$, which relates the different contributions to the quadratically divergent quantum corrections to the squared Higgs mass Δm_H^2 , into a naturalness parameter

$$\mu = -\frac{\Delta m_H^2|_{\text{NP}}}{\Delta m_H^2|_{\text{SM}}}, \quad \mu|_{\text{naturalness}} = 1. \quad (1.1)$$

Note that in the SM the top quarks contribute the strongest to the UV-sensitivity of the Higgs self-energy. As a first illustration for testing naturalness, we will analyze fermionic top partners which carry QCD colors, as predicted by little Higgs models [12–14].¹ Similar colored top partners exist in the so-called composite Higgs models as well [15, 16]. We define a simplified model consisting of the SM top fields and a pair of vector-like fermionic top-partners, which are either singlets or doublets under $SU(2)_w$. Then we show that the naturalness sum rule leads to an especially simple form after electroweak symmetry breaking, at leading order in the Higgs vacuum expectation value (VEV)

$$a_T = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_T^2}\right), \quad (1.2)$$

here m_T is the top partner mass and the relation involves at leading order only the couplings of the Higgs to a pair of top quarks λ_t and a pair of top-partners a_T . This suggests that, in order to test the naturalness sum rule up to $\mathcal{O}(v^2/m_T^2)$, only two measurements are necessary, one for each of the couplings λ_t and a_T . We then proceed to study how this relation can be measured to quantify the degree of cancellation in the contribution of the top sector to the quadratic divergence of the Higgs self energy in a future 100 TeV hadron collider [17, 18].

In section 2 we introduce a simplified model for naturalness, by extending the SM top sector with a pair of vector-like top partners, and derive relation from the requirement of naturalness. Based on this discussion we design a detector study dedicated to test naturalness of fermionic top partners at a 100 TeV collider in section 3. Finally we conclude in section 4.

2 A simplified model for naturalness

In the SM, the UV-sensitivity of the Higgs self-energy originates from three major sources which, in order of decreasing magnitude, are the coupling to top quarks, electroweak gauge bosons and the Higgs boson itself. In a typical natural theory, the contributions of top quarks, EW gauge bosons and the Higgs boson are expected to be canceled by their respective partner particles. In this work we consider a top sector with fermionic partner particles, which carry QCD colors and could be produced copiously in a hadron collider, if their masses are not too heavy. These top partners are vector-like and massive, thus allowing a quartic coupling with $H^\dagger H$ which enables the cancellation of the top quadratic divergence in the Higgs self-energy. We focus on the cases where the vector-like top partners carry the same EW charges as either the right-chiral top quark u_3^c [14, 19] or the

¹The case of scalar top partners as predicted by e.g. supersymmetry will be pursued in a future work.

left-chiral top quark $q_3 = (d_3, u_3)$ [20]. In the case where the vector-like pair (U^c, U) is a singlet under $SU(2)_w$ the effective Lagrangian for the non-linear coupling of the top sector to the Higgs scalar H can be expanded to

$$\begin{aligned} \mathcal{L}_U = & u_3^c \left(c_0 f U + c_1 H^\dagger q_3 + \frac{c_2}{2f} |H|^2 U + \frac{c_3}{6f^2} |H|^2 H^\dagger q_3 + \dots \right) \\ & + U^c \left(\widehat{c}_0 f U + \widehat{c}_1 H^\dagger q_3 + \frac{\widehat{c}_2}{2f} |H|^2 U + \frac{\widehat{c}_3}{6f^2} |H|^2 H^\dagger q_3 + \dots \right) + \text{h.c.} \end{aligned} \quad (2.1)$$

The mass dimension in these couplings is given by the symmetry breaking scale f and we imply $|H|^2 = H^\dagger H$. The parameter c_i and \widehat{c}_i are free and for odd-dimensional couplings they can be complex. A linear combination of u_3^c and U^c together with U becomes massive with a top partner mass of

$$m_{T'} = f c, \quad c = \sqrt{c_0^2 + \widehat{c}_0^2}. \quad (2.2)$$

Whereas, the orthogonal combination remains massless and acquires a Yukawa coupling to the Higgs doublet. In terms of these mass eigenstates

$$t'^c = \frac{\widehat{c}_0 u_3^c - c_0 U^c}{c}, \quad t' = q_3, \quad (2.3a)$$

$$T'^c = \frac{\widehat{c}_0 U^c + c_0 u_3^c}{c}, \quad T' = U, \quad (2.3b)$$

the Lagrangian (2.1) becomes

$$\begin{aligned} \mathcal{L}_{T'} = & m_{T'} T'^c T' + \lambda_{t'} H^\dagger t'^c t' + \lambda_{T'} H^\dagger T'^c t' + \frac{\alpha_{t'}}{2m_{T'}} |H|^2 t'^c T' + \frac{\alpha_{T'}}{2m_{T'}} |H|^2 T'^c T' \\ & + \frac{\beta_{t'}}{6m_{T'}^2} |H|^2 H^\dagger t'^c t' + \frac{\beta_{T'}}{6m_{T'}^2} |H|^2 H^\dagger T'^c t' + \mathcal{O}(H^4) + \text{h.c.}, \end{aligned} \quad (2.4)$$

where the couplings are given by

$$\lambda_{t'} = \frac{\widehat{c}_0 c_1 - c_0 \widehat{c}_1}{c}, \quad \lambda_{T'} = \frac{c_0 c_1 + \widehat{c}_0 \widehat{c}_1}{c}, \quad (2.5a)$$

$$\alpha_{t'} = \widehat{c}_0 c_2 - c_0 \widehat{c}_2, \quad \alpha_{T'} = c_0 c_2 + \widehat{c}_0 \widehat{c}_2, \quad (2.5b)$$

$$\beta_{t'} = (\widehat{c}_0 c_3 - c_0 \widehat{c}_3) c, \quad \beta_{T'} = (c_0 c_3 + \widehat{c}_0 \widehat{c}_3) c. \quad (2.5c)$$

The case in which the vector-like pair (Q^c, Q) consists of $SU(2)_w$ doublets can be written as

$$\begin{aligned} \mathcal{L}_Q = & \left(Q^c c_0 f + u_3^c c_1 H^\dagger + Q^c \frac{c_2}{2f} |H|^2 + u_3^c \frac{c_3}{6f^2} |H|^2 H^\dagger + \dots \right) q_3 \\ & + \left(Q^c \widehat{c}_0 f + u_3^c \widehat{c}_1 H^\dagger + Q^c \frac{\widehat{c}_2}{2f} |H|^2 + u_3^c \frac{\widehat{c}_3}{6f^2} |H|^2 H^\dagger + \dots \right) Q + \text{h.c.} \end{aligned} \quad (2.6)$$

The rotation into mass eigenstates leads to a Lagrangian identical to (2.4) with the replacement of

$$T'^c \leftrightarrow T', \quad t'^c \leftrightarrow t'. \quad (2.7)$$

Although the following discussion is based on Lagrangian (2.1) it can also be applied to Lagrangian (2.6) and our results can easily be adjusted to correspond to that case.

The contribution of Lagrangian (2.4) to the Higgs potential can be computed using the Coleman-Weinberg potential [21] and the quadratically divergent part is

$$V_{\text{quad}}^T = \frac{\Lambda^2}{16\pi^2} \text{tr } \mathcal{M}^2, \quad \mathcal{M}^2 = \mathcal{M}(H)^\dagger \mathcal{M}(H), \quad (2.8)$$

where $\mathcal{M}(H)$ is the fermion mass matrix with the Higgs field H as a background field, and Λ is the cutoff scale of the theory. In the basis (t'^c, T'^c) and (t', T') the mass matrix reads up to quadratic order in the Higgs field

$$\mathcal{M}(H) = \begin{pmatrix} 0 & 0 \\ 0 & m_{T'} \end{pmatrix} + \begin{pmatrix} \lambda_{t'} & 0 \\ \lambda_{T'} & 0 \end{pmatrix} H + \begin{pmatrix} 0 & \alpha_{t'} \\ 0 & \alpha_{T'} \end{pmatrix} \frac{|H|^2}{2m_{T'}} + \mathcal{O}(H^3). \quad (2.9)$$

The absence of quadratically divergent contributions in the Higgs mass

$$\Delta m_{\text{quad}}^2 = \frac{1}{2} \left. \frac{\partial^2 V_{\text{quad}}^T}{\partial |H|^2} \right|_{H=0} \equiv 0, \quad (2.10)$$

is then equivalent to the requirement that the coefficient of the $|H|^2$ term in $\text{tr } \mathcal{M}^2$ vanishes

$$\text{tr } \mathcal{M}^2 = \text{constant} + \mathcal{O}(H^3). \quad (2.11)$$

Several comments follow from this equation. First, due to the structure of the matrix (2.9), no linear term and hence no tadpole contribution arises. Second, $\alpha_{t'}$ only contributes to $|H|^4$ terms and does not enter into the cancellation of quadratic divergences in the Higgs mass. Finally, requiring the coefficients of the $|H|^2$ term to vanish leads to the naturalness sum rule

$$\alpha_{T'} = -|\lambda_{T'}|^2 - |\lambda_{t'}|^2. \quad (2.12)$$

It is worthwhile pointing out that the $\lambda_{T'}$ term contributes to the quadratic divergences with the same sign as the SM top quark, and the cancellation hinges entirely on the existence of the four-point coupling $\alpha_{T'}$ which must have a negative sign. In terms of the parameter in the Lagrangians of the gauge eigenstates (2.1) and (2.6) this naturalness sum rule reads

$$|c_1|^2 + c_0 c_2 = -|\widehat{c}_1|^2 - \widehat{c}_0 \widehat{c}_2. \quad (2.13)$$

The best known examples of models described by Lagrangian (2.4) and subject to the naturalness sum rule (2.12) are little Higgs models. Some typical examples are presented in table 1.² The collective symmetry breaking in the Yukawa sector of little Higgs models yields the relations

$$|c_1|^2 = -c_0 c_2, \quad |\widehat{c}_1|^2 = -\widehat{c}_0 \widehat{c}_2, \quad (2.14)$$

²In table 1 we have also considered mirror twin Higgs models [24], although their phenomenology differs significantly from the one in little Higgs theories under consideration. See also the discussion at the end of this section.

Model	Coset	SU(2)	c_0	c_1	c_2	\hat{c}_0	\hat{c}_1	\hat{c}_2
Toy model	$\frac{\text{SU}(3)}{\text{SU}(2)}$ [22]	1	λ_1	$-\lambda_1$	$-\lambda_1$	λ_2	0	0
Simplest	$\left(\frac{\text{SU}(3)}{\text{SU}(2)}\right)^2$ [23]	1	λ	$-\lambda$	$-\lambda$	λ	λ	$-\lambda$
Littlest Higgs	$\frac{\text{SU}(5)}{\text{SO}(5)}$ [14]	1	λ_1	$-\sqrt{2}i\lambda_1$	$-2\lambda_1$	λ_2	0	0
Custodial	$\frac{\text{SO}(9)}{\text{SO}(5)\text{SO}(4)}$ [20]	2	y_1	$\frac{i}{\sqrt{2}}y_1$	$-\frac{1}{2}y_1$	y_2	0	0
T -parity invariant	$\frac{\text{SU}(3)}{\text{SU}(2)}$ [19]	1	λ	$-\lambda$	$-\lambda$	$-\lambda$	$-\lambda$	λ
T -parity invariant	$\frac{\text{SU}(5)}{\text{SO}(5)}$ [19]	1	λ	$-\sqrt{2}i\lambda$	-2λ	$-\lambda$	$-\sqrt{2}i\lambda$	2λ
Mirror twin Higgs	$\frac{\text{SU}(4)\text{U}(1)}{\text{SU}(3)\text{U}(1)}$ [24]	1	0	$i\lambda_t$	0	λ_t	0	$-\lambda_t$

Table 1. Parameter of the couplings between top quarks, top partners and the Higgs scalar in the gauge eigenstate basis as defined in Lagrangian (2.1) for some representative models. In the column labeled SU(2) we indicate if the top partners are singlets or embedded into doublets under SU(2)_w and hence if this model corresponds to the Lagrangian (2.1) or (2.6).

as can easily be checked in table 1. This ensures at one-loop level the cancellation of the quadratic divergences from the top sector according to the condition (2.13). A common but by no means necessary additional restriction is to couple only one of the top partners to the Higgs field by setting $\hat{c}_1 = \hat{c}_2 = \dots = 0$, which leads to

$$\frac{\beta_{t'}}{\beta_{T'}} = \frac{\alpha_{t'}}{\alpha_{T'}} = \frac{\lambda_{t'}}{\lambda_{T'}} = \frac{\hat{c}_0}{c_0}. \quad (2.15)$$

For phenomenological reasons it can be desirable to introduce T -parity [25, 26]. When T -parity is implemented in the top sector [19] the top partner particles are T -odd, while SM particles are T -even. Therefore the mixing terms have to vanish $\beta_{T'} = \alpha_{t'} = \lambda_{T'} = 0$. Before rotation into the mass eigenstates this corresponds to the condition $\hat{c}_i = \mp c_i$ for even and odd i , respectively.

Finally, expanding the Higgs field around its VEV³

$$H = (0, v)^T + \frac{1}{\sqrt{2}}(h^+, h)^T, \quad v \approx 174 \text{ GeV}, \quad (2.16)$$

and rotating the fermion fields to linear order in v into mass eigenstates

$$t^c = t'^c + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right), \quad t = t' - \lambda_{T'}^* \frac{v}{m_{T'}} T' + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right), \quad (2.17a)$$

$$T^c = T'^c + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right), \quad T = T' + \lambda_{T'} \frac{v}{m_{T'}} t' + \mathcal{O}\left(\frac{v^2}{m_{T'}^2}\right), \quad (2.17b)$$

³In theories where the Higgs arises as a pseudo-Nambu-Goldstone boson from a spontaneously broken symmetry, the VEV of the Higgs differs slightly from the SM VEV of 174 GeV. The difference is formally of order of v^2/m_T^2 , similar to terms that have been neglected during the derivation of the naturalness sum rule.

leads to the Lagrangian

$$\begin{aligned} \mathcal{L}_T = & m_T T^c T + \lambda_t v t^c t + \frac{\lambda_t}{\sqrt{2}} h t^c t + \frac{\lambda_T}{\sqrt{2}} h T^c t + \frac{a_t v}{\sqrt{2} m_T} h t^c T + \frac{a_T v}{\sqrt{2} m_T} h T^c T \quad (2.18) \\ & + \frac{\alpha_t}{4 m_T} h^2 t^c T + \frac{\alpha_T}{4 m_T} h^2 T^c T + \frac{b_t v}{4 m_T^2} h^2 t^c t + \frac{b_T v}{4 m_T^2} h^2 T^c t + \mathcal{O}\left(h^3, \frac{v^2}{m_T^2}\right) + \text{h.c.} . \end{aligned}$$

The couplings m_T , $\lambda_{t,T}$ and $\alpha_{t,T}$ are up to linear order in v/m_T identical to their primed counterparts, and we have collected the higher order correction in appendix A. The new VEV induced couplings are

$$a_t = \alpha_{t'} + \lambda_{T'}^* \lambda_{t'} , \quad a_T = \alpha_{T'} + |\lambda_{T'}|^2 , \quad (2.19a)$$

$$b_t = \beta_{t'} - \alpha_{t'} \lambda_{T'} , \quad b_T = \beta_{T'} - \alpha_{T'} \lambda_{T'} . \quad (2.19b)$$

The naturalness sum rule (2.12) leads at leading order in v to the remarkable relation

$$a_T = -|\lambda_t|^2 + \mathcal{O}\left(\frac{v^2}{m_T^2}\right) . \quad (2.20)$$

Though the naturalness cancellation in Little Higgs models applies at one-loop only, the high-loop corrections to the sum rule are suppressed by extra loop factors. They are negligibly small compared to the uncertainty in this relation unless $m_T \gg \mathcal{O}(1)$ TeV. Additionally, in the simplified model under consideration higher dimensional operators encoding new physics at the cutoff scale $\Lambda \sim 4\pi f$ lead to terms of higher order in Lagrangian (2.1) and (2.6). Our calculations show that these terms do not influence the relation in (2.20).

The naturalness sum rule (2.20) has significant implications on tests of the naturalness sum rule at colliders. Naively, the relation (2.12) suggests that three measurements, one for each of the couplings $\lambda_{t'}$, $\lambda_{T'}$ and $\alpha_{T'}$, are necessary in order to test naturalness. However, as $\lambda_{t'}$, $\lambda_{T'}$ and $\alpha_{T'}$ are not defined in the basis of mass eigenstates after electroweak symmetry breaking, the separate reconstruction of their values becomes rather challenging. Previously it was suggested [22] to test the naturalness relation (2.12) by measuring the $\lambda_{T'}$ coupling via the top partner decay $T \rightarrow th$, and extracting the information of $\alpha_{T'}$ with the relation

$$\alpha_{T'} = \lambda_{T'} \frac{m_{T'}}{f} . \quad (2.21)$$

However, the proposed strategy is limited by two observations. First this relation is not generic for all little Higgs models. The $\lambda_{T'}$ term for example can be eliminated by imposing T -parity in the top sector [19]. Second, it implies that in order to extracting the value of the coupling $\alpha_{T'}$ it is necessary to measure the decay constant f . In contrast, the naturalness sum rule (2.20) has the virtue of being generic for all models which can be described by the simplified model under consideration. Especially it is independent of the assumption on T -parity. It is exact up to order v/m_T , and involves only two free parameters, the coupling of the Higgs to two top quarks and to two top partners. Notably, this does not involve the measurement of the decay constant f . In conclusion the relation (2.20) greatly simplifies testing the naturalness sum rule.

Although, the naturalness sum rules (2.12), (2.13) and (2.20) are derived for vector-like top partners in little Higgs models, they can also be applied to mirror twin Higgs models [24]. In this case, top quarks carry QCD charge while top partners carry mirror QCD charge, which forbids mixing between them. This is ensured by the condition that $c_0 = \widehat{c}_1 = c_2 \equiv 0$, which yields the relations $\beta_{T'} = \alpha_{t'} = \lambda_{T'} = 0$. Note the similarity to the case of little Higgs models with T -parity. However, in this case the naturalness sum rule (2.13) is reduced to

$$|c_1|^2 = -\widehat{c}_0 \widehat{c}_2. \quad (2.22)$$

Although, the collider kinematics of colorless top partners differs significantly from that of colored top partners, we list twin Higgs models as one possibility in table 1. However, the collider analysis in the following section applies only to models with colored top partners such as little Higgs models.

3 Collider analysis

In this section we assume that one of the searches for fermionic top partners [7–11] has been successful and that the mass m_T of the newfound particle has been determined. Based on the discussions above, the measurements of the couplings of a Higgs with a pair of top quarks and a pair of top partners constitute the two cornerstones for testing the naturalness sum rule. The former measurement has been extensively discussed in the literature, for searches at the LHC [27–32], at a future e^+e^- collider [33–35], and at a 100 TeV hadron collider [36]. We will focus on the measurement of the Higgs coupling to two top partners a_T via $T^c T h$ production at a 100 TeV hadron collider. The naturalness parameter (1.1) applied to this analysis reads

$$\mu_t = -\frac{a_T}{\lambda_t^2} + \mathcal{O}\left(\frac{v^2}{m_T^2}\right). \quad (3.1)$$

If the naturalness sum rule (2.12) is satisfied μ_t is up to an order of v^2/m_T^2 equal to one. In the following we assume the particle content and interactions of the SM augmented by a colored fermionic top partner subject to Lagrangian (2.18) without T -parity. The analysis proposed here is blind to the sign of a_T and we leave the removal of this degeneracy for a future publication.

3.1 Simulation of signal and background

In little Higgs models, fermionic top partners decay mainly into heavy quarks and bosons $B = W^\pm, Z, h$. The equivalence theorem ensures that in the limit $f \gg v$ the branching ratios (BR) relate according to [37]

$$\text{BR}(T \rightarrow th) \simeq \text{BR}(T \rightarrow tZ) \simeq \frac{1}{2} \text{BR}(T \rightarrow Wb) \simeq 25\%. \quad (3.2)$$

This relation receives a correction of order v/m_T from the a_t term in

$$\mathcal{L}_T \supset \frac{1}{\sqrt{2}} \left(\lambda_T T^c t + \frac{a_t v}{m_T} t^c T \right) h + \text{h.c.} \quad (3.3)$$

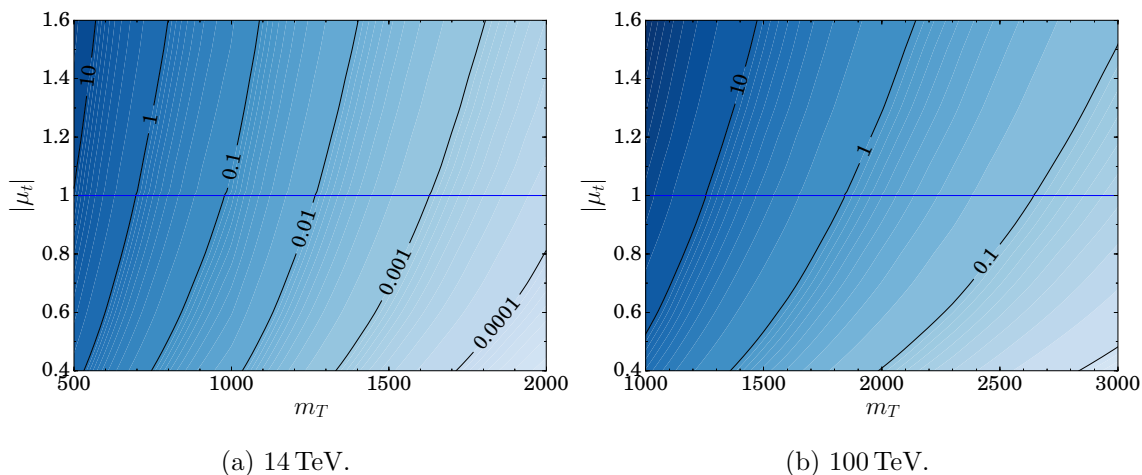


Figure 1. Cross section of the signal process $\bar{T}Th$ at 14 TeV **a** and 100 TeV **b** in fb as a function of the top partner mass m_T and the absolute value of the naturalness parameter $|\mu_t|$. The horizontal line at $|\mu_t| = 1$ indicates natural models. Note, that the production of $\bar{T}Th$ is not sensitive to the sign of μ_t , hence all figures depend on $|\mu_t|$ instead of μ_t .

For simplicity we ignore this uncertainty in the current analysis by fixing the BRs to the values in relation (3.2).

In order to be more consistent with the majority of the literature on collider phenomenology we will switch the notation from using Weyl fermions (T^c, T) to Dirac fermions (\bar{T}, T) for the remaining part of this section. We abbreviate heavy neutral bosons by $B^0 = Z, h$ and assume that jets j also include b -jets. The decay channels in this notation are

$$pp \rightarrow \bar{T}Th \rightarrow \begin{cases} \bar{t}B^0tB^0h \rightarrow \bar{t}tjj(\bar{b}b), & (3.4a) \\ \bar{t}B^0bW^+h \rightarrow \bar{t}bjj(\bar{b}b), & (3.4b) \\ \bar{b}W^\pm bW^\mp h \rightarrow \bar{b}l\nu bj(\bar{b}b). & (3.4c) \end{cases}$$

Here we assume that the associated Higgs is unboosted and decays into two separate bottom jets, while the other bosons are boosted and decay hadronically within the cone radius of a single jet. Furthermore, we require the top quarks to decay either fully or semi-leptonically.

We have implemented Lagrangian (2.18) together with the coupling which ensure the BRs (3.2) in FeynRules 2.3 [38]. This enables us to generate events with MadGraph 2.4 [39] and decay heavy particles with MadSpin. Light particle decay and showering is done in Pythia 6.4 [40] followed by the detector simulation Delphes 3.3 [41].

We have calculated the LO production cross section of the $\bar{T}Th$ process with MadGraph, and present it for 14 TeV and 100 TeV as a function of the top partner mass m_T and the naturalness parameter μ_t in figure 1. In particular, we neglect the Higgs coupling to two gluons, which is induced at one-loop level and formally a next-to-leading order contribution. However, when $|\mu_t|$ is small, the tree-level LO production rate becomes small and the one-loop contribution arising from the Higgs coupling to two gluons could become dominant. For this reason we choose to focus on the region of parameter space where $|\mu_t| > 0.4$. We have checked that in this case the tree-level production rate still dominates. For simplicity

	Cross section [fb]	
	semi-leptonic	full-leptonic
p_T	3150	526
$n(l_{\text{isol}})$	1315	111

Table 2. Cross section of the background process $\bar{t}tjjjj$ at 100 TeV in fb. During event generation we require at least two quarks to be bottom or charm quarks with $p_T > 40$ GeV. The other two quarks are required to be boosted with $p_T > 400$ GeV. The resulting cross sections are given in the row labeled p_T . Before the analysis we require the presence of at least one and two isolated leptons for semi- and full hadronic events, respectively. The isolation criterion is given at the end of section 3.1. We give the cross section after this pre-cut in the row labeled $n(l_{\text{isol}})$.

we have also assumed the top Yukawa coupling λ_t takes the value known from the SM, up to corrections of order v^2/m_T^2 .

In order to reduce the hadronic background we focus on the decay channel (3.4a) which involves two top quarks. The dominant electroweak background for this signal consists of a pair of top quarks produced in association with three bosons. In this case each top partner is mimicked by the combination of one top quark with one electroweak boson. However, the top pair production in association with multiple quarks has a much larger cross section. If we consider the mis-identification of jets we have to distinguish the cases in which pairs of jets mimic reconstructed objects from the cases in which a single jet is mistaken for a single boosted object. On the one hand, the decay products of heavy top-partner are generically boosted and tend to be detected as a single jet. On the other hand, the Higgs is produced with a low transverse momentum and its decay products will be detected separately. In this study we consider only Higgs decays into bottom quarks. In order to reduce the size of the top pair plus jet background we require the two leading jets to be boosted enough to lie within a jet cone of $\Delta R \approx 0.5$

$$p_T(j) > 400 \text{ GeV} \approx \frac{2m_B}{\Delta R(j)}. \tag{3.5}$$

We expect these jets to mimic the bosons originating from the top partner decay. Additionally, a combination of two softer jets can be misidentified as being the Higgs boson. As we require the Higgs boson signature to be b -like we restrict these jets to come from c or b quarks. Therefore, the main background in this analysis is top pair production in association with four jets where the two leading ones are boosted while the other two are b -like. We have generated this background with `MadGraph` and have required the jet transverse momentum to be $p_T > 400$ and 40 GeV for the two leading and the remaining jets, respectively. We show the LO cross sections for semi- and full leptonic decays of the top quarks in table 2. As a cut on the jet number is only a weak discriminator the top-pair production with more than four jets will contribute to the background. Because the strong coupling constant appears in a higher power and due to the increased number of jets which leads to a larger phase space this higher order background will be suppressed with respect to the leading contribution. We do not consider any k -factor in this analysis.

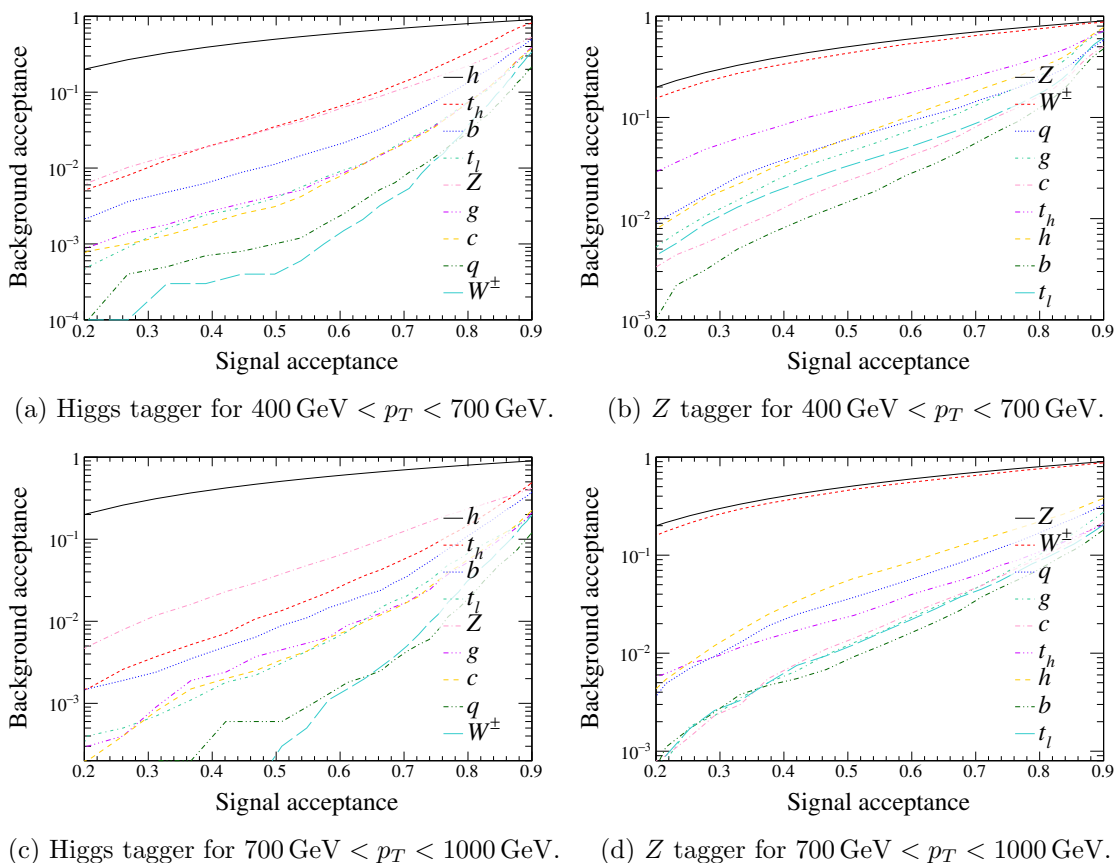


Figure 2. ROC curves of the signal and background acceptance for the Higgs tagger in **a** and **c** as well as for the Z -boson tagger in **b** and **d**. All processes are generated at 100 TeV via pair-production. (We are using an effective model in order to produce Higgs boson pairs.) During production and tagging we have ensured that the transverse momenta are constrained to 400–700 GeV for **a** and **b** and to 700–1000 GeV for **c** and **d**. As the jet cone size is $\Delta R = 0.5$, we expect the bosons of the SM and the top quarks to lie within a single jet for $p_T \gtrsim 400$ and 700 GeV, respectively.

During the analysis we cluster jets with the anti- k_T algorithm using a minimal transverse momentum of 40 GeV and a jet cone size of 0.5. Additionally we require the two leading jets to be boosted more than $p_T > 400$ GeV. We require at least one and two leptons for the semi- and full-leptonic analysis, respectively. Leptons must be isolated with an isolation radius of $\Delta R > 0.3$ and a maximal transverse momentum ratio of 0.2. The transverse momentum ratio is defined between the lepton and other cell activity within the isolation cone. If the lepton transverse momentum is larger than 100 GeV, we do not require it to be isolated [42]. Finally, we disregard events with less than five jets.

3.2 Event reconstruction

We reconstruct the signal signatures with multiple boosted decision trees (BDTs). Each of them is optimized to tag or reconstruct a single physical object, such as bottom or top quark, Higgs or Z boson and top partners. Such an object can consist of a sub-jet, a single

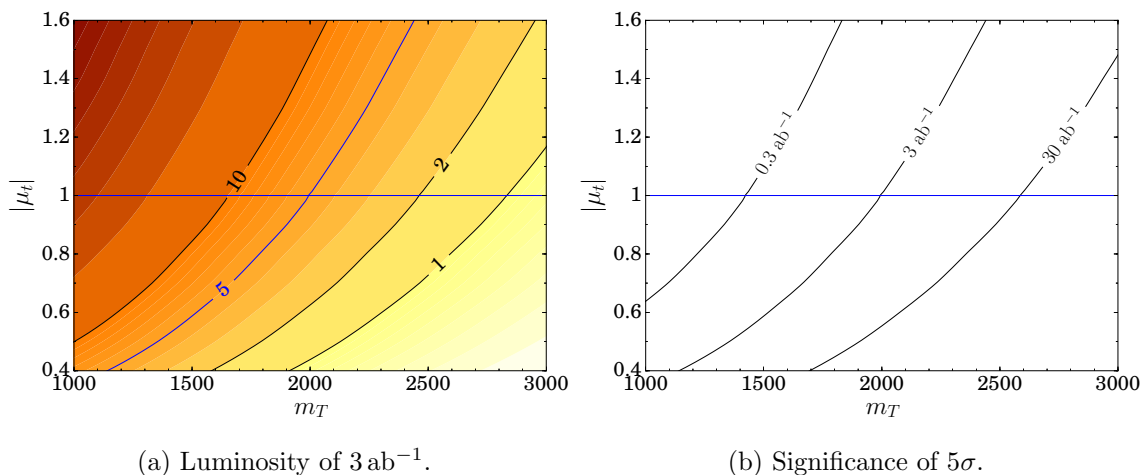


Figure 3. Discovery reach defined as $Z(b|b+s)$ for pair production of top partners in association with one Higgs boson at 100 TeV. We present the reaches for a fixed luminosity of 3 ab^{-1} in figure a and for a fixed significance of 5σ with luminosities of $0.3, 3, 30 \text{ ab}^{-1}$ in figure b.

jet or be combined from multiple jets. This Boosted Collider Analysis (BoCA) 0.3 [43] has been introduced in [44, 45] where we also discuss the efficiency of the bottom and top taggers. BoCA uses the TMVA library [46] of the ROOT 6.06 framework [47] together with FastJet 3.2 [48] for jet clustering. Here we introduce additionally taggers dedicated to Higgs and Z boson tagging. For boosted bosons we demonstrate their background mis-identification as a function of a given signal rate, the so called Receiver Operating Characteristic (ROC) curves in figure 2. We show the results for two different ranges of transverse momentum, which correspond to the cases in which the top quarks are boosted enough to end up in one and two jets, respectively. The Higgs tagger utilizes the displaced vertices of the bottom decay products. The strongest discriminators within these taggers are the boson mass and the pull [49] between its components. Furthermore, we train a neutral boson tagger to detect both of these particles, which allows us to reconstruct top partners from tops and neutral bosons. During this reconstruction we cater towards the boostness of the decay products by using the available substructure information. In order to suppress the $\bar{t}t + 3j$ and the $\bar{t}t + 5j$ backgrounds we require the neutral bosons from the top partner decays to be boosted with a cone size of $\Delta R < 0.5$ while the associated Higgs boson has to be un-boosted with a cone size of $\Delta R > 0.5$. Finally, we reconstruct the $\bar{T}Th$ signature by combining two top partner tagger with a Higgs boson tagger. The signal is separated from the background with a single cut on the final BDT result.

3.3 Results

For n observed events the significance can be defined as the log-likelihood ratio [50]

$$Z(x|n) = \sqrt{-2 \ln \frac{L(x|n)}{L(n|n)}}, \quad (3.6)$$

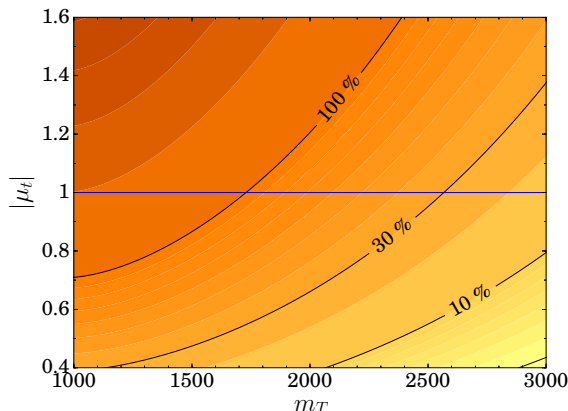


Figure 4. Ratio of signal over background s/b .

where x is the number of events predicted by the hypothesis which is tested. For counting experiments the likelihood function is given by the Poisson probability

$$L(x|n) = \frac{x^n e^{-x}}{n!}. \tag{3.7}$$

For the exclusion of the signal hypothesis $b+s$ we require $Z(b+s|n) \geq 2$ and for the discovery of a signal by exclusion of the background only hypothesis b we require $Z(b|n) \geq 5$. As we are projecting to future experiments we replace the event number n with the prediction for an alternative hypothesis. Hence, we are using $Z(b|b+s) \geq 5$ for the discovery of a model and $Z(b+s|b+s|_{\text{nat}}) \geq 2$ for the exclusion of an unnatural model.

In figure 3, we present the discovery reach against the background-only hypothesis of top partners in the $\bar{T}Th$ production channel at 100 TeV. This may not be the most sensitive channel for the search of fermionic top partners, but it shows the effectiveness of our analysis for testing naturalness. For natural theories we see that the discovery reach for top partners coupled to a Higgs boson can be pushed up to $\sim 1.4, 2, 2.5$ TeV, with $0.3, 3, 30 \text{ ab}^{-1}$ of data, respectively. In comparison, the ATLAS [7–9] and CMS [10] experiments have excluded fermionic top partner with $m_T < 780$ GeV, given BRs compatible with (3.2). The systematic errors are not included in our analysis. Instead, we present the expected signal over background ratio in figure 4 which indicates that the contour with $s/b = 10\%$ could extend to above 2.5 TeV for natural theories.

In the following we introduce two measures for the sensitivity of testing the naturalness sum rule. First, we calculate the exclusion limit of unnatural theories with $|\mu_t| \neq 1$, against the assumption that the observation at the collider is consistent with the prediction of a theory with $|\mu_t| = 1$. The exclusion limits of unnatural theories are presented in figure 5. With 30 ab^{-1} of data, unnatural theories with $||\mu_t| - 1| > 0.1$ can be excluded up to $m_T \sim 2.2$ TeV.

Second, we calculate the precision of measuring the naturalness parameter, defined as the uncertainty at one sigma confidence level

$$\delta |\mu_t| = \sqrt{\left(-\frac{1}{\lambda_t^2} \delta a_T\right)^2 + \left(2\frac{a_T}{\lambda_t^3} \delta \lambda_t\right)^2}. \tag{3.8}$$

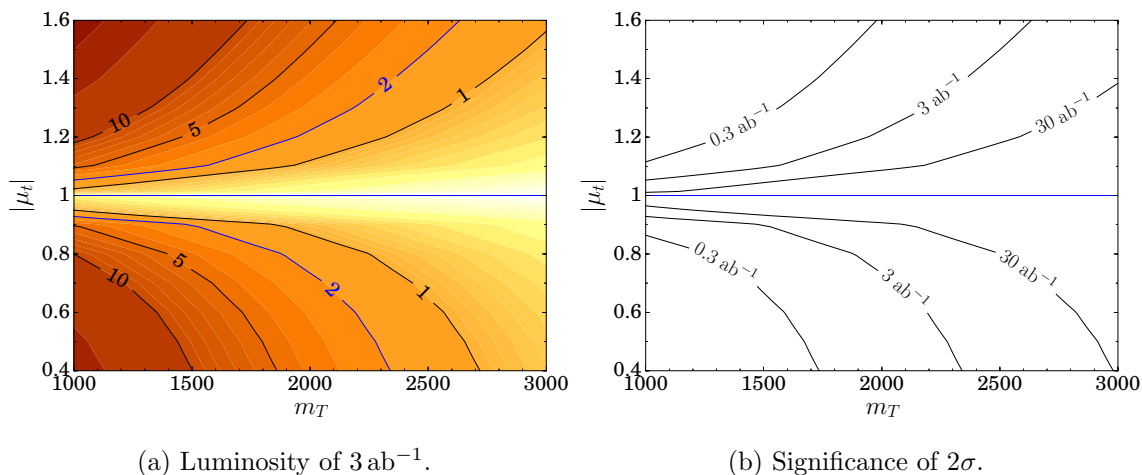


Figure 5. Exclusion limit for unnatural theories defined by $Z(b + s|b + s|_{\text{nat}})$ as a function of the top partner mass. Based on $\bar{T}Th$ production at 100 TeV. For a luminosity of 3 ab^{-1} in figure a and for $0.3, 3, 30 \text{ ab}^{-1}$ and a fixed significance of 2σ in figure b.

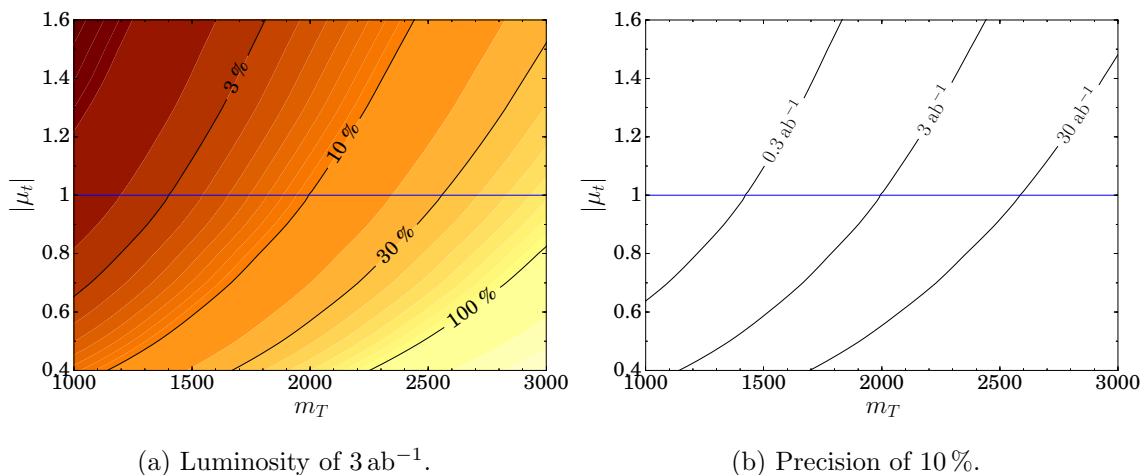


Figure 6. Precision of measuring the absolute value of the naturalness parameter $|\mu_t|$ at 3 ab^{-1} in figure a, and the reach of measuring with a precision of 10% at $0.3, 3, 30 \text{ ab}^{-1}$ in figure b. Here we assume that the top Yukawa coupling can be perfectly measured.

The precision of the coupling a_T can be estimated with the relation

$$\delta a_T = \frac{a_T}{2Z(b|n)}. \quad (3.9)$$

In figure 6 we present the precision under the assumption of a perfectly measured top Yukawa coupling $\delta\lambda_t = 0$. Figure 6a shows that in natural theories the naturalness parameter could be measured with a precision of 10% up to a top partner mass of $m_T \sim 2 \text{ TeV}$, using 3 ab^{-1} of data. Figure 6b shows that with $0.3, 3, 30 \text{ ab}^{-1}$ of data a precision of 10% could be achieved for top partners in natural theories with masses up to $\sim 1.4 \text{ TeV}$, $\sim 2 \text{ TeV}$ and above 2.5 TeV , respectively. Note that the definition of the precision (3.8)

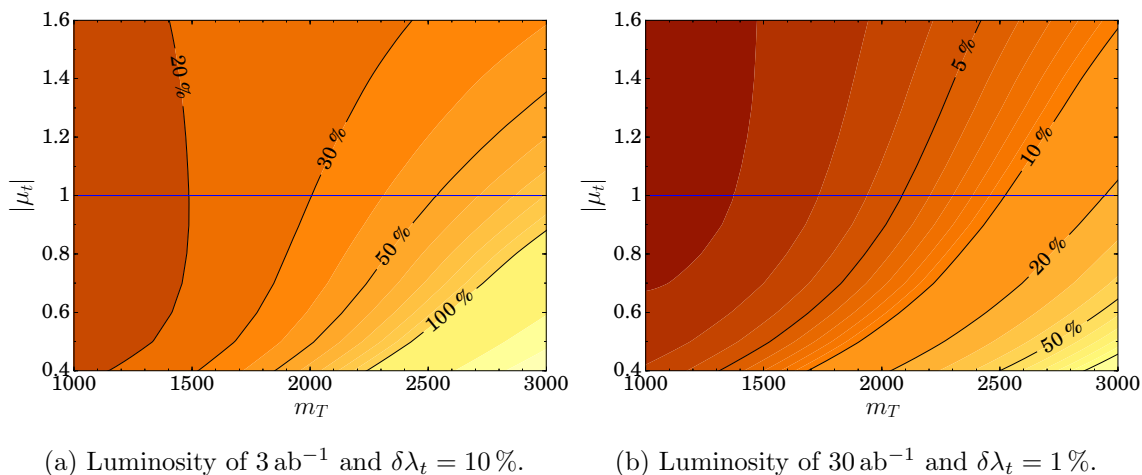


Figure 7. Precision of measuring the absolute value of the naturalness parameter $|\mu_t|$ at 100 TeV with 3 and 30 ab^{-1} of data. We assume the uncertainty of measuring λ_t to be 10% and 1%, respectively.

has the consequence that curves for $Z(b|n) = 5$ presented in figure 3 are approximately identical to the curves for $\delta|\mu_t|/|\mu_t| = 10\%$ shown in figure 6.

In figure 6 we have neglected the uncertainty on the measurement of the top Yukawa coupling. However, the measurement of the top Yukawa coupling is non-trivial and will be an important task at a 100 TeV hadron collider [36]. In figure 7, we show the precision of measuring the naturalness parameter when the uncertainty of measuring λ_t is included. We consider two cases. In figure 7a we assume a luminosity of 3 ab^{-1} at 100 TeV and an uncertainty of 10% achievable in future λ_t measurements at the HL-LHC [51, 52]. In figure 7b we consider 30 ab^{-1} and an uncertainty of 1% potentially reachable at a future 100 TeV collider [36]. Although a precision of 10% for the measurement of the naturalness parameter could still be pushed up to $m_T \sim 2.5 \text{ TeV}$, we do see a significant impact from $\delta\lambda_t$, particularly for small m_T . Therefore, it is crucial to improve the precision of measuring λ_t , in order to test naturalness with a higher precision.

4 Discussion and conclusion

The naturalness problem plays an extremely important role in driving particle physics. While, searches for partner particles predicted by theories of naturalness are actively conducted at the LHC, a further step is necessary to actually test the naturalness of these particles. Once discovered, it must be ensured that the newly found particle is not an impostor, but a particle with the right properties to cancel the quadratic UV-sensitivity of the Higgs self-energy.

Using the fermionic top sector as an example, we show how the naturalness sum rule can be measured and what sensitivity reach can be achieved at a 100 TeV hadron collider. We developed a simplified model for vector-like fermionic top partners and derived the naturalness sum rule in this general setup. Remarkably, we found that the naturalness sum

rule only involves couplings of the Higgs to a pair of top quarks and a pair of top partners, and is exact up to an order of v^2/m_T^2 . The derived condition is rather generic, covering representative theories of naturalness, including for example little Higgs models both with and without T -parity and mirror twin Higgs models. In particular, the necessary measurements do not involve the measurement of the decay constant f . The 100 TeV collider analyses based on the $\bar{T}Th$ production in the little Higgs models indicates that at 30 ab^{-1} :

- Theories which predict a $|\mu_t|$ deviating from the naturalness assumption by more than 10% which corresponds to $||\mu_t| - 1| > 0.1$ could be excluded for a top partner mass of up to $\sim 2.2 \text{ TeV}$.
- A precision of 10% for the measurement of $|\mu_t|$ could be achieved with a top partner mass of up to $\sim 2.5 \text{ TeV}$, given the precision of measuring the top Yukawa coupling of some percent.

We expect that the reach of this measurement can be further improved. Recall, that in this analysis presented in this article we only consider decays of $\bar{T}Th$ via a pair of top quarks and require them to decay fully or semi-leptonically. However, a larger branching fraction originates from the decay via one top quark and one bottom quark, that is $\bar{T}Th \rightarrow \bar{t}B^0bW^+h$ or $\bar{T}Th \rightarrow tB^0\bar{b}W^-h$. Additionally, the Higgs boson accompanying the top partners are required to decay to a pair of bottom quarks. In reality, some of its other decay modes may lead to a stronger background suppression. We would expect that these channels bring non-trivial improvements to the sensitivity of testing naturalness.

But, in this analysis we have neglected the precision with which the top partner mass can be measured (e.g., by mass reconstruction in the TT events). This yields an additional uncertainty on the relation between the signal rate and the coupling α_T and hence a contribution to $\delta\alpha_T$. An accurate estimation of the influence of this factor will be necessary. Additionally, we did not take into account systematic errors in the analysis. The systematic errors could receive multiple contributions, e.g., high-order corrections to the QCD background, PDF uncertainty, collider performance of future collider, etc. A reliable estimation is beyond the scope of this article due to the lack of input information. So we would postpone its considerations to future studies. At last, we would like to stress again that the $\bar{T}Th$ production is insensitive to the sign of the naturalness parameter and hence causes a degeneracy problem, despite its important role in testing naturalness. In order to break this degeneracy and exclude the theories with $\mu_t < 0$, new strategies such as indirect detection needs to be introduced.

Given the crucial role of the naturalness problem in driving particle physics, it is worthwhile to extend the analysis pursued in this article to other contexts. On the one hand, a complete cancellation of quadratic UV-sensitivity in the Higgs self-energy in little Higgs models requires the introduction of additional partner particles for gauge and Higgs bosons, which have to fulfill the corresponding naturalness sum rules. On the other hand, other theories of naturalness, most notably supersymmetry, rely on partner particles with opposite statistics in comparison to their corresponding SM particles, yielding a sum rule

as well. Some relevant discussions can be found in, e.g., ref [53–55]. A full exploration of these possibilities will be deferred to future work.

Acknowledgments

J. Hajer and T. Liu would like to thank J. Bernon, Y.-Y. Li and J. Shu for helpful discussions, while I. Low acknowledges the hospitality at Hong Kong University of Science and Technology, where part of this work was performed. J. Hajer and T. Liu are supported by the Collaborative Research Fund (CRF) under Grant № HUKST4/CRF/13G. T. Liu is also supported by the General Research Fund (GRF) under Grant № 16312716. Both the CRF and GRF grants are issued by the Research Grants Council of Hong Kong S.A.R.. H. Zhang was supported by the U.S. DOE under Contract № DE-SC0011702, as well as by IHEP under Contract № Y6515580U1 and IHEP Innovation Grant contract № Y4545171Y2. I. Low was supported in part by the U.S. Department of Energy under contracts № DE-AC02-06CH11357 and № DE-SC 0010143. C.-R. Chen was supported in part by the Ministry of Science and Technology of Taiwan under Grant № MOST-105-2112-M-003-010-MY3.

A Higher order corrections

In the main part of our discussion we have neglected higher order correction originating in the mass diagonalization after electroweak symmetry breaking. In order to make good for this omission we add the following quartic terms to Lagrangian (2.1)

$$\Delta\mathcal{L}_U = \frac{c_4}{4!f^3} |H|^4 u_3^c U + \frac{\widehat{c}_4}{4!f^3} |H|^4 U^c U + \text{h.c.} . \quad (\text{A.1})$$

The rotation into mass eigenstates (2.3) leads to the following additional terms in Lagrangian (2.4)

$$\Delta\mathcal{L}_{T'} = \frac{\gamma_{t'}}{4!m_{T'}^3} |H|^4 t'^c T' + \frac{\gamma_{T'}}{4!m_{T'}^3} |H|^4 T'^c T' + \text{h.c.} . \quad (\text{A.2})$$

The mass eigenstate parameter (2.5) at this order are in terms of gauge eigenstate parameter is given by

$$\gamma_{t'} = (\widehat{c}_0 c_4 - c_0 \widehat{c}_4) c^2, \quad \gamma_{T'} = (c_0 c_4 + \widehat{c}_0 \widehat{c}_4) c^2. \quad (\text{A.3})$$

The rotation (2.17) of the right handed fermion fields up to cubic order in v reads

$$t^c = t'^c - T'^c \frac{v^2}{2m_{T'}^2} (\alpha_{t'} + 2\lambda_{t'}^* \lambda_{T'}) + \mathcal{O}\left(\frac{v^4}{m_{T'}^4}\right), \quad (\text{A.4a})$$

$$T^c = T'^c + t'^c \frac{v^2}{2m_{T'}^2} (\alpha_{t'} + 2\lambda_{T'}^* \lambda_{t'}) + \mathcal{O}\left(\frac{v^4}{m_{T'}^4}\right). \quad (\text{A.4b})$$

At the same time the left handed fields are rotated by

$$t = t' - T' \frac{v}{m_{T'}} \lambda_{T'}^* - t' \frac{v^2}{2m_{T'}^2} |\lambda_{T'}|^2 - T' \frac{v^3}{6m_{T'}^3} \left(\beta_{T'}^* + 3\lambda_{t'}^* \alpha_{t'} - 3\lambda_{T'}^* \left(\alpha_{T'} + |\lambda_{T'}|^2 - 2|\lambda_{t'}|^2 \right) \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right), \quad (\text{A.5a})$$

$$T = T' + t' \frac{v}{m_{T'}} \lambda_{T'} - T' \frac{v^2}{2m_{T'}^2} |\lambda_{T'}|^2 + t' \frac{v^3}{6m_{T'}^3} \left(\beta_{T'} + 3\alpha_{t'} \lambda_{t'} - 3\lambda_{T'} \left(\alpha_{T'} + |\lambda_{T'}|^2 - 2|\lambda_{t'}|^2 \right) \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right). \quad (\text{A.5b})$$

The resulting quadratic correction to the top partner mass (2.2) is

$$m_T = m_{T'} \left(1 + \frac{v^2}{2m_{T'}^2} \left(\alpha_{T'} + |\lambda_{T'}|^2 \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right) \right). \quad (\text{A.6})$$

The Yukawa couplings (2.5a) are altered to

$$\lambda_t = \lambda_{t'} + \frac{v^2}{2m_{T'}^2} \left(\beta_{t'} - 3 \left(\alpha_{t'} \lambda_{T'} + \lambda_{t'} |\lambda_{T'}|^2 \right) \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right), \quad (\text{A.7a})$$

$$\lambda_T = \lambda_{T'} + \frac{v^2}{2m_{T'}^2} \left(\beta_{T'} + \alpha_{t'} \lambda_{t'} - \lambda_{T'} \left(2 \left(\alpha_{T'} - |\lambda_{t'}|^2 \right) + |\lambda_{T'}|^2 \right) \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right). \quad (\text{A.7b})$$

While the quadratic couplings (2.5b) up to this order are

$$\alpha_t = \alpha_{t'} + \frac{v^2}{2m_{T'}^2} \left(\gamma_{t'} + 2\lambda_{T'}^* \left(\beta_{t'} - \alpha_{T'} \lambda_{t'} \right) \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right), \quad (\text{A.8a})$$

$$\alpha_T = \alpha_{T'} + \frac{v^2}{2m_{T'}^2} \left(\gamma_{T'} + \alpha_{t'}^2 + \alpha_{T'}^2 + 2 \left(\lambda_{T'}^* \beta_{T'} + \alpha_{t'} \lambda_{t'}^* \lambda_{T'} \right) \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right). \quad (\text{A.8b})$$

The correction to the VEV induced terms (2.19a) up to quadratic order are

$$\Delta a_t = -\frac{v^2}{m_{T'}^2} \left(\frac{\gamma_{t'}}{3} + \frac{\beta_{T'}^* \lambda_{t'}}{3} + \lambda_{T'}^* \left(\beta_{t'} - \alpha_{t'} \lambda_{T'} - \lambda_{t'} \left(\alpha_{T'} + |\lambda_{T'}|^2 \right) \right) \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right), \quad (\text{A.9a})$$

$$\Delta a_T = -\frac{v^2}{m_{T'}^2} \left(\frac{\gamma_{T'}}{3} + \frac{\beta_{T'}^* \lambda_{T'}}{3} + \lambda_{T'}^* \left(\beta_{T'} - 2\alpha_{T'} \lambda_{T'} \right) - |\lambda_{T'}|^4 \right) + \mathcal{O} \left(\frac{v^4}{m_{T'}^4} \right). \quad (\text{A.9b})$$

We have refrained from writing down the quadratic corrections to the cubic couplings (2.5c) and (2.19b) as they involve terms of order H^5 .

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] G. 't Hooft, *Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking*, *NATO Sci. Ser. B* **59** (1980) 135 [[INSPIRE](https://arxiv.org/abs/hep-th/8011157)].

- [2] ATLAS collaboration, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys. Lett. B* **716** (2012) 1 [[arXiv:1207.7214](#)] [[INSPIRE](#)].
- [3] CMS collaboration, *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, *Phys. Lett. B* **716** (2012) 30 [[arXiv:1207.7235](#)] [[INSPIRE](#)].
- [4] H.E. Haber and G.L. Kane, *The search for supersymmetry: probing physics beyond the Standard Model*, *Phys. Rept.* **117** (1985) 75 [[INSPIRE](#)].
- [5] D.B. Kaplan and H. Georgi, *SU(2) \times U(1) breaking by vacuum misalignment*, *Phys. Lett.* **136B** (1984) 183 [[INSPIRE](#)].
- [6] D.B. Kaplan, H. Georgi and S. Dimopoulos, *Composite Higgs scalars*, *Phys. Lett.* **136B** (1984) 187 [[INSPIRE](#)].
- [7] ATLAS collaboration, *Search for production of vector-like top quark pairs and of four top quarks in the lepton-plus-jets final state in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *ATLAS-CONF-2016-013*, CERN, Geneva Switzerland, (2016).
- [8] ATLAS collaboration, *Search for top squarks in final states with one isolated lepton, jets and missing transverse momentum in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector*, *Phys. Rev. D* **94** (2016) 052009 [[arXiv:1606.03903](#)] [[INSPIRE](#)].
- [9] ATLAS collaboration, *Search for new physics using events with b-jets and a pair of same charge leptons in 3.2 fb^{-1} of pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *ATLAS-CONF-2016-032*, CERN, Geneva Switzerland, (2016).
- [10] CMS collaboration, *Search for pair production of vector-like T quarks in the lepton plus jets final state*, *CMS-PAS-B2G-16-002*, CERN, Geneva Switzerland, (2016).
- [11] T. Golling et al., *Physics at a 100 TeV pp collider: beyond the Standard Model phenomena*, *CERN Yellow Report* (2017) 441 [[arXiv:1606.00947](#)] [[INSPIRE](#)].
- [12] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *Electroweak symmetry breaking from dimensional deconstruction*, *Phys. Lett. B* **513** (2001) 232 [[hep-ph/0105239](#)] [[INSPIRE](#)].
- [13] N. Arkani-Hamed, A.G. Cohen, E. Katz, A.E. Nelson, T. Gregoire and J.G. Wacker, *The minimal moose for a little Higgs*, *JHEP* **08** (2002) 021 [[hep-ph/0206020](#)] [[INSPIRE](#)].
- [14] N. Arkani-Hamed, A.G. Cohen, E. Katz and A.E. Nelson, *The littlest Higgs*, *JHEP* **07** (2002) 034 [[hep-ph/0206021](#)] [[INSPIRE](#)].
- [15] R. Contino, Y. Nomura and A. Pomarol, *Higgs as a holographic pseudo-Goldstone boson*, *Nucl. Phys. B* **671** (2003) 148 [[hep-ph/0306259](#)] [[INSPIRE](#)].
- [16] K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model*, *Nucl. Phys. B* **719** (2005) 165 [[hep-ph/0412089](#)] [[INSPIRE](#)].
- [17] FCC collaboration, A. Ball et al., *Future Circular Collider study hadron collider parameters*, *FCC-ACC-SPC-0001*, CERN, Geneva Switzerland, (2014).
- [18] CEPC-SPPC STUDY GROUP collaboration, *CEPC-SPPC preliminary conceptual design report 1. Physics and detector*, http://cepc.ihep.ac.cn/preCDR/main_preCDR.pdf, (2015).
- [19] H.-C. Cheng, I. Low and L.-T. Wang, *Top partners in little Higgs theories with T-parity*, *Phys. Rev. D* **74** (2006) 055001 [[hep-ph/0510225](#)] [[INSPIRE](#)].
- [20] S. Chang, *A ‘littlest Higgs’ model with custodial SU(2) symmetry*, *JHEP* **12** (2003) 057 [[hep-ph/0306034](#)] [[INSPIRE](#)].

- [21] S.R. Coleman and E.J. Weinberg, *Radiative corrections as the origin of spontaneous symmetry breaking*, *Phys. Rev. D* **7** (1973) 1888 [[INSPIRE](#)].
- [22] M. Perelstein, M.E. Peskin and A. Pierce, *Top quarks and electroweak symmetry breaking in little Higgs models*, *Phys. Rev. D* **69** (2004) 075002 [[hep-ph/0310039](#)] [[INSPIRE](#)].
- [23] D.E. Kaplan and M. Schmaltz, *The little Higgs from a simple group*, *JHEP* **10** (2003) 039 [[hep-ph/0302049](#)] [[INSPIRE](#)].
- [24] G. Burdman, Z. Chacko, R. Harnik, L. de Lima and C.B. Verhaaren, *Colorless top partners, a 125 GeV Higgs and the limits on naturalness*, *Phys. Rev. D* **91** (2015) 055007 [[arXiv:1411.3310](#)] [[INSPIRE](#)].
- [25] H.-C. Cheng and I. Low, *TeV symmetry and the little hierarchy problem*, *JHEP* **09** (2003) 051 [[hep-ph/0308199](#)] [[INSPIRE](#)].
- [26] H.-C. Cheng and I. Low, *Little hierarchy, little Higgses and a little symmetry*, *JHEP* **08** (2004) 061 [[hep-ph/0405243](#)] [[INSPIRE](#)].
- [27] V. Drollinger, T. Müller and D. Denegri, *Searching for Higgs bosons in association with top quark pairs in the $H^0 \rightarrow b\bar{b}$ decay mode*, [hep-ph/0111312](#) [[INSPIRE](#)].
- [28] T. Plehn, G.P. Salam and M. Spannowsky, *Fat jets for a light Higgs*, *Phys. Rev. Lett.* **104** (2010) 111801 [[arXiv:0910.5472](#)] [[INSPIRE](#)].
- [29] P. Artoisenet, P. de Aquino, F. Maltoni and O. Mattelaer, *Unravelling $t\bar{t}h$ via the matrix element method*, *Phys. Rev. Lett.* **111** (2013) 091802 [[arXiv:1304.6414](#)] [[INSPIRE](#)].
- [30] CMS collaboration, *Search for the associated production of the Higgs boson with a top-quark pair*, *JHEP* **09** (2014) 087 [*Erratum ibid.* **10** (2014) 106] [[arXiv:1408.1682](#)] [[INSPIRE](#)].
- [31] ATLAS collaboration, *Search for $H \rightarrow \gamma\gamma$ produced in association with top quarks and constraints on the Yukawa coupling between the top quark and the Higgs boson using data taken at 7 TeV and 8 TeV with the ATLAS detector*, *Phys. Lett. B* **740** (2015) 222 [[arXiv:1409.3122](#)] [[INSPIRE](#)].
- [32] CMS collaboration, *Search for a Standard Model Higgs boson produced in association with a top-quark pair and decaying to bottom quarks using a matrix element method*, *Eur. Phys. J. C* **75** (2015) 251 [[arXiv:1502.02485](#)] [[INSPIRE](#)].
- [33] LHC/LC STUDY GROUP collaboration, G. Weiglein et al., *Physics interplay of the LHC and the ILC*, *Phys. Rept.* **426** (2006) 47 [[hep-ph/0410364](#)] [[INSPIRE](#)].
- [34] M. Klute, R. Lafaye, T. Plehn, M. Rauch and D. Zerwas, *Measuring Higgs couplings at a linear collider*, *EPL* **101** (2013) 51001 [[arXiv:1301.1322](#)] [[INSPIRE](#)].
- [35] A. Arbey et al., *Physics at the e^+e^- linear collider*, *Eur. Phys. J. C* **75** (2015) 371 [[arXiv:1504.01726](#)] [[INSPIRE](#)].
- [36] M.L. Mangano, T. Plehn, P. Reimitz, T. Schell and H.-S. Shao, *Measuring the top Yukawa coupling at 100 TeV*, *J. Phys. G* **43** (2016) 035001 [[arXiv:1507.08169](#)] [[INSPIRE](#)].
- [37] F. del Aguila, L. Ametller, G.L. Kane and J. Vidal, *Vector like fermion and standard Higgs production at hadron colliders*, *Nucl. Phys. B* **334** (1990) 1 [[INSPIRE](#)].
- [38] A. Alloul, N.D. Christensen, C. Degrande, C. Duhr and B. Fuks, *FeynRules 2.0 — a complete toolbox for tree-level phenomenology*, *Comput. Phys. Commun.* **185** (2014) 2250 [[arXiv:1310.1921](#)] [[INSPIRE](#)].

- [39] J. Alwall et al., *The automated computation of tree-level and next-to-leading order differential cross sections and their matching to parton shower simulations*, *JHEP* **07** (2014) 079 [[arXiv:1405.0301](#)] [[INSPIRE](#)].
- [40] T. Sjöstrand, S. Mrenna and P.Z. Skands, *PYTHIA 6.4 physics and manual*, *JHEP* **05** (2006) 026 [[hep-ph/0603175](#)] [[INSPIRE](#)].
- [41] DELPHES 3 collaboration, J. de Favereau et al., *DELPHES 3, a modular framework for fast simulation of a generic collider experiment*, *JHEP* **02** (2014) 057 [[arXiv:1307.6346](#)] [[INSPIRE](#)].
- [42] C. Brust, P. Maksimovic, A. Sady, P. Saraswat, M.T. Walters and Y. Xin, *Identifying boosted new physics with non-isolated leptons*, *JHEP* **04** (2015) 079 [[arXiv:1410.0362](#)] [[INSPIRE](#)].
- [43] J. Hajer, *BoCA: Boosted Collider Analysis*, <https://github.com/BoostedColliderAnalysis/BoCA>, (2015) [[Zenodo](#)].
- [44] J. Hajer, Y.-Y. Li, T. Liu and J.F.H. Shiu, *Heavy Higgs bosons at 14 TeV and 100 TeV*, *JHEP* **11** (2015) 124 [[arXiv:1504.07617](#)] [[INSPIRE](#)].
- [45] N. Craig, J. Hajer, Y.-Y. Li, T. Liu and H. Zhang, *Heavy Higgs bosons at low $\tan\beta$: from the LHC to 100 TeV*, *JHEP* **01** (2017) 018 [[arXiv:1605.08744](#)] [[INSPIRE](#)].
- [46] A. Hocker et al., *TMVA — Toolkit for Multivariate Data Analysis*, *PoS(ACAT)040* [[physics/0703039](#)] [[INSPIRE](#)].
- [47] R. Brun and F. Rademakers, *ROOT: an object oriented data analysis framework*, *Nucl. Instrum. Meth. A* **389** (1997) 81 [[INSPIRE](#)].
- [48] M. Cacciari, G.P. Salam and G. Soyez, *FastJet user manual*, *Eur. Phys. J. C* **72** (2012) 1896 [[arXiv:1111.6097](#)] [[INSPIRE](#)].
- [49] J. Gallicchio and M.D. Schwartz, *Seeing in color: jet superstructure*, *Phys. Rev. Lett.* **105** (2010) 022001 [[arXiv:1001.5027](#)] [[INSPIRE](#)].
- [50] G. Cowan, K. Cranmer, E. Gross and O. Vitells, *Asymptotic formulae for likelihood-based tests of new physics*, *Eur. Phys. J. C* **71** (2011) 1554 [*Erratum ibid.* **C 73** (2013) 2501] [[arXiv:1007.1727](#)] [[INSPIRE](#)].
- [51] CMS collaboration, *CMS at the high-energy frontier. contribution to the update of the european strategy for particle physics*, *CMS-NOTE-2012-006*, CERN, Geneva Switzerland, (2012).
- [52] ATLAS collaboration, *Physics at a high-luminosity LHC with ATLAS (update)*, *ATL-PHYS-PUB-2012-004*, CERN, Geneva Switzerland, (2012).
- [53] G. Burdman, M. Perelstein and A. Pierce, *Large Hadron Collider tests of a little Higgs model*, *Phys. Rev. Lett.* **90** (2003) 241802 [*Erratum ibid.* **92** (2004) 049903] [[hep-ph/0212228](#)] [[INSPIRE](#)].
- [54] M. Blanke, D. Curtin and M. Perelstein, *SUSY-Yukawa sum rule at the LHC*, *Phys. Rev. D* **82** (2010) 035020 [[arXiv:1004.5350](#)] [[INSPIRE](#)].
- [55] S. Chang, C. Kilic and T. Okui, *Measuring top squark interactions with the Standard Model through associated production*, *Phys. Rev. D* **84** (2011) 035015 [[arXiv:1105.1332](#)] [[INSPIRE](#)].