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TESTING OF UNIT ROOTS AND OTHER FRACTIONALLY INTEGRATED HYPOTHESES IN THE PRESENCE OF STRUCTURAL BREAKS

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Abstract: Tests for unit roots and other nonstationary hypotheses that were proposed by Robinson (1994) are applied in this article to the Nelson and Plosser's (1982) series. The tests can be expressed in a way allowing for structural breaks under both the null and the alternative hypotheses. When applying the tests to the same dataset as in Perron (1989) we observe that our results might be consistent with them when testing the nulls of trend-stationarity or a unit-root. However, we also observe that fractionally integrated hypotheses may be plausible alternatives in this context of structural breaks at a known period of time.

KEY WORDS: Long memory; Unit roots; Structural breaks.

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A major debate concerning the dynamics properties of macroeconomic time series has been going on since the seminal work of Nelson and Plosser (1982). In that paper, the traditional view that the series were stationary around a deterministic function of time was challenged. Using statistical techniques developed by Dickey and Fuller (1979, 1981), they found no strong against unit roots in US historical annual time series. However, the implication of structural change on unit-root tests which take no account of this possibility attracted the attention of Perron (1989, 1993), who found that the 1929 crash and the 1973 oil price shock were a cause of nonrejection of the unit-root hypothesis, and that when these were taken into account, a deterministic trend model was preferable. This question was also pursued by authors such as Christiano (1992), Krol (1992), Serletis (1992), Demery and Duck (1992), Mills ((1994) and Ben-David and Papell (1994) among others. Christiano (1992) argued that the date of the break should be treated as unknown, and suggested that tests for a structural break are themselves biased in favour of nonrejection. He proposed tests based on bootstrap critical values, coming to different conclusions from Perron (!989). Similarly, Zivot and Andrews (1992) allowed the structural break to be endogenous, finding less conclusive evidence against unit roots than did Perron (1989). Banerjee et al. (1992) also considered this problem, proposing sequential statistics based on the full sample, and a sequence of regressors indexed by a 'break' date. Using these techniques, they failed to reject the unit-root hypotheses in the real output in five industrialized countries (including the United States) but found evidence of stationarity around a shifted trend for Japan.

In this paper I propose tests for unit roots and other fractionally integrated hypotheses in the presence of structural breaks under both the null and the alternative hypotheses. The tests are due to Robinson (1994) and are very general in the sense that they allow us to test roots not only at the zero frequency but at any frequency on the interval [0, π]. One advantage of these tests is that the limiting distribution is standard, unlike most unit-root tests (with or without structural breaks), where a non-standard asymptotic distribution is obtained and critical values must be computed numerically on a case-by-case basis through Monte Carlo simulations.

The paper is organised as follows: Section 1 specifies the null and alternative hypotheses. Section 2 describes the test and its limit distribution. Section 3 applies the tests to the Nelson and Plosser (1982)'s dataset, and finally, Section 4 contains some concluding remarks.

1. NULL AND ALTERNATIVE HYPOTHESES

Slight variations in Robinson (1994) leads to the regression model

$$y_t = \beta_1' z_t + \beta_2' SB(T_b)_t + x_t , \qquad t = 1, 2, \ldots , \tag{1}$$

where y_t is the time series we observe; z_t is a (k_1 x 1) vector of deterministic regressors, which may include, for example, an intercept, a linear time trend or seasonal dummies; $SB(T_b)_t$ is a (k_2 x 1) vector of regressors related to the structural break at a known period of time T_b, and it can be specified in terms of an exogenous change in the level; or a change in the rate of growth; or in both of them. Under the alternative hypothesis we suppose that x_t in (1) satisfies

$$\rho(L;\theta)\, x_t = u_t , \qquad t = 1, 2, \ldots, \tag{2}$$

$$x_t = 0 \qquad t \leq 0, \tag{3}$$

where ρ is a prescribed function of the backshift operator L ($Lx_t = x_{t-1}$) and the p-dimensional parameter vector θ, and u_t is a covariance stationary sequence with zero mean and weak parametric autocorrelation. Following Robinson (1994), we specify ρ(L; θ) as:

$$\rho(L;\theta) = (1-L)^{d_1+\theta_{i_1}} \; (1+L)^{d_2+\theta_{i_2}} \;\; x \prod_{j=3}^{h} (1 - 2\cos w_j\, L + L^2)^{d_j + \theta_{i_j}} \tag{4}$$

for given h; for given real distinct numbers d_1, d_2, … , d_h, where for each j, $\theta_{ij} = \theta_l$ for some l, and for each l there is at least one j such that, $\theta_{ij} = \theta_l$; thus $h \geq p$. Under the null hypothesis defined as

$$H_o : \theta = 0, \tag{5}$$

(4) becomes

$$\rho(L) = (1-L)^{d_1} \; (1+L)^{d_2} \; x \prod_{j=3}^{h} (1 - 2\cos w_j \, L + L^2 \,)^{d_j} \; . \tag{6}$$

This specification allows us to consider particular cases of interest, for example, the I(1) model, when $\rho(L) = (1 - L)$; the I(2) when $\rho(L) = (1 - L)^2$; quarterly I(1) if $\rho(L) = (1 - L^4)$, etc. In the empirical application carried out in Section 3 we concentrate on cases where $\rho(L;\theta) = (1 - L)^{d+\theta}$ for different real values of d. That is, we look at fractional null and alternative hypotheses, with the singularity at the spectrum occurring at the zero frequency.

We can see from this previous setting that the model in (1) - (3) is very general, including under both the null and the alternative hypotheses, deterministic components, (like a linear time trend or seasonal dummies); unit or fractional roots of arbitrary order anywhere on the unit circle in the complex plane; and structural breaks at a known period of time in levels and rates of growth. The following section describes the test statistic and its limiting distribution.

2. THE TEST STATISTIC

Suppose we observe y_t and the deterministic regressors in z_t and $SB(T_b)_t$ for t=1, 2, … ,T_b, … ,T}. It is assumed that the I(0) u_t in (2) have parametric autocorrelation, such that u_t has spectral density f, which is a given function of frequency and of unknown parameters, specifically,

$$f(\lambda;\tau;\sigma^2) = \frac{\sigma^2}{2\pi}\, g(\lambda;\tau) \qquad -\pi < \lambda \le \pi ,$$

where the positive scalar σ^2 and the (qx1) vector τ are unknown, but g is of known form. More generally, g has to be a reasonably smooth function, implying that u_t in (2) is I(0), and thus does not have long memory.

We wish to test the null hypothesis (5) in the model given by (1) – (3), for given real numbers $d_1, d_2, \dots, d_h$, and given h. We first form

$$w_t = \rho(L) z_t + \rho(L)\, SB(T_b)_t ,$$

taking

$$z_t = SB(T_b)_t = 0 \quad for \ \ t \le 0.$$

The least squares estimate of $\beta = (\beta_1', \beta_2')'$ and residuals are

$$\bar{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t' \right)^{-1} \sum_{t=1}^{T} w_t\, \rho(L)\, y_t$$

$$\bar{u}_t = \rho(L)\, y_t - \bar{\beta}'\, w_t , \qquad t = 1,2,....,$$

and the periodogram of $\bar{u}_t$ is:

$$I(\lambda) = \left| \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} \bar{u}_t\, e^{i t \lambda} \right|^2 .$$

Unless g is a completely known function (e.g., $g \equiv 1$, as when u_t is white noise), we have to estimate the nuisance parameter vector τ. The estimate must be a Gaussian one, that is, it must have the same limit distribution as the efficient maximum likelihood estimate based on the assumption that $u_1, \dots u_T$ is Gaussian. One estimate which fits naturally into our frequency-domain setting is

$$\bar{\tau} = \arg\min_{T} \ \sigma^2(\tau), \qquad (7)$$

where the minimization is carried out over a suitable subset of R^q, and

$$\sigma^2(\tau) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\tau)^{-1} I_{\bar{u}}(\lambda_j); \qquad \lambda_j = \frac{2\pi j}{T}.$$

In view of the preceding remarks and the $T^{1/2}$-consistency of (7) under suitable regularity conditions, any estimate differing from (7) by $o_p(T^{-1/2})$ will produce a test with the classical large-sample properties. Now we form:

$$\bar{a} = \frac{-2\pi}{T}\sum_j^* \psi(\lambda_j)\, g(\lambda_j;\bar{\tau})^{-1} I_{\bar{u}}(\lambda_j),$$

$$\bar{A} = \frac{2}{T}\left(\sum_j^* \psi(\lambda_j)\psi(\lambda_j)' - \sum_j^* \psi(\lambda_j)\bar{\varepsilon}(\lambda_j)' x \left(\sum_j^* \bar{\varepsilon}(\lambda_j)\bar{\varepsilon}(\lambda_j)'\right)^{-1} x \sum_j^* \bar{\varepsilon}(\lambda_j)\,\psi(\lambda_j)'\right)$$

where

$$\psi(\lambda_j) = \operatorname{Re}\left(\frac{\partial}{\partial\theta}\log\rho(e^{i\lambda_j})\right) \quad and \quad \bar{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\bar{\tau}),$$

and the sums on * are over $\lambda_j \in M$, where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1), l=1,\ldots s\}$ such that ρ_l, $l = 1,\ldots s < \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$. Thus, we omit the contribution from the finitely many λ_j in an open λ_1-neighbourhood of any of the ρ_l.

The test statistic, which is derived via LM principle, is:

$$\bar{R} = \frac{T}{\bar{\sigma}^4}\bar{a}'\bar{A}^{-1}\bar{a} = \bar{r}'\bar{r}; \qquad \bar{r} = \frac{T^{1/2}}{\bar{\sigma}^2}\bar{A}^{-1/2}\bar{a}, \tag{8}$$

where $\bar{\sigma}^2 = \sigma^2(\bar{\tau})$. Under the null hypothesis (5), Robinson (1994) established under regularity conditions that

$$\bar{R} \to_d \chi_p^2 \quad as \quad T \to \infty. \tag{9}$$

The conditions on u_t in (8) are far more general than Gaussianity, with a moment condition only of order 2 required. An approximate 100α%- level test of (5) against alternatives

$$H_a: \theta \neq 0, \tag{10}$$

will be given by the rule:

Reject H_o *if* $\bar{R} > \chi^2_{p,\alpha}$, where the probability that χ^2_p exceeds $\chi^2_{p,\alpha}$ is α. If p = 1, an approximate one-sided $100\alpha\%$ level test of (5) against alternatives

$$H_a: \theta > 0 \tag{11}$$

rejects H_o if $\bar{r} > z_\alpha$, where the probability that a standard normal variate exceeds z_α is α, and conversely, a test of (5) against the alternatives

$$H_a: \theta < 0 \tag{12}$$

rejects H_o if $\bar{r} < -z_\alpha$. As this rule indicates, we are in a classical large sample testing situation for reasons described by Robinson (1994), who also showed that the above tests are efficient against local alternatives of form: H_1: $\theta = \delta T^{-1/2}$ for $\delta \neq 0$. Finally, we stress that the null χ^2 distribution of $\bar{R}$ holds across a broad class of exogenous regressors, and thus, including those in (1). On the other hand, in most of the tests for unit roots with or without structural breaks, the null limit distribution can vary with features of the regressors (see, e.g., Schmidt and Phillips, 1992).

3. AN EMPIRICAL APPLICATION

In this section we examine the presence of structural breaks in the Nelson and Plosser's (1982) series. These are fourteen US macroeconomic variables measured in historical annual data. We use the same dataset as in Perron (1989), i.e., we look at all except the unemployment series, with data ending in 1970. Alternatively we could have used an extended version of this dataset, ending in 1982, however, in order to obtain better comparisons with Perron's (1989) results, we have decided to use exactly the same dataset. The starting date is 1860 for consumer price index and industrial production; 1869 for velocity; 1871 for stock prices; 1889 for GNP deflator and money stock; 1890 for employment and unemployment rate; 1900 for interest rate, real wages and wages; and

1909 for nominal and real GNP and GNP per capita. As in Nelson and Plosser (1982) and Perron (1989), all except the interest rate are transformed to natural logarithms.

Denoting any of the series y_t, for eleven of the Nelson and Plosser's (1982) series, we employ throughout the model

$$y_t = \beta_1 + \beta_2 t + \beta_3 DU(T_b)_t + x_t, \quad t = 1, 2, ... \tag{13}$$

$$(1 - L)^{d+\theta} x_t = u_t, \quad t = 1, 2, ... \tag{14}$$

where $DU(T_b)_t = 1 \, I \, (t > T_b)$ and $T_b = 1929$. Thus, if d = 1, under H_o (5), the model becomes, for t > 1,

$$y_t = \beta_2 + y_{t-1} + \beta_3 DL(T_b)_t + u_t, \quad t = 2, 3, ... \tag{15}$$

where $DL(T_b)_t = 1 \, I \, (t = T_b + 1)$, which corresponds to the null 'model A' in Perron (1989). Similarly, with d = 0, if we cannot reject the null, the model becomes

$$y_t = \beta_1 + \beta_2 t + \beta_3 DU(T_b)_t + u_t, \quad t = 1, 2, ... \tag{16}$$

which is the alternative 'model A' in Perron (1989).

For the remaining two series, which are real wages and common stock prices, we will employ throughout the model

$$y_t = \beta_1 + \beta_2 t + \beta_3 DU(T_b)_t + \beta_4 DT(T_b) + x_t, \quad t = 1, 2, ... \tag{17}$$

$$(1 - L)^{d+\theta} x_t = u_t, \quad t = 1, 2, ... \tag{18}$$

with $DT(T_b)_t = t \, I \, (t > T_b)$ which, under H_o (5), becomes the null 'model C' in Perron (1989), i.e.,

$$y_t = \beta_2 + y_{t-1} + \beta_3 DL(T_b)_t + \beta_4 DU(T_b)_t + u_t, \quad t = 2, 3, ... \tag{19}$$

when d = 1, and its alternative 'Model C',

$$y_t = \beta_1 + \beta_2 t + \beta_3 DU(T_b)_t + \beta_4 DT(T_b)_t + u_t, \quad t = 1, 2, ... \tag{20}$$

when d = 0.

We have chosen these particular specifications in view of Perron's (1989) results. He found evidence of unit roots of the form as in (15) for consumer prices, velocity and interest rate; evidence of trend-stationary representations as in (16) for nominal, real and real per capita GNP, industrial production, employment, GNP deflator, wages and money stock; and finally, evidence of (20) for stock prices and real wages. However, we present the results not merely for the nulls when d = 0 and d = 1 but when d = 0, 0.25, 0.50, …(0.25) …, 2.00, thus including also tests for stationarity (d = 0.50) and for I(2) (d = 2), as well as other possibilities.

Table 1 reports the results for $\bar{r}$ in (8) with white noise u_t. The first point to note in this table is that the values of $\bar{r}$ are, for each of the series, monotonically decreasing with the value of d. This monotonic decrease is to be expected, given correct specification and adequate sample size, of any reasonable statistics, given that they are one-sided statistics. Thus, for example, we would wish that if H_o is rejected for d = 0.75 against alternatives of form: H_a: $\theta > 0$, an even more significant result in this direction would be obtained when testing H_o with d = 0.50. We also observe that the null hypothesis is, for all series, rejected when d = 0. Thus, the trend-deterministic approach is clearly rejected in all the series in favour of alternatives with a positive order of integration. Also, when d = 0.25 and d = 0.50, the null is always rejected. When d = 0.75, we see that H_o is not rejected for industrial production, stock prices and real wages, and the unit root null hypothesis is not rejected for these series along with real, nominal and real per capita GNP, employment and velocity. For the remaining five series, (GNP deflator, consumer prices, wages, money stock and interest rates), higher orders of integration are observed. This is particularly clear in cases of consumer prices and money stock where H_o is rejected even for d = 1.25 in favour of alternatives with d > 1.25. The results across this table provide strong evidence against the trend-stationary representations advocated by Perron (1989), and give evidence in favour of unit roots in at least eight of the Nelson and Plosser series.

(Table 1 about here)

Perron (1989), argued that given that the Great Crash did not occur instantaneously, but lasted several years, an autoregressive structure should be considered for the disturbance term. Thus, in Tables 2 and 3 we calculate the test statistics imposing autoregressions in u_t. Table 2 reports the results for AR(1) u_t. We observe in this table that the values of d where the null hypothesis is not rejected are much smaller than in Table 1. Thus, when d = 0, H_o is not rejected in ten out of the thirteen series presented. We see that consumer prices, money stock and velocity are the only series where d must be greater than zero. We observe several non-rejection values when d = 0.25 and also when d = 0.50 and 0.75, however, the unit root null hypothesis is now rejected in eight of the series, observing non-rejection values only for GNP deflator, money stock, consumer prices, velocity and interest rate. The latter three series are those in which the unit root null hypothesis was not rejected in Perron (1989). Therefore, the results in this table might be consistent with those in Perron (1989) though we also observe non-rejection values when d is between 0 and 1.

(Tables 2 and 3 about here)

Finally, in Table 3 we present the results supposing that u_t follows an AR(2) process. Higher order autoregressions were also performed but the results were fairly similar to those given in Table 3. When d = 0, the null hypothesis of trend-stationarity representations is not rejected in eight series. Perron (1989) found evidence against this hypothesis only for consumer prices, velocity and interest rates. We observe that when d = 0, H_o (5) is rejected in these three series along with GNP deflator and money. In all these series the null hypothesis is also rejected when d = 0.25. When d = 1, the null hypothesis is not rejected for the same five series as in Table 2, i.e., GNP deflator, consumer prices, velocity, money and interest rates. Thus, once more, the results are consistent with those in Perron (1989) when testing the nulls with d = 0 and d = 1, however, we observe that the lowest statistics are obtained in most of the series when d is greater than zero but smaller than one. We see that d = 0 gives the lowest statistics for real and nominal GNP; d = 0.25

for industrial production, employment, common stock prices and real wages; d = 0.75 for GNP per capita, wages and velocity; d = 1 for consumer prices, money and interest rate; and finally, d = 1.25 for GNP deflator.

4. CONCLUSIONS

A particular version of the tests of Robinson (1994) for testing unit roots and other nonstationary hypotheses has been proposed in this article for testing the order of integration of time series in the presence of structural breaks under both the null and the alternative hypotheses. These tests have a standard limit distribution and thus, do not require case by case evaluation of critical values based on Monte Carlo simulations. Also the tests include as particular cases, the null and alternative models used in Perron (1989) for testing the trend-stationary/unit root representations in Nelson and Plosser series. We should mention here that the emphasis given in the literature to the trend-stationary/unit-root representations may have rather obscured the fact that these are extremely specialized forms of nonstationarity. Our approach allows the testing of various other forms of nonstationarity, in particular, testing for fractional processes of any degree of integration.

The tests were applied to the Nelson and Plosser series, including, as in Perron (1989), a break due to the Great Crash in 1929. The results vary substantially depending on how we model the I(0) disturbances. Thus, if u_t is white noise, the trend-stationary representations are clearly rejected in all the series, with the orders of integration fluctuating between 0.75 and 1.75. The unit root hypothesis is not rejected in eight series, and for the remaining five, higher orders of integration are observed. Including autoregressions in u_t, the orders of integration are much smaller, ranging between 0 and 1 in practically all the series. We see that the I(0) and I(1) specifications are not rejected for the same series as in Perron (1989), however, we observe that orders of integrations greater than zero but smaller than one are more plausible alternatives in some cases.

We can conclude by saying that the tests of Robinson (1994) can be used for testing the order of integration of time series in the presence of structural breaks. A logical follow-up step should be to try to perform the tests allowing the break-date to be unknown. One possibility might be choosing the date which produces the smallest statistic for a given model. Research on this precise topic is now under way.

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TABLE 1: $\bar{r}$ in (13) / (17) and (14) with white noise u_t.										
		Values of d								
Series	SB(T_b)	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Real GNP	DU	7.30	5.30	3.62	2.16	**0.85**	**-0.29**	**-1.25**	-2.01	-2.60
Nominal GNP	DU	5.85	4.28	3.53	2.85	**1.69**	**0.36**	**-0.77**	**-1.66**	-2.33
GNP p. capita	DU	8.26	6.20	4.21	2.41	**0.91**	**-0.32**	**-1.30**	-2.07	-2.65
Industrial prod	DU	9.49	5.36	2.16	**0.03**	**-1.48**	-2.59	-3.41	-3.99	-4.42
Employment	DU	10.51	7.47	4.62	2.51	**0.85**	**-0.51**	**-1.58**	-2.40	-3.00
GNP deflator	DU	10.38	8.63	7.50	5.87	3.39	**0.88**	**-1.04**	-2.31	-3.14
C.P.I.	DU	20.28	19.39	15.88	10.23	5.71	2.53	**0.33**	**-1.11**	-2.03
Wages	DU	7.16	6.08	5.35	4.32	2.67	**0.87**	**-0.61**	**-1.69**	-2.45
Money stock	DU	10.84	9.08	7.50	6.34	4.86	2.88	**0.88**	**-0.73**	-1.99
Velocity	DU	19.66	16.17	9.88	3.43	**-0.25**	-1.98	-2.96	-3.61	-4.11
Interest rate	DU	10.01	9.78	9.24	6.75	2.69	**-0.70**	-2.69	-3.68	-4.19
Stock prices	DU + DT	9.54	7.76	4.12	**1.90**	**0.24**	**-0.93**	**-1.91**	-2.88	-3.67
Real wages	DU + DT	5.30	4.23	2.94	**1.49**	**0.09**	**-1.12**	-2.09	-2.82	-3.32

DU = 1 I (t > T_b) and DT = t I (t > T_b); T_b = 1929; All the series are transformed to natural logs except interest rate. In bold: Non-rejection values of the null hypothesis (5) at the 95% significance level.

TABLE 2: $\bar{r}$ in (13) / (17) and (14) with AR(1) u_t.										
		Values of d								
Series	SB(T_b)	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Real GNP	DU	**1.74**	**0.70**	**-0.87**	-2.26	-2.70	-2.72	-2.72	-2.75	-2.84
Nominal GNP	DU	**0.79**	**-0.48**	**-1.15**	**-1.68**	-2.28	-2.54	-2.64	-2.72	-2.82
GNP p. capita	DU	**1.88**	**0.79**	**-0.55**	-2.08	-2.56	-2.65	-2.69	-2.76	-2.87
Industrial prod	DU	**0.18**	**-1.25**	**-1.80**	-2.25	-2.65	-3.05	-3.45	-3.84	-4.19
Employment	DU	**1.70**	**0.80**	**-0.96**	-2.07	-2.46	-2.70	-2.93	-3.19	-3.45
GNP deflator	DU	**0.33**	**-1.72**	-2.47	-2.28	**-1.64**	**-1.50**	**-1.81**	-2.24	-2.67
C.P.I.	DU	6.45	3.99	2.17	**1.80**	**-0.16**	-2.02	-2.87	-3.42	-3.71
Wages	DU	**1.22**	**-0.09**	**-0.72**	**-1.35**	-1.96	-2.22	-2.48	-2.75	-2.99
Money stock	DU	2.66	**1.30**	**0.43**	**-0.16**	**-1.56**	-2.24	-2.19	-2.36	-2.67
Velocity	DU	2.26	**1.69**	**0.55**	**-0.004**	**-1.86**	-2.68	-3.03	-3.22	-3.44
Interest rate	DU	**0.08**	**-0.75**	**-1.00**	**-1.24**	**-1.28**	-2.61	-2.66	-2.87	-2.99
Stock prices	DU + DT	**1.46**	**-0.89**	-2.18	-2.38	-2.46	-2.41	-2.33	-2.65	-3.32
Real wages	DU + DT	**1.19**	**-1.15**	-3.00	-2.57	-2.25	-2.20	-2.38	-2.67	-2.95

DU = 1 I (t > T_b) and DT = t I (t > T_b); T_b = 1929; All the series are transformed to natural logs except interest rate. In bold: Non-rejection values of the null hypothesis (5) at the 95% significance level.

TABLE 3: $\bar{r}$ in (13) / (17) and (14) with AR(2) u_t.										
		Values of d								
Series	SB(T_b)	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Real GNP	DU	**-0.65**	**-0.77**	**-1.03**	**-1.61**	-2.23	-2.62	-2.84	-2.97	-3.03
Nominal GNP	DU	**-0.46**	**-0.46**	**-0.59**	**-0.93**	-1.97	-2.17	-2.59	-2.82	-2.91
GNP p. capita	DU	**-1.26**	**-1.10**	**-1.01**	**-1.46**	-2.09	-2.53	-2.80	-2.96	-3.07
Industrial prod	DU	**-1.10**	**-0.87**	**-1.19**	**-1.62**	-1.98	-2.23	-2.49	-2.77	-3.09
Employment	DU	**-1.40**	**-0.51**	**-0.68**	**-1.16**	-1.98	-2.02	-2.19	-2.44	-2.66
GNP deflator	DU	6.49	8.28	6.84	3.23	**-1.90**	**-1.87**	-2.16	-2.37	-2.54
C.P.I.	DU	2.42	2.03	**1.06**	**0.80**	**0.15**	**-0.74**	**-1.58**	-2.35	-2.84
Wages	DU	**-1.40**	**-1.47**	**-1.17**	**-0.95**	-1.96	-1.97	-2.02	-2.38	-2.65
Money stock	DU	-3.93	-2.48	-2.00	**-1.77**	**-1.69**	**-1.70**	**-1.78**	-1.98	-2.13
Velocity	DU	-6.48	-8.87	**-1.13**	**-0.02**	**-1.75**	-2.85	-3.32	-3.44	-3.51
Interest rate	DU	9.09	2.82	2.62	3.36	**-1.13**	**-1.18**	**-1.35**	-1.98	-2.03
Stock prices	DU + DT	**-1.01**	**-0.65**	**-0.82**	**-1.15**	-2.11	-2.34	-2.45	-3.56	-3.45
Real wages	DU + DT	**1.73**	**-1.55**	-2.10	-2.89	-3.01	-3.21	-3.45	-3.47	-3.48

DU = 1 I (t > T_b) and DT = t I (t > T_b); T_b = 1929; All the series are transformed to natural logs except interest rate. In bold: Non-rejection values of the null hypothesis (5) at the 95% significance level.