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Testing the Expectations Hypothesis with Survey Forecasts: The Impacts of Consumer Sentiment and the Zero Lower Bound in an I(2) CVAR [post-print]

Joshua Stillwagon
Trinity College, joshua.stillwagon@trincoll.edu

Testing the Expectations Hypothesis with Survey Forecasts: The Impacts of Consumer Sentiment and the Zero Lower Bound in an I(2) CVAR

by Josh R. Stillwagon[a,b]

[a]Department of Economics, Trinity College, Hartford, CT 06106
[b]INET Center for Imperfect Knowledge Economics, University of Copenhagen
joshua.stillwagon@trincoll.edu

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Monthly interest rate forecasts from nearly 50 major financial institutions are used to examine the expectations hypothesis at the short end of the term structure for the Canadian T-bill market and Libor markets in the US, UK, and Switzerland. Using CVARs, the term premium is found to move inversely with consumer sentiment in all four samples at the 1% level. Extension to the polynomial CVAR also suggests that a fall in the interest rate raises the premium, at least temporarily. This is interpreted as arising from the decreasing upside potential for bond price movements related to the zero lower bound.

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1 Introduction

The expectations hypothesis (EH) predicts that longer term interest rates should be an average of expected shorter term rates over the same period, perhaps plus a constant risk premium. This proposition has been frequently rejected in empirical work however (Fama and Bliss 1987, Campbell and Shiller 1991). It is not clear though whether this rejection is occurring because of a time-varying risk premium, violations of the rational expectations hypothesis (REH), or both; since the empirical work represents a joint hypothesis of REH and a constant premium.[1] This is an important question for interpreting movements in the yield curve; for practitioners and monetary authorities attempting to gauge the efficacy of their forward guidance. We would like to know to what extent longer term rates reflect expectations about shorter term rates and to what extent they reflect a risk premium. This is even more important given the evidence of international spillover propagating through expectations (Belke, Beckmann, and Kühl 2011, 2012).

One innovative approach to illuminate this question is to use survey data to dissect the problem into separate, more manageable pieces (Friedman 1980, Froot 1989, and MacDonald and Macmillan 1994).[2] Previous studies using interest rate forecasts have rejected REH, demonstrating the importance of testing the EH with survey data, and have also found evidence of a time-varying risk premium.[3]

Much of the expectations hypothesis literature, using survey data or otherwise, focuses on Treasury rates, often in the US. The financial crisis however has renewed focus on other markets for credit aside from simply federal funds. This study will examine the expectations hypothesis using survey data on the short end of the maturity spectrum for the T-bill market in Canada, as

[1] Guidolin and Thornton (2008) for example suggest the EH is rejected because of the unpredictability of short term rates. Cochrane and Piazzesi (2005) by contrast interpret predictable excess returns as real interest rate risk premia since their estimates are correlated with unemployment and inflation.

[2] A similar line of research has occurred in the foreign exchange market in examination of the forward discount anomaly, which also represents an inability to account for expected excess returns. See Dominguez (1986), Froot and Frankel (1989) and Cavaglia, Verschoor, and Wolff (1994); and MacDonald (2000) for a survey including FX, bonds, and stocks.

[3] In addition to those studies mentioned, Gourinchas and Tornell (2004), Piazzesi and Schneider (2008), and Bachetta, Mertens and van Wincoop (2009) reject REH in bond markets.

well as the Libor markets in Switzerland, the UK, and the US. The Libor is of course an important benchmark interest rate, impacting derivative markets more broadly, and is consequently important to study in its own right. Using the cointegrated vector autoregression (CVAR), the "pure" EH, assuming no premium, is rejected in all four samples; while the EH allowing for a constant premium is rejected as a stationary equilibrium for all three Libor samples.

Given evidence of a time varying risk premium, it would be valuable to examine the sources of its fluctuations. Previous studies have typically not tested which factors can account for movements in the expected excess returns found in survey data.[4] Theoretical work argues that the premium will depend on the macroeconomic outlook, and indeed empirical evidence suggests the premium is counter-cyclical.[5] Much of the empirical work though assumes REH, confounding inference due to the joint hypothesis, and either omits direct impacts of macro factors or relies on latent factor analysis to capture the agglomerated counter-cyclical driver of the risk premium (essentially proxying for macroeconomic expectations). This work will again attempt to capture individuals' expectations more directly (in this case about the future state of the economy); using measures of consumer sentiment from the OECD also obtained through survey.

Even for the Canadian T-bill sample, which cannot reject a constant premium, the model fit is greatly improved by allowing the premium to vary with consumer sentiment. The term premium appears to move inversely with consumer sentiment in all four samples, significant at the 1% level. This suggests that increased economic pessimism increases the preference for shorter maturity assets and consequently the premium on longer term holdings.

Lastly, this work will also examine a conception of risk developed in Frydman and Goldberg's (2007, 2012) imperfect knowledge economics (IKE) gap model, which has found support in currency and stock markets, applied now to term structure premia. Incidentally, the gap conception of risk was inspired by Keynes's (1936) discussion of liquidity preference, or the reason individuals would prefer to hold cash rather than interest bearing bonds. The intuition of the gap model, contextualized here to the term structure,

[4]An exception in bond markets is Wright (2011) who relates risk premia to inflation uncertainty. Exceptions in currency markets include Frydman and Goldberg (2007) and Stillwagon (2013).

[5]See for example Cochrane and Piazzesi (2005), Wachter (2006), Sarno, Thornton and Valente (2007), and Ludvigson and Ng (2009).

is that as the interest rate falls, there is an increasing skew to the potential movement of bond prices (given the zero lower bound), or "more to fear than to hope" as Keynes put it. Investors therefore demand a greater premium to compensate for the greater downside risk and lower upside risk.

Keynes suggested though that what matters is not necessarily the level of the interest rate per se, but rather its divergence from what is viewed as a relatively safe rate given recent experience. This suggests that the effects may be temporary. A fall in the interest rate would lead individuals to perceive an increasing negative skew to potential bond price movements, since the interest rate is now lower relative to recent levels than prior. Eventually though, this lower level would become the recent experience and viewed as the "new normal" safe rate causing the effect on the premium to fade. These transitory effects on the premium from a change in the level of the interest rate can be examined more explicitly in the medium-run relations of the Polynomial, or I(2), CVAR (Johansen 1997, Juselius 2006). Support for this hypothesis is found and extension to the I(2) model greatly improves the model fit and precision of the estimates, and provides a much more intricate examination of the driving and adjustment dynamics.

To summarize the results more succinctly, it appears that term premia at short maturities for both T-bill and Libor markets move inversely with consumer sentiment. Further, given consumer sentiment, a decrease in expected short term rates will decrease the longer term rate, but less than one for one since the premium will rise, at least temporarily, as interest rates fall relative to the perceived "safe" rate based on recent experience.

These results have some important implications for both policymakers and practitioners. At a basic level, they suggest that a steeper yield curve is less of a bullish sign when consumer sentiment and interest rates are lower. They also suggest that increases in short term rates as the economy improves will have a muted effect on long term rates, particularly when arising from the zero lower bound. Conventional monetary policy then may have limited ability to impact overheating real estate markets, which has been a growing concern in many countries. This then offers a new rationale for macroprudential policies to dampen asset bubbles, in addition to the usual argument that monetary policy is too blunt of a tool given its effect on all sectors of the economy. Lastly, the testing reveals that near-I(2) persistent trends in interest rates and sentiment are empirically relevant. Consequently, any analysis including interest rates or consumer sentiment would benefit from using the polynomial CVAR to allow for this possibility.

2 The Liquidity Premium and Consumption Risk

The risk premium here is defined as the difference between the current return on an n-period bond (i_t^n) minus the expected return from rolling over shorter maturity bonds over the same period $E(rr)_t$ (referred to hereafter as the expected rollover return). The expected rollover return is an average of the shorter term spot rate $i_t^{n/k}$ and forecasts of the future shorter term rates $E(i_{t+xn/k}^{n/k})$ where n/k is the maturity and k represents the number of consecutive bonds needed to span the lifetime of the longer term n-year bond.

$$\begin{aligned} rp_t &\equiv i_t^n - E(rr)_t \text{ where} \\ E(rr)_t &\equiv [i_t^{n/k} + E(i_{t+n/k}^{n/k}) + .. + E(i_{t+n-2n/k}^{n/k}) + E(i_{t+n-n/k}^{n/k})]/k \end{aligned} \tag{1}$$

To make this a bit more concrete, in this data set, given the forecasted maturities and forecast horizons available, i_t^n is a one-year interest rate, while the expected rollover return is an average of the spot 90-day interest rate, and the three, six, and nine-month forecast of the 90-day rate, which is the forecasted return if four 90-day bonds are held consecutively.[6]

The immediate question then is what, if anything, accounts for time variation in this premium. Previous work suggests that individuals should be compensated for Macroeconomic risks. Wachter (2006) adapts the Campbell and Cochrane (1999) consumption based premium model for equities to the bond market where the premium moves inversely with consumption and expected future consumption, relative to some slowly moving habit level of consumption. Empirical work corroborates that predictable excess bond returns are countercyclical (Cochrane and Piazzesi 2005, Ludvigson and Ng 2009). Following the same motivation that led this paper to use survey data to directly capture expectations about future interest rates, this work will capture expectations about future consumption and the state of the economy more directly, using survey results on consumer sentiment from the OECD, rather than attempting to proxy for this with past or future movements in macro fundamentals assuming REH.

[6] See section four for more details about the data.

$$i_t^n - E(rr)_t = \beta * CS_t + \mu + \varepsilon_t \tag{2}$$

Equation two is the general equation of focus for the empirical work in the I(1) CVAR. The premium is predicted to fluctuate over time inversely with consumer sentiment, CS_t, so $\beta < 0$.

3 The Liquidity Premium and the Gap Model

A second notion of risk, exemplified by Frydman and Goldberg's (2004, 2013) imperfect knowledge economics (IKE) gap model, is that individuals perceive risk to be related to how far the asset price has moved, in one direction or the other, relative to some benchmark value.[7] This hypothesis has found support when tested against survey data measures of the risk premium in currency and equity markets (Frydman and Goldberg 2007, 2011). Incidentally, their model was inspired by Keynes's discussion of the bond market, specifically relating to liquidity preference, or the reason individuals would choose to hold cash rather than interest bearing bonds.

> A rate of interest [much lower than the benchmark rate], leaves more to fear than to hope, and offers, at the same time, a running yield which is only sufficient to offset a very small measure of fear [of capital loss] (Keynes, 1936, p.202).

This conception of risk was examined briefly by Tobin (1958), which also developed the basic portfolio balance model, but while the discipline seized on the latter, it has largely overlooked the former. Keynes's discussion would tend to suggest that the risk premium between longer and shorter maturity bonds (approaching the zero maturity of cash) would tend to rise as the level of the longer term interest rate falls, as a result of the increasing negative skew to potential capital gains (increasing fear and decreasing hope). If the premium is a function of the level of the interest rate, we have:

[7] The derivation of the IKE gap model relies on an adaptation of Prospect Theory (Kahneman and Tversky 1979) and heterogeneous expectations, though these do not enter the empirical work herein. Further, the IKE model predicts a role for asset supplies relative to wealth shares, though given such data is not readily available, this work will abstract from that aspect of the model.

$$i_t^n - E(rr)_t = \beta * i_t^n + \mu + \varepsilon_t \tag{3}$$

Where $\beta < 0$. Regrouping the terms and re-normalizing on i_t^n, we have

$$i_t^n = \frac{1}{1-\beta} E(rr)_t + \mu + \varepsilon_t \tag{4}$$

This view of risk predicts then that the coefficient on the expected rollover return should be less than one. As the expected future short rates rise, the longer term rate rises, but less than one for one since the premium falls. Keynes emphasized though that the zero lower bound was not the only benchmark of relevance.

> What matters is not the absolute level of r but the degree of its divergence from what is considered a fairly safe level of r

In determining what is a fairly safe level, Keynes felt individuals would rely on the historical record "[u]nless reasons are believed to exist why future experience will be very different from past experience" and that following changes in the level of the interest rate, eventually "having become accustomed to each successive reduction" the liquidity function of the public would adjust to viewing the recently prevailing level as appropriate and safe. This suggests the effect on the premium will be only transitory, which can be examined in the I(2) CVAR.

Of course the model of Frydman and Goldberg (2007) is partial equilibrium in nature. A more expansive theoretical model would allow for feedback between asset prices, the real economy, and monetary policy. Feedback dynamics are allowed for and observable in the empirical modeling however.

4 Data Set and Visual Inspection

The particularly novel variable in this work is the expected return from rolling-over three month interest rates (Libor or T-bill depending on the sample) over a one year period (referred to as the expected rollover return). The survey data comes from FX4casts which provides monthly reports of three, six, and twelve-month forecasts for a selection of interest rates going back to October 2001. The "consensus" forecast is a geometric mean of the contributors' forecasts which includes 48 major banks and consulting firms (see appendix for a complete listing). The countries used in this study are the US, UK, Canada, and Switzerland.

Given the forecasted interest rates and time horizons of the forecasts, it is feasible to investigate the connection between the one-year rate and the expected return from rolling over the three-month rate four consecutive times over the same one-year period. The expected rollover return then is the average of the current three-month rate, its three-month forecast, six-month forecast, and an interpolated nine-month forecast. The interpolation is conducted using a cubic spline, as well as a simple linear interpolation. The results are very robust to either though.

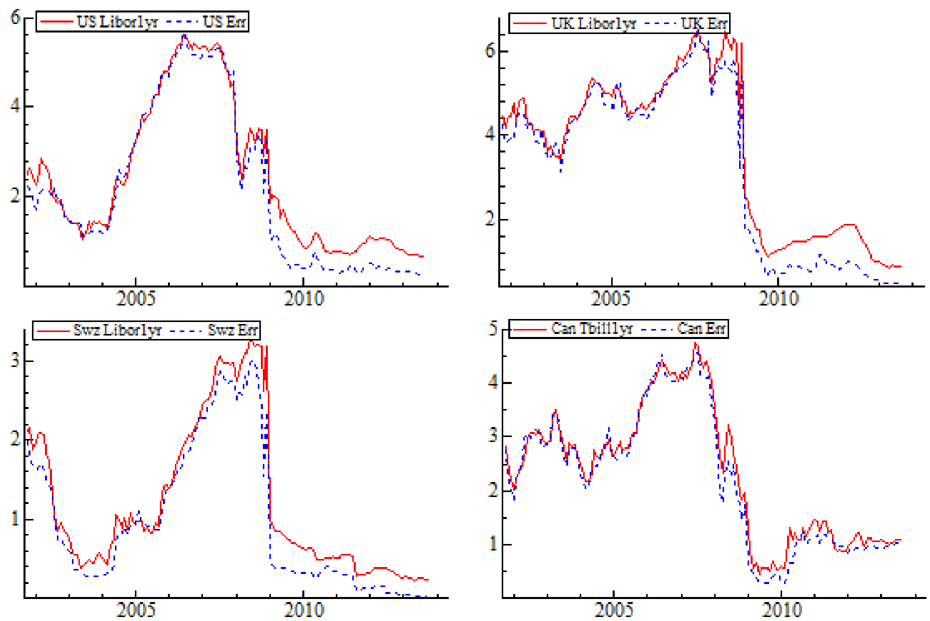
This data provides an examination of only the very short end of the term structure (as did MacDonald and Macmillan 1994), but is still of interest for several reasons. The first is that Froot (1989) found the most evidence of a time-varying premium at the short end. It is worthwhile to update this examination (particularly with a sample including the Great Recession) and to investigate the source of the fluctuations in the premium. Guidolin and Thornton (2008) meanwhile suggest that the rejection of the EH arises precisely because of the lack of predictability at the short end of the yield curve. We can now test the EH completely abstracting from this confounding influence by using subjective forecasts. Further, the near term horizons are of particular interest to both practitioners and policy makers. Wright (2011) by contrast uses forecasts at the five and 10-year horizon reported twice a year. The lower frequency necessitates a panel, whereas here there is sufficient power to examine each country individually. Estimating individual time series appears to be important given the notable coefficient variation seen in sections 7 and 10.

The other variables in the information set are the spot one-year Libor or T-bill rate, again provided by FX4casts to coincide with the timing of the forecasts, and consumer sentiment as measured by the OECD's consumer opinion survey: confidence indicator.[8] At this point it is informative to

[8] For more information see OECD (2013) "Main Economic Indicators - complete database", Main Economic Indicators (database).

The results are robust to using other measures of sentiment. For those countries with a monthly measure from the European Commission available on the FRED database (all excluding Switzerland) the correlation with the OECD measure exceeds 0.98 over the study's sample. Similarly, the University of Michigan measure for the US has a correlation examine the data visually. First we will view the co-movement between the expected rollover return and the one-year interest rate.

Figure 1: The one year interest rate (in red) and the expected rollover return (in blue dashes)

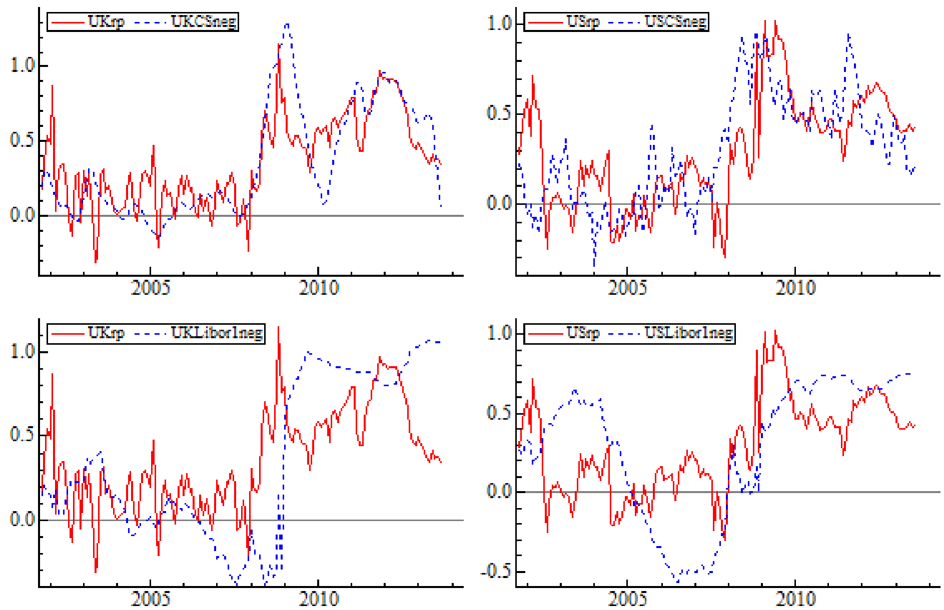


We can see that the two series tend to co-move together quite closely, but that there is in general an expected excess return for the longer maturity asset. This premium appears to notably increase during the Great Recession, though not quite to the same degree for the Canadian sample. This may be indicative of a difference between the T-bill and Libor markets where the latter has a larger and more variable premium seemingly.

Next we will more specifically examine the premium or difference between the two series in the graphics above. Given that the hypothesis is that the premium should move inversely with consumer sentiment and the level of the interest rate, it is helpful to invert these two measures to more clearly see the co-movement. The scale for the second series is adjusted to match the mean and range of the premium.

with the OECD measure exceeding 0.98 as well.

Figure 2: UK and US Risk Premium and the Negative of Consumer Sentiment and the Interest Rate

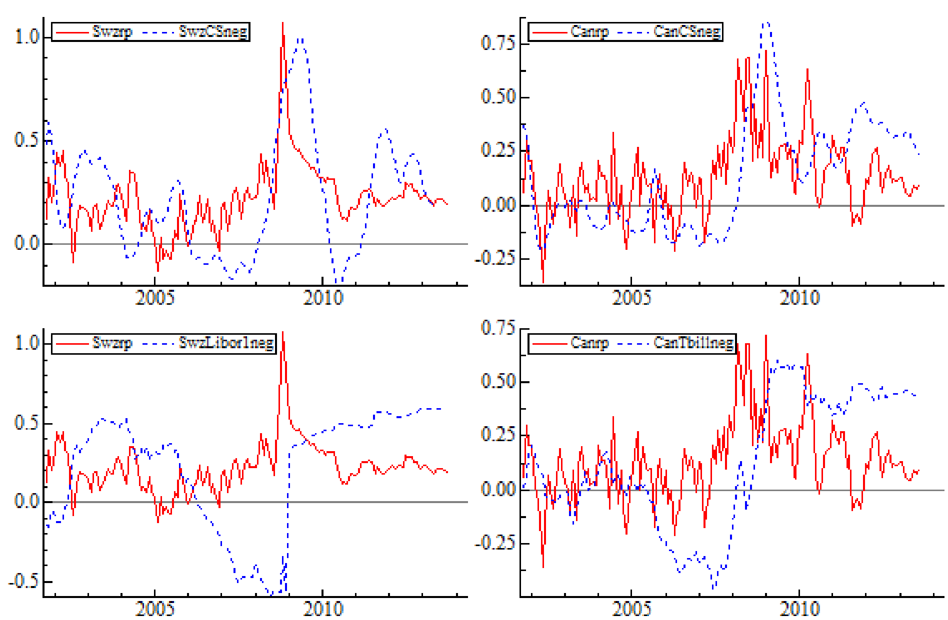


Again it is clear that the risk premium increased during the Great Recession, shown in red for the UK in the two graphs on the left and for the US in the two graphs on the right. Now though we can see that it moves quite closely with the inverse of consumer sentiment (shown as the blue dashed line in the top two graphs for the UK and US respectively). There are two perhaps notable divergences though for the UK sample. In 2009, consumer sentiment revives and the premium does not decrease to the same extent, and then again in 2012 consumer sentiment recovers and the premium does not decline to quite the same extent. We can see that the two periods in which the premium does not decline as much as the negative of consumer sentiment (in 2009 and 2012) are concurrent to declines in the interest rate (shown inverted with the blue dashed line for the two lower graphs). In both cases, the improvement in consumer sentiment (which appears to lower the premium) is offset partially by a decline in the interest rate (which appears to raise the premium all else equal).

The ability of these two variables to collectively account for the general movements in the premium is quite striking. It is worth noting though that the premium experiences much more short-term fluctuation than either. This may be connected to the differenced processes of the two variables, so for example perhaps the change in the interest rate is also relevant, which we can examine in the polynomial cointegration framework.

The time variation in the term premium is quite significant in each sample with a range exceeding a full percentage point even at the very short end of the term structure. In figure 3, it is clearer now that there was a shift in the average level of the premium beginning in late 2007 even for the Canadian T-bill sample (shown as the red line in the two right graphs of figure 3); coinciding with the escalating financial crisis, lower interest rates, and deteriorating consumer sentiment. This higher premium has gradually subsided towards the pre-recession average in each sample, as consumer sentiment has improved, albeit with some volatility in both variables. The connection between the risk premium and these factors will be more rigorously tested below.

Figure 3: Swiss and Canadian Risk Premium and the Negative of Consumer Sentiment and the Interest Rate



5 Approach to Testing and Identification

The CVAR model (Johansen 1988, 1991) extends the error-correction model (ECM) of Engle and Granger (1987) to allow for a systems approach with multiple, simultaneous cointegrating relations. The data is ordered in terms of the levels of persistence.[9] The ECM for a VAR(2) model, i.e. including two lags, can be represented generically as:

$$\Delta x_t = \Gamma \Delta x_{t-1} + \Pi x_{t-1} + \mu_t + \varepsilon_t \tag{5}$$

where the vector $x_t' = [i_t^n, E(rr)_t, CS_t]$ includes the one year interest rate, the expected rollover return, and consumer sentiment. The Π matrix is just a Granger causal reformulation of the covariances, while Γ represents the coefficients of the short-run dynamics. μ_t represents the constant term, and ε_t is an i.i.d. error term. If the variables in the information set are integrated of order one (or I(1)), the unit roots imply that the matrix Π is not full rank and can be decomposed into an α vector and a β' vector. The β' vector describes the linear combinations of the variables which become stationary; representing an equilibrium relationship which can be interpreted very similarly to standard regression results. The α vector meanwhile describes the error-correction indicating which variables are endogenous and how quickly they adjust back to equilibrium.

The CVAR is designed to allow the data to "speak freely" in terms of the rank (number of relationships in the information set), and causality (which variables are error-correcting and which are weakly exogenous driving the equilibrium), rather than constraining the data from the outset with untested assumptions.[10] The specification testing indicates four lags to address serial-correlation and a rank of one with two common stochastic trends. The roots of the companion matrix suggest there is some persistence in the differenced process for the Swiss sample, which would require extension to the I(2) CVAR to examine, though this only slightly alters the interpretation in the I(1) model (Juselius 2006). Identification in the CVAR is achieved by imposing the coefficient restriction used to create the premium $i_t^n - E(rr)_t$ which is a

[9] See Johansen (1995) and Juselius (2006) for book-length treatments of the CVAR model.

[10] This term "speak freely" comes from Hoover, Johansen, and Juselius (2006). See also Hendry and Mizon (1993) for more on the general-to-specific methodology.

symmetry restriction between the longer term rate and expected rollover return, normalized to one for both, interpretable as a no risk-adjusted arbitrage condition.

6 Tests of the Expectations Hypothesis

We can conduct tests of the "pure", or risk neutral, expectations hypothesis, which states that the longer term rate is exactly equal to the expected shorter term rates with no premium, as over-identifying restrictions that $\mu = 0$ and that consumer sentiment has no independent influence.

$$i_t^n = E(rr)_t + \varepsilon_t \tag{6}$$

Table 1: Tests of the Risk Neutral Expectations Hypothesis

$\mu = 0$	p-value
Canada	0.004
Switzerland	0.000
UK	0.004
US	0.012

The p-value tests whether the remaining error term in equation 4 is stationary. This is testing whether the one year rate is identical to the average of expected three month rates over the same year. Given all of the p-values are below .05, we can reject the null of a zero premium in all samples and conclude that there is indeed a premium. Next we can test the null that the premium is constant.

$$i_t^n = E(rr)_t + \mu + \varepsilon_t \tag{7}$$

Table 2: Tests of a Constant Term Premium

	μ	p-value
Canada	0.119 [5.058]	0.185
Switzerland	0.196 [4.740]	0.000
UK	0.388 [3.900]	0.009
US	0.229 [3.256]	0.021

The constant term would be interpreted here as the constant level of the premium. The t-values are reported in parentheses below the coefficient estimates. In the three Libor samples, the p-values signify that the relationship in equation 5 is non-stationary, which we can interpret as implying that the premium is non-constant and those estimates are in fact spurious, and not t-distributed.

These relationships could likely be made stationary by including a sufficient number of mean shifts, as unaddressed breaks lead to over-rejection of cointegration, but the breaks would imply time-variation in the premium similarly to rejecting cointegration without the breaks. In the Canadian T-bill sample, we cannot reject that the premium is stationary, but, as will be seen, it does appear to co-move inversely with consumer sentiment in a statistically significant way and the model fit improves greatly.

7 The Liquidity Premium and Consumer Sentiment

Next we can test whether the premium is related to consumer sentiment by examining whether its inclusion improves the p-value of the relationship, and whether the coefficient on consumer sentiment is significantly less than zero as implied by the hypothesized counter-cyclical movements in the premium. Equation 2, which is estimated here, is shown again below.

$$i_t^n = E(rr)_t + \beta * CS_t + \mu + \varepsilon_t \quad (2)$$

Table 3: Tests for an Equilibrium with Consumer Sentiment

	β	μ	p-value
Canada	-0.055 [-2.901]	5.661 [2.965]	0.743
Switzerland	-0.327 [-8.384]	33.054 [8.434]	0.684
UK	-0.253 [-8.083]	25.681 [8.192]	0.075
US	-0.019 [5.445]	1.958 [6.118]	0.499

As can be seen, the p-value testing for a stationary, cointegrating relationship has increased in each case. It is now clear that the premium is cointegrated with consumer sentiment (forming a stationary equilibrium) with a negative effect of consumer sentiment that is statistically significant at the 1% level. The unity coefficient on the expected rollover return has now been recovered in essence (it cannot be rejected, and is often far from rejection) which Dai and Singleton (2002) note is an important criterion for a term premium model to resolve the expectations puzzle.

While we only just slightly fail to reject the UK equilibrium as cointegrating, it was previously rejected at the 1% level and the effect of consumer sentiment is highly significant with a coefficient implying that a fall in consumer sentiment by one point (as normalized by the OECD) increases the premium by approximately 25 basis points. This is economically significant as the premium can vary by more than a full percentage point over the samples even at the very short end of the term structure. We could not reject a constant premium for the Canadian T-bill sample, but by adding consumer sentiment the p-value increases dramatically from 0.185 to 0.743. The p-value can be viewed similarly to an adjusted R-squared, where inclusion of an irrelevant variable would actually lower it.

The hallmark of the IKE approach is the allowance for non-routine structural change. Frydman and Goldberg (2007, 2011) argue that in a world of imperfect knowledge, individuals will rationally adjust their forecast strategies over time in ways which cannot be fully pre-specified in advance, both reflecting and engendering change in the process underpinning price determination. While unaddressed breaks would tend to lessen the probability of establishing cointegration, to ensure the reliability of the estimates a max test for beta constancy is conducted. This is a recursive sup test based on the idea of Nyblom (1989) and similar to those most prevalent in the literature (Bai and Perron 1998) but applicable to the CVAR (Hansen and Johansen 1999). Juselius (2006) notes that there is ubiquitously some degree of parameter instability present in any model. In each case though, both the "X" series, associated with the full model, and the "R" series, associated with the reduced form including only the cointegrating relations, do not exceed the critical value indicated by the 1.00 marker, suggesting that the estimates are relatively constant over the sample. One possible reason for this is in fact the use of survey data, since it circumvents structural change occurring due to revisions in forecasting strategies about future interest rates and overall macroeconomic sentiment. Wright (2011) also argues that the use of survey data could overcome the presence of breaks attributable to learning.

Figure 4: US Max Test for Beta Constancy

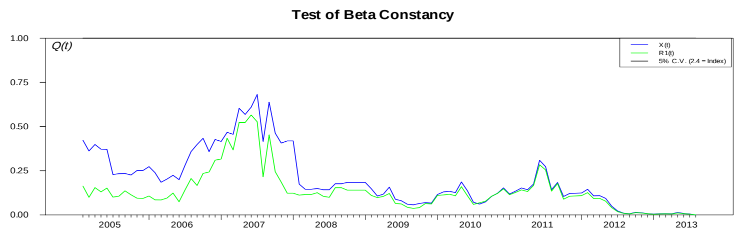


Figure 5: UK Max Test for Beta Constancy

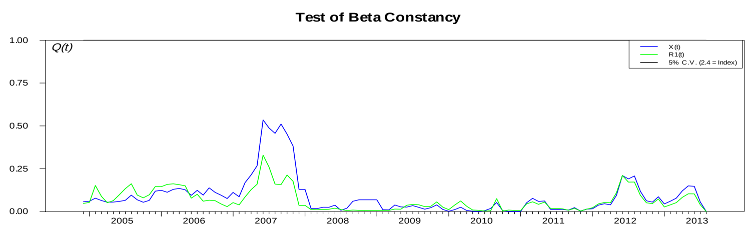


Figure 6: Swiss Max Test for Beta Constancy

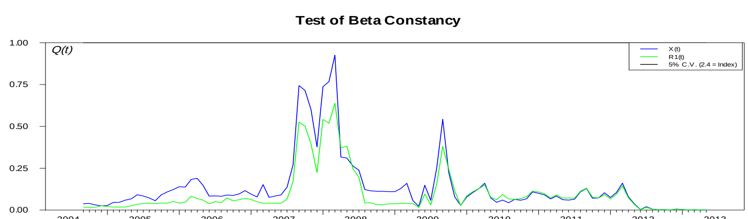
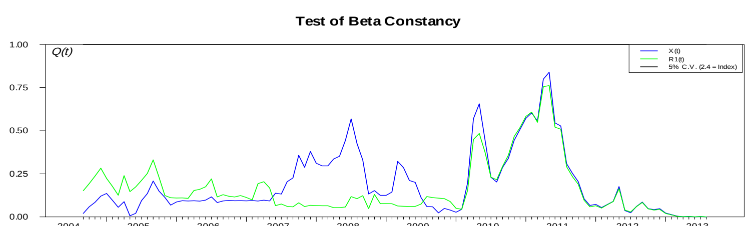


Figure 7: Canadian Max Test for Beta Constancy



Lastly, we can examine the error correction. A significant α coefficient represents an endogenous variable and the magnitude represents the percentage of the disequilibrium which the variable corrects within each monthly period. For example if $\beta > 0$, an α coefficient of -0.5 indicates that the variable is making up 50% of the disequilibrium each period.

Table 4: Error Correction between the Premium and Sentiment

	Δi_t^n	$\Delta E(rr)_t$	ΔCS_t
Canada	-0.362 [-3.313]	0.068 [0.545]	-0.185 [-2.827]
Switzerland	0.038 [1.488]	0.067 [2.604]	-0.077 [-7.114]
UK	-0.045 [-0.515]	0.139 [1.567]	-0.121 [-2.554]
US	-0.113 [-1.974]	0.106 [1.894]	1.212 [0.624]

In the US sample, it appears to be both the longer term rate and expectations about short term rates which are adjusting. In the other three samples, consumer sentiment appears to be adjusting along with the longer term rate in the Canadian sample and the expectations of shorter term rates in the Swiss sample. This seems consistent with previous findings of international spillovers through expectations highlighted by Belke, Beckmann, and Kühl (2011, 2012) since the driving effects of sentiment appear to be emanating from the US which was the epicenter of the financial crisis. The adjustment process will be further elaborated upon in the I(2) model.

8 Polynomial Cointegration

As mentioned previously, the I(2) model may be useful to examine the transitory impact of a change in the interest rate on the term premium. Some of the results for the I(1) model, particularly for the Swiss sample, also suggest some persistence in the differenced process, which can be addressed by extension to the I(2) model. Without loss of generality, the I(2) model can be discussed in terms of acceleration rates, changes and levels for a VAR(3) specification:

$$\Delta^2 x_t = \Gamma_1 \Delta^2 x_{t-1} + \Gamma \Delta x_{t-1} + \Pi x_{t-1} + \mu_0 + \mu_1 t + \varepsilon_t \tag{8}$$

The hypothesis that x_t is I(1) is a reduced rank condition on $\Pi = \alpha\beta'$ where α, β are $p \times r$, where p is the number of included variables in the information set and r is the rank. The hypothesis that x_t is I(2) is an additional reduced rank restriction on Γ of $\alpha'_\perp \Gamma \beta_\perp = \xi\eta'$ where ξ, η are $(p-r) \times s_1$ and s_1 is the number of I(1) common stochastic trends. The first condition is related to the variables in levels while the second is related to the variables in first differences. The interpretation of this second I(2) condition is that the differenced series also contain unit roots (Johansen 1997, Juselius 2006).

The model yields $(p - r - s1) = s2$ number of I(2) trends and medium-run relations with cointegration occurring through some linear combination of the differenced variables. The model also implies that $s2$ of the r relations will be polynomially cointegrating where the levels of the variables combine to I(1) and then combine again with the differenced processes of the near I(2) variables to produce a trend stationary equilibrium condition $\beta' x_t + \delta' \Delta x_t$ (Rahbek and Parulo 1999). These dynamic equilibria consequently lead to more elaborate driving and adjustment processes as described in sections 10.2 and 10.3. Following Rahbek, Kongsted and Jorgensen (1999) the trend is restricted to the cointegrating relationship to prevent undesirable quadratic and cubic trends, however the estimation is not conducted using the two-step procedure, but rather through MLE with an LR test found to have superior small sample properties (Nielsen and Rahbek 2007). This test and other rank diagnostics, presented in the appendix, suggest that there is one polynomially cointegrating relationship in each sample and therefore one I(1) and one I(2) trend.

To return to equation 4, if the interest rate changes are persistent, it and the expected rollover return could cointegrate not to I(0) but to I(1), CI(2,1), and then would combine with the differences to produce a stationary, polynomially cointegrating equilibrium. In this case, the gap effect could manifest additionally or alternatively in the differenced process of the polynomially cointegrating relations or the medium-run relations between the differences. An increase in the expected rollover return would appear to increase the longer term rate one-for-one in the long-run, but in the medium-run it is partially offset by the falling premium resulting from the rising interest rate.

$$i_t^n = \frac{1}{(1-\beta_1)} E(rr)_t + \beta_2 CS_t + \delta_1 \Delta i_t^n + \delta_2 \Delta E(rr)_t + \delta_3 \Delta CS_t + \varepsilon_t \sim I(0) \quad (9)$$

$$\Delta i_t^n = \beta_{1\perp}\Delta E(rr)_t + \beta_{2\perp}\Delta CS + v_t \sim I(0) \quad (10)$$

The hypothesis of the gap effect in the case would be $\beta_1 < 0$ or if $\beta_1 = 0$ then δ_2 and/or $\beta_{1\perp} < 0$.

9 Testing for Persistent Changes in the Variables: Tests of a Known Vector in $\tilde{\tau}$

In order to test whether the individual variables undergo persistent changes, or exhibit near-I(2) behavior, we can test for a known vector b in $\tilde{\tau}$ (which represents the relationships in both $\beta'$and $\beta_\perp$). The null hypothesis is that the variable being tested is at most trend I(1), conditional on Δx_t. Given all of the variables were rejected as I(0) in the I(1) model, a failure to reject this hypothesis would imply that the variable is I(1). If the known vector in $\tilde{\tau}$ is rejected, we conclude that the changes in the variable experience significant persistence, that is near-I(2) behavior.

Juselius often emphasizes that when one discusses a variable as being near I(2) or even near I(1) we are not discussing an inherent property of the variable, but rather a statistical approximation over the sample (2006, 2014). Saying a variable appears to behave as near I(2) is simply saying that the shocks have a persistent effect on both the levels and growth rates of the variables, though not necessarily infinitely lived. Ignoring this additional persistence ignores important aspects of the data and the system dynamics.

Juselius (2013, 2014) demonstrates the importance of conducting tests for near-I(2) persistence in a multivariate framework, showing that the standard univariate unit root tests have low power to discriminate between an I(1) and I(2) trend when the signal-to-noise ratio is low, meaning the variance of the I(1) component is much greater than the variance of the I(2) trend, as she often finds is the case. The p-values of this test are presented below.

Table 5: Test of Variables being at most I(1)

	Err_t	i_t	CS_t	$i_t - Err_t$
US	0.184	0.060	0.007	0.140
UK	0.672	0.627	0.099	0.364
CAN	0.005	0.007	0.027	0.571
SWZ	0.007	0.000	0.000	0.000

Consumer sentiment can be rejected as at most I(1) in the US and Swiss samples at the 1% level, in the Canadian sample at the 5% level, and the UK sample at the 10% level. The interest rate can also be rejected as at most I(1) in both the Canadian and Swiss sample at the 1% level and for the US sample at the 10% level, consistent with the persistent trending nature observed in figure 1. Only in the case of the UK do the interest rate and expected rollover return appear to be clearly I(1). While it is true that tests of non-stationarity are influenced by the sample length, similar results have been obtained with much longer samples (Juselius 2006, 2013, and Johansen et al. 2010). Further, interest rates have clearly exhibited much longer term trends, first rising consistently from the Great Depression and then falling consistently following the Volcker disinflation to our current lows.

10 Testing the Two Risk Premium Theories as Restrictions on β

First we will focus on the impact of consumer sentiment in the I(2) model. This can be seen in the β vector of the tables below. The trend has been restricted to prevent variation in the premium unrelated to changes in consumer sentiment or interest rates.

Table 6: US Polynomially Cointegrating Relation

US	i_t	Err_t	CS_t	p-value
β	1.000	-1.000	0.013 [24.041]	0.646
δ	-0.013	-0.017	-0.287	
α	-0.152 [-5.805]	-0.052 [-2.041]	6.406 [7.556]	

Table 7: UK Polynomially Cointegrating Relation

UK	i_t	Err_t	CS_t	p-value
β	1.000	-0.918 [-8.831]	0.086 [92.612]	0.865
δ	-0.110	-0.156	0.638	
α	-0.200 [-2.565]	-0.024 [-0.257]	-0.174 [-3.160]	

Table 8: Canadian Polynomially Cointegrating Relation

CAN	i_t	Err_t	CS_t	p-value
β	1.000	-1.000	0.068 [221.898]	0.657
δ	0.829	0.785	0.648	
α	-0.352 [-7.098]	-0.293 [-5.113]	-0.149 [-5.091]	

Table 9: Swiss Polynomially Cointegrating Relation

SWZ	i_t	Err_t	CS_t	p-value
β	1.000	-1.000	0.162 [489.795]	0.914
δ	-0.350	-0.263	-0.533	
α	0.100 [2.130]	0.068 [2.927]	0.068 [-6.990]	

In all cases the relationships are highly stationary with p-values ranging from .646-.914 demonstrating that they provide a good characterization of an equilibrium relationship. The effect of consumer sentiment in all cases has the hypothesized negative sign. Of particular note is that the effect of consumer sentiment is very precisely estimated in each sample with t-values of approximately 24, 93, 222, and 490! This is partially a result of the polynomially cointegrated parameters being t^2 consistent (or super super consistent) and also of the trend restriction.[11]

In three of the four samples we cannot reject the homogeneity restriction on the interest rate and expected rollover return. Dai and Singleton (2002) emphasize that if properly controlling for the risk premium, one should be able to recover the unity coefficient. The issue is a bit more complicated if the risk premium is actually a function of the level of the interest rate, but only in the case of the UK do we obtain a rather non-stationary equilibrium with the restriction that an increase in the expected rollover return increases the longer term interest rate one-for-one in the long-run, after controlling for consumer sentiment. This is particularly interesting since the UK sample was also the only case where the interest rates did not exhibit persistent changes according to table 1. As discussed in section 8, if the interest rates are I(1) then the effect of the zero lower bound has to manifest in the levels of the relationship (meaning the coefficient of unity cannot be recovered), whereas if the interest rates are more persistent than I(1), this effect could be observed instead in the differenced processes.

[11] Tests for parameter constancy have not yet been extended to the I(2) model. Since cointegration has been established though, breaks are less of a concern.

The Canadian sample has the largest estimates of the delta coefficients, and has a sign compatible with a fall in the interest rate increasing the premium. The other zero lower bound effects will be seen in the medium-run relationships between differences. This is seemingly consistent with the fact that the Canadian sample has the least persistent premium (see Figure 1, and Tables 2 and 5) likely a by-product of the difference between T-bill and Libor markets where default risk is more of a concern for the latter (Collin-Dufresne and Solnik 2001).

10.1 Medium-Run Relations

The $\beta_{\perp}$ vector is a relationship which cointegrates from order two to order one, CI(2,1), and becomes stationary only through differencing. This can be interpreted as a medium-run relationship. Again the sign of the coefficients on ΔErr_t and ΔCS_t have been reversed to more easily interpret them as right hand side variables. If both the interest rates and premium are > I(1) we may expect the effect of the zero lower bound to manifest in the medium-run relations in differences.

Table 10: Medium-Run Relations

	Δi_t	ΔErr_t	ΔCS_t	$Trend$
$US\ \beta_{\perp}$	1.000	-1.001	-0.096	0.040
$UK\beta_{\perp}$	1.000	-1.099	-0.100	0.023
$CAN\ \beta_{\perp}$	1.000	-0.831	2.475	0.019
$SWZ\ \beta_{\perp}$	1.000	-1.086	-0.529	0.021

Here both the effects of the expected rollover return and consumer sentiment have alternated sign from what was observed in the longer-run β vector. While the exact magnitudes across vectors cannot be compared or combined (since they are individually normalized), combining the qualitative information from β with the coefficients on ΔErr_t in the $\beta_{\perp}$ and δ vectors suggests that while an increase in the expected short term rates increases the longer maturity interest rate it is partially offset by a falling premium, consistent with the IKE gap model. A nice feature of these results is the fact that reverse causation is not a concern. An exogenous increase in the premium would tend to increase the interest rate, but here we find the opposite correlation.

Similar to the effect of consumer sentiment, the zero lower bound has the smallest impact in the US sample. This could be related to the US reserve currency and safe haven status. Worsening economic conditions tend to lower both the interest rate and consumer sentiment, but the upward force this exerts on the premium appears to be partially mitigated by the "flight to quality" towards US assets. Alternatively, it may be a by-product of quantitative easing. The Fed is conducting its purchases not with risk-adjusted arbitrage or the profit motive in mind, but rather with the precise intention of lowering the liquidity premium.[12]

10.2 Multi-Tier Error-Correction

One interesting aspect of the I(2) CVAR is that it allows for more elaborate dynamics of adjustment, as there is a multi-tier error correction to the medium-run relations, where the acceleration rates can adjust to the changes in the variables, and to the long-run relations, where the changes adjust to the equilibrium in levels (as in the standard error-correction model). Error-correction in the medium-run is implied by $\alpha_{ij}\delta_{ij} < 0$; meaning that if the alpha coefficient and delta coefficient have opposite signs for a given significant variable and a given cointegrating relation then its acceleration rates are adjusting to the changes in the variables. Long-run error correction is implied by $\delta_{ij}\beta_{ij} > 0$ meaning if the delta and beta coefficients share the same sign for a given significant variable and a given cointegrating relationship, then its changes are adjusting to the relationship in levels.

The only medium-run adjustment we observe is in consumer sentiment (for three of the four samples excluding Canada), while both interest rates are error-increasing in the medium-run, which can be interpreted as a positive feedback dynamic, meaning for example that if interest rates have been rising people expect them to continue to do so even if they have reached the equilibrium value. Meanwhile, in the longer-run, the one-year interest rate is still error-increasing in three of the four samples (excluding Switzerland where consumer sentiment is error-increasing in the longer-run). The long-run adjustment, while sample dependent, appears to be primarily occurring through consumer sentiment and expectations about shorter-term interest rates. This is seemingly consistent with the behavioral models of short-run extrapolation and long-run mean reversion in expectations (see Barberis and Thaler 2003 for a survey). The adjustment should be interpreted a bit ten-

[12] See Caballero and Krishnamurthy (2008) for a model of flight to quality, and Krishnamurthy and Vissing-Jorgensen (2011) for an analysis of the effects of QE.

tatively though, as there is a "reporting lag" in that by the time the surveys on expected interest rates and consumer sentiment are collected, individuals have already acted on those expectations and influenced the interest rate. Therefore, what appears as adjustment in expectations/ sentiment, and error-increasing behavior on the part of the interest rate could in fact be the reverse. This is not detrimental to the major conclusion of the paper however that the premium moves inversely with consumer sentiment and the level of and/or change in the interest rate.

10.3 The Source of Interest Rate Persistence: I(1) and I(2) Trends

The polynomially cointegrated VAR also provides more elaborate dynamics of the driving forces or MA representation. The alpha orthogonal vectors $\alpha_\perp$ indicate the sources of the common trends. The variables which are significant have unexpected shocks, represented by the acceleration rates, which accumulate over time driving the equilibrium. The $\alpha'_{\perp 1}$ vector represents the I(1) trend. Of particular interest is the $\alpha'_{\perp 2}$ vector which represents the I(2) trend, where the unexpected shocks double accumulate, producing the additional persistence beyond I(1) leading to persistent changes in the variables.

Table 11: I(1) Common Stochastic Trends

	$\Delta\Delta Err_t$	$\Delta\Delta i_t$	$\Delta\Delta CS_t$
$US\ \alpha'_{\perp 1}$	0.322 [9.195]	0.365 [8.148]	0.011 [3.050]
$UK\ \alpha'_{\perp 1}$	0.501 [3.171]	0.102 [0.553]	-0.186 [-1.815]
$CAN\ \alpha'_{\perp 1}$	0.022 [1.209]	0.029 [1.959]	-0.114 [-5.948]
$SWZ\ \alpha'_{\perp 1}$	0.225 [2.347]	0.167 [1.668]	0.332 [12.104]

The results in terms of the common stochastic trends are quite intuitive and also quite consistent across the samples. The I(1) trend in table 11 can be interpreted as shocks to the expected future short-term interest rates, given the expected rollover return and interest rate share the same sign, and often nearly identical coefficients. The I(2) trend meanwhile can be viewed as a shock to the risk premium, given the interest rate and the expected rollover return possess opposites signs, and again in several cases quite similar coefficients as seen in table 12.

Table 12: I(2) Common Stochastic Trends

	$\Delta\Delta Err_t$	$\Delta\Delta i_t$	$\Delta\Delta CS_t$
$US\ \alpha'_{\perp 2}$	1.000 $[NA]$	-0.882 $[-14.075]$	-0.013 $[-3.560]$
$UK\ \alpha'_{\perp 2}$	0.561 $[3.786]$	-0.939 $[-3.026]$	1.000 $[NA]$
$CAN\ \alpha'_{\perp 2}$	1.000 $[NA]$	-0.827 $[-4.887]$	-0.016 $[-0.082]$
$SWZ\ \alpha'_{\perp 2}$	-0.760 $[-2.999]$	1.000 $[NA]$	0.011 $[0.247]$

11 Conclusion

This work uses survey data on traders' interest rate forecasts to test the expectations hypothesis of the term structure and finds clear evidence of a time-varying risk premium in four markets. The use of survey data eliminates the potential for ambiguity that existed in previous studies which examine ex post excess bond returns, due to their joint hypothesis nature concerning the risk premium and REH. Further, it identifies two significant factors which impact the magnitude of the risk premium. The first is overall consumer sentiment about the prospects for the broader economy, analogous to Keynes's "animal spirits". A simple correlation between the premium and consumer sentiment is itself useful for interpreting the enigmatic movements in the yield curve. Despite the "reporting lag" mentioned however there is evidence that consumer sentiment is Granger causing changes in the premium. The economic interpretation is that increased pessimism appears to increase aversion to the greater risk inherent in longer term bonds, and thus increases the equilibrium premium.

The second factor which appears to impact the premium is the level of and/or changes in the interest rate. This is consistent with the IKE gap model; the intuition being that the increasing skew to potential bond price movements from a lower interest rate causes investors to demand a greater premium. This was primarily observed in the medium-run relations of the I(2) model, indicating that these effects are transitory, suggesting, as Keynes argued, that what matters is not merely how far the interest rate is from zero but rather how far it is from recent levels. The near-I(2) persistence was found at very high significance levels, suggesting a need to continue conducting estimation using the I(2) CVAR in future work. This framework provided additional benefits in terms of the model fit, precision of the estimates, and a more intricate examination of the driving and adjustment dynamics.

These results provide useful information to policymakers and practitioners attempting to interpret movements in the yield curve, as they imply that the deviations from the expectations hypothesis will grow as consumer sentiment and interest rates decline. There is of course much work left to be done though in understanding term premia, aside from the contributions made here and those previously mentioned. Theoretical work incorporating feedbacks between these dynamics of the term premium and the real economy could be revelatory. Ideally, this study could be expanded to other maturities and markets as survey data availability will allow. It would also be of interest to apply the effect of consumer sentiment, the IKE gap model, and polynomial cointegration to ex post returns enabling longer samples. Lastly, there are undoubtedly other determinants of the subjective term premium left to be discovered, which can be tested with less ambiguity through the use of survey data, and possible non-linearities to those factors already suggested which may be fruitful to investigate.

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Appendix

Contributors to FX4casts Consensus Forecasts: Allied Irish Bank, ANZ Bank, Bank of America/Merrill Lynch, Bank of New York Mellon, Barclays Capital, Bayerische Landesbank, BNP Paribas, Canadian Imperial Bank of Commerce, Credit-Agricola, Citigroup, Commerzbank, Credit Suisse-First Boston, Danske Bank, Deka, Deutsche Bank, DnBNOR, Economist Intelligence Unit, Goldman Sachs, Handels Banken, HSBC, IHS Global Insight, ING Bank, Intesa Sanpaolo, JP Morgan Chase, Julius Baer, Lloyds TSB, Macquarie Capital Securities, Moody's Economy.com, Morgan Stanley, National Australia Bank, Nomura, Nordea, Rabobank, Royal Bank of Canada, Royal Bank of Scotland, Scotia Bank, SEB, Societe Generale, Standard Chartered, Suntrust, Swedbank, Tokyo-Mitsubishi UFJ, Toronto Dominion, UBS Warburg, UnicreditHVB, Vontobel, Wachovia, Westpac.

Determining the Two Reduced Rank Conditions

In order to determine the two reduced rank conditions in the I(2) model, three criteria are relied upon: the roots of the companion matrix, graphics of the polynomially cointegrating relations, and a likelihood ratio test. For the latter, the number of cointegrating relations r and the number of I(2) trends s_2 among the common stochastic trends $(p-r)$ can be inferred using the maximum likelihood procedure of Nielsen and Rahbek (2007). The trace test is computed as a joint hypothesis for all possible combinations of r and s_2. The tests are nested within rows (for a given rank) and nested in the last column (corresponding to the I(1) model where there are no s_2 trends). The results should be interpreted as beginning with the most restricted model in the upper-left ($r = 0$, $s_1 = 0$, $s_2 = p-r$) continuing until the end of the row, and then moving down to the next row and testing from left to right again, proceeding until the first failure to reject. The p-values are presented below. It is worth noting that the rank test occurs under the auxiliary hypothesis of no economic null. Given the expectations hypothesis, and the co-movement seen in figure 1 between the expected rollover return and the interest rate, we should expect a rank of at least one.

Figure 8: US Polynomially Cointegrating Relation

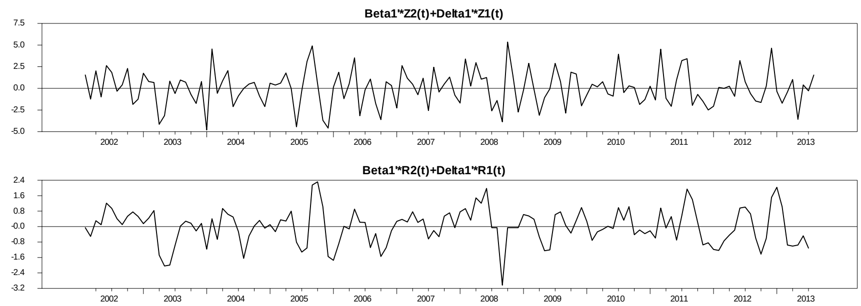

Table 13: US I(2) Rank Test

	$s2=3$	$s2=2$	$s2=1$	$s2=0$
$r=0$	0.000	0.000	0.000	0.049
$r=1$		0.000	0.252	0.742
$r=2$			0.860	0.923

Table 14: US Roots of the Companion Matrix

	Root 1	*Root* 2	*Root* 3	*Root* 4
$r=3$	0.975	0.975	0.664	0.664
$r=2$	1.00	0.939	0.664	0.664
$r=1$	1.00	1.00	0.795	0.648
$r=0$	1.00	1.00	1.00	0.641

For the US, the conclusion appears to be r=1 and s2=1. The rank test only borderline rejects the case of r=0 with s2=0, but again this choice of rank is a non-starter and the graphic of the first cointegrating relationship appears quite stationary. The moduli of the largest roots, constituting the $p-r$ common stochastic trends, demonstrate that a rank of one eliminates all of the very large roots (>0.9) though there remains one moderately large root (.795), seemingly indicating some still remaining persistence in the system, perhaps related to the differenced process, as indicated by the rank tests conclusion of there being one I(2) trend.

Examining the three criteria in conjunction yields the same conclusion for the UK sample as in the US sample, though it is less clear from the rank test alone. The roots of the companion matrix indicate three large roots, which could be consistent with a rank of zero, or a rank of one with one I(2) trend. A rank of one eliminates the very large roots, but one moderately large root remains. The first cointegrating relation also appears stationary. This information collectively leads to a rank of one with one I(2) trend, which does indeed have the highest p-value in the rank test among those for a rank of one or zero (whereas a rank of two is inconsistent with the observed roots).

Figure 9: UK Polynomially Cointegrating Relation

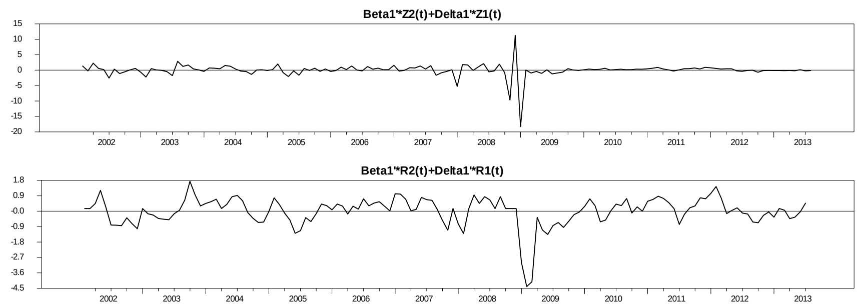

Table 15: UK I(2) Rank Test

	$s2 = 3$	$s2 = 2$	$s2 = 1$	$s2 = 0$
$r = 0$	0.000	0.000	0.419	0.323
$r = 1$		0.533	0.725	0.709
$r = 2$			0.957	1.000

Table 16: UK Roots of the Companion Matrix

	Root 1	*Root* 2	*Root* 3	*Root* 4
$r = 3$	0.992	0.901	0.901	0.573
$r = 2$	1.00	0.903	0.903	0.574
$r = 1$	1.00	1.00	0.801	0.563
$r = 0$	1.00	1.00	1.00	0.766

The criteria for the Swiss case again leads to the same ultimate conclusion of a rank of one with one I(2) trend, evident based on the rank test, whereas the other possible candidates would leave a greater number of unaddressed large roots (for example a rank of two with one I(2) trend or a rank of one with no I(2) trend, the latter of which is inconsistent with the evidence of I(2) dynamics found earlier in the paper).

Figure 10: Swiss Polynomially Cointegrating Relation

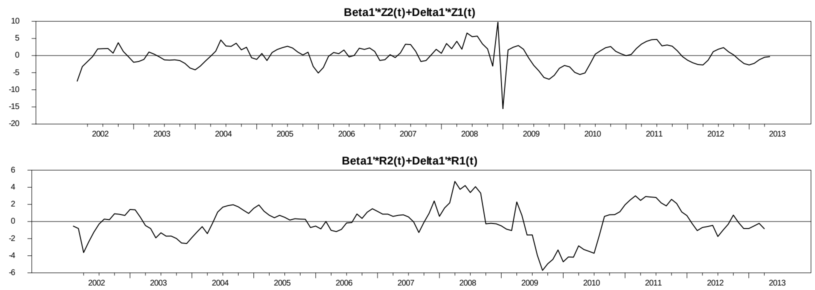

Table 17: Swiss I(2) Rank Test

	$s2 = 3$	$s2 = 2$	$s2 = 1$	$s2 = 0$
$r = 0$	0.000	0.000	0.000	0.000
$r = 1$		0.000	0.079	0.559
$r = 2$			0.374	0.999

Table 18: Swiss Roots of the Companion Matrix

	Root 1	*Root* 2	*Root* 3	*Root* 4
$r = 3$	1.006	0.911	0.911	0.532
$r = 2$	1.000	0.908	0.908	0.532
$r = 1$	1.00	1.00	0.907	0.907
$r = 0$	1.00	1.00	1.00	0.897

Lastly, the Canadian case is again not as cut and dry across the criteria, but the ultimate deduction seems quite well justified. The rank test does not reject until a rank of two with zero I(2) trends. This specification appears to leave two moderately sized roots remaining however (of approximately .85). More conclusively though, the analysis in sections 5 and 7 demonstrate statistically significant I(2) dynamics, so we would want to include one I(2) trend at the least. This again suggests a rank of one with one I(2) trend.

Figure 11: Canadian Polynomially Cointegrating Relation

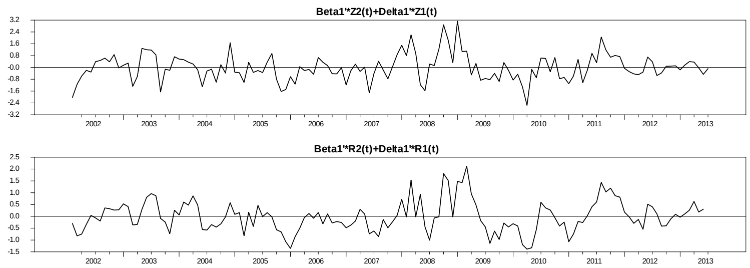

Table 19: Canadian I(2) Rank Test

	$s2 = 3$	$s2 = 2$	$s2 = 1$	$s2 = 0$
$r = 0$	0.000	0.000	0.000	0.000
$r = 1$		0.000	0.001	0.016
$r = 2$			0.033	0.469

Table 20: Canadian Roots of the Companion Matrix

	Root 1	*Root* 2	*Root* 3	*Root* 4
$r = 3$	0.949	0.845	0.845	0.339
$r = 2$	1.00	0.852	0.852	0.588
$r = 1$	1.00	1.00	0.743	0.743
$r = 0$	1.00	1.00	1.00	0.647