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Tests for Exponentiality: A Comparative Study

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Abstract The exponential distribution is considered in situtations where intervals between events are considered as well as where a skewed distribution is appropriate. The exponential distribution also plays key role in survival analysis. Goodness-of-fit for exponentiality is crucial as, in the natural sciences, some of the commonly used distributions such as gamma and Weibull distributions are just translated versions of the exponential distributions. Several well known exponentiality tests are reviewed. A power comparison is performed using simulation.

Keywords: *Ahsanullah characterization, Gini statistic, Pietra statistic, Rossberg characterization*

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1. Introduction

Assessing whether a data set is in compliance with a postulated distribution is termed as goodness-of-fit of a distribution. Tests for exponentiality is the general term for goodness-of-fit tests for exponential distributions. An exhaustive list of references for goodness-of-fit tests for exponential distributions are given in N. Balakrishnan and Asit P. Basu [1], K. Yu. Volkova [2], A. P. Rogozhnikov and B. Yu. Lemeshko [3], and the references there in.

In this paper, we implement several exponentiality tests, such as, Test for exponentiality based on Ahsanullah characterization [4,5], Atkinson test for exponentiality [6], Cox and Oakes test for exponentiality [7], Cramervon Mises test for exponentiality [7], Deshpande test for exponentiality [8], Test for exponentiality of Epps and Pulley ([7] section 2.8.1), Epstein test for exponentiality [9], Frozini test for exponentiality [10], test for exponentiality based on the Gini statistic [11], Gnedenko F-test of exponentiality [9], Harris modification of Gnedenko F-test [9], Hegazy-Green test for exponentiality [12], Hollander-Proshan test for exponentiality [13], Kimber-Michael test [14] and [15], Kochar test for exponentiality [16], Kolmogorov-Smirnov test ([7] section 2.1) forexponentiality, Lorenz test for exponentiality [17], Moran test for exponentiality [18,19], Pietra statistic [9], exponentiality based on Rossberg characterization [2], Shapiro-Wilk test for exponentiality [20], WE test for exponentiality [9], Wong and Wong test for exponentiality [9], and Anderson-Darling test for exponentiality [21].

2. Tests Descriptions

2.1. Test for Exponentiality Based on Ahsanullah Characterization

The test is based on the following statistic by Alexey Novikov and Ruslan Pusev,

$$I_n = \int_0^\infty \left(H_n(t) - G_n(t)\right) dF_n(t),$$

where F_n is the empirical distribution function,

$$H_n(t) = \frac{1}{n^2} \sum_{i,j=1}^{n} 1\left\{\left|X_i - X_j\right| < t\right\}, t \geq 0,$$

$$G_n(t) = \frac{1}{n^2} \sum_{i,j=1}^{n} 1\left\{2\min\left(X_i - X_j\right) < t\right\}, t \geq 0.$$

Under exponentiality, one has

$$\sqrt{n} \xrightarrow{\mathcal{D}} \mathcal{N}\left(0, \frac{647}{4725}\right), \textit{as } n \to \infty$$

[4,5]. This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *AHTT* and *AHTS*, respectively.

2.2. Atkinson Test for Exponentiality

The Atkinson test for exponentiality is based on the following statistic [6],

$$T_n = \sqrt{n} \left| \frac{\left(n^{-1} \sum_{i=1}^{n} X_i^p\right)^{1/p}}{\bar{X}} - \left(\Gamma(1+p)\right)^{1/p} \right|.$$

The statistic is asymptotically (absolute) normal,

$$T_n(P) \xrightarrow{\mathcal{D}} \left|\mathcal{N}\left(0, \sigma^2(p)\right)\right|, \textit{ as } n \to \infty,$$

where,

$$\sigma^2(p) = \left(\Gamma(1+p)\right)^{2/p} \left(-1 - \frac{1}{p^2} + \frac{\Gamma(1+2p)}{p^2 \Gamma^2(1+p)}\right).$$

This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *AKTT* and *AKTS*, respectively.

2.3. Test for Exponentiality of Cox and Oakes

The Cox and Oakes test is a test for the composite hypothesis of exponentiality [7]. The test statistic is,

$$CO_n = n + \sum_{j=1}^{n} (1 - y_j) \log(Y_j),$$

where $Y_j = X_j / \bar{X}$. $(6/n)^{1/2} (CO_n / \pi)$ is asymptotically standard normal [7]. This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *COTT* and *COTS*, respectively.

2.4. Cramer-von Mises Test for Exponentiality

The Cramer-von Mises test for exponentiality (Henze and Meintanis [7] is based on the following statistic,

$$\omega_n^2 = \int_0^\infty \left(F_n(x) - (1 - \exp(-x)) \right)^2 \exp(-x) dx,$$

where F_n is the empirical distribution function of the scaled data $Y_j = X_j / \bar{X}$. The p-value is computed by Monte Carlo simulation. Here we represent the test as *CMTS*.

2.5. Deshpande Test for Exponentiality

Deshpande test for the composite hypothesis of exponentiality [8], is based on the following statistic,

$$J = \frac{1}{n(n-1)} \sum_{i \neq j} 1\{x_i > b x_j\}.$$

Under exponentiality, one has

$$\sqrt{n}\left(J - \frac{1}{b+1} \right) \xrightarrow{\mathcal{D}} \mathcal{N}(0, 4\zeta_1), \text{ as } n \to \infty,$$

where

$$\zeta_1 = \frac{1}{4} \left(\begin{array}{l} 1 + \dfrac{b}{b+2} + \dfrac{1}{2b+1} + \dfrac{2(1-b)}{b+1} \\ - \dfrac{2b}{b^2 + b + 1} - \dfrac{4}{(b+1)^2} \end{array} \right).$$

This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *DPTT* and *DPTS*, respectively.

2.6. Test for Exponentiality of Epps and Pulley

The test statistic is

$$EP_n = (48n)^{1/2} \left(\frac{1}{n} \sum_{j=1}^{n} \exp(-Y_j) - \frac{1}{2} \right),$$

where $Y_j = X_j / \bar{X}$. EP_n is asymptotically standard normal ([7] section 2.8.1). This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *EPTT* and *EPTS*, respectively.

2.7. Epstein Test for Exponentiality

The test [9] is based on the following statistic:

$$EPS_n = \frac{2n\left(\log\left(n^{-1} \sum_{i=1}^{n} D_i \right) - n^{-1} \sum_{i=1}^{n} \log(D_i) \right)}{1 + (n+1)/(6n)},$$

where

$$D_i = (n - i + 1)\left(X_{(i)} - X_{(i-1)} \right),$$

$X_0 = 0$, $X_{(1)} \leq \cdots \leq X_{(n)}$ are order statistics. Under exponentiality, EPS_n is approximately distributed as a chi-square with $n-1$ degrees of freedom. This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *ESTT* and *ESTS*, respectively.

2.8. Frozini Test for Exponentiality

The Frozini test for exponentiality is based on the following statistic [10],

$$B_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left| 1 - \exp\left(-X_{(i)} / \bar{X} \right) - \frac{i - 0.5}{n} \right|.$$

The p-value is computed by Monte Carlo simulation. Here we represent the test as *FRTS*.

2.9. Test for Exponentiality Based on the Gini Statistic

The test is based on the Gini statistic,

$$G_n = \frac{\sum_{i,j=1}^{n} \left| X_i - X_j \right|}{2n(n-1)\bar{X}}.$$

Under exponentiality, the normalized statistic $(12(n-1))^{1/2} (G_n - 0.5)$ is asymptotically standard normal [11]. This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *GSTT* and *GSTS*, respectively.

2.10. Gnedenko F-test of Exponentiality

The test [9] is based on the following statistic,

$$Q_n(R) = \frac{\sum_{i=1}^{R} D_i / R}{\sum_{i=R+1}^{n} D_i / (n-R)},$$

where

$$D_i = (n-i+1)\left(X_{(i)} - X_{(i-1)}\right),$$

$X_{(0)} = 0, \quad X_{(1)} \le ... \le X_{(n)}$ are order statistics. Under exponentiality, $Q_n(R)$ follows an F distribution with $2R$ and $2(n-R)$ degrees of freedom. Here we represent the test as *GNTT*.

2.11. Harris modification of Gnedenko F-test

The test [9] is based on the following statistic,

$$Q_n(R) = \frac{\left(\sum_{i=1}^{R} D_i + \sum_{i=n-R+1}^{n} D_i\right)/2R}{\sum_{i=R+1}^{n-R} D_i/(n-2R)},$$

where

$$D_i = (n-i+1)\left(X_{(i)} - X_{(i-1)}\right),$$

$X_{(0)} = 0, \quad X_{(1)} \le ... \le X_{(n)}$ are order statistics. Under exponentiality, $Q_n(R)$ follows an F distribution with $4R$ and $2(n-2R)$ degrees of freedom. Here we represent the test as *HGTT*.

2.12. Hegazy-Green Test for Exponentiality

The Hegazy-Green test for exponentiality [12] is based on the following statistic,

$$T_1 = n^{-1}\sum_{i=1}^{n}\left|X_{(i)} + \ln\left(1 - \frac{i}{n+1}\right)\right|.$$

The p-value is computed by Monte Carlo simulation. Here we represent the test as *HFTS*.

2.13. Hegazy-Green Test for Exponentiality

The Hegazy-Green test for exponentiality [12] is based on the following statistic,

$$T_2 = n^{-1}\sum_{i=1}^{n}\left(X_{(i)} + \ln\left(1 - \frac{i}{n+1}\right)\right)^2.$$

The p-value is computed by Monte Carlo simulation. Here we represent the test as *HRTS*.

2.14. Hollander-Proshan Test for Exponentiality

The test [13] is based on the following statistic,

$$J_n = \frac{1}{n(n-1)(n-2)}\sum_{i \ne j,k;\, j<k} 1\left\{x_i > x_j + x_k\right\}.$$

Under exponentiality, one has

$$\sqrt{n}\left(J_n - \frac{1}{4}\right) \xrightarrow{\mathcal{D}} \mathcal{N}\left(0, \frac{5}{432}\right), \; as\ n \to \infty.$$

This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *HPTT* and *HPTS*, respectively.

2.15. Kimber-Michael Test for Exponentiality

The Kimber-Michael test [14,15] for exponentiality is based on the following statistic,

$$D = \max_i \left|r_i - s_i\right|,$$

where

$$s_i = \frac{2}{\pi}\arcsin\sqrt{1 - \exp\left(-X_{(i)}/\bar{X}\right)},$$

$$r_i = \frac{2}{\pi}\arcsin\sqrt{(i-0.5)/n}.$$

The p-value is computed by Monte Carlo simulation. Here we represent the test as *KMTS*.

2.16. Kochar Test for Exponentiality

The Kochar test for exponentiality [16] is based on the following statistic,

$$T = \sqrt{\frac{108n}{17}}\,\frac{\sum_{i=1}^{n} J\left(i/(n+1)\right)X_{(i)}}{\sum_{i=1}^{n} X_i},$$

where

$$J(u) = 2(1-u)\left(1 - \log(1-u)\right) - 1.$$

The statistic T is asymptotically standard normal. This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *KCTT* and *KCTS*, respectively.

2.17. Kolmogorov-Smirnov Test for Exponentiality

The Kolmogorov-Smirnov test ([7] section 2.1) for exponentiality is based on the following statistic,

$$KS_n = \sup_{x \ge 0}\left|F_n(x) - \left(1 - \exp(x)\right)\right|,$$

where, F_n is the empirical distribution function of the scaled data $Y_j = X_j/\bar{X}$. The p-value is computed by Monte Carlo simulation. Here we represent the test as *KSTS*.

2.18. Lorenz Test for Exponentiality

The Lorenz test for exponentiality [17] for exponentiality is based on the following statistic,

$$L = \sum_{i=1}^{np} X_{(i)} / \sum_{i=1}^{n} X_{(i)}.$$

The statistic $\sqrt{n}\left(L-p-(1-p)\log(1-p)\right)$ is asymptotically standard normal. This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *LZTT* and *LZTS*, respectively.

2.19. Moran Test for Exponentiality

The Moran test for exponentiality [18,19] is based on the following statistic,

$$T_n^+ = \gamma + \frac{1}{n}\sum_{i=1}^{n}\log\frac{X_i}{\bar{X}},$$

where, where γ is Euler-Mascheroni constant. The statistic is asymptotically normal,

$$\sqrt{n}T_n^+ \xrightarrow{\mathcal{D}} \mathcal{N}\left(0, \frac{\pi^2}{6}-1\right), \text{ as } n \to \infty.$$

This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *MRTT* and *MRTS*, respectively.

2.20. Test for Exponentiality Based on the Pietra statistic

The test is based on the following Pietra statistic [9],

$$P_n = \sum_{i=1}^{n}\frac{\left|X_i-\bar{X}\right|}{2n\bar{X}}.$$

The p-value is computed by Monte Carlo simulation. Here we represent the test as *PSTS*.

2.21. Test for Exponentiality Based on Rossberg Characterization

The test is based on the following statistic [2],

$$S_n = \int_0^{\infty}\left(H_n(t)-G_n(t)\right)dF_n(t),$$

where F_n is the empirical distribution function,

$$H_n(t) = \left(C_n^3\right)^{-1}\sum_{1\le i<j<k\le n} 1\left\{X_{2,\{i,j,k\}} - X_{1,\{i,j,k\}} < t\right\},$$
$$t \ge 0,$$

$$G_n(t) = \left(C_n^2\right)^{-1}\sum_{1\le i<j\le n} 1\left\{\min\left(X_i - X_j\right) < t\right\}, t \ge 0.$$

Here $X_{2,\{i,j,k\}}, s = 1,2,$ denotes the s_{th} order statistic of X_i, X_j, X_k. The p-value is computed from the limit null distribution. Under exponentiality, one has

$$\sqrt{n}S_n \xrightarrow{\mathcal{D}} \mathcal{N}\left(0, \frac{52}{1125}\right), \text{ as } n \to \infty.$$

This test can be implemented either using the asymptotic normal distribution or using the simulated distribution under the exponentiality assumption. Here we represent them as *RCTT* and *RCTS*, respectively.

2.22. Shapiro-Wilk Test for Exponentiality

The Shapiro-Wilk test for exponentiality [20] is based on the following statistic,

$$W = \frac{n\left(\bar{X}-X_{(1)}\right)^2}{(n-1)\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2}.$$

The p-value is computed by Monte Carlo simulation. Here we represent the test as *SWTS*.

2.24. Wong and Wong Test for Exponentiality

The Wong and Wong test for the composite hypothesis of exponentiality [9] is based on the following statistic,

$$Q = \frac{X_{(n)}}{X_{(1)}},$$

where $X_{(1)}$ and $X_{(n)}$ are the smallest and the largest order statistics respectively. The p-value is computed by Monte Carlo simulation. Here we represent the test as *WWTS*.

2.25. Anderson-Darling Upper Tail Test

Anderson-Darling upper tail test is based on the statistic [21]

$$LA = -n - 2\sum_{i=1}^{n}\left\{\begin{array}{l}\frac{2i-1}{2n}\log Z_i \\ +\left(1+\frac{2i-1}{2n}\right)\log\left(1-Z_i\right)\end{array}\right\},$$

where $Z_i = 1-e^{-X_i}$ and X_i is the i^{th} ordered measurement in the sample. The p-value is computed by Monte Carlo simulation. Here we represent the test as *ADTS*.

3. Power Comparison

Samples are generated from standard exponential distribution to check whether the proportions of rejections match with the respective levels of siginificances. And samples are generated from some selected alternative distributions, such as, standard Uniform (0, 1), standard half-normal, Weibull (1.0, 1.4), Gamma (2.0, 1.0), Beta (2.0, 1.0), Gamma (0.5, 1.0), and Log Normal (0.0, 0.8), distributions to investigate how powerful the tests are. We have considered sample sizes, 12, 20, and 28. Levels of significances considered are 1%, 5%, and 10%. In each cases, 10,000 samples are considered.

Table 1. Abbreviations of the Tests

AHTT	Ahsanullah characterization test using normal approximation
AHTS	Ahsanullah characterization test using simulation
AKTT	Atkinson test using normal approximation
AKTS	Atkinson test using simulation
COTT	Cox and Oakes test using normal approximation
COTS	Cox and Oakes test using simulation
CMTS	Cramer-von Mises test using simulation
DPTT	Deshpande test using normal approximation
DPTS	Deshpande test using simulation
EPTT	Epps and Pulley test using normal approximation
EPTS	Epps and Pulley test using simulation
ESTT	Epstein test using normal approximation
ESTS	Epstein test using simulation
FRTS	Frozini test using simulation
GSTT	Gini statistic test using normal approximation
GSTS	Gini statistic test using simulation
GNTT	Gnedenko F-test
HGTT	Harris-Gnedenko F-test
HFTS	Hegazy-Green test using simulation
HRTS	Hegazy-Green alternative test using simulation
HPTT	Hollander-Proshan test using normal approximation
HPTS	Hollander-Proshan test using simulation
KMTS	Kimber-michael test using simulation
KCTT	Kochar test using normal approximation
KCTS	Kochar test using simulation
KSTS	Kolmogorov-Smirnov test using simulation
LZTT	Lorenz test using normal approximation
LZTS	Lorenz test using simulation
MRTT	Moran test using normal approximation
MRTS	Moran test using simulation
PSTS	Pietra statistic test using simulation
RCTT	Rossberg characterization test using normal approximation
RCTS	Rossberg characterization test using simulation
SWTS	Shapiro-Wilk test using simulation
WETS	WE test using simulation
WWTS	Wong and Wong test using simulation
ADTS	Anderson-Darling test using simulation

Table 2. Test Performance Summary

	Worst	Good	Better	Best
Table 3	AHTS HPTT LZTT RCTT	AHTT AKTT DPTT KCTT MRTT	PSTS RCTS	AKTS COTS CMTS DPTS EPTT EPTS ESTT ESTS FRTS GSTT GSTS GNTT HGTT HFTS HRTS HPTS KMTS KCTS KSTS LZTS MRTS SWTS WETS WWTS ADTS
Table 4	AHTS HFTS HRTS HPTT, LZTT RCTT SWTS	AKTT AKTS COTT COTS DPTT DPTS EPTT EPTS FRTS GSTT GSTS GNTT HPTS KMTS PSTS WETS	AHTT KCTT ADTS	KCTT
Table 5	AHTS HFTS HRTS LZTT RCTT SWTS		AHTT KCTT	KCTT
Table 6	AHTS HFTS HRTS HPTT LZTT RCTT SWTS WWTS	AKTT COTT COTS DPTT DPTS EPTT EPTS FRTS GSTT GSTS GNTT HPTS KCTS LZTS MRTT MRTS PSTS WETS ADTS	AHTT KCTT	KCTT
Table 7	AHTS ESTT ESTS HPTT LZTT RCTT SWTS WWTS	AKTT AKTS COTT COTS DPTS EPTT EPTS FRTS GSTT GSTS GNTT HPTS KCTS LZTS MRTT MRTS RCTS WETS	DPTT KCTT PSTS ADTS	AHTT HFTS HRTS
Table 8	AHTS HFTS HRTS LZTT RCTT SWTS WWTS	HGTT HPTT AKTT AKTS COTT CMTS ESTT ESTS KMTS KSTS	AHTT COTS DPTT DPTS EPTT EPTS FRTS GSTT GSTS GNTT HPTS KCTT KCTS LZTS MRTS PSTS RCTS WETS ADTS	AHTT DPTT EPTS FRTS GSTT GSTS HPTS KCTT PSTS WETS and ADTS
Table 9	AHTS GNTT HGTT HFTS HRTS HPTT LZTT PSTS RCTT ADTS	AKTT AKTS DPTT DPTS EPTT EPTS FRTS GSTT GSTS HPTS KCTS KSTS RCTS SWTS WETS	CMTS KMTS LZTS WWTS COTT COTS MRTT MRTS	MRTT MRTS
Table 10	AHTS ESTT ESTS HPTT KMTS LZTT RCTT SWTS WWTS	DPTS FRTS HFTS MRTS RCTS	DPTT KCTT	AHTT

Table 3. Rejection Proportions for Standard Exponential Data

	$n=12$			$n=20$			$n=28$			
$Tests$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	
$AHTT$	0.0865	0.1754	0.2455	0.0568	0.1375	0.1895	0.0448	0.1107	0.1669	⊗
$AHTS$	0.0107	0.0499	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗ ⊗
$AKTT$	0.0058	0.0223	0.0605	0.0056	0.0313	0.0745	0.0073	0.0343	0.0833	⊗
$AKTS$	0.0094	0.0492	0.0972	0.0104	0.0457	0.0982	0.0098	0.0516	0.0992	
$COTT$	0.0078	0.0279	0.0753	0.0085	0.0387	0.0837	0.0077	0.0381	0.0894	
$COTS$	0.0109	0.0498	0.0986	0.0113	0.0510	0.0969	0.0089	0.0472	0.0982	
$CMTS$	0.0109	0.0476	0.1037	0.0101	0.0500	0.1004	0.0108	0.0509	0.1025	
$DPTT$	0.0381	0.0128	0.1419	0.0235	0.0744	0.1266	0.0197	0.0635	0.1156	⊗
$DPTS$	0.0116	0.0585	0.1104	0.0096	0.0513	0.1085	0.0115	0.0518	0.1112	
$EPTT$	0.0059	0.0338	0.0855	0.0057	0.0423	0.0922	0.0066	0.0423	0.0962	
$EPTS$	0.0108	0.0486	0.0973	0.0089	0.0502	0.0997	0.0087	0.0506	0.0984	
$ESTT$	0.0070	0.0442	0.0917	0.0078	0.0473	0.0920	0.0079	0.0419	0.0898	
$ESTS$	0.0108	0.0538	0.0972	0.0119	0.0546	0.1059	0.0091	0.0538	0.1003	
$FRTS$	0.0097	0.0477	0.1017	0.0090	0.0529	0.1003	0.0106	0.0499	0.1055	
$GSTT$	0.0088	0.0453	0.0979	0.0081	0.0490	0.0984	0.0080	0.0468	0.1010	
$GSTS$	0.0115	0.0460	0.0986	0.0083	0.0506	0.0988	0.0085	0.0501	0.1008	
$GNTT$	0.0092	0.0503	0.0991	0.0088	0.0496	0.0988	0.0091	0.0484	0.1044	
$HGTT$	0.0094	0.0505	0.1063	0.0095	0.0502	0.1045	0.0094	0.0536	0.1018	
$HFTS$	0.0104	0.0534	0.0967	0.0092	0.0502	0.0998	0.0106	0.0500	0.0965	
$HRTS$	0.0125	0.0527	0.0958	0.0069	0.0501	0.0945	0.0090	0.0491	0.0968	
$HPTT$	0.0000	0.0005	0.0107	0.0000	0.0000	0.0033	0.0000	0.0000	0.0003	⊗⊗
$HPTS$	0.0113	0.0414	0.1032	0.0097	0.0525	0.0984	0.0097	0.0497	0.1020	
$KMTS$	0.0102	0.0473	0.1080	0.0126	0.0537	0.1013	0.0113	0.0486	0.1005	
$KCTT$	0.0335	0.2043	0.3477	0.0251	0.1465	0.2657	0.0235	0.1179	0.2173	⊗
$KCTS$	0.0106	0.0443	0.0968	0.0073	0.0483	0.0988	0.0093	0.0513	0.0994	
$KSTS$	0.0108	0.0486	0.1061	0.0096	0.0521	0.0967	0.0101	0.0533	0.1055	
$LZTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗⊗
$LZTS$	0.0106	0.0485	0.1035	0.0113	0.0500	0.1014	0.0093	0.0538	0.1026	
$MRTT$	0.0108	0.0309	0.0717	0.0119	0.0395	0.0835	0.0094	0.0376	0.0877	⊗
$MRTS$	0.0100	0.0507	0.0968	0.0125	0.0522	0.0963	0.0091	0.0476	0.0977	
$PSTS$	0.0001	0.0511	0.1018	0.0101	0.0480	0.1021	0.0100	0.0518	0.0984	
$RCTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗⊗
$RCTS$	0.1574	0.1632	0.1031	0.0116	0.0500	0.1003	0.0105	0.0553	0.1014	
$SWTS$	0.0109	0.0446	0.0995	0.0079	0.0480	0.0946	0.0080	0.0465	0.1023	
$WETS$	0.0106	0.0466	0.0952	0.0074	0.0502	0.1033	0.0090	0.0537	0.0994	
$WWTS$	0.0090	0.0486	0.0955	0.0111	0.0478	0.0922	0.0073	0.0497	0.1016	
$ADTS$	0.0104	0.0516	0.0985	0.0092	0.0493	0.1080	0.0087	0.0513	0.0952	

⊗ on the right panel indicates very low power, robust against the alternative considered. The more ⊗s the lower the power.

Table 4. Rejection Proportions for Standard Uniform Data

		$n=12$			$n=20$			$n=28$			
	$Tests$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	
⋆⋆	$AHTT$	0.5802	0.7404	0.8101	0.7164	0.8453	0.8972	0.8070	0.9093	0.9368	
	$AHTS$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
⋆	$AKTT$	0.0016	0.1928	0.4305	0.0925	0.5195	0.7389	0.2872	0.7591	0.8807	
⋆	$AKTS$	0.0013	0.1907	0.3759	0.0733	0.4193	0.6368	0.1395	0.6477	0.8082	
⋆	$COTT$	0.0005	0.1935	0.3971	0.0892	0.4401	0.6272	0.2331	0.6234	0.7672	
⋆	$COTS$	0.1488	0.3321	0.4558	0.3050	0.5211	0.6515	0.3932	0.6888	0.7780	
	$CMTS$	0.0002	0.0488	0.1501	0.0359	0.2264	0.4167	0.1246	0.4517	0.6438	
⋆	$DPTT$	0.3926	0.5221	0.5714	0.5298	0.6814	0.7596	0.6541	0.7998	0.8453	
⋆	$DPTS$	0.1542	0.3515	0.4739	0.3044	0.5248	0.6804	0.4444	0.7062	0.7987	
⋆	$EPTT$	0.0366	0.3417	0.5476	0.2408	0.6431	0.8016	0.4724	0.8264	0.9090	
⋆	$EPTS$	0.1764	0.4042	0.5451	0.4243	0.6575	0.7830	0.5660	0.8389	0.8999	
	$ESTT$	0.0509	0.1777	0.2898	0.0992	0.2839	0.4337	0.1583	0.4067	0.5526	
	$ESTS$	0.0308	0.1182	0.1954	0.0754	0.2096	0.3075	0.1146	0.3154	0.4311	
⋆	$FRTS$	0.2022	0.4499	0.6035	0.4547	0.6919	0.8208	0.6032	0.8652	0.9214	
⋆	$GSTT$	0.1878	0.4364	0.5736	0.4454	0.7183	0.8262	0.6513	0.8711	0.9229	
⋆	$GSTS$	0.1950	0.4327	0.5846	0.4702	0.6990	0.8166	0.6287	0.8731	0.9229	
⋆	$GNTT$	0.2597	0.5159	0.6519	0.4786	0.7240	0.8278	0.6442	0.8528	0.9115	
	$HGTT$	0.0149	0.0596	0.1121	0.0156	0.0672	0.1253	0.0182	0.0657	0.1193	
	$HFTS$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	$HRTS$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	$HPTT$	0.0000	0.0657	0.3323	0.0000	0.0232	0.3010	0.0000	0.0096	0.2694	⊗
⋆	$HPTS$	0.1979	0.4320	0.5723	0.4404	0.6493	0.7731	0.5736	0.8244	0.8842	
⋆	$KMTS$	0.0137	0.1836	0.3714	0.2850	0.6679	0.8314	0.7208	0.9290	0.9700	
⋆⋆⋆	$KCTT$	0.5699	0.9034	0.9614	0.8234	0.9706	0.9914	0.9322	0.9930	0.9982	
	$KCTS$	0.2451	0.5037	0.6527	0.5835	0.7911	0.8821	0.7621	0.9416	0.9654	
	$KSTS$	0.0237	0.1206	0.2344	0.1051	0.3206	0.4803	0.2425	0.5212	0.6878	
	$LZTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	$LZTS$	0.1697	0.3597	0.4909	0.3672	0.5593	0.6854	0.4857	0.7370	0.8092	
	$MRTT$	0.0003	0.1182	0.3165	0.0373	0.3174	0.5093	0.1265	0.4747	0.6403	
	$MRTS$	0.1327	0.2923	0.4036	0.2460	0.4345	0.5530	0.3016	0.5761	0.6713	
⋆	$PSTS$	0.2489	0.5117	0.6568	0.4926	0.7162	0.8284	0.6109	0.8606	0.9166	
	$RCTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	$RCTS$	0.0608	0.1505	0.2369	0.1106	0.2491	0.3778	0.1621	0.3643	0.4778	
	$SWTS$	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
⋆	$WETS$	0.2152	0.4681	0.6152	0.5188	0.7557	0.8636	0.6989	0.9272	0.9578	
	$WWTS$	0.0035	0.0162	0.0304	0.0025	0.0136	0.0288	0.0026	0.0157	0.0249	
⋆⋆	$ADTS$	0.3265	0.5825	0.7416	0.6084	0.8155	0.8998	0.7446	0.9326	0.9640	

⊗ on the right panel indicates very low power, robust against the alternative considered.
★ on the left panel indicates large power and the more ★s the larger the power.

Table 5. Rejection Proportions for Standard Half Normal Data

		$n=12$			$n=20$			$n=28$			
	Tests	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	
★★	AHTT	0.2686	0.4248	0.5290	0.3012	0.4572	0.5603	0.3229	0.5070	0.6030	
	AHTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	AKTT	0.0001	0.0402	0.1560	0.0055	0.1198	0.2775	0.0237	0.2116	0.3912	
	AKTS	0.0014	0.0561	0.1479	0.0024	0.0996	0.2102	0.0098	0.1245	0.3051	
	COTT	0.0003	0.0519	0.1712	0.0074	0.1256	0.2722	0.0314	0.2080	0.3637	
	COTS	0.0346	0.1208	0.2096	0.0605	0.1731	0.2845	0.1111	0.2549	0.3776	
	CMTS	0.0002	0.0116	0.0496	0.0022	0.0477	0.1219	0.0127	0.0825	0.1956	
	DPTT	0.1500	0.2421	0.2950	0.1639	0.2705	0.3693	0.2001	0.3353	0.4171	
	DPTS	0.0401	0.1269	0.2215	0.0592	0.1720	0.2634	0.0962	0.2256	0.3324	
	EPTT	0.0041	0.0940	0.2264	0.0248	0.1838	0.3448	0.0629	0.2850	0.4491	
	EPTS	0.0366	0.1247	0.2219	0.0748	0.1957	0.3222	0.1151	0.2914	0.4290	
	ESTT	0.0101	0.0553	0.1133	0.0117	0.0604	0.1236	0.0159	0.0646	0.1304	
	ESTS	0.0125	0.0558	0.1088	0.0120	0.0536	0.1030	0.0150	0.0517	0.1088	
	FRTS	0.0368	0.1381	0.2469	0.0770	0.2076	0.3327	0.1231	0.2779	0.4111	
	GSTT	0.0327	0.1262	0.2226	0.0703	0.2093	0.3316	0.1150	0.3067	0.4299	
	GSTS	0.0357	0.1291	0.2234	0.0787	0.1985	0.3222	0.1203	0.3019	0.4260	
	GNTT	0.0479	0.1838	0.3065	0.0879	0.2659	0.4173	0.1299	0.3575	0.5093	
	HGTT	0.0104	0.0520	0.1041	0.0097	0.0501	0.1006	0.0110	0.0539	0.1083	
	HFTS	0.0000	0.0002	0.0013	0.0000	0.0001	0.0003	0.0000	0.0001	0.0001	⊗
	HRTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	HPTT	0.0000	0.0067	0.0795	0.0000	0.0003	0.0324	0.0000	0.0000	0.0160	
	HPTS	0.0388	0.1246	0.2191	0.0653	0.1829	0.2866	0.1042	0.2475	0.3700	
	KMTS	0.0012	0.0240	0.0636	0.0077	0.0539	0.1346	0.0220	0.0973	0.1897	
★★	KCTT	0.1650	0.5363	0.7252	0.2370	0.5983	0.7709	0.3083	0.6606	0.8041	
	KCTS	0.0380	0.1309	0.2285	0.0805	0.2100	0.3405	0.1208	0.3024	0.4568	
	KSTS	0.0030	0.0354	0.0888	0.0161	0.0783	0.1603	0.0340	0.1203	0.2338	
	LZTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	LZTS	0.0349	0.1216	0.2104	0.0612	0.1804	0.2862	0.1042	0.2584	0.3656	
	MRTT	0.0015	0.0325	0.1426	0.0027	0.0940	0.2270	0.0176	0.1598	0.2939	
	MRTS	0.0350	0.1202	0.2042	0.0552	0.1533	0.2616	0.0937	0.2242	0.3241	
	PSTS	0.0649	0.2053	0.3376	0.1090	0.2989	0.4492	0.1602	0.3874	0.5502	
	RCTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	RCTS	0.0232	0.0853	0.1479	0.0270	0.1013	0.1812	0.0479	0.1339	0.2143	
	SWTS	0.0003	0.0020	0.0084	0.0000	0.0005	0.0018	0.0000	0.0000	0.0007	⊗
	WETS	0.0425	0.1331	0.2264	0.0785	0.2060	0.3347	0.1265	0.3060	0.4607	
	WWTS	0.0055	0.0268	0.0518	0.0049	0.0215	0.0509	0.0050	0.0224	0.0529	
	ADTS	0.0683	0.2143	0.3576	0.1201	0.3285	0.4849	0.1789	0.4272	0.6034	

⊗ on the right panel indicates very low power, robust against the alternative considered.
★ on the left panel indicate large power and the more ★s the larger the power.

Table 6. Rejection Proportions for Weibull (1,1.4) Data

		$n=12$			$n=20$			$n=28$			
	Tests	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	
★★	AHTT	0.4050	0.6018	0.6886	0.5003	0.6862	0.7724	0.5756	0.7596	0.8461	
	AHTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★	AKTT	0.0010	0.0788	0.2466	0.0159	0.2430	0.4539	0.0724	0.4039	0.6270	
	AKTS	0.0018	0.0791	0.2098	0.0021	0.2114	0.3324	0.0156	0.3102	0.4906	
★	COTT	0.0000	0.0989	0.2851	0.0265	0.2884	0.4954	0.0980	0.4557	0.6627	
★	COTS	0.0644	0.2038	0.3400	0.1369	0.3981	0.5271	0.2802	0.5236	0.6823	
	CMTS	0.0001	0.0205	0.0804	0.0054	0.1255	0.2221	0.0348	0.2244	0.3890	
★	DPTT	0.2375	0.3698	0.4197	0.2976	0.4706	0.5685	0.3948	0.5749	0.6745	
★	DPTS	0.0565	0.2057	0.3233	0.1047	0.3086	0.4376	0.2321	0.4628	0.5939	
★	EPTT	0.0069	0.1647	0.3380	0.0625	0.3387	0.5199	0.1496	0.4868	0.6776	
★	EPTS	0.0622	0.2090	0.3358	0.1201	0.3764	0.4990	0.2571	0.5046	0.6657	
	ESTT	0.0107	0.0560	0.1105	0.0159	0.0698	0.1353	0.0176	0.0833	0.1589	
	ESTS	0.0119	0.0523	0.1079	0.0121	0.0566	0.1153	0.0170	0.0562	0.1193	
★	FRTS	0.0662	0.2154	0.3533	0.1170	0.3880	0.4925	0.2327	0.4843	0.6339	
★	GSTT	0.0571	0.2076	0.3180	0.1308	0.3600	0.4928	0.2328	0.4974	0.6513	
★	GSTS	0.0568	0.2012	0.3217	0.1120	0.3648	0.4861	0.2445	0.4980	0.6610	
★	GNTT	0.0578	0.2266	0.3624	0.1185	0.3444	0.5022	0.1772	0.4504	0.6255	
	HGTT	0.0177	0.0864	0.1568	0.0250	0.1120	0.1877	0.0324	0.1267	0.2235	
	HFTS	0.0000	0.0024	0.0111	0.0000	0.0018	0.0070	0.0000	0.0013	0.0068	⊗
	HRTS	0.0000	0.0000	0.0012	0.0000	0.0000	0.0004	0.0000	0.0000	0.0002	⊗
	HPTT	0.0000	0.0166	0.1334	0.0000	0.0017	0.0668	0.0000	0.0000	0.0423	⊗
★	HPTS	0.0583	0.1894	0.3136	0.1011	0.3387	0.4612	0.2241	0.4651	0.6137	
	KMTS	0.0006	0.0219	0.0664	0.0065	0.0817	0.1356	0.0281	0.1166	0.2260	
★★★	KCTT	0.2325	0.6521	0.8070	0.3484	0.7228	0.8528	0.4435	0.7796	0.8981	
★	KCTS	0.0472	0.1816	0.3038	0.0919	0.3269	0.4468	0.2107	0.4515	0.6080	
	KSTS	0.0074	0.0594	0.1274	0.0360	0.1665	0.2398	0.0770	0.2466	0.3912	
	LZTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★	LZTS	0.0547	0.1802	0.3098	0.1217	0.3524	0.4614	0.2338	0.4702	0.6138	
★	MRTT	0.0000	0.0583	0.2369	0.0092	0.2325	0.4500	0.0592	0.3959	0.6162	
★	MRTS	0.0631	0.2007	0.3266	0.1285	0.3766	0.5123	0.2708	0.5013	0.6499	
★	PSTS	0.0915	0.2935	0.4566	0.1888	0.4811	0.6240	0.3120	0.5981	0.7580	
	RCTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	RCTS	0.0446	0.1429	0.2347	0.1016	0.2569	0.3723	0.1476	0.3768	0.4931	
	SWTS	0.0001	0.0035	0.0116	0.0000	0.0012	0.0032	0.0000	0.0002	0.0009	⊗
★	WETS	0.0575	0.1952	0.3195	0.1083	0.3451	0.4589	0.2306	0.4598	0.6097	
	WWTS	0.0006	0.0036	0.0103	0.0004	0.0032	0.0082	0.0003	0.0036	0.0066	⊗
★	ADTS	0.0922	0.3026	0.4785	0.1957	0.5068	0.6498	0.3326	0.6298	0.7951	

⊗ on the right panel indicates very low power, robust against the alternative considered.
★ on the left panel indicates large power and the more ★s the larger the power.

Table 7. Rejection Proportions for Gamma (2,1) Data

		$n=12$			$n=20$			$n=28$			
	Tests	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	
★★★	*AHTT*	0.5320	0.7158	0.8088	0.6622	0.8292	0.8965	0.7796	0.9047	0.9454	
	AHTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★	*AKTT*	0.0012	0.1138	0.3423	0.0329	0.3573	0.5956	0.1382	0.5597	0.7531	
★	*AKTS*	0.0015	0.1276	0.3015	0.0066	0.3002	0.5015	0.0445	0.4532	0.6273	
★	*COTT*	0.0002	0.1483	0.4032	0.0507	0.4379	0.6800	0.2039	0.6619	0.8324	
★	*COTS*	0.1010	0.2972	0.4609	0.2892	0.5551	0.7105	0.4625	0.7229	0.8448	
	CMTS	0.0002	0.0276	0.1189	0.0167	0.1950	0.3743	0.0927	0.3951	0.5646	
★★	*DPTT*	0.3243	0.4707	0.5309	0.4417	0.6257	0.7311	0.5946	0.7569	0.8329	
★	*DPTS*	0.1009	0.2783	0.4158	0.2096	0.4890	0.6243	0.3638	0.6502	0.7705	
★	*EPTT*	0.0131	0.2245	0.4358	0.1105	0.4557	0.6537	0.2558	0.6324	0.7870	
★	*EPTS*	0.0944	0.2610	0.4301	0.2370	0.5016	0.6374	0.3888	0.6669	0.7802	
	ESTT	0.0124	0.0647	0.1215	0.0197	0.0851	0.1667	0.0262	0.1070	0.1911	⊗
	ESTS	0.0114	0.0491	0.0976	0.0139	0.0628	0.1293	0.0248	0.0780	0.1287	⊗
★	*FRTS*	0.0937	0.2917	0.4471	0.2082	0.4972	0.6426	0.3567	0.6656	0.7674	
★	*GSTT*	0.0830	0.2664	0.4101	0.2071	0.4732	0.6219	0.3590	0.6387	0.7626	
★	*GSTS*	0.0794	0.2547	0.4191	0.2102	0.4849	0.6225	0.3836	0.6558	0.7681	
★	*GNTT*	0.0696	0.2486	0.4043	0.1341	0.3988	0.5730	0.2225	0.5243	0.6929	
	HGTT	0.0274	0.1169	0.2142	0.0482	0.1643	0.2892	0.0616	0.2244	0.3559	
★★★	*HFTS*	0.6895	0.9005	0.9460	0.9150	0.9839	0.9938	0.9777	0.9977	0.9994	
★★★	*HRTS*	0.3849	0.7019	0.8321	0.6095	0.8732	0.9436	0.7763	0.9499	0.9854	
	HPTT	0.0000	0.0224	0.1842	0.0000	0.0030	0.1268	0.0000	0.0006	0.0836	⊗
★	*HPTS*	0.0844	0.2481	0.4040	0.2046	0.4829	0.6307	0.3626	0.6540	0.7705	
	KMTS	0.0003	0.0246	0.0774	0.0159	0.0887	0.1629	0.0376	0.1586	0.2351	
★★	*KCTT*	0.2843	0.7087	0.8534	0.4232	0.7814	0.9015	0.5516	0.8486	0.9289	
★	*KCTS*	0.0801	0.2234	0.3722	0.1636	0.4125	0.5466	0.2995	0.5682	0.6802	
	KSTS	0.0107	0.0871	0.1743	0.0629	0.2404	0.3680	0.1450	0.3957	0.5341	
	LZTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★	*LZTS*	0.0807	0.2523	0.3874	0.1905	0.4593	0.6176	0.3797	0.6429	0.7685	
★	*MRTT*	0.0000	0.0869	0.3479	0.0209	0.3842	0.6562	0.1458	0.6325	0.8260	
★	*MRTS*	0.1052	0.3020	0.4679	0.2994	0.5613	0.7213	0.4618	0.7418	0.8514	
★★	*PSTS*	0.1400	0.3755	0.5474	0.2643	0.5813	0.7478	0.4637	0.7382	0.8535	
	RCTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★	*RCTS*	0.0724	0.2331	0.3370	0.1920	0.4084	0.5772	0.3438	0.6005	0.7154	
	SWTS	0.0005	0.0057	0.0182	0.0001	0.0021	0.0079	0.0003	0.0015	0.0034	⊗
★	*WETS*	0.0918	0.2433	0.3911	0.1915	0.4349	0.5690	0.3282	0.5686	0.6803	
	WWTS	0.0000	0.0004	0.0011	0.0000	0.0001	0.0010	0.0000	0.0000	0.0006	⊗
★★	*ADTS*	0.1524	0.3898	0.5678	0.2991	0.5917	0.7553	0.4827	0.7524	0.8636	

⊗ on the right panel indicates very low power, robust against the alternative considered.
★ on the left panel indicates large power and the more ★s the larger the power.

Table 8. Rejection Proportions for Beta (2,1) Data

		$n=12$			$n=20$			$n=28$			
	Tests	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	
★★★★	*AHTT*	0.9950	0.9993	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
	AHTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★★	*AKTT*	0.0584	0.9041	0.9872	0.9269	0.9985	1.0000	0.9990	1.0000	1.0000	
★★	*AKTS*	0.0035	0.8951	0.9804	0.7923	0.9974	0.9997	0.9890	1.0000	1.0000	
★★	*COTT*	0.0514	0.8899	0.9797	0.9088	0.9963	0.9999	0.9957	0.9998	1.0000	
★★★	*COTS*	0.8487	0.9610	0.9861	0.9898	0.9980	0.9999	0.9994	1.0000	1.0000	
★★	*CMTS*	0.0493	0.5963	0.8416	0.8063	0.9794	0.9970	0.9852	0.9997	1.0000	
★★★★	*DPTT*	0.9586	0.9857	0.9906	0.9977	0.9994	0.9999	1.0000	1.0000	1.0000	
★★★	*DPTS*	0.8085	0.9504	0.9810	0.9794	0.9976	0.9993	0.9992	1.0000	1.0000	
★★★	*EPTT*	0.5733	0.9699	0.9950	0.9872	0.9995	1.0000	0.9999	1.0000	1.0000	
★★★★	*EPTS*	0.9039	0.9827	0.9936	0.9963	0.9998	1.0000	1.0000	1.0000	1.0000	
★★	*ESTT*	0.3294	0.6445	0.7902	0.6781	0.8979	0.9571	0.8836	0.9766	0.9929	
★★	*ESTS*	0.2610	0.5269	0.6775	0.6119	0.8143	0.9101	0.8430	0.9585	0.9801	
★★★★	*FRTS*	0.9194	0.9878	0.9970	0.9982	0.9998	1.0000	1.0000	1.0000	1.0000	
★★★★	*GSTT*	0.9028	0.9872	0.9967	0.9983	0.9998	1.0000	1.0000	1.0000	1.0000	
★★★★	*GSTS*	0.9246	0.9867	0.9965	0.9978	0.9998	1.0000	1.0000	1.0000	1.0000	
★★★	*GNTT*	0.8310	0.9633	0.9865	0.9851	0.9979	0.9998	0.9989	1.0000	1.0000	
★	*HGTT*	0.1638	0.4010	0.5574	0.3211	0.6044	0.7329	0.4759	0.7442	0.8502	
	HFTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	HRTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★	*HPTT*	0.0000	0.7220	0.9711	0.0000	0.7539	0.9932	0.0000	0.7954	0.9984	
★★★★	*HPTS*	0.9192	0.9854	0.9969	0.9978	0.9996	0.9999	1.0000	1.0000	1.0000	
★★	*KMTS*	0.3566	0.8741	0.9716	0.9892	0.9997	1.0000	1.0000	1.0000	1.0000	
★★★★	*KCTT*	0.9959	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
★★★	*KCTS*	0.9457	0.9913	0.9982	0.9992	1.0000	1.0000	1.0000	1.0000	1.0000	
★★	*KSTS*	0.3321	0.6997	0.8542	0.8403	0.9740	0.9944	0.9772	0.9994	0.9997	
	LZTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★★★	*LZTS*	0.8718	0.9643	0.9857	0.9913	0.9987	0.9996	0.9996	1.0000	1.0000	
	MRTT	0.0000	0.7837	0.9497	0.7523	0.9867	0.9984	0.9775	0.9992	0.9999	
★★★	*MRTS*	0.8219	0.9403	0.9719	0.9732	0.9946	0.9990	0.9976	0.9996	1.0000	
★★★★	*PSTS*	0.9252	0.9874	0.9960	0.9957	0.9998	1.0000	1.0000	1.0000	1.0000	
	RCTT	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★★★	*RCTS*	0.5618	0.7800	0.8790	0.8682	0.9749	0.9900	0.9800	0.9975	0.9993	
	SWTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
★★★★	*WETS*	0.9276	0.9888	0.9971	0.9986	0.9999	1.0000	1.0000	1.0000	1.0000	
	WWTS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0006	⊗
★★★★	*ADTS*	0.9655	0.9961	0.9997	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000	

⊗ on the right panel indicates very low power, robust against the alternative considered.
★ on the left panel indicate large power and the more ★s the larger the power.

Table 9. Rejection Proportions for Gamma (0.5,1) Data

		$n = 12$			$n = 20$			$n = 28$			
	$Tests$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	
	$AHTT$	0.0093	0.0138	0.0229	0.0013	0.0353	0.1422	0.0116	0.2281	0.4145	
	$AHTS$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
⋆	$AKTT$	0.1923	0.3170	0.4038	0.3483	0.5092	0.5975	0.4900	0.6596	0.7304	
⋆	$AKTS$	0.1743	0.3368	0.4086	0.3040	0.4993	0.5638	0.4242	0.6306	0.6848	
⋆⋆⋆	$COTT$	0.3914	0.5269	0.6016	0.5919	0.7352	0.7843	0.7435	0.8507	0.8841	
⋆⋆⋆	$COTS$	0.3384	0.5555	0.6315	0.5494	0.7392	0.7996	0.6762	0.8497	0.8890	
⋆⋆	$CMTS$	0.2879	0.5430	0.6333	0.4761	0.6975	0.7801	0.6010	0.8081	0.8705	
⋆	$DPTT$	0.1461	0.4081	0.4952	0.2695	0.5751	0.6793	0.4283	0.7028	0.8048	
⋆	$DPTS$	0.2167	0.4044	0.4924	0.3973	0.6140	0.7117	0.5604	0.7596	0.8206	
⋆	$EPTT$	0.1916	0.3313	0.4247	0.3453	0.5184	0.6102	0.4819	0.6664	0.7404	
⋆	$EPTS$	0.1847	0.3861	0.4860	0.3441	0.5479	0.6555	0.4436	0.6810	0.7601	
	$ESTT$	0.1502	0.3127	0.4243	0.1960	0.3889	0.4976	0.2315	0.4414	0.5645	
	$ESTS$	0.1133	0.2421	0.3392	0.1745	0.3224	0.3982	0.1891	0.3814	0.4796	
⋆	$FRTS$	0.1972	0.3445	0.4394	0.3447	0.5307	0.6214	0.4592	0.6808	0.7492	
⋆	$GSTT$	0.1778	0.3719	0.4880	0.3243	0.5450	0.6523	0.4650	0.6825	0.7669	
⋆	$GSTS$	0.1966	0.3866	0.4846	0.3552	0.5516	0.6532	0.4564	0.6852	0.7605	
	$GNTT$	0.0007	0.0067	0.0128	0.0001	0.0028	0.0047	0.0001	0.0004	0.0017	⊗
	$HGTT$	0.0207	0.0688	0.1243	0.0158	0.0652	0.1081	0.0157	0.0624	0.1030	⊗
	$HFTS$	0.0000	0.0004	0.0006	0.0000	0.0000	0.0000	0.0000	0.0002	0.0001	⊗
	$HRTS$	0.0006	0.0035	0.0067	0.0004	0.0020	0.0021	0.0001	0.0001	0.0018	⊗
	$HPTT$	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
⋆	$HPTS$	0.1946	0.3935	0.5127	0.4018	0.6161	0.7034	0.5595	0.7588	0.8166	
⋆⋆	$KMTS$	0.3115	0.5557	0.6518	0.4977	0.7222	0.8029	0.6429	0.8302	0.8792	
	$KCTT$	0.0025	0.0305	0.0731	0.0268	0.0873	0.1558	0.0900	0.2187	0.2987	
⋆	$KCTS$	0.1388	0.3272	0.4214	0.2545	0.4663	0.5721	0.3538	0.5816	0.6851	
⋆	$KSTS$	0.2174	0.4404	0.5385	0.3666	0.5771	0.6835	0.4763	0.7015	0.7871	
	$LZTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
⋆⋆	$LZTS$	0.2451	0.4493	0.5177	0.4105	0.6130	0.7066	0.5355	0.7545	0.7966	
⋆⋆⋆	$MRTT$	0.4622	0.5819	0.6507	0.6569	0.7731	0.8172	0.7940	0.8797	0.9063	
⋆⋆⋆	$MRTS$	0.3671	0.5671	0.6655	0.5872	0.7609	0.8232	0.7142	0.8679	0.9042	
	$PSTS$	0.0001	0.0024	0.0049	0.0000	0.0003	0.0016	0.0000	0.0002	0.0002	⊗
	$RCTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
⋆	$RCTS$	0.1281	0.2688	0.3781	0.2945	0.5101	0.6224	0.4673	0.6862	0.7739	
⋆	$SWTS$	0.1044	0.2805	0.4160	0.1762	0.4044	0.5499	0.2436	0.5216	0.6653	
⋆	$WETS$	0.1018	0.2571	0.3644	0.1478	0.3588	0.4757	0.2120	0.4543	0.5780	
⋆⋆	$WWTS$	0.2758	0.5236	0.6319	0.3040	0.6068	0.7364	0.3492	0.6749	0.7925	
	$ADTS$	0.0000	0.0025	0.0043	0.0000	0.0001	0.0012	0.0000	0.0001	0.0000	⊗

⊗ on the right panel indicates very low power, robust against the alternative considered.
⋆ on the left panel indicates large power and the more ⋆s the larger the power.

Table 10. Rejection Proportions for Log Normal (0, 0.8) Data

		$n = 12$			$n = 20$			$n = 28$			
	$Tests$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$	
⋆⋆⋆	$AHTT$	0.4678	0.6555	0.7576	0.5727	0.7795	0.8705	0.6913	0.8575	0.9170	
	$AHTS$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	$AKTT$	0.0061	0.0762	0.2269	0.0179	0.1807	0.3447	0.0493	0.2524	0.4142	
	$AKTS$	0.0066	0.0793	0.2031	0.0087	0.1325	0.2805	0.0278	0.1873	0.3313	
	$COTT$	0.0021	0.1001	0.2888	0.0225	0.2600	0.4606	0.0870	0.3799	0.5615	
	$COTS$	0.0701	0.2193	0.3297	0.1126	0.3352	0.4907	0.1873	0.4508	0.5703	
	$CMTS$	0.0049	0.0308	0.1101	0.0080	0.1332	0.2977	0.0450	0.2775	0.4925	
⋆⋆	$DPTT$	0.2477	0.3902	0.4468	0.3213	0.5154	0.6381	0.4527	0.6423	0.7389	
⋆	$DPTS$	0.0712	0.2100	0.3383	0.1200	0.3546	0.5188	0.1994	0.5079	0.6573	
	$EPTT$	0.0105	0.1425	0.2926	0.0445	0.2364	0.3851	0.0935	0.2962	0.4408	
	$EPTS$	0.0639	0.1855	0.2793	0.0940	0.2493	0.3741	0.1289	0.3012	0.4245	
	$ESTT$	0.0141	0.0627	0.1238	0.0208	0.0869	0.1601	0.0267	0.1100	0.2017	⊗
	$ESTS$	0.0113	0.0519	0.1017	0.0150	0.0616	0.1191	0.0186	0.0768	0.1369	⊗
⋆	$FRTS$	0.0645	0.2076	0.3383	0.0936	0.3308	0.5040	0.1736	0.4429	0.6203	
	$GSTT$	0.0489	0.1639	0.2693	0.0888	0.2466	0.3620	0.1353	0.3044	0.4266	
	$GSTS$	0.0559	0.1666	0.2607	0.0817	0.2333	0.3640	0.1198	0.2951	0.4221	
	$GNTT$	0.0342	0.1273	0.2325	0.0413	0.1699	0.2860	0.0596	0.2009	0.3300	
	$HGTT$	0.0462	0.1798	0.2989	0.0846	0.2638	0.4141	0.1315	0.3612	0.5178	
⋆	$HFTS$	0.1331	0.3025	0.4295	0.2052	0.4235	0.5677	0.2772	0.5440	0.6912	
	$HRTS$	0.1059	0.2238	0.3260	0.1435	0.2878	0.4178	0.1963	0.3689	0.5078	
	$HPTT$	0.0000	0.0112	0.1197	0.0000	0.0012	0.0610	0.0000	0.0000	0.0343	⊗
	$HPTS$	0.0606	0.1760	0.2723	0.0929	0.2962	0.4656	0.1603	0.4343	0.5968	
	$KMTS$	0.0070	0.0321	0.0811	0.0095	0.0541	0.1117	0.0166	0.0712	0.1394	⊗
⋆⋆	$KCTT$	0.1674	0.4708	0.6369	0.1806	0.4580	0.6111	0.2019	0.4463	0.5816	
	$KCTS$	0.0461	0.1356	0.2188	0.0606	0.1785	0.2828	0.0846	0.2078	0.3117	
	$KSTS$	0.0163	0.0809	0.1701	0.0360	0.1803	0.3135	0.0819	0.2855	0.4541	
	$LZTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	$LZTS$	0.0487	0.1557	0.2595	0.0795	0.2432	0.3976	0.1417	0.3575	0.4945	
	$MRTT$	0.0000	0.0556	0.2598	0.0091	0.2504	0.4951	0.0653	0.4229	0.6465	
⋆	$MRTS$	0.0771	0.2373	0.3462	0.1337	0.3930	0.5564	0.2130	0.5452	0.6758	
	$PSTS$	0.0826	0.2397	0.3563	0.1142	0.3142	0.4735	0.1618	0.3802	0.5147	
	$RCTT$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
⋆	$RCTS$	0.0650	0.2307	0.3676	0.1596	0.4400	0.6011	0.2712	0.6272	0.7734	
	$SWTS$	0.0293	0.0838	0.1431	0.0391	0.0938	0.1531	0.0492	0.1103	0.1685	⊗
	$WETS$	0.0639	0.1618	0.2550	0.0759	0.2053	0.3142	0.1070	0.2301	0.3360	
	$WWTS$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	⊗
	$ADTS$	0.0784	0.2447	0.3583	0.1070	0.3090	0.4460	0.1538	0.3552	0.4913	

⊗ on the right panel indicates very low power, robust against the alternative considered.
⋆ on the left panel indicates large power and the more ⋆s the larger the power.

In Table 3, when samples are taken from the standard exponential distribution (null distribution), the levels of significances are closely estimated for the tests AKTS, COTS, CMTS, DPTS, EPTT, EPTS, ESTT, ESTS, FRTS, GSTT, GSTS, GNTT, HGTT, HFTS, HRTS, HPTS, KMTS, KCTS, KSTS, LZTS, MRTS, SWTS, WETS, WWTS, and ADTS. The tests PSTS and RCTS are also closely estimating the levels of significances with exceptions for smaller samples. Tests AHTT, AKTT, DPTT, KCTT, and MRTT are not estimating the levels of significances closely. Tests AHTS, HPTT, LZTT, and RCTT are wrongly estimating the levels of significances always as zeros or very close to zeros.

In Table 4, when samples are taken from standard uniform distribution, the worst performing tests are, AHTS, HFTS, HRTS, HPTT, LZTT, RCTT, and SWTS, their rejection proportions are zeros or close to zeros. Tests AKTT, AKTS, COTT, COTS, DPTT, DPTS, EPTT, EPTS, FRTS, GSTT, GSTS, GNTT, HPTS, KMTS, PSTS, and WETS, have some decent powers at least for higher samples. Tests AHTT, KCTT, and ADTS have high powers, among them KCTT has highest powers for all sample sizes and for all levels considered.

In Table 5, when samples are taken from standard half normal distribution, the worst performing tests are, AHTS, HFTS, HRTS, LZTT, RCTT, and SWTS, their rejection proportions are zeros. Tests AHTT, and KCTT have high powers, between them KCTT has highest powers for higher sample sizes and for higher levels of significances.

In Table 6, when samples are taken from Weibull (1.0,1.4) distribution, the worst performing tests are, AHTS, HFTS, HRTS, HPTT, LZTT, RCTT, SWTS, and WWTS, their rejection proportions are zeros or close to zeros. Tests AKTT, COTT, COTS, DPTT, DPTS, EPTT, EPTS, FRTS, GSTT, GSTS, GNTT, HPTS, KCTS, LZTS, MRTT, MRTS, PSTS, WETS, and ADTS have some decent powers at least for higher samples and higher levels of significances. Tests AHTT, and KCTT have high powers, between them KCTT has highest powers for higher sample sizes and for higher levels of significances.

In Table 7, when samples are taken from Gamma (2.0,1.0) distribution, the worst performing tests are, AHTS, ESTT, ESTS, HPTT, LZTT, RCTT, SWTS, and WWTS, their rejection proportions are zeros or close to zeros. Tests AKTT, AKTS, COTT, COTS, DPTS, EPTT, EPTS, FRTS, GSTT, GSTS, GNTT, HPTS, KCTS, LZTS, MRTT, MRTS, RCTS, and WETS have some decent powers at least for higher samples and higher levels of significances. Tests DPTT, KCTT, PSTS, and ADTS have some good powers at least for higher samples and higher levels of significances. Tests AHTT, HFTS, and HRTS have high powers, among them HFTS has highest powers for higher sample sizes and for higher levels of significances.

In Table 8, when samples are taken from Beta(2.0,1.0) distribution, the worst performing tests are, AHTS, HFTS, HRTS, LZTT, RCTT, SWTS, and WWTS, their rejection proportions are zeros or close to zeros. Tests HGTT and HPTT have some decent powers at least for higher samples and higher levels of significances. Tests AKTT, AKTS, COTT, CMTS, ESTT, ESTS, KMTS, and KSTS, have high powers at least for higher samples and higher levels of significances. Tests AHTT, COTS, DPTT, DPTS, EPTT, EPTS, FRTS, GSTT, GSTS, GNTT, HPTS, KCTT, KCTS, LZTS, MRTS, PSTS, RCTS, WETS, and ADTS have very high powers, among them AHTT, DPTT, EPTS, FRTS, GSTT, GSTS, HPTS, KCTT, PSTS, WETS, and ADTS have highest powers irrespective of sample sizes and levels of significances.

In Table 9, when samples are taken from Gamma (0.5,1.0) distribution, the worst performing tests are, AHTS, GNTT, HGTT, HFTS, HRTS, HPTT, LZTT, PSTS, RCTT, and ADTS, their rejection proportions are zeros or close to zeros. Tests AKTT, AKTS, DPTT, DPTS, EPTT, EPTS, FRTS, GSTT, GSTS, HPTS, KCTS, KSTS, RCTS, SWTS, and WETS have some good powers at least for higher samples and higher levels of significances. Tests CMTS, KMTS, LZTS, and WWTS have good powers at least for higher samples and higher levels of significances. Tests COTT, COTS, MRTT, and MRTS, have high powers, among them MRTT and MRTS have highest powers irrespective of sample sizes and levels of significances.

In Table 10, when samples are taken from Log Normal(0,0.8) distribution, the worst performing tests are, AHTS, ESTT, ESTS, HPTT, KMTS, LZTT, RCTT, SWTS, and WWTS, their rejection proportions are zeros or close to zeros. Tests DPTS, FRTS, HFTS, MRTS, and RCTS have some decent powers at least for higher samples and higher levels of significances. Tests DPTT and KCTT have good powers at least for higher samples and higher levels of significances. Only AHTT have some high powers irrespective of sample sizes and levels of significances.

4. Concluding Remarks

Overall worst performing tests are Ahsanullah characterization test using simulation (AHTS), Hegazy-Green test using simulation (HFTS), Hegazy-Green alternative test using simulation (HRTS), Lorenz test using normal approximation (LZTT), Rossberg characterization test using normal approximation (RCTT), Shapiro-Wilk test using simulation (SWTS), and Wong and Wong test using simulation (WWTS). These tests do not estimate the levels of significances correctly and have very low powers against the alternatives considered here.

Ahsanullah characterization test using normal approximation (AHTT) and Kochar test using normal approximation (KCTT) have high powers except for Gamma distribution with parameters 0.5 and 1.0. A few other tests also show higher powers but for some selected alternatives.

References

[1] Balakrishnan, N. and Basu, Asit P. *The Exponential Distribution: Theory, Methods and Applications*. Gordon and Breach Publishers, The Netherlands, 1995.

[2] Volkova, K.Yu. "On Asymptotic Efficiency of Exponentiality Tests Based on Rossberg's Characterization". *Journal of Mathematical Sciences*, 167(4), 486-494, 2010.

[3] Rogozhnikov, A. P. and Lemeshko, B. Yu. "A Review of Tests for Exponentiality". *11th International Conference on Actual Problems of Electronics Instrument Engineering*, 159-166, 2012.

[4] Volkova, K.Yu. and Nikitin, Ya. Yu. "Exponentiality tests based on Ahsanullahs characterization and their efficiency". *Zap. Nauchn. Sem. POMI*, 412, 69-87, 2013.

[5] Volkova, K.Yu. and Nikitin, Ya. Yu. “Exponentiality tests based on Ahsanullahs characterization and their efficiency". *Journal of Mathematical Sciences*, 204(1), 42-54, 2015.

[6] Mimoto, N. and Zitikis, R. “The Atkinson index, the Moran statistic, and testing exponentiality". *J. Japan Statist. Soc.,* 38, 187-205, 2008.

[7] Henze, N. and Meintanis, S.G. “Recent and classical tests for exponentiality: a partial review with comparisons". *Metrika*, 61, 29-45, 2005.

[8] Deshpande, J.V. “A class of tests for exponentiality against increasing failure rate average alternatives". *Biometrika*, 70, 514-518, 1983.

[9] Ascher, S. “A survey of tests for exponentiality". *Communications in Statistics Theory and Methods*, 19, 1811-1825, 1990.

[10] Frozini, B.V. “On the distribution and power of a goodness-of-fit statistic with parametric and nonparametric applications". *Goodness-of-fit,* (Ed. by Revesz P., Sarkadi K., Sen P.K.) Amsterdam-Oxford-New York: North-Holland, 133-154, 1987.

[11] Gail, M.H. and Gastwirth, J.L. “A scale-free goodness-of-fit test for the exponential distribution based on the Gini statistic". *CJ. R. Stat. Soc. Ser. B*, 40(3), 350-357, 1978.

[12] Hegazy, Y. A. S. and Green, J. R. “Some new goodness-of-fit tests using order statistics". *Journal of the Royal Statistical Society. Series C* (Applied Statistics), 24, 99-308, 1975.

[13] Hollander M., Proshan F. “Testing whether new is better than used". *Ann. Math. Stat.*, 43, 1136-1146, 1972.

[14] Kimber, A.C. “Tests for the exponential, Weibull and Gumbel distributions based on the stabilized probability plot". *Biometrika*, 72, 661-663, 1985.

[15] Michael, J.R. “The stabilized probability plot". *Biometrika*, 70, 11-17, 1983.

[16] Kochar, S.C. “Testing exponentiality against monotone failure rate average". *Communications in Statistics Theory and Methods*, 14, 381-392, 1985.

[17] Gail, M.H. and Gastwirth, J.L. “A scale-free goodness-of-fit test for the exponential distribution based on the Lorenz curve". *Journal of the American Statistical Association*, 73, 787-793, 1978.

[18] Moran, P.A.P. “The random division of an interval Part II". *Journal of the Royal Statistical Society. Series B* (Methodological), 13, 147-150, 1951.

[19] Tchirina, A.V. “Bahadur efficiency and local optimality of a test for exponentiality based on the Moran statistics". *Journal of Mathematical Sciences*, 127(1), 1812-1819, 2005.

[20] Shapiro, S.S. and Wilk, M.B. “An analysis of variance test for the exponential distribution (complete samples)". *Technometrics*, 14, 355-370, 1972.

[21] Anderson, T. W. and Darling, D. A. “Asymptotic theory of certain goodness of fit criteria based on stochastic processes". *Annals of Mathematical Statistics*, 23, 193-212, 1952.