Web-based supporting materials for

Tests for goodness of fit in ordinal logistic regression models

⋮

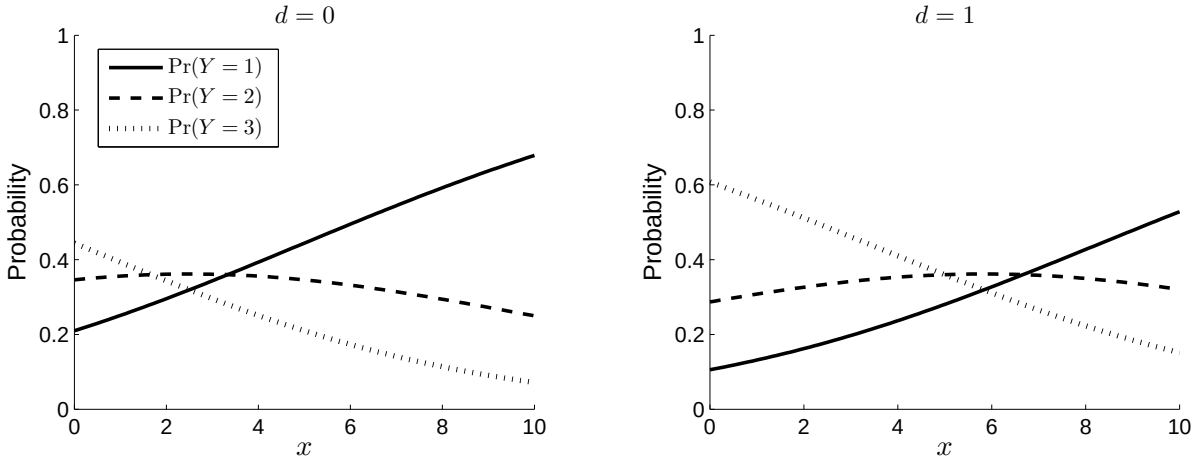

Figure 1: Distribution of response probabilities for the **adjacent-category model** with three response levels (Equation 12).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	0.73	4.78	9.76	0.66	4.52	9.23	1.06	5.45	10.2	0.88	4.73	10.1
C_{10}	0.78	4.33	9.48	0.68	4.36	9.19	0.95	4.97	10.0	0.83	4.71	9.73
C_{12}	0.64	4.31	9.35	0.72	4.31	9.01	0.90	5.12	10.2	0.87	4.94	9.63
Lipsitz	1.79	7.21	13.7	1.25	6.12	11.6	1.47	5.86	10.7	1.08	5.49	10.7
PR (χ^2)	1.11	6.20	12.4	1.00	5.47	11.0	1.03	5.93	11.3	1.15	5.48	10.5
PR (D^2)	1.64	7.50	14.0	1.14	6.06	11.8	1.16	6.09	11.7	1.19	5.51	10.6
x is normally distributed: $N(5, 3)$												
C_8	0.94	4.43	9.47	0.91	4.87	10.1	0.86	4.89	10.7	1.13	5.13	9.99
C_{10}	0.71	4.33	9.56	0.74	4.55	9.69	0.96	4.75	9.94	0.94	4.96	10.2
C_{12}	0.76	4.67	9.55	0.78	4.59	9.73	1.00	4.81	9.96	1.01	4.70	9.52
Lipsitz	1.32	6.98	13.0	1.25	6.06	11.4	1.17	5.15	10.7	1.14	5.43	10.3
PR (χ^2)	0.99	6.06	12.1	1.28	5.92	12.1	1.11	5.12	10.9	1.22	6.07	11.6
PR (D^2)	1.73	7.29	13.9	1.45	6.49	12.9	1.19	5.36	11.2	1.22	6.15	11.7

Table 1: Simulated rejection rates (%) under the null hypothesis with the **adjacent-category model** and three response levels (Equation 12).

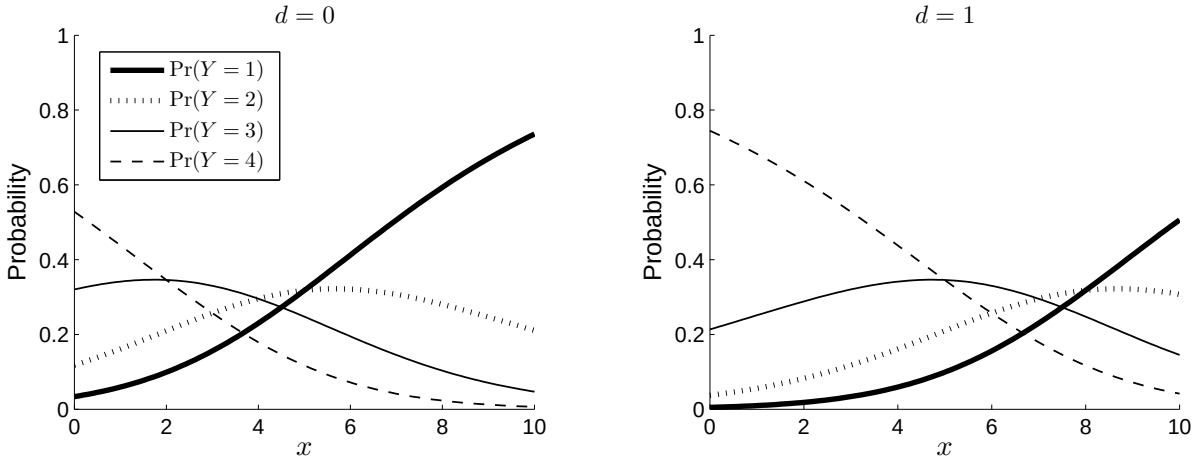

Figure 2: Distribution of response probabilities for the **adjacent-category model** with four response levels (Equation 13).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	1.43	5.26	10.3	1.07	5.18	10.2	1.00	4.98	9.77	0.89	4.91	9.95
C_{10}	1.58	5.72	10.3	1.26	5.53	10.4	1.04	4.82	9.55	1.03	4.97	9.92
C_{12}	1.84	5.99	10.6	1.44	5.41	10.1	1.16	4.71	9.60	0.98	5.18	10.2
Lipsitz	1.64	6.60	12.3	1.54	6.81	12.7	1.32	6.24	11.6	1.06	4.90	10.4
PR (χ^2)	1.09	5.36	10.6	1.09	5.20	10.7	1.10	5.58	10.8	1.09	5.17	10.7
PR (D^2)	1.75	7.26	14.2	1.36	6.69	13.0	1.40	6.42	12.3	1.15	5.58	11.2
x is normally distributed: $N(5, 3)$												
C_8	1.68	5.39	9.69	1.21	5.05	9.92	1.05	5.02	9.53	1.18	5.01	10.3
C_{10}	1.76	5.74	9.83	1.34	5.32	10.0	1.32	5.01	10.0	1.13	5.25	10.2
C_{12}	2.01	5.90	10.0	1.45	5.58	9.97	1.24	5.34	10.2	1.24	5.10	10.7
Lipsitz	1.43	6.49	12.1	1.39	6.20	11.7	0.94	5.64	10.6	1.02	5.15	10.5
PR (χ^2)	0.88	5.50	10.7	1.16	5.50	11.1	1.41	5.76	11.5	1.11	5.27	10.6
PR (D^2)	1.36	7.49	14.3	1.53	7.01	13.5	1.63	6.78	12.6	1.19	5.40	11.2

Table 2: Simulated rejection rates (%) under the null hypothesis using with the **adjacent-category model** and four response levels (Equation 13).

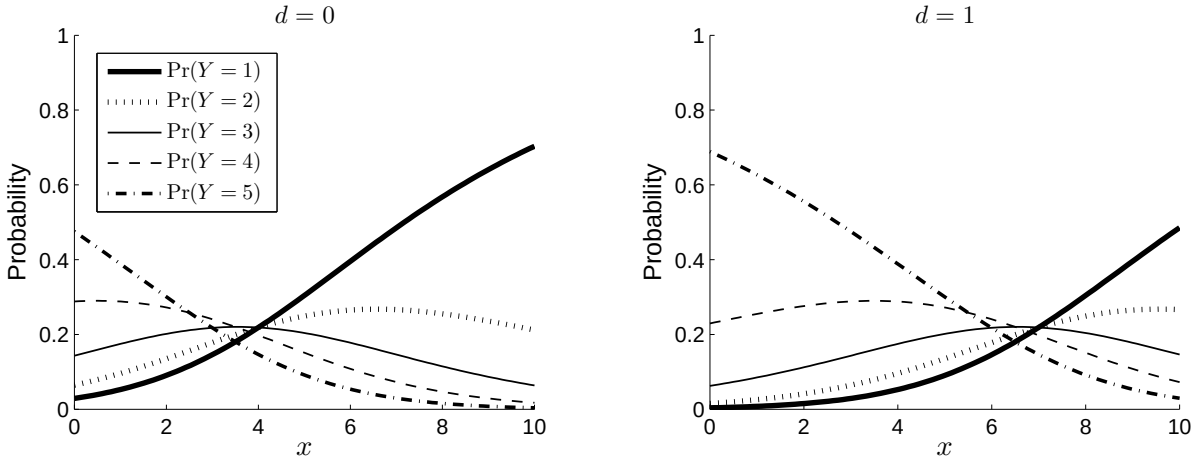

Figure 3: Distribution of response probabilities for the **adjacent-category model** with five response levels (Equation 14).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	1.59	5.50	10.0	1.35	5.05	9.97	0.93	4.75	9.20	1.11	4.85	9.29
C_{10}	1.89	5.80	10.1	1.31	4.96	9.97	1.05	4.42	9.10	1.21	4.86	9.65
C_{12}	1.90	6.05	10.4	1.34	5.46	10.1	1.13	4.88	9.50	1.05	5.20	10.5
Lipsitz	1.38	6.09	11.6	1.35	6.39	11.6	1.03	5.02	10.2	1.21	5.42	10.6
PR (χ^2)	1.05	5.18	10.3	1.13	5.12	10.3	1.00	5.47	10.8	0.81	5.32	10.5
PR (D^2)	1.83	8.17	15.1	1.72	6.97	13.4	1.25	6.63	12.9	1.02	5.73	11.4
x is normally distributed: $N(5, 3)$												
C_8	1.64	5.33	9.79	1.07	5.06	10.4	1.18	4.98	9.91	0.91	5.03	10.0
C_{10}	1.77	5.68	9.96	1.27	5.38	10.5	1.14	5.01	9.56	1.04	5.08	10.2
C_{12}	2.01	5.82	10.2	1.57	5.90	10.7	1.26	4.98	9.87	1.07	5.27	10.1
Lipsitz	1.45	6.12	12.0	1.36	6.10	11.8	1.23	5.58	11.1	1.12	5.25	10.1
PR (χ^2)	1.07	5.37	10.8	1.01	5.33	10.7	0.93	5.11	10.6	1.13	5.35	10.6
PR (D^2)	1.69	8.41	15.7	1.44	7.34	14.2	1.18	6.22	12.3	1.23	5.75	11.0

Table 3: Simulated rejection rates (%) under the null hypothesis with the **adjacent-category model** and five response levels (Equation 14).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	2.05	5.77	10.2	1.53	5.69	10.3	1.28	4.93	10.0	1.10	5.10	9.82
C_{10}	2.17	6.26	10.6	1.97	6.01	10.5	1.31	5.10	9.83	1.21	4.95	10.0
C_{12}	2.76	6.34	10.4	2.34	6.62	11.2	1.32	4.89	9.45	1.02	5.40	10.2
Lipsitz	1.79	7.66	14.0	1.66	7.67	14.5	1.25	5.98	11.7	1.38	5.73	10.8
PR (χ^2)	*	*	*	*	*	*	*	*	*	*	*	*
PR (D^2)	*	*	*	*	*	*	*	*	*	*	*	*

Table 4: Simulated rejection rates (%) under the null hypothesis with the **adjacent-category model**, four response levels, and **three continuous covariates** (Equation 15).

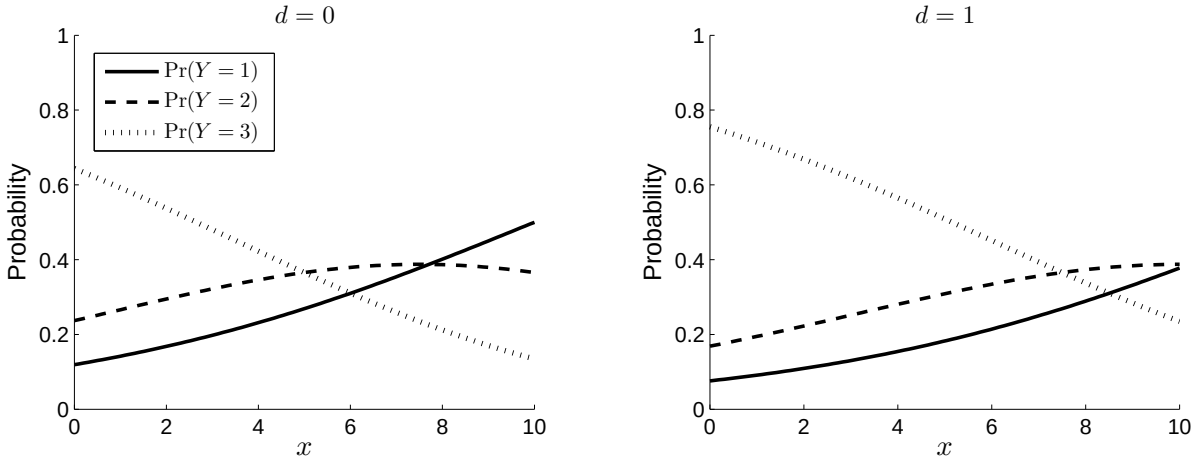

Figure 4: Distribution of response probabilities for the **constrained continuation-ratio model** with three response levels (Equation 16).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	0.73	4.16	8.86	0.94	4.94	9.70	0.88	4.58	9.36	0.93	4.90	9.83
C_{10}	0.73	4.04	9.21	0.77	4.60	9.59	0.84	4.92	10.2	1.01	4.69	9.87
C_{12}	0.75	4.13	9.14	0.98	4.49	9.29	0.84	4.44	9.02	0.93	4.65	9.84
Lipsitz	1.56	7.57	13.4	1.36	6.41	12.3	1.27	5.97	11.0	1.10	4.92	10.3
PR (χ^2)	0.89	5.71	11.3	1.13	5.63	11.8	1.18	5.58	11.3	1.17	5.91	11.3
PR (D^2)	1.38	7.14	13.5	1.43	6.27	12.4	1.20	5.88	11.6	1.23	6.01	11.5
x is normally distributed: $N(5, 3)$												
C_8	0.83	4.90	9.59	1.05	5.00	10.4	0.94	4.53	9.50	0.95	5.01	10.2
C_{10}	0.79	4.55	9.68	0.98	4.90	10.5	0.89	4.88	10.0	0.93	4.77	9.74
C_{12}	0.76	4.52	9.31	0.86	4.89	9.82	1.01	4.74	9.62	0.82	4.81	9.61
Lipsitz	1.82	6.96	12.8	1.53	6.64	12.4	1.20	5.30	10.9	1.12	5.39	10.3
PR (χ^2)	0.98	5.97	12.1	1.19	6.20	12.1	1.15	5.99	11.4	1.13	5.72	11.5
PR (D^2)	1.56	7.52	14.0	1.53	6.80	13.0	1.21	6.10	11.6	1.18	5.78	11.6

Table 5: Simulated rejection rates (%) under the null hypothesis with the **constrained continuation-ratio model** and three response levels (Equation 16).

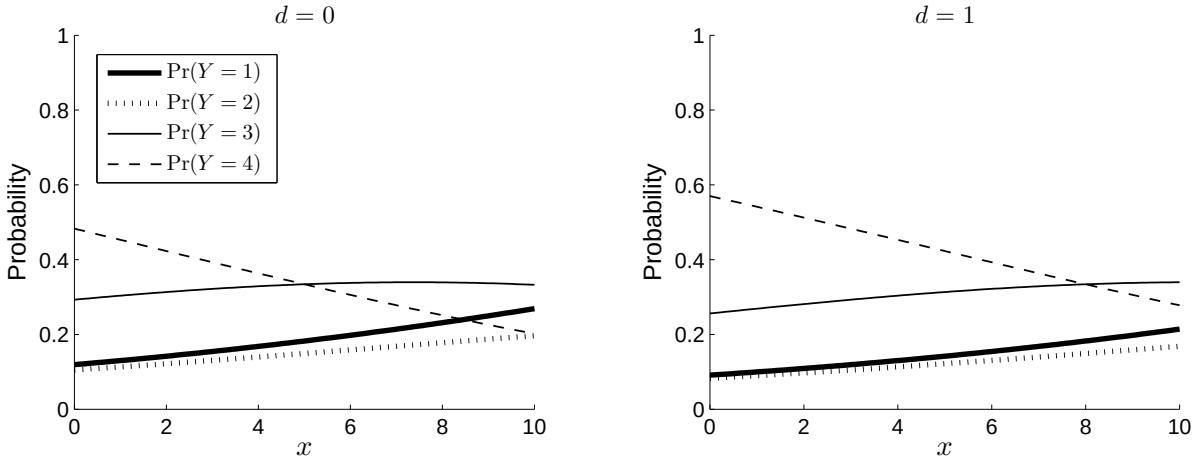

Figure 5: Distribution of response probabilities for the **constrained continuation-ratio** model with four response levels (Equation 17).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	0.66	4.10	8.83	0.96	5.35	10.0	1.23	4.98	9.54	0.93	4.65	9.72
C_{10}	0.84	4.51	9.18	1.01	4.97	9.88	0.94	4.86	9.67	0.91	4.69	9.88
C_{12}	0.66	3.97	9.00	0.99	5.05	9.64	0.73	4.99	9.82	1.04	4.86	9.66
Lipsitz	1.44	6.32	12.1	1.51	6.85	12.6	0.90	5.72	10.7	1.05	5.15	10.5
PR (χ^2)	0.98	5.38	11.0	1.15	5.50	10.8	1.05	5.59	11.0	1.23	5.74	11.2
PR (D^2)	1.99	8.43	15.3	1.59	6.52	12.3	1.26	6.05	11.7	1.31	5.94	11.6
x is normally distributed: $N(5, 3)$												
C_8	0.95	4.85	10.0	0.80	4.82	10.2	1.08	5.20	10.3	1.05	5.16	10.3
C_{10}	1.03	4.72	9.93	0.91	5.01	10.1	0.96	4.91	9.91	0.92	4.93	10.2
C_{12}	0.81	4.67	9.42	0.81	4.83	9.79	0.97	5.41	10.4	0.94	5.08	9.83
Lipsitz	1.65	6.54	12.5	1.35	6.49	12.4	1.26	5.80	11.6	1.16	5.42	10.8
PR (χ^2)	1.11	5.50	11.8	1.17	6.02	12.0	1.15	5.88	11.8	1.27	5.77	11.7
PR (D^2)	2.02	8.60	16.1	1.67	6.98	13.5	1.28	6.34	12.5	1.35	6.08	11.8

Table 6: Simulated rejection rates (%) under the null hypothesis with the **constrained continuation-ratio model** and four response levels (Equation 17).

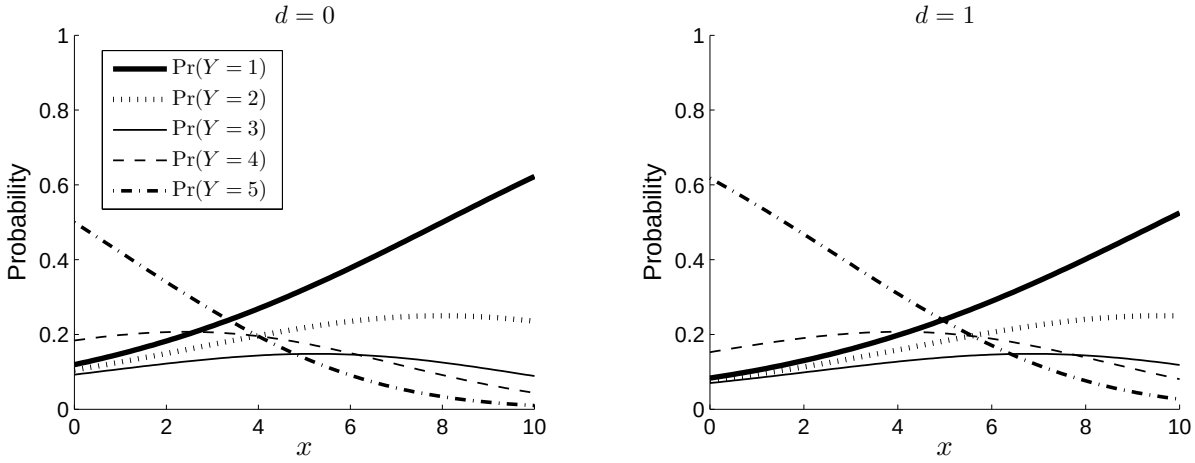

Figure 6: Distribution of response probabilities for the **constrained continuation-ratio model** with five response levels (Equation 18).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	1.08	4.76	9.83	0.68	4.79	9.56	0.86	5.03	9.94	1.03	4.78	9.43
C_{10}	0.91	4.72	9.42	0.75	4.68	9.55	0.91	4.65	9.37	1.06	4.86	9.76
C_{12}	1.00	4.88	9.74	0.98	4.70	9.48	0.97	4.84	9.61	0.86	4.42	9.35
Lipsitz	1.40	6.31	12.0	1.35	5.94	11.8	1.30	5.77	11.0	1.28	5.73	10.7
PR (χ^2)	0.99	5.34	11.4	0.90	5.37	10.6	1.08	5.76	10.8	1.35	5.89	11.2
PR (D^2)	2.17	9.98	17.4	1.73	7.02	13.0	1.39	6.43	12.0	1.44	6.26	11.7
x is normally distributed: $N(5, 3)$												
C_8	1.07	4.81	9.78	1.01	4.85	10.2	0.90	4.66	9.42	1.08	5.17	9.95
C_{10}	1.11	4.96	9.69	1.09	5.16	10.3	0.84	4.50	9.41	1.02	5.06	9.82
C_{12}	1.03	4.93	9.80	0.98	4.82	10.0	0.93	4.78	9.38	1.13	4.81	9.66
Lipsitz	1.37	6.13	12.0	1.37	6.60	12.8	1.21	5.64	11.0	1.16	5.69	10.8
PR (χ^2)	1.00	5.39	11.1	0.98	5.68	11.5	1.13	5.51	11.1	1.14	5.72	11.5
PR (D^2)	2.28	9.91	17.4	1.61	7.35	13.8	1.37	6.15	12.1	1.28	5.92	11.8

Table 7: Simulated rejection rates (%) under the null hypothesis with the **constrained continuation-ratio model** and five response levels (Equation 18).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	1.01	4.90	9.72	0.87	4.76	9.65	0.89	4.76	9.77	0.89	4.80	9.89
C_{10}	1.27	5.25	10.2	1.05	4.54	8.90	0.94	4.78	9.79	0.82	4.61	9.61
C_{12}	1.27	5.26	9.77	1.00	4.82	9.20	0.78	4.43	9.67	0.81	4.54	9.49
Lipsitz	1.47	6.10	11.8	1.50	6.49	12.7	1.16	5.22	10.4	1.17	5.53	10.2
PR (χ^2)	*	*	*	*	*	*	*	*	*	*	*	*
PR (D^2)	*	*	*	*	*	*	*	*	*	*	*	*

Table 8: Simulated rejection rates (%) under the null hypothesis with the **constrained continuation-ratio model**, four response levels, and **three continuous covariates** (Equation 19).

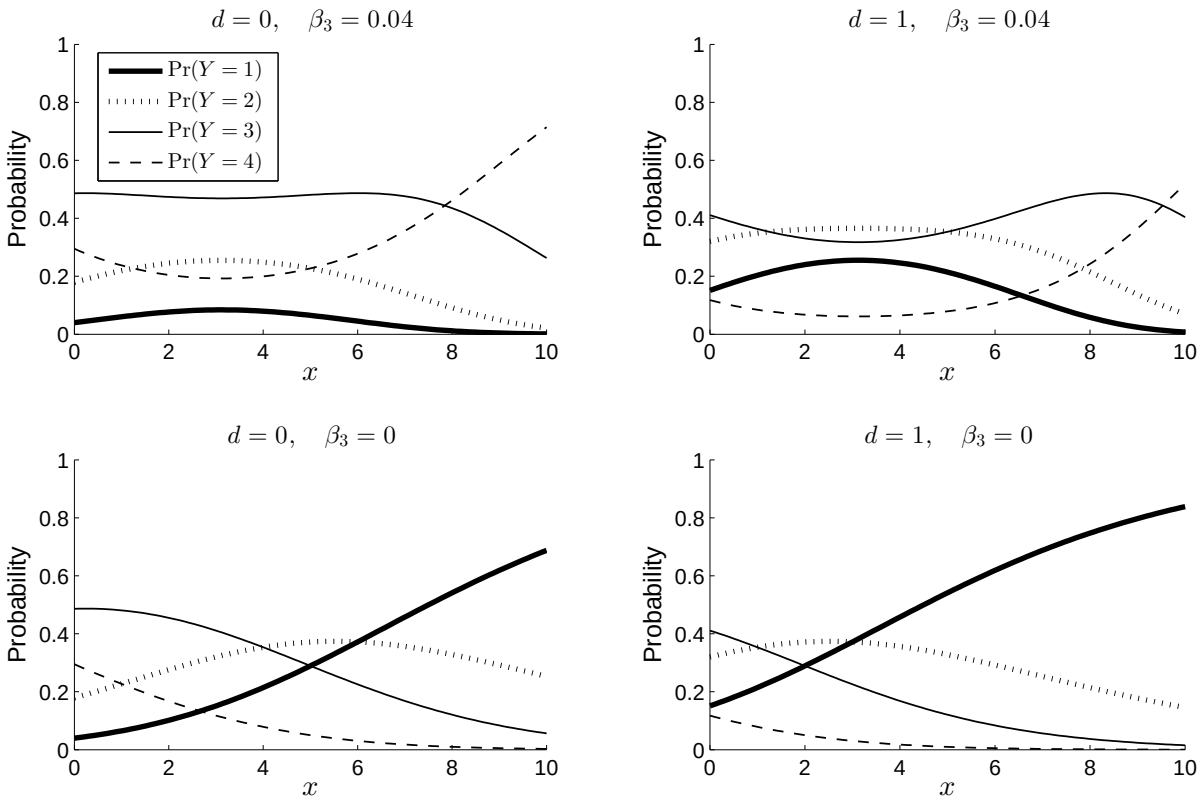

Figure 7: Upper plots: distribution of response probabilities for the correct **adjacent-category model** used to assess the power to detect a **missing quadratic term** when $\beta_3 = 0.04$ (Equation 20). Lower plots: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the upper and lower plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_3 = 0.02$												
C_8	1.28	6.00	11.0	2.38	8.12	15.0	4.66	15.9	25.4	14.7	33.9	46.6
C_{10}	1.30	5.78	11.2	2.14	7.90	14.6	4.54	15.6	25.9	14.0	34.0	47.3
C_{12}	1.04	5.58	11.2	1.78	7.82	14.5	4.20	14.6	23.7	14.0	32.5	45.7
Lipsitz	3.06	10.4	17.7	5.24	16.3	25.6	13.1	30.1	41.5	35.9	59.6	71.1
PR (χ^2)	1.08	5.00	10.5	1.12	5.26	11.3	1.08	5.78	11.4	1.06	5.82	11.9
PR (D^2)	1.60	7.56	14.3	1.48	7.08	13.8	1.34	6.54	12.7	1.16	6.08	12.3
$\beta_3 = 0.04$												
C_8	2.46	8.96	16.0	4.74	14.6	23.8	12.4	29.4	40.8	37.1	60.1	71.6
C_{10}	2.42	9.28	16.5	4.34	14.9	24.5	13.3	30.4	43.0	39.4	61.6	73.4
C_{12}	2.46	8.80	15.8	4.24	14.1	23.9	12.0	28.8	41.3	39.1	62.3	73.6
Lipsitz	4.90	14.7	23.6	13.5	31.2	43.2	31.5	53.7	66.1	66.9	83.4	89.7
PR (χ^2)	1.66	7.26	13.4	1.56	6.80	13.2	2.34	8.82	16.2	3.80	12.3	21.1
PR (D^2)	2.00	9.14	17.7	1.96	8.56	16.1	2.46	9.78	17.5	3.64	12.5	21.5
$\beta_3 = 0.06$												
C_8	4.46	11.8	18.2	10.1	23.3	33.7	25.5	47.0	58.6	68.3	84.7	91.0
C_{10}	4.36	11.3	17.5	10.1	22.2	32.5	24.4	45.7	58.1	68.0	85.7	91.3
C_{12}	4.84	10.6	16.8	8.60	20.7	29.7	22.5	43.5	56.7	66.7	84.6	90.9
Lipsitz	11.4	26.7	37.3	25.0	46.1	57.6	55.5	75.4	83.9	92.8	97.9	98.9
PR (χ^2)	5.16	12.4	18.9	6.34	15.9	23.9	9.56	24.1	36.1	23.1	45.4	58.6
PR (D^2)	2.36	10.0	18.0	4.22	14.1	23.7	8.04	23.3	36.5	22.1	45.5	60.2

Table 9: Simulated power (%) to detect a **missing quadratic term** with the **adjacent-category model** (Equation 20).

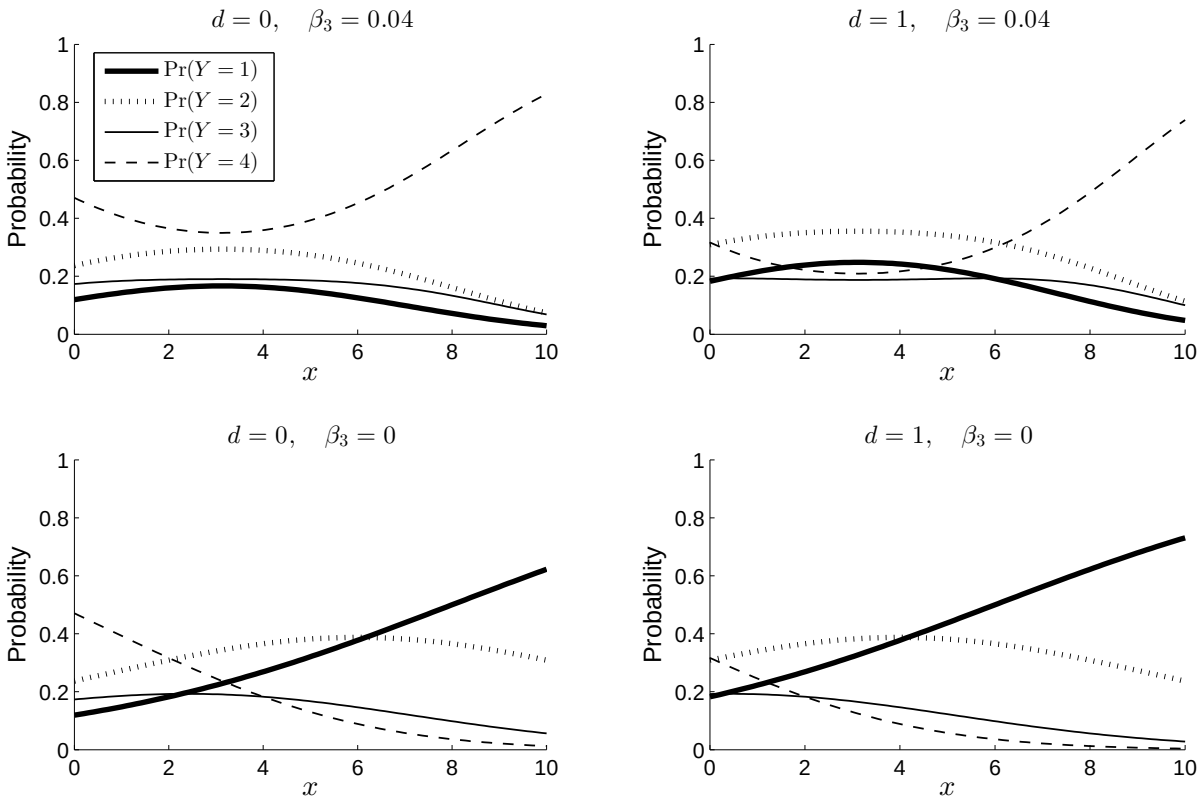

Figure 8: Upper plots: distribution of response probabilities for the correct **constrained continuation-ratio model** used to assess the power to detect a **missing quadratic term** when $\beta_3 = 0.04$ (Equation 21). Lower plots: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the upper and lower plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_3 = 0.02$												
C_8	1.08	5.08	10.4	1.16	6.22	12.8	2.78	10.4	17.5	5.86	19.2	29.4
C_{10}	1.18	5.00	9.72	1.16	6.62	12.8	2.86	9.80	17.5	5.74	18.2	29.0
C_{12}	0.86	4.54	9.32	1.34	6.24	12.1	2.76	9.56	16.8	5.68	18.3	28.5
Lipsitz	1.84	8.26	15.3	3.56	12.2	20.7	6.90	18.3	29.2	15.5	35.4	47.6
PR (χ^2)	0.96	5.36	10.7	0.92	5.06	10.7	0.94	5.52	11.3	1.08	5.42	11.0
PR (D^2)	1.84	7.02	13.5	1.24	6.02	11.6	1.10	6.04	11.8	1.20	5.60	11.2
$\beta_3 = 0.04$												
C_8	1.32	6.90	13.4	2.74	10.5	18.4	6.64	18.6	29.9	15.4	33.3	45.1
C_{10}	1.22	6.64	13.9	2.84	10.3	18.7	6.68	18.7	29.7	15.3	34.9	47.2
C_{12}	1.46	6.44	12.6	2.16	9.60	17.7	5.56	17.5	28.0	15.1	34.6	48.4
Lipsitz	4.56	13.9	22.4	7.86	21.1	31.4	15.3	33.5	45.5	35.6	57.4	69.4
PR (χ^2)	1.10	5.34	11.5	1.00	5.86	11.4	1.36	5.94	12.5	1.54	6.86	13.4
PR (D^2)	2.32	8.08	15.7	1.44	6.88	13.1	1.52	6.56	13.4	1.54	7.28	13.8
$\beta_3 = 0.06$												
C_8	1.52	7.26	14.2	4.64	16.2	26.5	18.6	39.8	52.1	59.3	79.3	87.2
C_{10}	1.56	6.80	12.7	4.16	13.9	23.4	15.4	34.7	47.4	53.2	74.5	83.6
C_{12}	1.24	5.72	11.3	3.46	12.2	21.2	12.7	31.8	44.1	47.7	71.6	81.4
Lipsitz	9.58	24.2	35.5	18.9	39.8	52.7	46.8	68.6	78.2	85.8	94.9	97.1
PR (χ^2)	1.22	5.64	11.3	1.48	6.42	13.0	1.94	8.22	15.0	2.82	11.6	21.0
PR (D^2)	2.52	9.98	19.3	2.40	9.98	18.2	2.46	10.5	18.5	3.48	13.8	23.3

Table 10: Simulated power (%) to detect a **missing quadratic term** with the **constrained continuation-ratio model** (Equation 21).

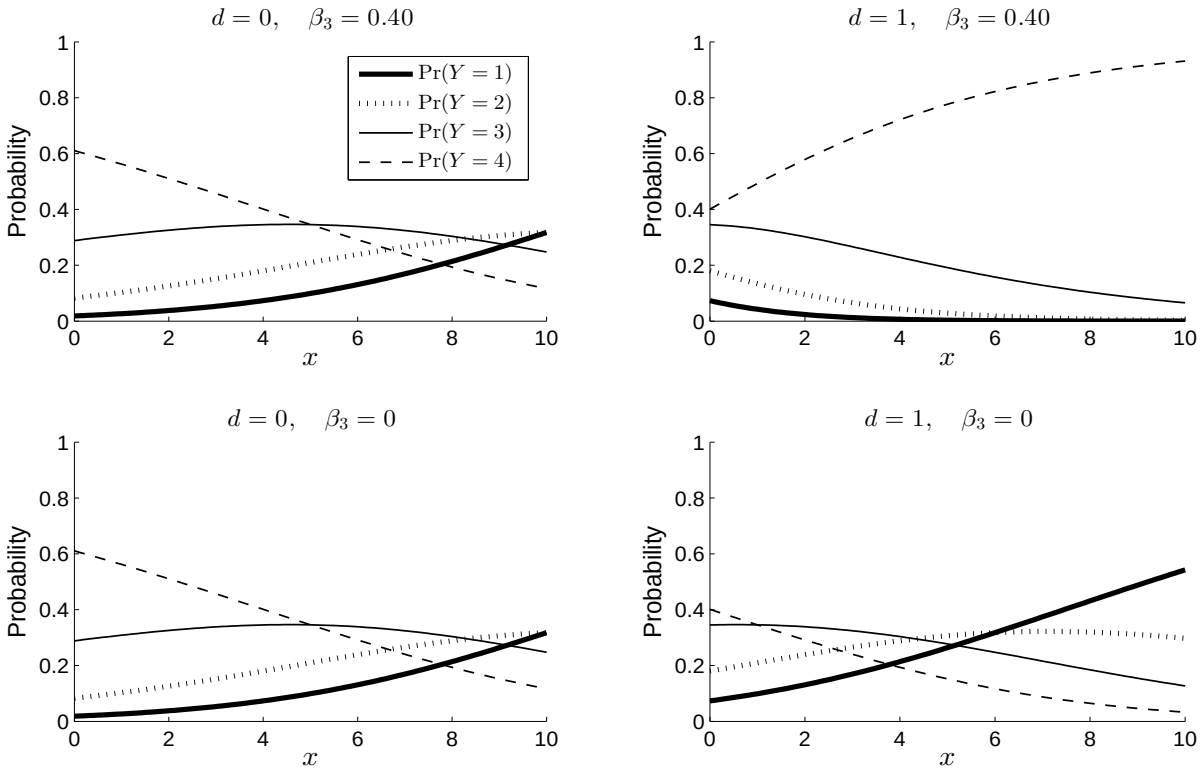

Figure 9: Upper plots: distribution of response probabilities for the correct **adjacent-category model** used to assess the power to detect a **missing interaction term between a continuous and a dichotomous covariate** when $\beta_3 = 0.4$ (Equation 22). Lower plots: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the upper and lower plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_3 = 0.2$												
C_8	5.90	17.4	27.6	19.0	40.3	53.2	62.8	82.8	89.7	98.1	99.6	99.8
C_{10}	4.84	15.8	25.7	17.3	36.7	49.9	57.0	78.1	86.6	97.1	99.2	99.6
C_{12}	5.28	15.5	25.3	15.1	33.8	46.8	52.2	74.8	83.6	95.5	98.9	99.5
Lipsitz	18.1	37.5	49.4	39.7	61.8	73.1	83.1	93.3	96.1	99.4	99.6	99.6
PR (χ^2)	11.6	29.9	42.6	33.3	57.3	69.8	78.5	91.3	95.4	99.4	99.9	100
PR (D^2)	15.8	36.4	49.9	38.2	61.7	72.6	79.7	92.0	95.5	99.5	99.9	100
$\beta_3 = 0.4$												
C_8	29.0	48.5	60.1	74.5	88.7	93.7	99.6	99.9	100	100	100	100
C_{10}	27.7	45.7	56.3	71.4	86.5	91.9	99.2	99.9	100	100	100	100
C_{12}	26.9	44.0	54.1	68.1	83.9	89.7	98.4	99.7	99.9	100	100	100
Lipsitz	58.6	79.4	85.9	87.6	92.7	93.9	96.1	96.1	96.1	97.0	97.0	97.0
PR (χ^2)	36.6	60.8	72.6	82.3	94.3	97.1	99.7	100	100	100	100	100
PR (D^2)	36.0	61.1	73.4	83.0	94.5	97.1	99.8	100	100	100	100	100
$\beta_3 = 0.6$												
C_8	42.9	59.3	68.1	86.8	93.7	96.1	99.9	100	100	100	100	100
C_{10}	40.8	56.2	64.3	85.0	92.7	95.3	99.8	100	100	100	100	100
C_{12}	41.3	54.7	62.8	82.2	90.9	94.1	99.7	99.9	100	100	100	100
Lipsitz	67.4	84.4	90.1	90.9	93.1	93.6	95.9	95.9	95.9	96.8	96.8	96.8
PR (χ^2)	46.7	67.9	77.9	90.3	96.5	98.2	99.9	100	100	100	100	100
PR (D^2)	41.7	66.0	77.1	88.3	96.2	98.1	99.9	100	100	100	100	100

Table 11: Simulated power (%) to detect a **missing interaction term between a continuous and a dichotomous covariate** with the **adjacent-category model** (Equation 22).

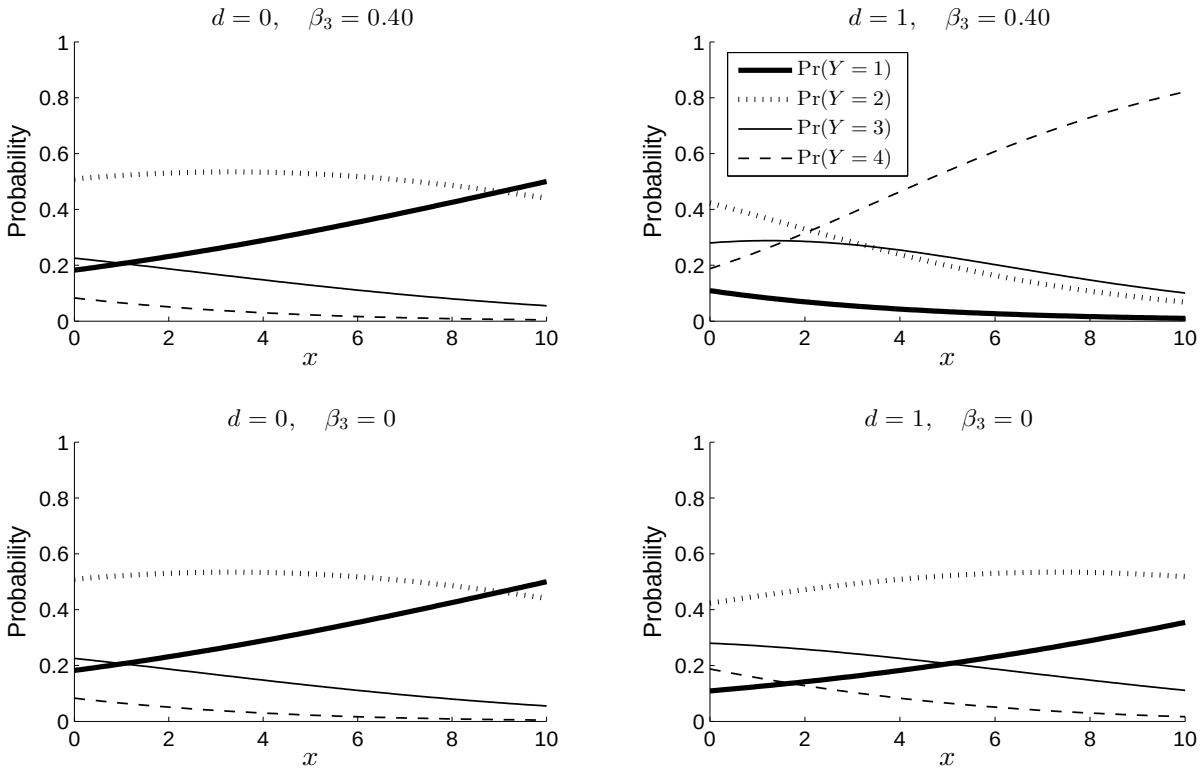

Figure 10: Upper plots: distribution of response probabilities for the correct **constrained continuation-ratio model** used to assess the power to detect a **missing interaction term between a continuous and a dichotomous covariate** when $\beta_3 = 0.4$ (Equation 23). Lower plots: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the upper and lower plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_3 = 0.2$												
C_8	2.56	9.54	16.8	5.94	17.5	28.3	18.9	40.9	54.3	63.3	83.3	90.5
C_{10}	2.46	9.04	15.5	4.80	15.3	25.3	16.3	36.5	50.7	57.5	78.3	87.0
C_{12}	2.44	8.62	14.6	4.18	13.9	23.8	13.6	33.3	45.9	51.4	74.0	83.7
Lipsitz	9.30	24.4	35.7	17.7	37.4	49.7	45.3	69.1	78.9	88.3	96.3	98.2
PR (χ^2)	4.94	15.8	26.1	12.1	29.7	42.6	33.9	58.3	70.9	78.8	91.5	95.4
PR (D^2)	6.80	19.4	30.7	14.7	33.5	46.5	37.8	61.6	73.4	79.9	92.1	95.7
$\beta_3 = 0.4$												
C_8	11.4	29.9	42.9	45.3	70.4	81.3	93.9	98.4	99.3	100	100	100
C_{10}	10.4	26.4	39.3	39.2	64.0	76.7	90.9	97.6	98.9	100	100	100
C_{12}	8.90	23.5	35.2	34.6	58.4	72.2	88.3	96.3	98.5	100	100	100
Lipsitz	44.5	68.9	79.4	79.8	92.4	96.3	99.5	99.9	100	100	100	100
PR (χ^2)	24.4	47.9	61.6	65.5	85.5	92.0	97.9	99.5	99.8	100	100	100
PR (D^2)	30.0	54.6	68.1	71.3	89.1	93.7	98.1	99.6	99.8	100	100	100
$\beta_3 = 0.6$												
C_8	21.1	45.1	58.2	75.5	91.1	95.7	99.8	100	100	100	100	100
C_{10}	19.3	40.5	53.6	69.8	88.5	94.1	99.6	100	100	100	100	100
C_{12}	16.2	35.2	48.8	64.1	84.8	92.1	99.5	99.9	100	100	100	100
Lipsitz	73.9	90.0	94.9	98.1	99.6	99.9	100	100	100	100	100	100
PR (χ^2)	41.6	69.0	80.6	89.9	97.5	99.0	100	100	100	100	100	100
PR (D^2)	52.8	77.6	86.3	93.3	98.6	99.3	100	100	100	100	100	100

Table 12: Simulated power (%) to detect a **missing interaction term between a continuous and a dichotomous covariate** with the **constrained continuation-ratio model** (Equation 23).

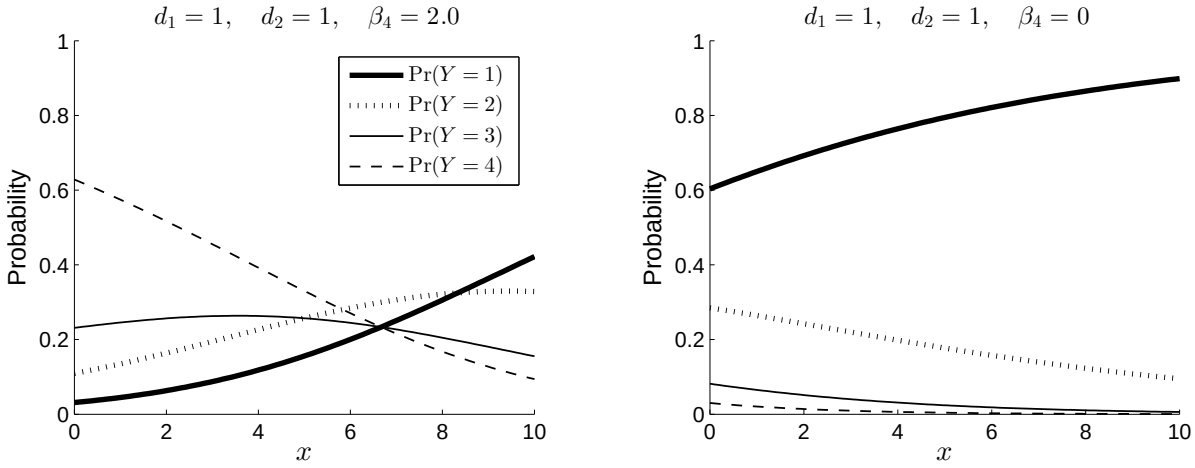

Figure 11: Left panel: distribution of response probabilities for the correct **adjacent-category model** used to assess the power to detect a **missing interaction term between two dichotomous covariates** when $d_1 = 1$, $d_2 = 1$, and $\beta_4 = 2.0$ (Equation 24). Right panel: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the left and right plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_4 = 1.0$												
C_8	3.94	5.98	7.54	5.76	8.78	11.2	5.88	8.90	12.0	6.36	10.3	13.9
C_{10}	4.32	6.12	7.70	6.14	8.92	11.6	6.52	9.68	12.5	6.82	10.8	14.2
C_{12}	4.74	6.86	8.28	6.44	9.44	11.9	7.34	10.3	13.0	7.16	11.3	14.9
Lipsitz	1.92	8.58	15.2	2.08	8.42	16.0	1.78	7.74	14.4	1.58	7.00	13.4
PR (χ^2)	5.16	8.34	10.6	7.98	12.8	16.7	10.3	19.1	25.7	20.1	35.7	46.8
PR (D^2)	0.22	1.20	2.76	0.78	3.40	7.92	2.72	9.98	17.4	11.5	29.6	41.9
$\beta_4 = 2.0$												
C_8	12.6	17.5	21.0	24.8	33.3	39.2	42.3	56.6	65.1	75.8	87.1	91.6
C_{10}	13.2	17.4	20.6	25.0	33.1	38.4	41.2	54.1	62.1	74.1	85.3	90.0
C_{12}	13.3	17.9	21.1	25.6	33.0	37.7	41.5	53.9	60.8	73.1	84.3	89.3
Lipsitz	10.5	26.3	38.2	20.1	40.2	53.1	47.1	68.2	78.0	85.6	93.8	96.2
PR (χ^2)	18.0	25.0	30.0	35.8	48.6	56.5	67.3	79.9	85.9	97.1	98.9	99.5
PR (D^2)	1.94	8.20	14.6	11.2	26.5	37.6	43.6	65.1	76.1	92.7	97.6	98.7
$\beta_4 = 3.0$												
C_8	29.3	38.8	45.0	60.5	71.5	77.0	93.4	96.3	97.6	100	100	100
C_{10}	29.4	37.0	42.5	57.7	68.2	73.6	88.4	93.6	95.7	99.8	99.9	100
C_{12}	29.7	37.0	42.0	58.0	67.1	72.9	90.7	94.6	96.0	99.9	100	100
Lipsitz	33.2	54.9	65.4	62.9	79.7	86.1	91.6	95.4	96.5	98.6	98.6	98.7
PR (χ^2)	41.6	52.9	59.7	76.6	84.8	88.8	97.9	98.9	99.5	100	100	100
PR (D^2)	8.30	21.8	31.8	42.1	63.0	73.0	89.7	95.8	97.5	99.9	100	100

Table 13: Simulated power (%) to detect a **missing interaction term between two dichotomous covariates** with the **adjacent-category model** (Equation 24).

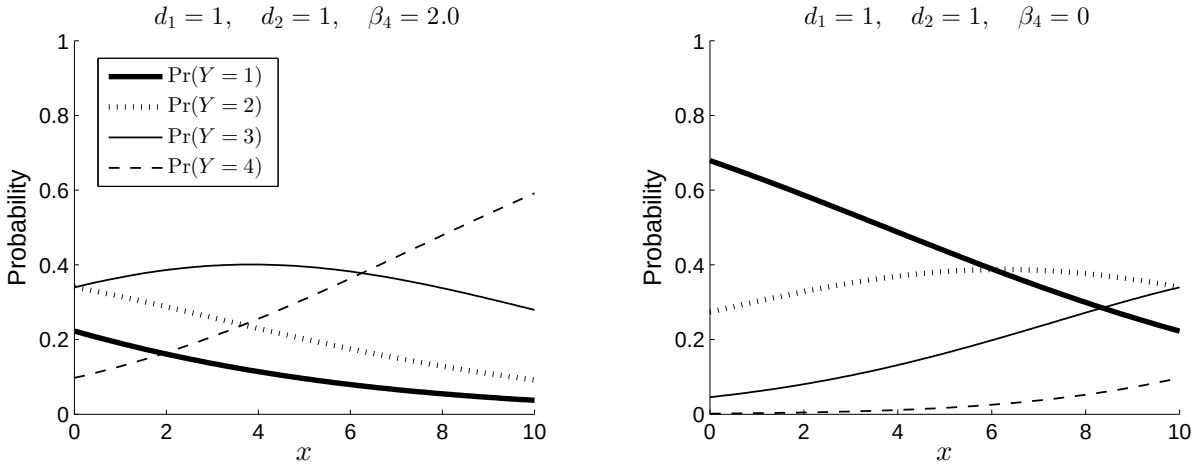

Figure 12: Left panel: distribution of response probabilities for the correct **constrained continuation-ratio model** used to assess the power to detect a **missing interaction term between two dichotomous covariates** when $d_1 = 1$, $d_2 = 1$, and $\beta_4 = 2.0$ (Equation 25). Right panel: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the left and right plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_4 = 1.0$												
C_8	3.00	6.72	10.7	2.86	7.28	12.2	3.64	10.5	16.4	4.56	13.9	22.4
C_{10}	3.16	6.66	10.7	3.42	7.58	12.8	3.34	9.78	15.8	4.44	13.5	20.8
C_{12}	3.16	7.02	10.8	3.22	7.92	12.3	3.44	9.54	15.7	4.06	12.3	19.8
Lipsitz	2.14	8.76	16.5	2.94	11.5	18.8	4.14	13.7	22.4	8.20	22.1	33.0
PR (χ^2)	4.48	10.9	16.5	6.18	17.0	26.0	16.2	35.6	47.7	47.7	71.3	80.9
PR (D^2)	3.06	10.4	18.7	5.66	17.8	28.2	16.6	37.6	51.1	50.3	73.7	83.0
$\beta_4 = 2.0$												
C_8	3.16	7.38	11.9	3.54	8.36	13.7	2.50	8.20	13.9	2.50	8.40	15.1
C_{10}	3.24	7.12	11.5	3.94	8.46	13.8	2.74	8.54	15.0	2.96	9.22	15.7
C_{12}	3.82	7.58	12.1	4.06	9.20	14.0	3.14	9.08	15.2	3.18	10.0	17.1
Lipsitz	3.32	10.6	18.4	3.36	10.4	17.9	3.22	10.6	18.2	3.76	11.2	19.1
PR (χ^2)	14.5	31.5	43.7	44.1	68.1	78.9	92.1	98.1	99.0	100	100	100
PR (D^2)	16.1	37.1	52.0	49.3	72.6	82.9	93.7	98.4	99.2	100	100	100
$\beta_4 = 3.0$												
C_8	5.10	10.9	17.0	7.50	16.6	23.8	12.8	26.2	35.0	25.0	42.4	53.6
C_{10}	5.12	11.5	17.2	7.80	16.9	24.6	15.1	28.2	37.8	29.6	46.8	57.4
C_{12}	5.48	11.3	16.7	7.50	17.2	24.1	14.7	29.4	38.9	33.1	50.6	60.5
Lipsitz	10.5	21.6	30.9	15.2	29.8	39.4	27.4	43.8	52.9	47.0	64.0	72.7
PR (χ^2)	41.6	67.7	78.5	92.6	98.2	99.4	100	100	100	100	100	100
PR (D^2)	51.2	76.5	85.3	95.4	99.0	99.6	100	100	100	100	100	100

Table 14: Simulated power (%) to detect a **missing interaction term between two dichotomous covariates** with the **constrained continuation-ratio model** (Equation 25).

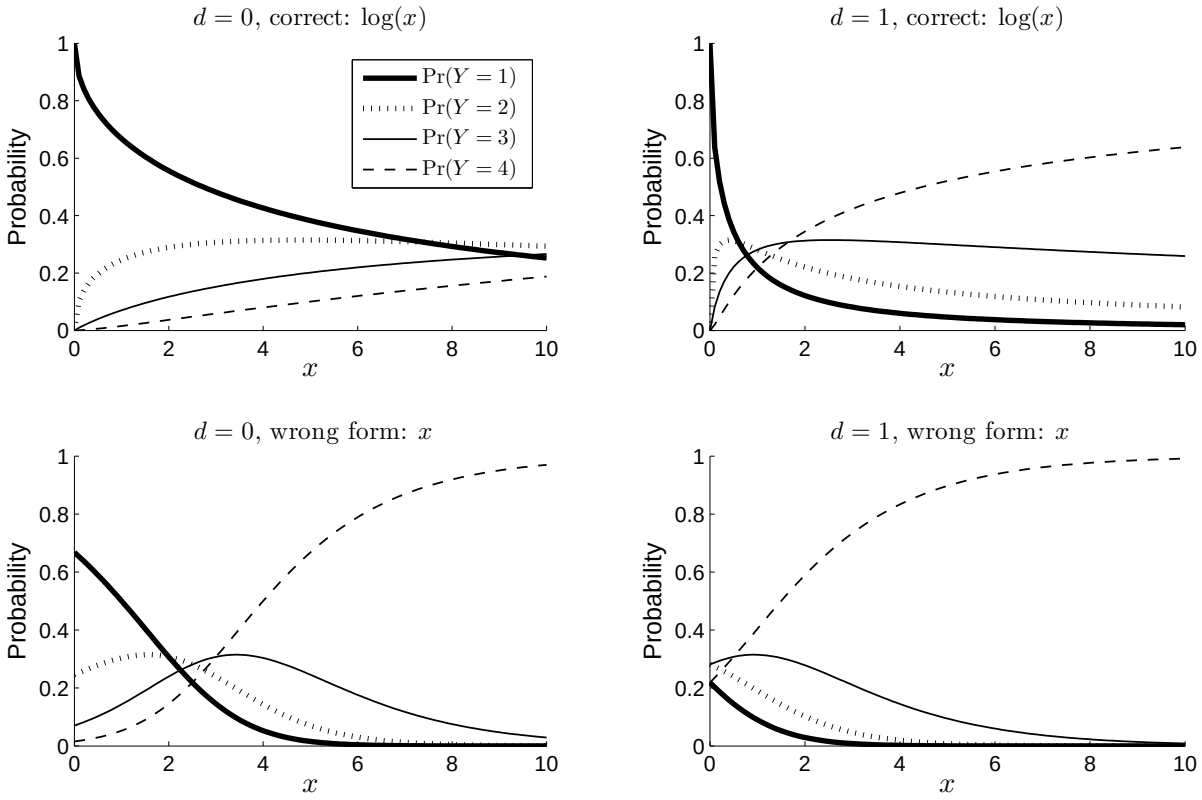

Figure 13: Upper plots: distribution of response probabilities for the correct **adjacent-category model** used to assess the power to detect a **wrongly fitted functional form** (Equation 26). Lower plots: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the upper and lower plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
C_8	1.70	7.02	13.0	2.36	9.84	17.3	6.90	20.3	30.9	21.7	44.0	57.5
C_{10}	1.56	6.06	12.4	2.64	9.52	17.3	7.70	20.5	31.9	23.8	45.8	59.4
C_{12}	1.86	6.80	12.4	2.96	10.5	17.9	7.60	20.2	30.7	23.5	46.3	58.7
Lipsitz	3.74	12.1	19.9	7.72	21.4	32.6	21.1	41.3	53.9	54.9	76.0	84.6
PR (χ^2)	1.22	6.62	13.0	1.62	7.54	13.8	2.48	9.82	17.4	4.52	14.6	24.6
PR (D^2)	1.76	8.90	16.7	2.00	9.08	16.0	2.80	10.5	18.3	4.48	14.6	24.6

Table 15: Simulated power (%) to detect a **wrongly fitted functional form** with the **adjacent-category model** (Equation 26).

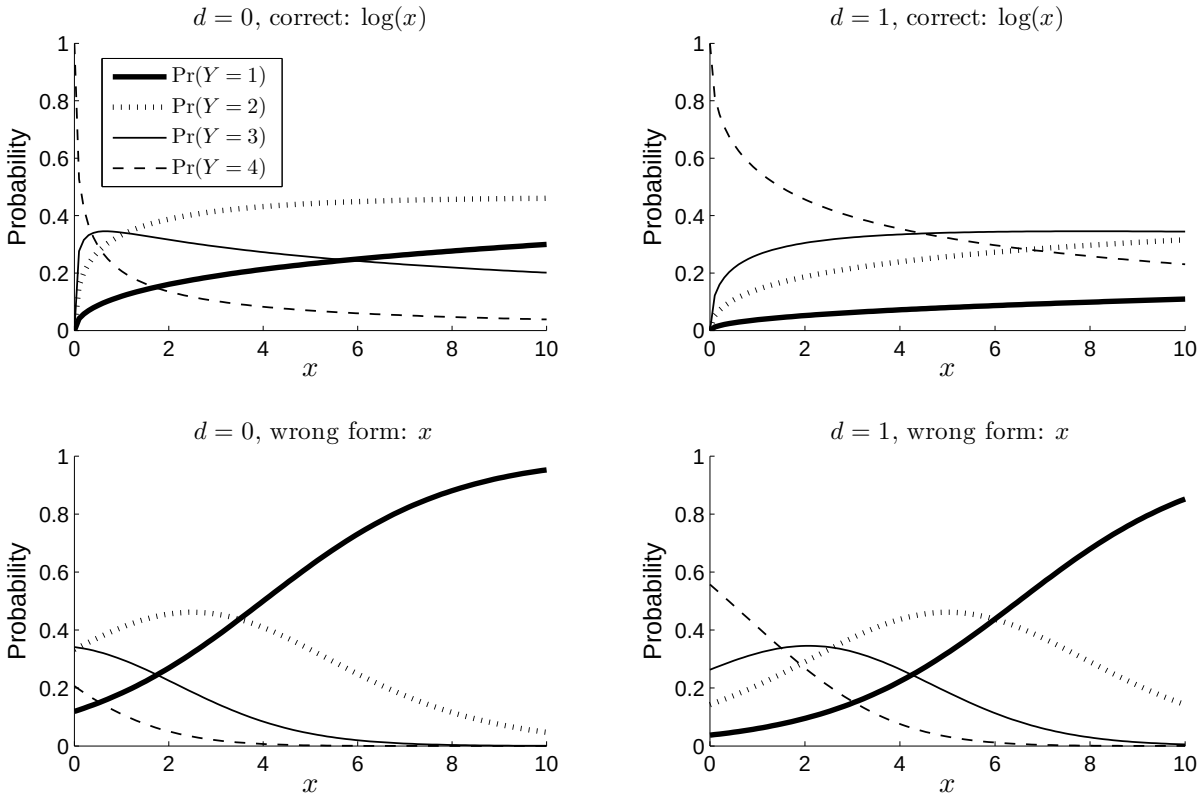

Figure 14: Upper plots: distribution of response probabilities for the correct **constrained continuation-ratio model** used to assess the power to detect a **wrongly fitted functional form** (Equation 27). Lower plots: the distribution of response probabilities for the hypothetical situation where the fitted model is the correct model. The actual fitted model has response probabilities somewhere between the upper and lower plots.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
C_8	1.30	5.28	10.6	1.82	6.98	12.8	2.92	10.4	18.0	7.44	21.0	31.4
C_{10}	1.08	5.92	11.0	1.68	7.56	13.6	3.22	11.1	18.9	8.34	24.0	35.8
C_{12}	1.40	5.58	10.2	2.24	8.40	14.6	3.12	11.3	19.7	9.12	23.8	36.7
Lipsitz	2.74	9.72	16.9	4.46	14.6	23.4	8.60	21.5	32.5	22.6	44.5	58.1
PR (χ^2)	0.98	5.50	11.1	1.48	6.32	12.4	1.16	6.42	12.7	2.12	7.86	13.8
PR (D^2)	1.76	8.02	15.3	1.96	8.46	15.1	1.40	7.36	13.6	2.28	8.12	14.0

Table 16: Simulated power (%) to detect a **wrongly fitted functional form** with the **constrained continuation-ratio model** (Equation 27).

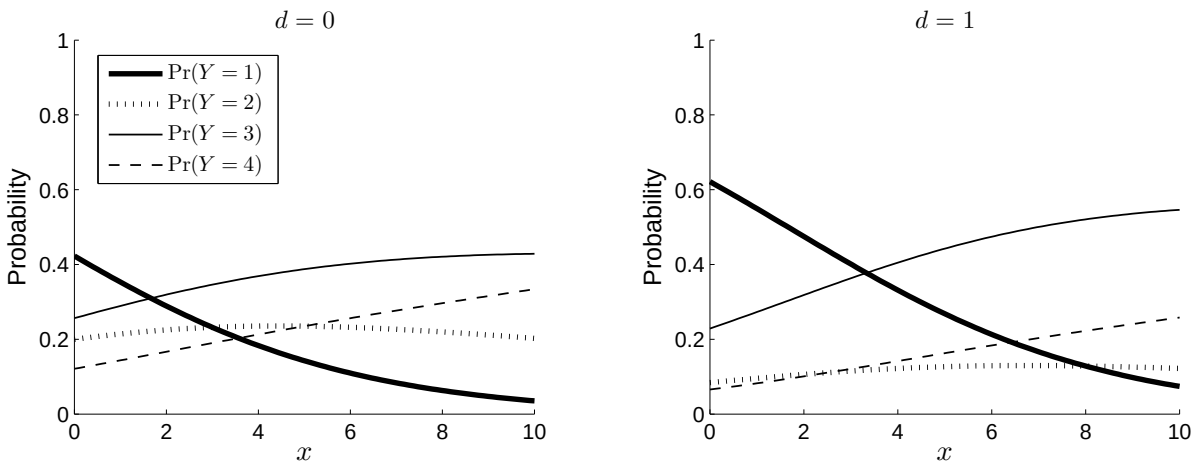

Figure 15: Distribution of response probabilities for the multinomial model in Equations 28–30. This model is used to assess the power to detect an **unordered response variable** for both the **adjacent-category and constrained continuation-ratio models**.

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
C_8	3.14	11.2	19.1	7.16	20.6	32.0	20.8	43.2	56.3	59.6	80.0	87.8
C_{10}	2.44	10.2	18.1	5.66	18.2	29.1	17.4	37.6	50.6	52.7	75.0	83.9
C_{12}	2.56	9.54	17.5	5.36	16.9	27.4	15.4	34.3	46.6	47.4	71.3	81.1
Lipsitz	1.52	6.12	12.1	1.90	7.32	14.4	2.18	8.58	15.3	3.78	11.1	19.6
PR (χ^2)	11.3	28.5	41.7	32.4	56.0	69.3	72.5	88.8	94.1	98.7	99.7	99.9
PR (D^2)	14.9	34.0	46.8	33.8	57.8	70.2	73.1	89.4	94.4	98.7	99.8	99.9

Table 17: Simulated power (%) to detect an **unordered response variable** with the **adjacent-category model** (Equations 28–30).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
C_8	5.46	17.1	28.2	18.4	40.2	53.6	55.7	78.0	86.9	96.4	99.2	99.8
C_{10}	4.96	15.9	26.4	15.4	36.3	49.0	48.7	72.3	82.5	94.7	98.6	99.4
C_{12}	4.26	14.1	24.1	13.0	32.1	45.6	43.2	67.2	78.6	92.2	97.8	99.1
Lipsitz	2.42	8.76	15.3	2.84	10.4	17.8	3.58	11.2	19.0	6.58	18.0	27.6
PR (χ^2)	19.0	41.7	55.4	51.5	75.0	85.1	92.5	97.9	99.1	100	100	100
PR (D^2)	24.7	47.6	60.0	55.0	77.6	86.4	93.3	98.0	99.2	100	100	100

Table 18: Simulated power (%) to detect an **unordered response variable** with the **constrained continuation-ratio model** (Equations 28–30).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	1.20	5.18	10.1	0.88	5.30	10.8	1.56	5.66	11.6	1.86	7.70	14.8
C_{10}	1.06	4.54	9.60	0.86	4.98	10.0	1.26	6.00	11.3	1.42	6.88	13.9
C_{12}	0.96	4.86	9.70	1.10	4.80	9.42	1.38	6.04	11.4	1.26	7.16	13.0
Lipsitz	1.88	6.74	12.8	1.84	7.72	13.7	1.70	7.38	14.0	2.64	9.22	16.8
PR (χ^2)	1.04	5.24	10.4	0.88	5.90	12.4	1.64	7.64	13.4	2.28	9.12	16.3
PR (D^2)	2.14	9.40	18.1	1.90	8.86	16.3	2.00	8.66	14.6	2.48	9.54	16.7

Table 19: Simulated power (%) to detect a **missing independent covariate** with the **adjacent-category model** (Equation 31).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
x is uniformly distributed: $U(0, 10)$												
C_8	1.10	4.66	10.1	1.62	5.94	11.1	2.06	8.32	15.0	3.64	12.1	21.4
C_{10}	1.08	5.54	10.0	1.26	5.56	10.8	1.76	7.66	13.8	2.72	10.8	19.1
C_{12}	1.06	4.94	9.80	1.16	5.36	9.98	1.82	7.74	14.3	2.86	10.3	17.6
Lipsitz	1.58	6.82	12.7	1.48	6.72	13.5	1.50	6.18	12.1	1.32	6.02	11.3
PR (χ^2)	1.20	6.08	12.3	1.96	7.30	13.7	2.76	11.0	19.1	6.28	18.2	28.7
PR (D^2)	2.34	8.96	16.4	2.48	8.28	15.4	3.20	12.1	20.4	6.68	18.7	29.5

Table 20: Simulated power (%) to detect a **missing independent covariate** with the **constrained continuation-ratio model** (Equation 32).

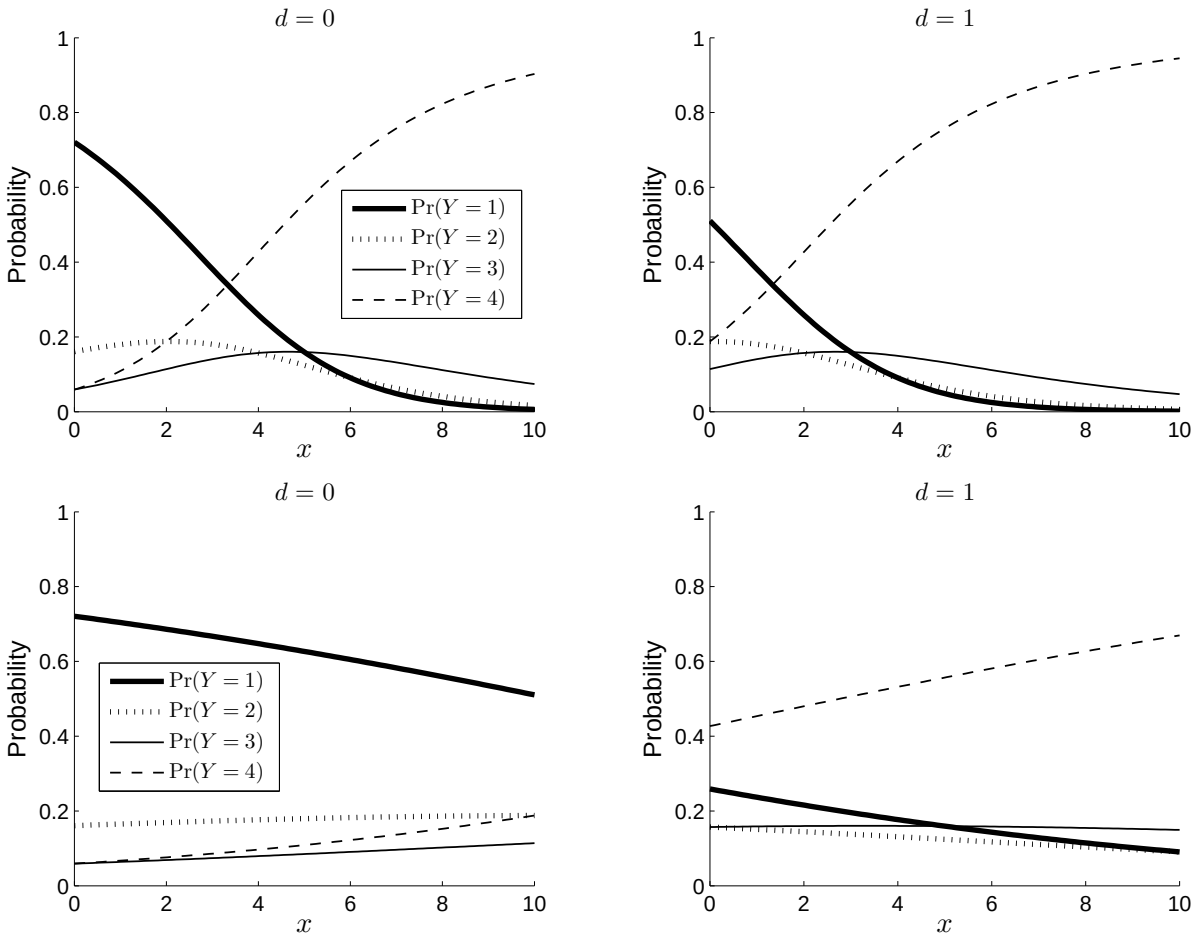

Figure 16: Distribution of response probabilities for the correct constrained continuation-ratio model used to assess **the power of the adjacent-category model to detect data generated by the constrained continuation-ratio model**. Upper plots: the continuous covariate is the most influential (Equation 33). Lower plots: the dichotomous covariate is the most influential (Equation 34).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_1 = 0.25, \beta_2 = 0.5$ *(the continuous covariate is the most influential)*												
C_8	4.42	11.8	18.7	7.48	20.7	30.9	25.2	48.7	61.9	71.1	88.1	93.5
C_{10}	4.30	11.2	18.3	7.10	17.9	28.1	21.2	42.8	56.6	63.5	82.9	90.1
C_{12}	4.44	11.3	18.1	6.34	17.1	26.4	18.0	38.7	51.1	58.2	79.4	87.4
Lipsitz	2.42	8.60	15.6	3.00	10.4	17.4	2.90	10.6	18.0	4.40	13.4	22.2
PR (χ^2)	4.18	13.2	22.3	9.96	27.0	40.0	31.9	56.6	69.6	77.4	91.2	95.3
PR (D^2)	6.04	18.7	31.1	14.2	34.8	48.9	37.5	62.8	74.7	80.3	92.6	96.3
$\beta_1 = 0.05, \beta_2 = 1.0$ *(the dichotomous covariate is the most influential)*												
C_8	1.46	6.24	11.9	2.46	10.0	19.0	7.58	22.8	34.8	29.4	52.6	66.2
C_{10}	1.86	6.16	12.3	2.80	11.0	18.8	6.84	20.6	31.7	24.7	47.5	61.2
C_{12}	1.42	6.32	12.6	2.38	9.94	17.8	6.06	18.7	29.8	20.8	43.3	57.3
Lipsitz	2.10	7.98	14.4	2.36	9.56	16.6	2.06	8.02	14.5	1.40	8.00	14.0
PR (χ^2)	2.60	11.2	20.0	7.06	21.4	33.3	21.6	44.3	57.4	57.4	79.3	86.8
PR (D^2)	5.16	17.1	26.7	9.02	24.1	36.5	22.8	45.6	58.4	58.8	79.9	87.1

Table 21: Simulated power (%) to detect **data generated from the constrained continuation-ratio model** (Equations 33 and 34) **by the adjacent-category model**.

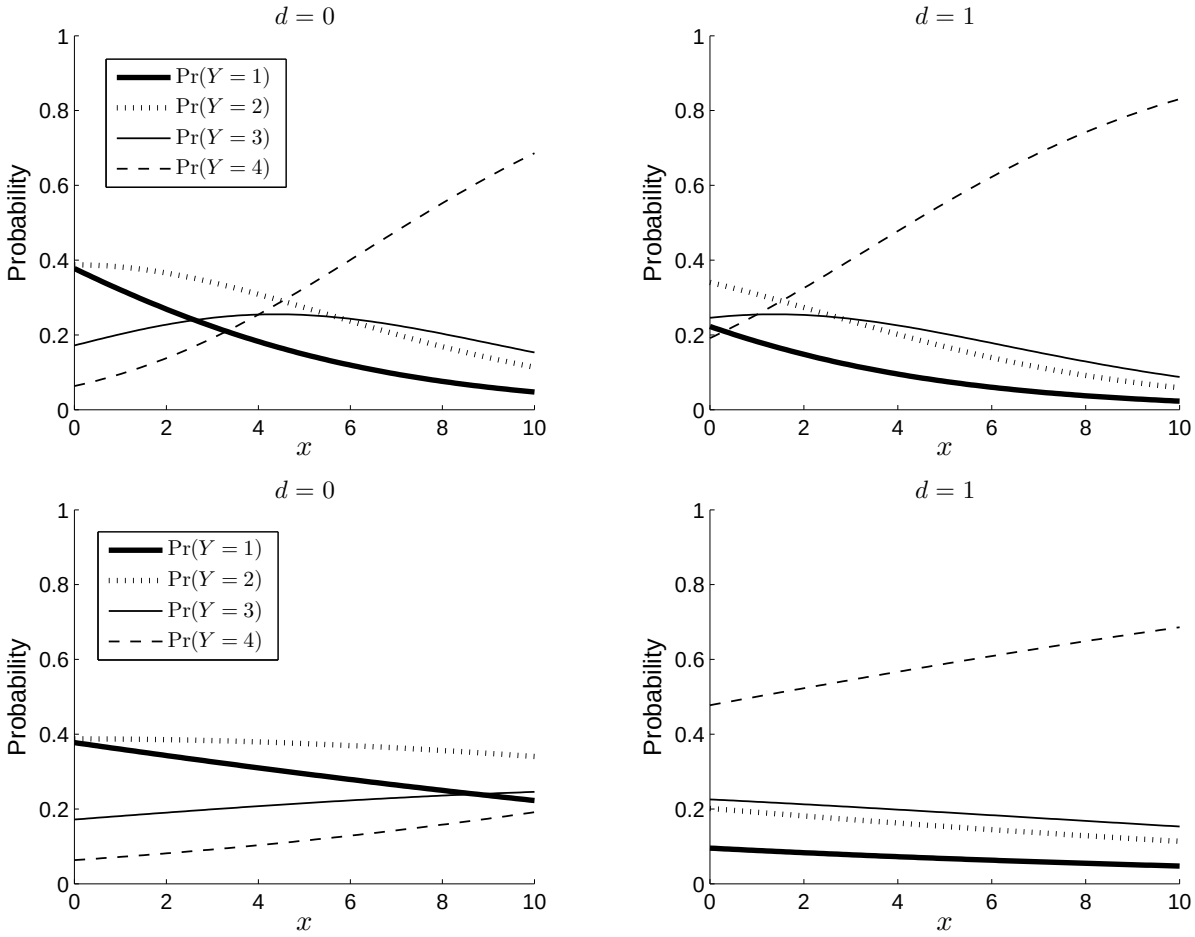

Figure 17: Distribution of response probabilities for the correct adjacent-category model used to assess **the power of the constrained continuation-ratio model to detect data generated by the adjacent-category model**. Upper plots: the continuous covariate is the most influential (Equation 35). Lower plots: the dichotomous covariate is the most influential (Equation 36).

	$n = 100$			$n = 200$			$n = 400$			$n = 800$		
Significance level	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
$\beta_1 = 0.25, \beta_2 = 0.75$ *(the continuous covariate is the most influential)*												
C_8	4.02	11.2	18.4	5.58	16.3	24.9	11.2	26.5	37.7	28.7	51.1	64.0
C_{10}	4.70	12.2	18.3	5.62	14.9	23.9	9.96	24.8	35.9	25.2	47.2	59.5
C_{12}	4.42	11.7	18.4	5.56	15.0	23.4	8.88	23.7	34.3	22.0	44.6	56.8
Lipsitz	1.46	6.60	12.8	2.06	7.90	14.5	1.34	6.42	12.5	1.50	6.56	11.9
PR (χ^2)	3.56	12.2	21.4	6.06	17.2	27.7	14.6	32.0	44.5	34.9	59.7	71.3
PR (D^2)	3.54	13.9	23.5	5.40	17.0	27.2	13.4	31.3	43.1	33.0	58.1	70.5
$\beta_1 = 0.075, \beta_2 = 1.75$ *(the dichotomous covariate is the most influential)*												
C_8	2.78	9.22	15.7	4.14	13.1	21.6	10.2	25.6	37.1	28.4	51.5	64.6
C_{10}	2.80	9.50	16.2	4.00	11.9	20.5	8.62	22.7	34.5	24.7	47.6	60.4
C_{12}	2.64	8.84	16.0	3.76	12.4	19.7	8.36	21.9	32.8	22.2	43.6	56.6
Lipsitz	1.70	7.20	13.2	1.82	6.90	13.0	1.60	6.18	12.4	1.40	5.50	10.8
PR (χ^2)	5.28	15.0	24.7	9.00	23.9	35.7	23.6	44.5	56.5	55.7	77.6	85.2
PR (D^2)	5.96	17.2	26.9	8.58	23.6	35.0	22.3	43.0	55.5	53.9	76.3	84.7

Table 22: Simulated power (%) to detect **data generated from the adjacent-category model** (Equations 35 and 36) **by the constrained continuation-ratio model**.

Variable	Coefficient	Standard error	z	$\Pr > \lvert z \rvert$	95% CI
Age	-0.0095	0.027	-0.35	0.73	$(-0.063, 0.044)$
Age^2	0.033	0.014	2.30	0.021	$(0.0049, 0.062)$
Gender	0.17	0.097	1.79	0.073	$(-0.017, 0.37)$
Race	0.16	0.097	1.69	0.091	$(-0.026, 0.36)$
Emot	0.31	0.11	2.88	0.004	$(0.099, 0.52)$
Custd	-0.35	0.11	-3.25	0.001	$(-0.57, -0.14)$
$\widehat{\alpha}_1$	-1.67	0.15			
$\widehat{\alpha}_2$	-1.29	0.31			
$\widehat{\alpha}_3$	0.18	0.44			

Table 23: Results of fitting an adjacent-category model of Neuro on Age, Age^2, Gender, Race, Emot, and Custd; $n = 508$.

Variable	Coefficient	Standard error	z	$\Pr > \lvert z \rvert$	95% CI
Age	-0.011	0.046	-0.24	0.81	$(-0.10, 0.080)$
Age^2	0.057	0.025	2.31	0.021	$(0.0086, 0.11)$
Gender	0.26	0.16	1.60	0.11	$(-0.058, 0.57)$
Race	0.27	0.16	1.68	0.092	$(-0.045, 0.59)$
Emot	0.49	0.18	2.65	0.008	$(0.13, 0.84)$
Custd	-0.57	0.17	-3.31	0.001	$(-0.91, -0.23)$
$\widehat{\alpha}_1$	1.16	0.19			
$\widehat{\alpha}_2$	0.50	0.24			
$\widehat{\alpha}_3$	0.035	0.30			

Table 24: Results of fitting a constrained continuation-ratio model of Neuro on Age, Age^2, Gender, Race, Emot, and Custd; $n = 508$.

Model/test	Value of test statistic	DOF*	p-value
Adjacent-category model			
C_8	10.4	20	0.96
C_{10}	13.4	26	0.98
C_{12}	28.9	32	0.62
Lipsitz	7.06	9	0.63
PR(χ^2)	87.8	88	0.49
PR(D^2)	91.2	88	0.39
Constrained continuation-ratio model			
C_8	11.8	20	0.92
C_{10}	21.3	26	0.73
C_{12}	27.7	32	0.69
Lipsitz	12.7	9	0.18
PR(χ^2)	90.9	88	0.40
PR(D^2)	92.2	88	0.36

*DOF = χ^2 degrees of freedom.

Table 25: Results of applying the goodness-of-fit tests to adjacent-category and constrained continuation-ratio models of Neuro on Age, Age^2, Gender, Race, Emot, and Custd; $n = 508$.