The most restrictive convergence plot is usually the one for the propagation constant of the narrowest strip spacing at the lowest frequency. This is due to the field having least space in terms of wavelength to adjust to the structure. However, when the odd mode is weakly bound at high frequencies, a small change in the propagation constant changes the decay rate away from the strips dramatically. This greatly affects the power contained in the mode and hence the characteristic impedance. In this case the odd-mode impedance at high frequencies sets the number of modes required. Generally, there exists a relation between the number of modes and the $\epsilon_2/\epsilon_1 = \epsilon_2/\epsilon_3$ dielectric step. Larger steps require more modes since the field has a more complicated structure in which to conform.

Numerical instabilities occur in the determinant calculation for large matrices. A single-precision (64-bit word) Gaussian elimination routine with partial pivoting was found to be insufficient for some calculations. A double-precision (128-bit word) version was tried with surprisingly no improvement in stability. It was found that a single-precision Gaussian elimination routine with *full* pivoting dramatically improves the numerical stability. The full pivoting routine was found to slow the overall computation by a factor of approximately 2 over the partial pivoting routine.

IV. Conclusions

A mode-matching method is applied to the analysis of coupled-line Microslab waveguide. The method is appropriate for analysis only and generally requires a long-word-length computer due to the large matrices encountered. Numerical stability can be improved by using full pivoting for the Gaussian elimination routine. Design is facilitated by repeated analysis to generate design charts. Design charts are provided for GaAs/alumina Microslab implementations.

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Tests of Microstrip Dispersion Formulas

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***Abstract* — A set of published formulas for the frequency dependence of the microstrip effective relative dielectric constant $\epsilon_{re}(f)$ is tested relative to an assemblage of measured data values for this quantity chosen from the literature. The r.m.s. deviation of the predicted from the measured values ranged from 2.3 percent to 4.1 percent of the seven formulas for $\epsilon_{re}(f)$ tested. A formula due to Kirschning and Jansen [10] showed the lowest average deviation from measured values, although the differences between the predictions of their formula and others tested are of the order of the error limits of the comparison process. It is concluded that the results indicate the suitability of relatively simple analytical expressons for the computation for microstrip dispersion.**

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I. Introduction

The widespread use of microstrip transmission line for microwave circuit construction has created a need for accurate and practical computational algorithms for the values of microstrip line parameters. For this purpose, the microstrip line is modeled as an equivalent TEM system at the operating frequency. For this quasi-TEM model, a characteristic impedance Z_0 and an effective relative dielectric constant: $\epsilon_{re} = (c/v_p)^2$ are defined, where v_p/c is the velocity of the waves on microstrip line normalized to the free-space velocity of light. Because the wave fields exist in two dielectric media, namely the substrate dielectric material and the ambient air, hybrid modes propagate on microstrip line, and the wave velocity is frequency-dependent. A knowledge of the dispersive relative dielectric constant $\epsilon_{re}(f)$ is required for microstrip circuit design.

Although computer-aided design (CAD) programs are available with capability for generating microstrip line parameters, an extensive development has been made of closed-form expressions which allow rapid computation of the parameters without the necessity of setting up of a CAD file [1]–[3]. Veghte and Balanis have recently shown that the use of closed-form expressions for $\epsilon_{re}(f)$ within a computer program for the study of transient pulses on microstrip facilitated the calculation and saved computer time [4]. They reviewed the origin of several closed-form expressions appearing in the literature [5]–[9] and compared their respective predicted values for $\epsilon_{re}(f)$ over a wide frequency range. The expressions tested were shown to be in relatively close agreement to their predictions of dispersive behavior, but no comparison was made of these results with experimentally observed values of $\epsilon_{re}(f)$. In literature presentations of closed-form expressions for $\epsilon_{re}(f)$ [5]–[9], their predictions have typically been shown in comparison with a limited selection of experimental data points over a small range of substrate dielectric constants and microstrip line dimensions. It is the purpose of the present work to compare predicted values of $\epsilon_{re}(f)$ from several closed-form expressions with experimentally measured data chosen from a wide range of sources.

II. Closed-Form Expressions

The microstrip dispersion expressions tested are shown below in (1) to (6). The algebraic forms of these expressions as shown in (1) to (6) have been modified from the form of their original presentation to bring them into a similar formalism for comparison here. In these expressions, $\epsilon_{re}(0)$ is the zero-frequency, or quasi-static value of ϵ_{re}, ϵ_r is the substrate relative dielectric constant, h is substrate height, w is the microstrip line width, Z_0 its characteristic impedance, $\mu_0 = 4\pi/10^7$ henrys/meter, f is frequency, and c the velocity of light.

1) W. J. Getsinger [5]:

$$\epsilon_{re}(f) = \epsilon_{re}(0)\frac{1 + K_1 f_{ni}^2}{1 + G f_{ni}^2} \tag{1}$$

where

$$G = 0.6 + 0.009 Z_0$$
$$K_1 = (\epsilon_r / \epsilon_{re}(0)) G$$
$$f_{n1} = 2\mu_0 hf / Z_0.$$

2) M. V. Schneider [6]:

$$\epsilon_{re}(f) = \epsilon_{re}(0)\left[\frac{1+f_{n2}^2}{1+K_2 f_{n2}^2}\right]^2 \quad (2)$$

where

$$K_2 = \sqrt{\epsilon_{re}(0)/\epsilon_r}$$
$$f_{n2} = (4hf/c)\sqrt{\epsilon_r - 1}.$$

3) E. Yamashita *et al.* [7]:

$$\epsilon_{re}(f) = \epsilon_{re}(0)\left[\frac{1+K_3 f_{n3}^{1.5}}{1+f_{n3}^{1.5}}\right]^2 \quad (3)$$

where

$$K_3 \sqrt{\epsilon_r/\epsilon_{re}(0)}$$
$$fn_3 = (4^{1/3} hf/c)\sqrt{\epsilon_r - 1}\left\{0.5 + \left[1 + 2\log_{10}\left(1 + \frac{w}{h}\right)\right]^2\right\}.$$

4) Hammarstad–Jensen [2]:

$$\epsilon_{re}(f) = \epsilon_{re}(0)\frac{1+K_4 f_{n4}^2}{1+G_4 f_{n4}^2} \quad (4)$$

where

$$G_4 = \frac{\pi^2}{12}\frac{(\epsilon_r - 1)}{\epsilon_{re}(0)}\sqrt{\frac{Z_0}{60}}$$
$$K_4 = \epsilon_r G_4/\epsilon_{re}(0)$$
$$f_{n4} = 2\mu_0 hf/Z_0.$$

5) M. Kobayashi [8]:

$$\epsilon_{re}(f) = \epsilon_{re}(0)\left[\frac{1+f_{n5}^2}{1+K_5 f_{n5}^2}\right]^2 \quad (5)$$

where

$$K_5 = \sqrt{\epsilon_{re}(0)/\epsilon_r}$$
$$f_{n5} = (2\pi hf/c)\left(1 + \frac{w}{h}\right)\sqrt{\epsilon_r - \epsilon_r e(0)}\,/D_5$$
$$D_5 = \tan^{-1}\left(\epsilon_r \sqrt{\frac{\epsilon_{re}(0) - 1}{\epsilon_r - \epsilon_{re}(0)}}\right).$$

6) Pramanick–Bhartia [9]:

$$\epsilon_{re}(f) = \epsilon_{re}(0)\frac{1+f_{n6}^2}{1+K_6 f_{n6}^2} \quad (6)$$

where

$$K_6 = \epsilon_{re}(0)/\epsilon_r$$
$$f_{n6} = 2\mu_0 hf/Z_0.$$

7) Kirschning–Jansen [10]:

$$\epsilon_{re}(f) = \epsilon_{re}(0)\frac{1+K_7 P}{1+P} \quad (7)$$

TABLE I

$\epsilon_{re}(f)$ Source	R_E^2	Rms Error
Getsinger [5]	0.1056	3.0%
Schneider [6]	0.1975	4.1
Yamashita [7]	0.0900	2.7
Hammarstad [2]	0.0986	2.9
Kobayashi [8]	0.0735	2.5
Pramanick/Bhartia [9]	0.0758	2.5
Kirschning/Jansen [10]	0.0603	2.3

where

$$K_7 = \epsilon_r/\epsilon_{re}(0).$$

In addition,

$$P = P_1 P_2 [(0.1844 + P_3 P_4) 10 F_4]^{1.5763}$$
$$P_1 = 0.27488 + \left[0.6315 + 0.525/(1 + 0.157F_4)^{20}\right]\left(\frac{w}{h}\right) - 0.065683\exp(-8.7513 w/h)$$
$$P_2 = 0.33622[1 - \exp(-0.03442\epsilon_r)]$$
$$P_3 = 0.0363\exp\left(-4.6\frac{w}{h}\right)\left\{1 - \exp\left[-(F_4/3.87)^{4.97}\right]\right\}$$
$$P_4 = 1 + 2.751\left\{1 - \exp\left[-(\epsilon_r/15.916)^8\right]\right\}$$
$F_4 = fh$ in units of GHz–cm.

A total of 120 data points were arbitrarily selected from published measurements of $\epsilon_{re}(f)$ [11]–[18]. With these, the error measure R_E^2 given by

$$R_E^2 = \sum_{j=1}^{120}\left[(\epsilon_{re}(f)\,\mathrm{calc} - \epsilon_{re}(f)\,\mathrm{meas})/\epsilon_{re}(f)\,\mathrm{meas}\right]_j^2 \quad (8)$$

was calculated and summed over all 120 data points for each of the closed-form expressions (1)–(7). The results of this calculation are shown in Table I. The table shows the values of R_E^2 calculated from data, and the r.m.s. error per measurement, $\sqrt{R_E^2/120}$, in percent.

III. Measured Data

The experimentally measured data values utilized for the above comparison were obtained from [11]–[18]. In all of these sources, the dispersion data were presented graphically as plotted curves of ϵ_{re} or v_p/c versus frequency. An enlarged photocopy was made of each curve and data values were taken from these by measurement. Four data points were taken from each curve, spanning the given frequency range of measurement. The maximum range of frequencies available in the overall data set was from 1 to 50 GHz, corresponding to a range of bandwidth–substrate height products [18] from $fh = 0.1$ to 7.85 GHz–cm. The substrate relative dielectric constants included in the survey assumed values from 2.5 to 15.87. The ratios of line width to substrate height, w/h extended from 0.10 to 7.85.

IV. Discussion

It may be seen in Table I that there is relatively small difference in the error measure R_E^2 computed for the seven dispersion relations tested. This error includes the effects of measurement error in the original $\epsilon_{re}(f)$ measurements, graphical error in converting the measured data to the published curve, and transcription error in retrieving data graphically from the curve, all in addition to the errors of approximation of the formulas under test. It is apparent that the formula of Kirschning and Jansen, (7), is in best average agreement with the data. The use of (5) and (6), which are more compact and simpler for rapid calculation than (7), would probably provide a satisfactory solution for many applications [3]. In the present comparison, no account is taken of shielding or enclosure effects on microstrip dispersion, as was true of all of the reported measurement data.

The accuracy of the dispersion formulas tested relative to experimental data did not show any significant correlation with the values of the bandwidth–substrate height product fh of the data sources measured. A large amount of variation was found to exist among data sources relative to this parameter. A test calculation was made of the correlation coefficient between the product fh and the discrepancy $\sigma = (\epsilon_{re}(\text{calc}) - \epsilon_{re}(\text{meas}))/\epsilon_{re}(\text{meas})$, relative to the $\epsilon_{re}(f)$ values predicted by (7). The resulting correlation coefficients assumed values from $+0.7$ to -0.8, depending on the data source utilized. Other dispersion relations ((1)–(6)) showed a similar lack of correlation between fh and error in ϵ_{re}. This amount of variability is assumed to be due to inherent experimental error as well as to the random errors which enter in the process of data transfer involving a small published graph. Direct availability of numerical data values would allow a more precise evaluation of the dispersion equations. It is assumed, however, that due to the large number of data points utilized in the computation of Table I, the error values and the ordering indicated therein are reliable. Since the objective of the present work was to observe the performance of the given dispersion relations relative to available measurements, no attempt is made here to evaluate the relative merits of the derivations of these relations. Discussion of these points is provided in [9] and [19].

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