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Tetraquark states in the bottom sector and the status of the $Y_b(10890)$ state

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Abstract We have performed an exploratory study of bottom tetraquarks ($[bq\bar{b}\bar{q}]$; $q \in u, d$) in the diquark–antidiquark framework with the inclusion of spin hyperfine, spin–orbit and tensor components of the one gluon exchange interaction. Our focus here is on the $Y_b(10890)$ and other exotic states in the bottom sector. We have predicted some of the bottom counterparts to the charm tetraquark candidates. Our present study shows that if $Z_b(10610)$ and $Z_b(10650)$ are diquark–diantiquark states then they have to be first radial excitations only and we have predicted the $Z_b(10650)$ state as first radial excitation of tetraquark state X_b (10.143–10.230). We have identified X_b state with $J^{PC} = 1^{+-}/0^{++}$ as being the analog of $Z_c(3900)$. The observation of the X_b will provide a deeper insight into the exotic hadron spectroscopy and is helpful to unravel the nature of the states connected by the heavy quark symmetry. We particularly focus on the lowest P-wave $[bq][\bar{b}\bar{q}]$ states with $J^{PC} = 1^{--}$ by computing their leptonic, hadronic, and radiative decay widths to predict the status of the still controversial $Y_b(10890)$ state. Apart from this, we have also shown here the possibility of mixing of P-wave states. In the case of mixing of the 1^{--} state with different spin multiplicities, we found that the predicted masses of the mixed P states differ from the $Y_b(10890)$ state only by ± 20 MeV energy difference, which can be helpful to resolve further the structure of $Y_b(10890)$.

1 Introduction

A plethora of new kinds of states which have been observed recently has inspired extensive interest in revealing the underlying structure of these newly observed states. Exploration of these states will improve our understanding of non-perturbative QCD. In recent years significant experimental progress has been achieved regarding the discoveries of the bottomonium-like and charmonium-like charged manifestly exotic resonances $Z_b(10610)$, $Z_b(10650)$ [1–7], $Z_c(3900)$ [8–12], and $Z_c(4020/4025)$ [13–16]. Their production mechanism and decay rates are not compatible with a standard quarkonium interpretation. A huge effort in understanding the nature of these new states and in building a new spectroscopy is forthcoming.

In recent years strong experimental evidence from B and charm factories has been accumulating for the existence of exotic new quarkonia states, narrow resonances, called X, Y, Z particles, which do not seem to have a simple $q\bar{q}$ structure. Their masses and decay modes show that they contain a heavy quark–antiquark pair, but their quantum numbers are such that they must also contain a light quark–antiquark pair [17]. The theoretical challenge has been to determine the nature of these resonances. Their production mechanism, masses, decay widths, spin-parity assignments, and decay modes have been revisited recently [18–20]. The term exotica labels states which have an identical number of quarks and antiquarks but defy an ordinary meson classification. Many exotic states in the charm sector with $c\bar{c}$ content have been discovered by Belle and others [8,21]. There are most likely many more which are yet unknown and many of them should also reflect in the $b\bar{b}$ sector according to heavy quark symmetry. The non-discovery of the respective $b\bar{b}$ partners of the charmonium-like exotica would be even more enigmatic. The Belle collaboration has extended the study of the XYZ exotic state family to the bottomonium sector by claiming the observations of two exotica states in $\Upsilon(5S)$ decays [3]. The CMS experiment also searched for the bottomonium partner of $X(3872)$ at hadron colliders [22] in the $\Upsilon(1S)\pi\pi$ decay mode and found no evidence for the X_b state, while the ratio of the cross section X_b to $\Upsilon(2S)$ shows an upper limit in the range of (0.9–5.4) % at 95 % confidence level for X_b masses between 10–11 GeV. Those are the first upper limits on the production of a possible X_b state at a hadron collider. Currently there are pending, unanswered questions

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concerning the exotic spectroscopy in the heavy quark sectors especially in the bottom sector. To promote the endeavor of understanding the heavy exotic states, the exploration of the bottom sector is important. Motivated by the BaBar discovery of a large $Y(4260) \rightarrow \pi + \pi + J/\psi$ signal in the charmonium mass region, the Belle experiment has searched for a similar state in the bottomonium sector [23]. They observed partial decay widths $\Upsilon(5S) \rightarrow \pi + \pi + \Upsilon(nS)$ $(n = 1, 2, 3)$ associated with the peak in the $\pi + \pi + \Upsilon(nS)$ cross section hundreds of times larger than the theoretical predictions [1] and the corresponding measured rates for the $\Upsilon(4S)$ [24]. This observation suggests the presence of a new, non-conventional hadronic state in the bottom sector equivalent of the $Y(4260)$ of the charm sector with mass around 10.890 GeV [26] which is referred as $Y_b(10890)$ state. Indeed, there exist three candidates up to date, namely the states labeled $Y_b(10890)$, $Z_b(10610)$ and $Z_b(10650)$, observed by Belle [3]. Not only new states are waiting to be discovered but also the existence of $Y_b(10890)$ needs to be established or refuted. The $Y_b(10890)$ is a potential exotic state still remains to be confirmed since its observation first reported by the Belle collaboration [1,27]. Looking into the interest in this case, present study is particularly focus on the negative parity 1^{--} exotic states. Apart from its spin parity, the study of its dileptonic, hadronic, and radiative decay widths also help us to solve the puzzling features of this state. The interpretation of the hidden bottom four-quark state as a tetraquark exotic state has been advanced and has been studied in considerable detail [25,26,28–37]. The experimental search for tetraquark states is a very difficult problem, since exotic candidates are nothing but the resonances immersed in the excited hadron spectra and, moreover, they usually decay to several hadrons. Their mass and decay products put them in the category of quarkonia-like resonances but their masses do not fit into the conventional quark model spectrum of quark–antiquark mesons [38,39]. However, to confirm a new resonance it is necessary to study all its properties with high level of accuracy including its mass and width. In this work, we develop phenomenology to study some of the theoretical problems of multiquarks and predict multiquark bound states and resonances. In particular, as a benchmark, we study in detail the heavy-light antilight-antiheavy systems who are expected to produce tetraquarks. Despite the intense experimental attempts, these resonances are still mysterious and complicated and we still lack of a comprehensive theoretical framework. In particular, the most popular phenomenological models proposed to explain the internal structure of these particles are the compact tetraquark in the constituent diquark–antiquark picture and the loosely bound di-meson molecular picture. Following Gell-Mann's suggestion of the possibility of a diquark structure [40], various authors have introduced effective degrees of freedom of diquarks in order to describe tetraquarks as composed of a constituent diquark

and diantiquark using QCD sum rules [41,42]. This concept of diquark was even used to account for some experimental phenomena [43,44]. The authors of Refs. [45,46] studied the tetraquark systems in the diquark–antidiquark picture using the chromomagnetic interactions. In the same way Maiani et al. [47–50] also studied tetraquarks and pentaquarks systems by considering this concept of diquark. In their study they have included the spin–spin interactions. On the other hand Ebert et al. [25,51–53] employed the relativistic quark model based on the quasi-potential approach in order to find the mass spectra of hidden heavy tetraquark systems. Unlike Maiani et al., they ignored the spin–spin interactions inside the diquark and antidiquark. The presence of a coherent diquark structure within tetraquarks helps us to treat the problem of four-body to that of two-two-body interactions. In the present case, we employ the diquark and antidiquark picture in the beauty sector and compute the mass spectra of the diquark–antidiquarks $[bq\bar{b}\bar{q}]; q \in u, d$ in the ground and orbitally excited states with the inclusion of both $S = 0$ and $S = 1$ diquarks. We present the formalism of the study of hidden bottom tetraquark states in Sect. 2. In Sect. 3, we discuss the Y_b states and their decay properties. We conclude and discuss our findings in Sect. 4.

2 Theoretical framework

In this paper we shall take a different path and investigate different ways in which the experimental data can be reproduced. We have treated the four particle system as two-two-body systems interacting through effective potential of the same form of the two-body interaction potential of Eq. 1). The existence of exotic hadrons of the diquark–diantiquark pair called tetraquarks or diquarkonia is a problem which was foremost raised about 20 years ago and was used to describe scalar mesons below 1 GeV in 1977 by Jaffe [54–57]. He suggested the idea of strongly correlated two-quark–two-antiquark states to baryon–antibaryon channels where the MIT bag model is used to predict the quantum numbers and the masses of prominent states. There are two types of diquarks; one is $S = 0$, good (scalar) diquarks, and another one is $S = 1$, bad (vector) diquarks. We have available lattice results which favor the evidence of an attractive diquark (antidiquark) channel for the good diquarks (color antitriplet, flavor antisymmetric) with spin $S = 0$ in accordance with Jaffe's proposal. On the other hand there are no lattice results available for an attractive channel for the bad diquarks, i.e. with spin $S = 1$. Here, we use the fact that the effective QCD-lagrangian is independent of spin in the heavy quark limit and we incorporate the diquark with $S = 1$ also in computing the mass spectra. There are many methods to estimate the mass of a hadron, among which the phenomenological

potential model is a fairly reliable one, especially for heavy hadrons [58–61]. In the present study, the non-relativistic interaction potential we have used is the Cornell potential, which consists of a central term $V(r)$, which is just the sum of the Coulomb (vector) and linear confining (scalar) parts given by

$$V(r) = V_V + V_S = k_s \frac{\alpha_s}{r} + Ar + B, \tag{1}$$

$$k_s = -4/3 \quad \text{for} \quad q\bar{q},$$
$$= -2/3 \quad \text{for} \quad qq \quad \text{or} \quad \bar{q}\bar{q}. \tag{2}$$

The value of the α_s, the running coupling constant, is determined by [62–64]

$$\alpha_s(\mu^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\left(\ln\frac{\mu^2 + M_B^2}{\Lambda^2}\right)} \tag{3}$$

where $\mu = 2m_a m_b/(m_a + m_b)$, $\Lambda = 0.413$ GeV, M_B is the background mass, and n_f is number of flavors [62–64]. The model parameters we have used in the present study are the same as in Refs. [62–66]. The constituent quark masses employed here are $m_u = m_d = 0.33$ GeV and $m_b = 4.88$ GeV. The degeneracy of these exotic states is removed by including the spin-dependent part of the usual one gluon exchange potential [67–70]. The potential description extended to spin-dependent interactions results in three types of interaction terms such as the spin–spin, the spin–orbit, and the tensor part. Accordingly, the spin-dependent part V_{SD} is given by

$$V_{SD} = V_{SS}\left[\frac{1}{2}\left(S(S+1) - \frac{3}{2}\right)\right]$$
$$+ V_{LS}\left[\frac{1}{2}(J(J+1) - S(S+1) - L(L+1))\right]$$
$$+ V_T\left[12\left(\frac{(S_1.r)(S_2.r)}{r^2} - \frac{1}{3}(S_1.S_2)\right)\right]. \tag{4}$$

The coefficient of these spin-dependent terms of Eq. (3) can be written in terms of the vector (V_V) and scalar (V_S) parts of the static potential described in Eq. (1) as

$$V_{LS}^{ij}(r) = \frac{1}{2M_i M_j r}\left[3\frac{dV_V}{dr} - \frac{dV_S}{dr}\right], \tag{5}$$

$$V_T^{ij}(r) = \frac{1}{6M_i M_j}\left[3\frac{d^2 V_V}{dr^2} - \frac{1}{r}\frac{dV_S}{dr}\right], \tag{6}$$

$$V_{SS}^{ij}(r) = \frac{1}{3M_i M_j}\nabla^2 V_V = \frac{16\pi\alpha_s}{9M_i M_j}\delta^3(r). \tag{7}$$

Here M_i, M_j correspond to the masses of the respective constituting two-body systems. The Schrödinger equation with the potential given by Eq. (1) is numerically solved using the Mathematica notebook of the Runge–Kutta method [71] to obtain the energy eigenvalues and the corresponding wave functions.

2.1 The four-quark state in diquark–antidiquark picture

In this section, we calculate the mass spectra of tetraquarks with hidden bottom as the bound states of two clusters (Qq and $\bar{Q}\bar{q}$) ($Q = b$; $q = u, d$). We think of the diquarks as two correlated quarks with no internal spatial excitation. Because a pair of quarks cannot be a color singlet, the diquark can only be found confined into hadrons and used as effective degree of freedom. Heavy-light diquarks may be the building blocks of a rich spectrum of exotic states which cannot be fitted in the conventional quarkonium assignment. Maiani et al. [47] in the framework of the phenomenological constituent quark model considered the masses of hidden/open charm diquark–antidiquark states in terms of the constituent diquark masses with their spin–spin interactions included. We discuss the spectra in the framework of a non-relativistic hamiltonian including chromomagnetic spin–spin interactions between the quarks (antiquarks) within a diquark (antidiquark. Masses of diquark (antidiquark) states are obtained by numerically solving the Schrödinger equation with the respective two-body potential given by Eq. (1) and incorporating the respective spin interactions described by Eq. (3) perturbatively.

In the diquark–antidiquark structure, the masses of the diquark/diantiquark system are given by

$$m_d = m_Q + m_q + E_d + \langle V_{SD}\rangle_{Qq}, \tag{8}$$

$$m_{\bar{d}} = m_{\bar{Q}} + m_{\bar{q}} + E_{\bar{d}} + \langle V_{SD}\rangle_{\bar{Q}\bar{q}}. \tag{9}$$

Further, the same procedure is adopted to compute the binding energy of the diquark–antidiquark bound system as

$$M_{d-\bar{d}} = m_d + m_{\bar{d}} + E_{d\bar{d}} + \langle V_{SD}\rangle_{d\bar{d}}. \tag{10}$$

Here Q and q represents the heavy quark and light quark, respectively. In the present paper, d and $\bar{d}$ represent diquark and antidiquark, respectively. While E_d, $E_{\bar{d}}$, $E_{d\bar{d}}$ are the energy eigenvalues of the diquark, antidiquark, and diquark–antidiquark system, respectively. The spin-dependent potential (V_{SD}) part of the hamiltonian described by Eq. (3) has been treated perturbatively. Details of the computed results are listed in Tables 1, 2, 3, 4 and 5 for the low lying positive parity and negative parity states, respectively.

2.2 Mixing of P-wave states

In the limit of a heavy quark, the spin of the light and heavy degrees of freedom are separately conserved by the strong interaction. So hadrons containing a heavy quark can be simultaneously assigned the quantum numbers $S_{Q\bar{q}}$, $m_{Q\bar{q}}$, $\bar{Q}q$, and $m_{\bar{Q}q}$. Since the dynamics depends only on the spin

Table 1 Mass spectra of four-quark states in the diquark–antidiquark picture (for $L_1 = 0$, $L_2 = 0$) (in GeV)

S_d	L_d	$S_{\bar d}$	$L_{\bar d}$	J_d	$J_{\bar d}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	0	0	0	0	0	0	0^{++}	$^1S_0$	10.309	0.0	0.0	0.0	10.309
1	0	0	0	1	0	1	1^{+-}	$^3S_1$	10.316	0.0	0.0	0.0	10.316
1	0	1	0	1	1	0	0^{++}	$^1S_0$	10.323	−0.179	0.0	0.0	10.143
						1	1^{+-}	$^3S_1$	10.323	−0.089			10.233
						2	2^{++}	$^5S_1$	10.323	0.089			10.413

Table 2 Mass spectra of four-quark states in the diquark–antidiquark picture ($L_1 = 1$, $L_2 = 0$) (in GeV)

S_d	L_d	$S_{\bar d}$	$L_{\bar d}$	J_d	$J_{\bar d}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	1	0	0	1	0	1	1^{--}	$^1P_1$	10.917	0.0	0.0	0.014	10.931
1	1	0	0	0	0	0	0^{-+}	$^3P_0$	10.917	0.000	−0.0059	−0.0286	10.883
						1	1^{-+}	$^3P_1$		0.000	−0.0029	−0.011	10.921
						2	2^{-+}	$^3P_2$		0.000	0.0029	−0.0256	10.913
1	1	1	0	0	1	1	1^{--}	$^1P_1$	10.925	−0.019	0.0	−0.0233	10.882
				1	1	0	0^{-+}	$^3P_0$		−0.0095	−0.0059	−0.0467	10.862
						1	1^{-+}	$^3P_1$			−0.0029	−0.011	10.900
						2	2^{-+}	$^3P_2$			0.0029	−0.026	10.892
				2	1	1	1^{--}	$^5P_1$			−0.0088	−0.072	10.853
						2	2^{--}	$^5P_2$			−0.0029	0.0256	10.957
						3	3^{--}	$^5P_3$			0.006	−0.037	10.903

of the light degrees of freedom, the hadron will appear in degenerate multiplets of total spin S that can be formed from diquark and antidiquark and, accordingly, we can classify the states in the convenient way. In the present study, we find that the masses of orbitally excited states with relative angular momentum $L = 1$ and total spin $S = 0, 1, 2$, corresponding to $^1P_1$, $^3P_1$, $^5P_1$, and $^3P_0$, are close to each other in the mass region around 10.850–11.201 GeV. The importance of the linear combination of scalar and axial vector states was noted by Rosner [72] and he emphasized in the context of the constituent quark model the individual conservation of heavy and light degrees of freedom in the heavy quark systems. In the mass spectra shown in Table 2, two $^1P_1$ states with masses 10.931 GeV and 10.882 GeV and another $^5P_1$ state with mass 10.853 GeV are there. Similarly, in the mass spectra shown in Table 3, there are two $^1P_1$ states with masses 10.201 GeV and 10.145 GeV, respectively, a $^3P_1$ state with mass 11.163 GeV, and a $^5P_1$ state with mass 11.117 GeV. Generally mixing is done through

$$\begin{pmatrix} |P_J > \\ |P_J' > \end{pmatrix} = U^{-1} \begin{pmatrix} |\alpha > \\ |\beta > \end{pmatrix}.$$

Here U^{-1} is given by

$$U^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

So we have shown here the possibility that these states might be getting mixed up with each other to yield mixed states according to [72–74]

$$|P_J\rangle = \sqrt{\frac{2}{3}}|\alpha\rangle + \sqrt{\frac{1}{3}}|\beta\rangle, \tag{11}$$

$$|P_J'\rangle = -\sqrt{\frac{1}{3}}|\alpha\rangle + \sqrt{\frac{2}{3}}|\beta\rangle. \tag{12}$$

Here $|\alpha\rangle$ and $|\beta\rangle$ are states with the same parity. The $|P_J' >$ and $|P_J >$ are the lower and higher eigenstates, respectively, as given in Ref. [75]. For a finite mixing angle (or mixing probability 1) the masses of the $|P_J' >$ and $|P_J >$ states will lie only between the masses of the $|\alpha >$ and $|\beta >$ states. Accordingly, we get mixed states at 10.914 GeV and 10.898 GeV for mixing of the two states $^1P_1$ (10.931 GeV) and $^1P_1$ (10.882 GeV). Similarly for the mixing of $^1P_1$ (10.931) and $^5P_1$ (10.853), we obtained states at 10.905 GeV and 10.879 GeV and for the mixing of $^1P_1$ (10.882) and $^5P_1$ (10.853) states, we obtained mixed states at 10.871 GeV and 10.862 GeV. In the same way we obtained mixed states for other combinations also. These mixed states are listed in Table 6. The masses of 1^{--} mixed states lie very close to the 10.890 resonance. We have also computed the leptonic and hadronic and radiative decay widths for these mixed states.

Table 3 Mass spectra of four-quark states in the diquark–antiquark picture ($L_1 = 1$, $L_2 = 1$) (in GeV)

S_d	L_d	$S_{\bar d}$	$L_{\bar d}$	J_d	$J_{\bar d}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	1	0	1	1	1	0	0^{++}	$^1S_0$	10.843	0.0	0.0	0.0	10.843
						1	1^{+-}	$^1P_1$	11.187	0.0	0.0	0.0139	11.201
						2	2^{++}	$^1D_2$	11.348	0.0	0.0	0.002	11.350
1	1	0	1	0	1	1	1^{+-}	$^3S_1$	10.843	0.0	0.0	0.0	10.843
						0	0^{++}	$^3P_0$	11.188	0.0	-0.0059	-0.0278	11.154
				1	1	1	1^{+-}	$^3P_1$		0.0	-0.0029	0.0069	11.192
						2	2^{++}	$^3P_2$		0.0	0.0029	-0.0069	11.184
						1	1^{+-}	$^3D_1$	11.348	0.0	-0.0007	-0.003	11.344
				2	1	2	2^{++}	$^3D_2$		0.0	-0.00024	0.0015	11.350
						3	3^{+-}	$^3D_3$		0.0	0.00049	-0.00135	11.347
1	1	1	1	0	0	0	0^{++}	$^1S_0$	10.843	-0.174	0.0	0.0	10.640
				1		1	1^{+-}	$^3S_1$		-0.092	0.0	0.0	10.751
				2		2	2^{++}	$^5S_2$		0.092	0.0	0.0	10.936
				0	1	1	1^{+-}	$^1P_1$	11.188	-0.019	0.0	-0.023	11.145
				1	1	0	0^{++}	$^3P_0$		-0.0098	-0.0059	-0.046	11.126
						1	1^{+-}	$^3P_1$		-0.0098	-0.0029	-0.011	11.163
						2	2^{++}	$^3P_2$		-0.0098	0.0029	-0.025	11.155
				2	1	1	1^{+-}	$^5P_1$		0.0098	-0.0088	-0.071	11.117
						2	2^{++}	$^5P_2$		0.0098	-0.0029	0.0255	11.220
						3	3^{+-}	$^5P_3$		0.0098	0.0059	-0.0371	11.167
				0	2	2	2^{++}	$^1D_2$	11.348	-0.0072	0.0	-0.0033	11.338
				1	2	1	1^{+-}	$^3D_1$		-0.0036	0.007	-0.0057	11.339
						2	2^{++}	$^3D_2$		-0.0036	-0.0024	-0.0017	11.344
						3	3^{+-}	$^3D_3$		-0.0036	0.00049	-0.004	11.341
						0	0^{++}	$^5D_0$		0.0036	-0.0014	-0.017	11.333
				2	2	1	1^{++}	$^5D_1$		0.0036	-0.0012	-0.010	11.340
						2	2^{++}	$^5D_2$		0.0036	-0.0007	-0.0033	11.351
						3	3^{+-}	$^5D_3$		0.0036	0	0.0047	11.357
						4	4^{++}	$^5D_4$		0.0036	0.0009	-0.0074	11.346

Table 4 First radially excited mass spectra of four-quark states in the diquark–antiquark picture (for $L_1 = 0$, $L_2 = 0$) (in GeV)

S_d	L_d	$S_{\bar d}$	$L_{\bar d}$	J_d	$J_{\bar d}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	0	0	0	0	0	0	0^{++}	$^1S_0$	10.702	0.0	0.0	0.0	10.702
1	0	0	0	1	0	1	1^{+-}	$^3S_1$	10.709	0.0	0.0	0.0	10.709
1	0	1	0	1	1	0	0^{++}	$^1S_0$	10.716	-0.066	0.0	0.0	10.650
						1	1^{+-}	$^3S_1$	10.716	-0.033			10.683
						2	2^{++}	$^5S_1$	10.716	0.033			10.750

3 $Y_b(10890)$ state and its decay properties

The prominent exotic state $Y_b(10890)$ with $J^{PC} = 1^{--}$ was first observed by the Belle collaboration [1,4] and to date, it remains to be confirmed by independent experiments. The anomalously large production cross sections for $e^+e^- \to \Upsilon(1S; 2S; 3S)\pi^+\pi^-$ measured at $\Upsilon(5S)$ was not in good agreement with the line shape and production rates for the conventional $b\bar b$ $\Upsilon(5S)$) state. An important issue is whether the puzzling events seen by Belle stem from the decays of the $\Upsilon(5S)$ or from another particle, Y_b, having a mass close enough to the mass of the $\Upsilon(5S)$. This result motivated theorists to resolve the puzzling features of this peak, which lies approximately at mass 10.890 GeV. Cur-

Table 5 First radially excited mass spectra of four-quark states in the diquark–antidiquark picture ($L_1 = 1$, $L_2 = 0$) (in GeV)

S_d	L_d	$S_{\bar{d}}$	$L_{\bar{d}}$	J_d	$J_{\bar{d}}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	1	0	0	1	0	1	1^{--}	$^1P_1$	11.140	0.0	0.0	0.011	11.151
1	1	0	0	0	0	0	0^{-+}	$^3P_0$	11.140	0.000	-0.0047	-0.022	11.114
				1		1	1^{-+}	$^3P_1$		0.000	-0.0023	-0.0055	11.144
				2		2	2^{-+}	$^3P_2$		0.000	0.0023	-0.0055	11.137
1	1	1	0	0	1	1	1^{--}	$^1P_1$	11.148	-0.021	0.0	-0.018	11.108
				1	1	0	0^{-+}	$^3P_0$		-0.0108	-0.0047	-0.036	11.095
					1	1	1^{-+}	$^3P_1$			-0.0023	-0.0092	11.125
					2	2	2^{-+}	$^3P_2$			0.0023	-0.010	11.119
				2	1	1	1^{--}	$^5P_1$		0.0108	-0.007	-0.057	11.094
						2	2^{--}	$^5P_2$			-0.002	0.020	11.177
						3	3^{--}	$^5P_3$			0.004	-0.029	11.134

Table 6 Mixed P-wave states(in GeV))

J^{PC}	State	Mixed state
1^{--}	$^1P_1$ (10.931)	10.914 (P_J)
1^{--}	$^1P_1$ (10.882)	10.898 (P'_J)
1^{--}	$^1P_1$ (10.931)	10.905 (P_J)
1^{--}	$^5P_1$ (10.853)	10.879 (P'_J)
1^{--}	$^1P_1$ (10.882)	10.862 (P_J)
1^{--}	$^5P_1$ (10.853)	10.871 (P'_J)
0^{-+}	$^3P_0$ (10.883)	10.876 (P_J)
0^{-+}	$^3P_0$ (10.862)	10.868 (P'_J)

rently, there are two competing theoretical explanations: the tetraquark interpretation on the one hand [26,29–31] and the re-scattering model [76] on the other. The tetraquark model can explain the enhancement and the resonant structure via Zweig-allowed decay processes and coupling to intermediate states, while the re-scattering model is based on the decay $\Upsilon(5S)) \rightarrow B^*\bar{B}^*$ and a subsequent recombination of the B mesons. A detailed study of this state is available in the literature [26,28,29]. The state $\Upsilon(10890)$ is usually referred to as the $\Upsilon(5S)$, since its mass is close to the mass of the 5S state predicted by potential models. However, a different proposal has been put forward by the authors of Ref. [26], in which they call this state Y_b and this state is a P-wave tetraquark analogous to $Y(4260)$, though the current experimental situation regarding the peak around 10.890 GeV is still debatable. However, here we found three vector states with $J^{PC} = 1^{--}$, whose masses are around 10.890 GeV, i.e. 10.882GeV, 10.853 GeV, and 10.931 GeV. We also found 1^{--} mixed $^1P_1$ states to lie at 10.914 and 10.898 GeV. To resolve the Y_b further we have calculated the di-electronic, hadronic, and radiative decay widths of these states.

3.1 Leptonic decay width of $J^{PC} = 1^{--}$ state

In the conventional $b\bar{b}$ systems, the decay widths are determined by the wave functions at the origin for the ground state, while for the P-waves the derivations of these wave functions at the origin are used. We have used the same Van Royen–Weisskopf formula but with a slight modification. Since the tetraquark size is larger than that of quarkonia, to take into account the larger size of the tetraquark, we have modified the wave functions by including a quantity σ, a size parameter, whose value varies from $\sigma \in [\frac{1}{2}, \frac{\sqrt{3}}{2}]$ [77]. These tetraquark wave functions will affect the decay amplitudes, thereby influencing the decay rates. The partial electronic decay widths $\Gamma_{ee[bu]}$ and $\Gamma_{ee[bd]}$ of the tetraquark states Y_{bu} and Y_{bd} made up of diquarks and antidiquarks (for up quark and down quark, respectively) are given by the well-known Van Royen–Weisskopf formula for P-waves [78],

$$\Gamma(Y_{[bu]/[bd]} \longrightarrow e^+e^-) = \frac{24\alpha^2 \langle e_Q \rangle^2}{M_{Y_b^4}} \sigma^2 |R'_{11}(0)|^2. \quad (13)$$

Here, α is the fine structure coupling constant, $\sigma < 1$, and $\langle e_Q \rangle$ is the effective charge of Qq diquark system given by [79]

$$\langle e_Q \rangle = \left| \frac{m_Q e_q - m_q e_Q}{m_Q + m_q} \right|. \quad (14)$$

For computing the leptonic decay width, we have employed the numerically obtained radial solutions while the authors of Refs. [26,78] have used a value calculated by using the $Q\bar{Q}$-onia package [80], giving $|R'_{11}(0)|^2 = 2.067$ GeV^5. Our calculated results for leptonic decay widths for Y_{bu} and Y_{bd} are shown in Table 7 with the available theoretical data. Since all the vector 1^{--} states are P-waves, the value of $R'(0)$ will not change as the masses of the diquarks remain

Table 7 Di-leptonic decay widths (in keV)

State	$\Gamma_{ee[bl]}$	$\Gamma_{ee[bh]}$
$Y_b(10882)$	0.0251	0.123
$Y_b(10853)$	0.0254	0.125
$Y_b(10931)$	0.0246	0.121
$Y(10914)$ (mixed state)	0.02485	0.122
$Y(10898)$ (mixed state)	0.02499	0.1229
$Y(10905)$ (mixed state)	0.0249	0.1226
$Y(10879)$ (mixed state)	0.02517	0.1238
$Y(10862)$ (mixed state)	0.02532	0.1245
$Y(10871)$ (mixed state)	0.02524	0.1241
Others	0.09 ± 0.03 [26]	0.08 ± 0.03 [26]
		0.12 [82]

the same. Hence, the value of leptonic decay width does not change significantly as it only varies with the mass. However, in the case of mixed states the contributions from the radial wavefunctions will be noticeable.

3.2 Hadronic decay width of $J^{PC} = 1^{--}$ state

In this section, we have studied the hadronic decay of the 1^{--} P-wave $Y_b(10890)$ state. We discuss the two-body hadronic decays, i.e. $Y_b(q) \rightarrow B_q^*(k)\bar{B}_q^*(l)$. These are Zweig-allowed processes and involve essentially the quark rearrangements. For calculating dominant two-body hadronic decay widths of the 1^{--} $Y_b(10890)$ state, the vertices are given as [81]

$$Y_b \longrightarrow B\bar{B} = F(k^\mu - l^\nu),$$
$$Y_b \longrightarrow B\bar{B}^* = \frac{F}{M}\epsilon^{\mu\nu\rho\sigma}k_\rho l_\sigma,$$
$$Y_b \longrightarrow B^*\bar{B}^* = F(g^{\mu\rho}(q+l)^\nu - g^{\mu\nu}(k+q)^\rho$$
$$+ g^{\rho\nu}(q+k)^\mu),$$

and the corresponding decay widths are given by

$$\Gamma(Y_b \longrightarrow B\bar{B}) = \frac{F^2|\vec{k}|^3}{2M^2\pi}, \tag{15}$$

$$\Gamma(Y_b \longrightarrow B\bar{B}^*) = \frac{F^2|\vec{k}|^3}{4M^2\pi}, \tag{16}$$

$$\Gamma(Y_b \longrightarrow B^*\bar{B}^*)$$
$$= \frac{F^2|\vec{k}|^3(48|\vec{k}|^4 - 104M^2|\vec{k}|^2 + 27M^4)}{2\pi(M^3 - 4|\vec{k}|^2M)^2}. \tag{17}$$

Here $|\vec{k}|$ is the center of mass momentum given by

$$|\vec{k}| = \frac{\sqrt{M^2 - (M_k + M_l)^2}\sqrt{M^2 - (M_k + M_l)^2}}{2M}. \tag{18}$$

Here M is the mass of the decaying particle and M_k, M_l are the masses of the decay products. The decay constant F is a non-perturbative quantity and to evaluate it is beyond the scope of our approach. We adopted the same approach used in [26,82] and made an estimate using the known two-body decays of $\Upsilon(5S)$, which are described by the same vertices as given in [81]. To extract the value of F and $|\vec{k}|$, we have used the values of the decay widths for the decays $\Upsilon(5S) \rightarrow B_q(k)\bar{B}_q(l)$, $B_q(k)\bar{B}_q^*(l)$, $B_q^*(k)\bar{B}_q^*(l)$ from the Particle Data Group [83]. The extracted values of F and $|\vec{k}|$ are shown in Table 8 along with the decay width results. To take into account the different hadronic sizes of the tetraquarks we have included the quantity σ, which already was discussed earlier. The results for the hadronic decay widths are shown in Table 5, which differ from the corresponding PDG [83] values of $\Upsilon(5S)$. Out of these three P-wave states, the computed value of the hadronic decay width for $Y_b(10853)$ is of the order of 50 MeV as against the PDG value of 110 ± 13 MeV and consistent with the BELLE measurements. For the other two states we get higher values than they actually should have. So out of these three states, we predict only the state with mass 10.853 GeV as a $Y_b(10890)$ state.

3.3 Radiative decay width of $J^{PC} = 1^{--}$ state

We study the radiative decays of these states using the idea of vector meson dominance (VMD) which describes the interactions between photons and hadronic matter [84] and we hope that this will increase our insight in these tetraquark states. The transition matrix element for the radiative decay of $Y_b \rightarrow \chi_b + \gamma$ is given with the use of VMD by

$$< \chi_b \mid \gamma > = < \gamma \mid \rho > \frac{1}{m_\rho^2} < \chi_b\rho \mid Y_b > \tag{19}$$

and the decay width is given by

$$\Gamma(Y_b \rightarrow \chi_b + \gamma) = 2|A^2|(\frac{f_\rho}{m_\rho^2})^2 \frac{1}{8\pi M_{Y_b}^2} \frac{(\lambda)^{\frac{1}{2}}}{2M_{Y_b}}. \tag{20}$$

Here λ is the center of mass momentum and $f_\rho = 0.152$ GeV^2 [85,86]. Similarly, we have computed the radiative decay $Y_b \rightarrow \eta_b + \gamma$. The present results are shown in Table 9 with the available theoretical data. There is no experimental data available for the radiative decay of $Y_b(10890)$ and we look forward to see the experimental support in favor of our predictions.

4 Results and discussions

We have computed the mass spectra of hidden bottom four-quark states in the diquark–antidiquark picture which are

Table 8 Reduced partial hadronic decay widths and reduced total decay widths (in keV), the extracted value of the coupling constant F and the center of mass momentum $|\mathbf{k}|$

State	Decay mode	F	$|\vec{k}|$	Γ	$\frac{\Gamma}{\sigma^2}$	$\frac{\Gamma_{tot}}{\sigma^2}$
	$Y_{bq} \to B\bar{B}$	1.35	1.31	5.500	6.790	71.89
$Y_b(10882)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.22	11.87	14.66	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.11	40.86	50.44	
	$Y_{bq} \to B\bar{B}$	1.35	1.25	4.800	5.930	56.08
$Y_b(10853)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.15	10.00	12.34	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.04	30.63	37.81	
	$Y_{bq} \to B\bar{B}$	1.35	1.41	6.800	8.400	97.45
$Y_b(10931)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.32	14.91	18.40	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.23	57.23	70.65	
Mixed P states						
	$Y_{bq} \to B\bar{B}$	1.35	1.381	6.399	7.900	89.09
$Y(10914)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.29	13.96	17.23	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.19	51.81	63.96	
	$Y_{bq} \to B\bar{B}$	1.35	1.33	5.745	7.098	76.72
$Y(10898)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.23	12.13	14.98	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.13	44.27	54.65	
	$Y_{bq} \to B\bar{B}$	1.35	1.41	6.135	7.574	84.83
$Y(10905)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.32	13.34	16.47	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.23	49.24	60.79	
	$Y_{bq} \to B\bar{B}$	1.35	1.41	5.509	6.802	73.01
$Y(10879)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.32	11.59	14.31	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.23	42.06	51.90	
	$Y_{bq} \to B\bar{B}$	1.35	1.27	5.035	6.217	64.35
$Y(10862)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.17	10.51	12.98	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.06	36.58	45.17	
	$Y_{bq} \to B\bar{B}$	1.35	1.29	5.268	6.504	69.26
$Y(10871)$	$Y_{bq} \to B\bar{B}^*$	3.12	1.19	11.04	13.63	
	$Y_{bq} \to B^*\bar{B}^*$	0.92	1.09	39.80	49.14	

listed in Tables 1, 2, 3, 4, and 5. We have taken various combinations of the orbital and spin excitations to compute the mass spectra. The computed mass spectra are compared with other available theoretical results in Fig. 1. Apart from this we mainly have paid attention to the $Y_b(10890)$ state and have computed leptonic, hadronic, and radiative decay width of Y_b, which are listed in Tables 7, 8, and 9, respectively. Apart from this, we have also addressed mixing of 1^{--} P-waves which are also listed in the respective tables. The core of the present study is that the color diquark is handled as a constituent building block. We predicted some of the bottom tetraquark states as counterparts in the charm sector. It is necessary to highlight that the observation of the bottom counterparts to the new anomalous charmonium-like states is very important, since it will allow one to distinguish between different theoretical descriptions of these states. In this viewpoint, it would also be valuable to look for the analog in the bottom sector, as states related by heavy quark symmetry may

have universal behaviors. The predicted bottom counterparts are shown in Fig. 2 for better understanding. In the present study, we have noticed that the mass difference between the predicted $X_b(10233)$ and $\chi_{b1}(9892)$

$$M_{X_b} - M_{\chi_{b_1}} \sim 341 \, \text{MeV}, \tag{21}$$

which is of the same order of magnitude as the mass difference between $X(3872)$ and $\chi_{c_1}(3510)$ of the charm sector,

$$M_X - M_{\chi_{c_1}} \sim 360 \, \text{MeV}. \tag{22}$$

This kind of similarity between the charm and the bottom sector is very interesting. We found that the mass difference between $X_b(10143)$ and its first radially excited state $X_b(10650)$ states is $\sim 510 \, \text{MeV}$, similar to charmonia, which is about 590 MeV. In the same way, we found that the mass difference between $X_b(10233)$ and its first radially excited state $X_b(10683)$ states is $\sim 450 \, \text{MeV}$. So by taking the evidence from these results, we can say that the four-quark

Table 9 Radiative decay widths (in keV))

State	$\Gamma \to \chi_b + \gamma$	$\frac{\Gamma \to \chi_b + \gamma}{\Gamma \to \Upsilon + \pi^+ + \pi^-}$	$\Gamma \to \eta_b + \gamma$	$\frac{\Gamma \to \eta_b + \gamma}{\Gamma \to \Upsilon + \pi^+ + \pi^-}$
$Y_b(10882)$	0.173	0.293	0.247	0.418
$Y_b(10853)$	0.169	0.286	0.243	0.413
$Y_b(10931)$	0.179	0.304	0.252	0.427
Mixed P states				
$Y(10914)$	0.177	0.300	0.250	0.424
$Y(10898)$	0.175	0.296	0.248	0.421
$Y(10905)$	0.176	0.298	0.249	0.422
$Y(10879)$	0.172	0.292	0.246	0.418
$Y(10862)$	0.170	0.288	0.244	0.415
$Y(10871)$	0.171	0.291	0.245	0.416
Others [82]	–	0.3	–	0.5

Table 10 Interpretation of some first radially excited states

J^{PC}	State	First radial excitation	Exp
0^{++}	$X_b(10.143)$	$10.650[Z_b(10650)]$	10.652 ± 0.0025 [3]
1^{+-}	$X_b(10.233)$	$10.683[Z_b(10650)]$	
1^{--}	$Y_b(10.853)$	$11.095[Y_b(?)]$	
1^{--}	$Y_b(10.882)$	$11.108[Y_b(?)]$	
1^{--}	$Y_b(10.931)$	$11.151[Y_b(?)]$	

Fig. 1 Mass spectra of bottom tetraquark states (in GeV).

state in the bottom sector, analogous to the charm sector, should exist. We have predicted some of the radially excited states which are listed in Table 10. Accordingly, we predicted the $Z_b(10650)$ state as the first radial excitation of either the $X_b(10143)$ (0^{++}) state or the $X_b(10233)$ (1^{+-}) state. The authors of Ref. [87] studied the masses of the S-wave $[bq][\bar{b}\bar{q}]$ tetraquark states with the inclusion of chromomagnetic interaction and they predicted that the lowest $[bq][\bar{b}\bar{q}]$ tetraquark state appears at 10.167 GeV. This result is consistent with the results of Ref. [88] where, using the color-magnetic interaction with the flavor symmetry breaking corrections, the $[bq][\bar{b}\bar{q}]$ tetraquark states were predicted to be around 10.2–10.3 GeV. The same results are found by the authors of Refs. [36,37], who have used the QCD sum rule approach for the computation of the mass spectra of the $[bq][\bar{b}\bar{q}]$ tetraquark state. The authors of Ref. [41] have used different tetraquark $[bq][\bar{b}\bar{q}]$ currents and they have obtained $M_{X_b} = (10220 \pm 100)$ MeV, which is in complete agreement with the result of Ref. [42]. These predictions of the X_b state and its production rates in hadron–hadron collisions have indicated a promising prospect to find the X_b at hadron colliders, in particular the LHC, and we suggest our experimental colleagues to perform an analysis. Such an attempt will likely lead to the discovery of the X_b and thus enrich the list of exotic hadron states in the heavy bottom sector. The observation of the X_b will provide a deeper insight into the exotic
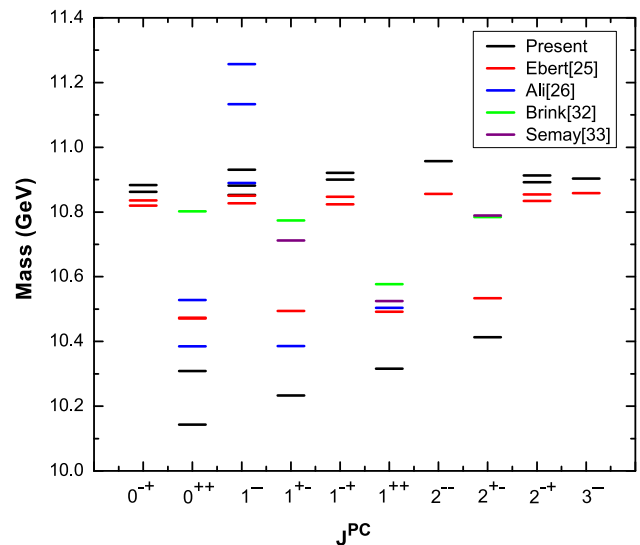
hadron spectroscopy and is helpful to unravel the nature of the states connected by the heavy quark symmetry. Similarly, there exist other radial excited states in the region 11.095–11.151 GeV, corresponding to 2P states. We look forward to see experimental searches for these states. The authors of Refs. [89,90] have predicted the $Z_b(10650)$ state as a di-mesonic molecular state in the ground state. From our present study, we suggest that if Z_b states are diquark–diantiquark states, then they are not the ground state of a bottomonium-like four-quark state but the first radially excited state of its ground state, which lies in the range 10.100–10.300 GeV, which is in agreement with the results reported by the authors of Ref. [91]. The same presumption was made by the authors of Refs. [48,49,60] to explain $Z(4430)$ state as an excitation of state $Z_c(3900)/Z_c(3885)$ in the charm sector. So in this conjecture, our prediction regarding $Z_b(10650)$ state is just a straightforward extension to the beauty sector and we observe that the $Z_b(10650)$ is also a radially excited state of a still unmeasured X_b state just like that of authors of Ref. [91], who predicted the $Z_b(10610)$ state as the radial excitation of $X_b(10100)$ such that the mass difference is $M_{Z_b}(10650) - M_{X_b(10143)} \sim 510$ MeV, which is very close to the mass difference of $\Upsilon(2S) - \Upsilon(1S) = 560$ MeV. To have a clear-cut picture of the discussion made regarding the bottom exotic states, the above discussed exotic states are displayed in Fig.2, with analogous states at the charm sector.

The comparison between the bottom tetraquark states and charm tetraquark states accentuates the resemblance between the presumptions made in the present study, namely the existence of $X_b(10143)$ as a ground state of $Z_b(10650)$, and the presumption related to the existence of ground state of $Z(4430)$ made in Refs. [48,49,60]. The presumption of bottom tetraquark states analogous to charm spectra should stim-
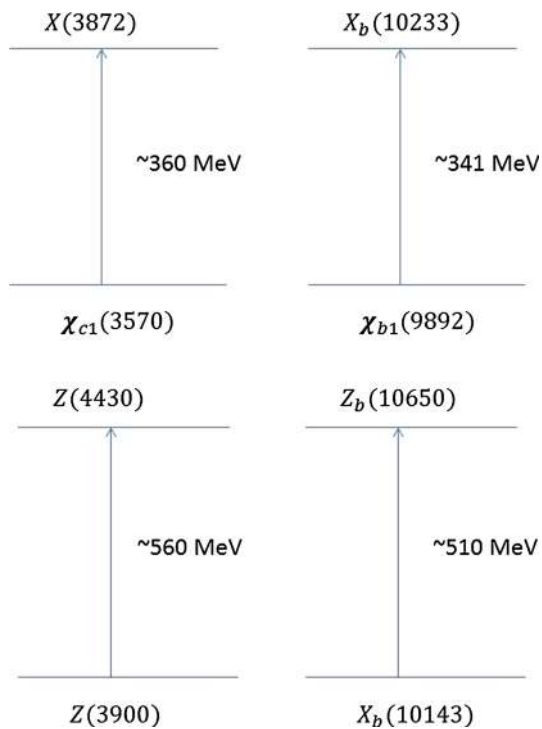
Fig. 2 Bottom tetraquark states analog of charm tetraquark states in the mass region of interest

ulate searches for these states in both the beauty and the charm sector within the mass range around 10 100–10 300 and 3500–3870 MeV, respectively. The search for these states would not only enable one to find an unobserved state as shown in Fig. 1 but also enable one to detect many more prominent states in these mass ranges. As the $Y_b(10890)$ state with quantum number 1^{--} is of our keen of interest, in this study we have predicted three P-wave 1^{--} states in the mass region around 10.850–10.931 GeV. We have observed that the P-wave state has mass 10.853 GeV as the Y_b state. The calculated partial electronic decay widths for the P-wave Y_b is about 0.03–0.12 keV, which is in agreement with the available experiment data [6] and other theoretical predictions [26,82]. Our present calculation shows that the leptonic width of Y_b is much lower than that of the width of the conventional state $\Upsilon(5S)(0.31 \pm 0.07$ keV) [83]. From this we can say that the $\Upsilon(10890)$ peak is different from the $\Upsilon(5S)$ and possibly is $Y_b(10890)$ only. We have also computed the two-body hadronic decays of Y_b. The total hadronic decay width is of the order of 50 MeV, which is lower than the total decay width of the $\Upsilon(5S) = 110$ MeV state. So this narrow width state $Y_b(10890)$ might be a tetraquark state only rather than the conventional $b\bar{b}$ state. We have also computed the radiative decay widths of Y_b, but due to lack of experimental results we cannot draw any concrete conclusion here. These results can be guidelines for future studies. In the absence of experimental data, we cannot draw any conclusion regarding

mixing of P-wave states but we expect that our results could be helpful to understand the structure of these states. Our computed masses of the 1^{--} mixed states, i.e. the $^1P_1$ and $^5P_1$ states, lie very close to the $Y(10890)$ state by at most an order of ± 20 MeV. So we look forward to see the experimental search for these states with very high precision as these states are very closely spaced. The experiments should have in principle the sensitivity to detect and also to explore the nature of such near-lying states. The present study of mixing is an attempt to signify its importance to further resolve the mystery of $Y_b(10890)$. If the status of $Y_b(10890)$ is confirmed then it will be a major step in the direction of testing the models and provide theorists with vital input to present a credible explanation of this new form of hadrons.

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