

TEXTURE AND MAGNETIC PROPERTIES

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The magnetic properties of ferromagnetic materials are closely related to the crystallographic texture, i.e. the orientation distribution of the crystallites, they are, however, not uniquely determined by the texture. Higher order “textural quantities” such as, for instance, orientation correlation functions are additionally needed for a more complete description.

The properties in saturation are uniquely determined by the texture. They can be expressed by a fourth—or sixth—order approximation of the texture function which can be obtained by “low resolution” texture measurements.

On the other hand, magnetic anisotropy measurements can only provide a low-resolution approximation to the texture function and not the complete function.

The distribution of magnetization directions can be described by a “magnetic texture” which can be calculated from pole figures measured by neutron diffraction using the method of superposed pole figures.

KEY WORDS Crystallographic texture, magnetic texture, magnetization energy, magnetostriction, induced anisotropy, neutron diffraction.

When we speak about magnetic materials, we usually mean ferromagnetic materials and within these, hard and soft materials are being distinguished. Furthermore, nearly all technologically used materials have a polycrystalline structure. Properties of polycrystalline materials, in general, depend on the properties of the individual crystallites and on the structural parameters of the polycrystal, i.e. the sizes, shapes, arrangements, and crystallographic orientations of the constitutive crystallites, and the crystals, in turn, may also contain lattice defects which may drastically change their properties compared with the properties of ideal crystals.

The basis of ferromagnetism is the spontaneous magnetization which is accompanied by a deformation of the crystal lattice, the magnetostriction. Both effects are strongly anisotropic i.e. they depend on the crystallographic direction in which they are being considered. The properties of a polycrystalline material will thus depend on the distribution of crystallographic orientations i.e. on the texture of the material.

The texture is defined by the orientation distribution function of the crystallites irrespective of their sizes, shapes and arrangements. If we consider the total magnetic energy of a big ideal single crystal, it is seen that this energy does not only depend on the crystallographic directions of the spontaneous magnetization directions but also on the size and shape of the crystal and the magnetic field distribution in the environment of the crystal. Hence, the texture is only one of several structural parameters which have an influence on the magnetic properties of a polycrystalline material. The sizes and shapes of the crystallites are two further parameters which have to be taken into account, and the environments of

the crystallites can be taken into account, in a first approximation, by the orientation correlation function which describes the orientation differences of neighbouring crystallites.

In this paper we shall restrict ourselves mainly to consider the influence of texture on the magnetic properties of a material.

THE CRYSTALLOGRAPHIC TEXTURE

In order to define the crystallographic orientation of a crystallite in a polycrystalline sample, we need two different coordinate systems, i.e. the crystal coordinate system K_B which consists of a triple of selected crystallographic directions e.g. [100], [010], [001] and the sample coordinate system K_A consisting of three sample directions e.g. rolling direction (RD), transverse direction (TD), and normal direction (ND) of a sheet. The crystal orientation is then defined by the rotation g which transforms the coordinate system K_A into K_B as shown in Figure 1

$$K_B = g \cdot K_A \quad (1)$$

The rotation g can be specified in several ways e.g. by a transformation matrix g_{ik} , by the rotation axis r and the rotation angle ω , or by the Euler angles φ_1 , Φ , φ_2

$$g = [g_{ik}] = \{r_{\theta\psi}, \omega\} = \{\varphi_1, \Phi, \varphi_2\} \quad (2)$$

The texture of a polycrystal can then be defined by the volume fraction of crystals having the orientation g within the angular limits dg

$$f(g) = \frac{dV/V}{dg} \quad (3)$$

The texture function must be invariant with respect to the crystal symmetry rotations g^c and it can be invariant with respect to certain sample symmetries g^s . Hence, it is

$$f(g^c \cdot g \cdot g^s) = f(g) \quad (4)$$

In the case of cubic symmetry there are 24 rotations g^c and in the case of orthorhombic sheet symmetry there are 4 rotations g^s , i.e. 96 symmetry operations.

Besides this basic distribution function, the ODF, two other deduced distribution functions are often considered, i.e. the pole distribution functions or pole figures and the axis distribution functions or inverse pole figures. If $y = \{\alpha, \beta\}$ is a specific sample direction and $h = \{\Theta, \gamma\}$ is a specific crystal direction then these

$$K_A \xrightarrow{g} K_B$$

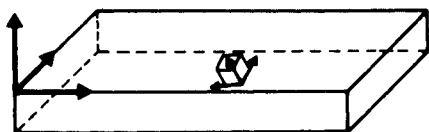


Figure 1 The orientation of a crystallite in a polycrystalline material is defined by the rotation g which transforms the sample coordinate system K_A into the crystal coordinate system K_B .

functions are defined by the volume fractions of crystals for which h is parallel to y

$$P_h(y) = \frac{dV(g)/V}{dy} \quad \begin{array}{l} \text{Pole figure} \\ h = \text{const} \end{array} \quad (5)$$

$$R_y(h) = \frac{dV(h)/V}{dh} \quad \begin{array}{l} \text{inverse pole figure} \\ y = \text{const} \end{array} \quad (6)$$

These functions are integrals over the ODF

$$P_h(y) = R_y(h) = \frac{1}{2\pi} \int_{h \parallel y} f(g) \cdot d\psi \quad (7)$$

Thereby the integral is to be taken over all those crystal orientations for which the sample direction y is parallel to the crystal direction h . The two directions are related to each other by the crystal orientation g

$$y = g \cdot h \quad y_i = g_{ik} \cdot h_k \quad (8)$$

Usually, the pole figures are considered for crystal directions h perpendicular to low index lattice planes (hkl). These pole figures can be measured by X-ray, neutron, or electron diffraction. They are thus the primary data of most of all experimental texture investigations. The texture function $f(g)$ is then obtained by solving Eq. (7). This procedure, shown schematically in Figure 2 is called pole

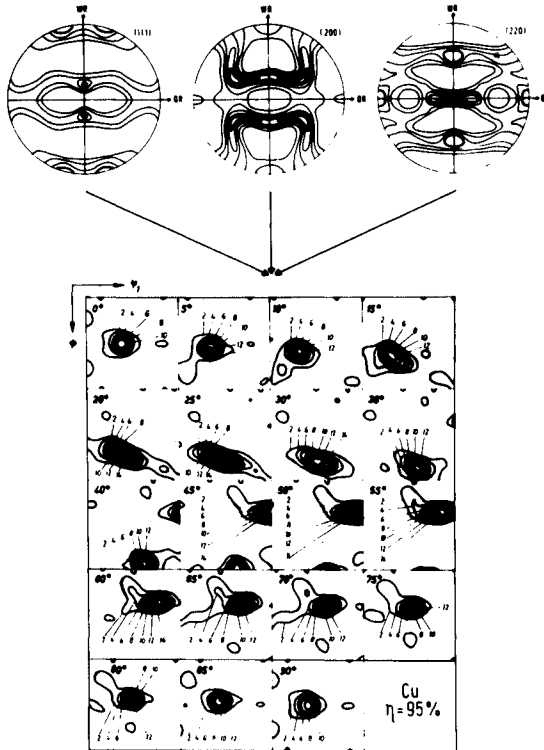


Figure 2 The orientation distribution function can be calculated from a sufficient number of experimentally measured pole figures.

figure inversion. It requires a certain number of measured pole figures which depends on the crystal symmetry and on the required angular resolving power in $f(g)$. The inverse pole figures, on the other hand, describe the distribution of crystal directions in a given sample direction y . They are thus closely related to mean values of physical properties in the considered sample direction. (For further details see e.g. Bunge, 1982; Bunge and Esling, 1986; Bunge, 1987).

GENERALIZATION OF THE CONCEPT OF TEXTURE

The environment of a crystal in the polycrystalline structure can, to a certain extent, be taken into account by the misorientation distribution function and the orientation correlation function. If two neighbouring crystals have the orientations g_1 and g_2 then the orientation "difference" or misorientation across their boundary is given by

$$\Delta g = g_2 \cdot g_1^{-1} \quad (9)$$

The misorientation distribution function MODF can be defined by the area fraction of boundaries across which the orientation difference Δg exists

$$F(\Delta g) = \frac{dA(\Delta g)/A}{d \Delta g} \quad (10)$$

If the positions of the crystals of the orientations g_1 and g_2 are statistically uncorrelated, then the area fraction $dA(\Delta g)/A$ of grainboundaries Δg depends only on the relative frequency of g_1 - and g_2 - grains i.e. on the texture. In this case the uncorrelated MODF is given by

$$F_u(\Delta g) = \oint f(\Delta g \cdot g) \cdot f(g) \cdot dg \quad (11)$$

This situation may arise, for instance, during primary recrystallization when independently formed nuclei grow until they impinge upon each other. If there is, however, a preference of certain orientation differences Δg , then an orientation correlation function may be defined by

$$\varphi(\Delta g) = \frac{F(\Delta g)}{F_u(\Delta g)} \quad (12)$$

The correlation function is one of a larger number of "generalized textural quantities" which are, in principle, independent of the texture.

The MODF and the correlation function must be invariant with respect to the crystal symmetries of both crystals. Hence it must be

$$F(g^{c_1} \cdot \Delta g \cdot g^{c_2}) = F(\Delta g) \quad (13)$$

$$\varphi(g^{c_1} \cdot \Delta g \cdot g^{c_2}) = \varphi(\Delta g) \quad (14)$$

In the case of cubic crystal symmetry, these are $24 \cdot 24 = 576$ symmetry operations (see e.g. Bunge and Weiland, 1988).

THE MAGNETIC TEXTURE

The definitions of the “crystallographic” texture can also be applied to describe the magnetic state of a polycrystalline, ferromagnetic material. If the crystal anisotropy is not too low, then the spontaneous magnetization M^s (without an external field) is parallel to one of several symmetrically equivalent crystal directions h_0^i shown in Figure 3a. If we include the actual magnetization direction in the crystal symmetry considerations, then the symmetry of a spontaneously magnetized crystal is lower than that of the non-magnetized crystal. In this case we may choose another crystal coordinate system K_B^M in each domain for example in such a way that the magnetization direction h_0^i becomes the x_3 -direction as is shown in Figure 3b. The magnetic orientation can then be defined by

$$K_B^M = g \cdot K_A \tag{15}$$

and the magnetic texture is given by the distribution function

$$f^M(g) = \frac{dV(g)/V}{dg} \tag{16}$$

In the case of cubic symmetry, for instance, the spontaneous magnetization may be in [001] direction which reduces the cubic symmetry to tetragonal. The so defined magnetic texture has the symmetries

$$f^M(g^M \cdot g) = f^M(g) \tag{17}$$

where g^M are, for instance, the tetragonal symmetry rotations. The magnetic texture will generally not obey the sample symmetry g^s defined in Eq. (4). The magnetic texture is related to the crystallographic texture by the condition

$$f(g) = \sum_{g^c} f^M(g^c \cdot g) \tag{18}$$

and in the demagnetized state, the magnetic texture f_0^M is identical with the crystallographic texture

$$f_0^M(g) = f(g) \tag{19}$$

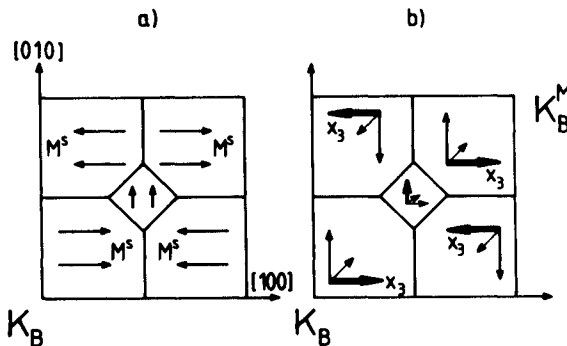


Figure 3 Definition of the magnetic texture.

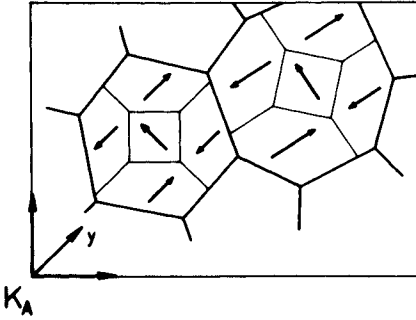


Figure 4 The distribution of magnetization directions with respect to the sample coordinate system K_A defines the magnetic pole figure.

Indeed, Eq. (19) may be considered as the definition of the demagnetized state. (This is a stronger condition than requiring only $M = 0$ at $H = 0$). The distribution of magnetization directions is given by the x_3 -pole figure of the magnetic texture $f^M(g)$, Figure 4,

$$P_{x_3}^M(y) = \frac{dV(y)/V}{dy} \quad (20)$$

which is related to the magnetic texture function $f^M(g)$ by the relationship equivalent to Eq. (7)

$$P_{x_3}^M(y) = \frac{1}{2\pi} \int_{x_3 \parallel y} f^M(g) \cdot d\psi \quad (21)$$

It should be mentioned that this function is not random in the demagnetized state of a textured material. Rather it is identical with the h_0 -pole figure of the crystallographic texture

$$P_{x_3}^M(y)_0 = P_{h_0}(y) \quad (22)$$

It should also be mentioned that the pole figure of the crystallographic, as well as of the magnetic texture, Eq. (7), and Eq. (21), are generally not centrosymmetric

$$P_h(-y) \neq P_h(+y) \quad (23)$$

This applies, of course, particularly to the distribution of magnetization directions

$$P_{x_3}^M(-y) \neq P_{x_3}^M(+y) \quad (24)$$

The component of the magnetization in the sample direction z is given by

$$M(z) = \oint P_{x_3}^M(y) \cdot \cos \langle y, z \rangle \cdot dy \quad (25)$$

as is shown schematically in Figure 5. This function is zero in the demagnetized state for any direction z .

The definition of the magnetic texture, Eq. (16), can also be applied in the case of weak external magnetic fields as long as the fields does not rotate the spontaneous magnetization out of the preferred crystallographic directions h_0^i . In this case the magnetic texture function $f^M(g, H)$ describes the magnetization state in the corresponding parts of the hysteresis loop, especially the initial magnetization and the remanence.

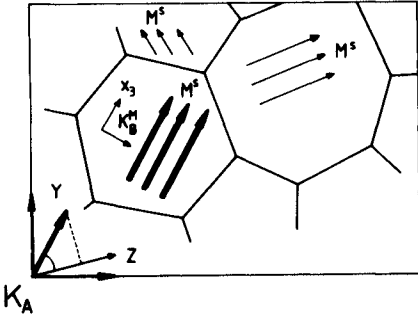


Figure 5 The component of the total magnetization vector in the sample direction z is given by an integral over the magnetic pole figure.

In higher external fields, the spontaneous magnetization will no longer be in the initial directions h_0^i . Then the definition of Eq. (16) can no longer be used. In this case, two other distribution functions may, however, be useful, i.e. the distribution of magnetization directions with respect to the sample coordinate system K_A and to the crystallographic coordinate system K_B as is shown in Figure 6

$$P^M(y) = \frac{dV(y)/V}{dy} \tag{26}$$

$$R^M(h) = \frac{dV(h)/V}{dh} \tag{27}$$

These functions are the magnetic pole figure and inverse pole figures respectively.

The component of the total magnetization in the sample direction z is still given by Eq. (25) with $P^M(y)$ replacing $P_{x_3}^M(y)$. Under the assumptions made above, it is for low magnetic fields

$$P^M(y) = P_{x_3}^M(y); \quad |H| \rightarrow 0 \tag{28}$$

whereas in the vicinity of saturation the magnetization direction is parallel to the field direction H . Then $R^M(h)$ is given by the inverse pole figure of the crystallographic texture defined in Eq. (6)

$$R^M(h) = R_H(h); \quad |H| \rightarrow \infty \tag{29}$$

In this approximation, the high-field state is completely determined by the crystallographic texture whereas the low-field state is, in general, not.

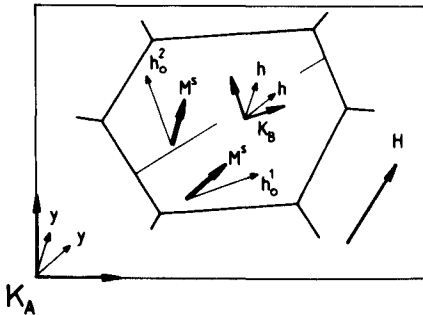


Figure 6 In higher magnetic fields the magnetization is rotated out of the directions h_0^i of easiest magnetization. The distribution of magnetization directions can be considered in the sample coordinate system K_A (magnetic pole figure) or in the crystal coordinate system (inverse pole figure).

THE MAGNETIZATION ENERGY

The total magnetization energy of a polycrystalline ferromagnetic material contains the crystal energy, stress energy, magnetization interaction energies and the field energy (see e.g. Chikazami, 1964). As far as the influence of texture is concerned, the crystal energy and the field energy can easily be considered whereas the stress energy and the interaction energy depend on other structural parameters, too, e.g. the correlation function defined in Eq. (12).

The crystal energy depends on the crystallographic direction of the magnetization $E^K(h)$ whereas the field energy is given by the cosine between magnetization and field direction. Hence, the total of these two energies is given by

$$E(g, h) = E^K(h) + |H| \cdot \cos \langle H, g \cdot h \rangle = \min \quad (30)$$

Thereby h is the crystallographic direction and $g \cdot h$ the sample direction of the spontaneous magnetization. The actual magnetization direction h minimizes Eq. (30). In zero field $H = 0$ Eq. (30) has several crystallography equivalent solutions

$$h = h_0^i; \quad H = 0 \quad (31)$$

In high magnetic fields, sufficient to saturation, the magnetization direction $y = g \cdot h$ is parallel to the external field H . In low magnetic fields Eq. (30) will generally have several (crystallographically non-equivalent) relative minima which shift continuously into h_0^i with $H \rightarrow 0$.

$$h^i = h^i(g, H) \quad (32)$$

In high fields, in the vicinity of saturation, only one of these directions will be left, i.e. that one which is nearest to the field direction

$$h_r = h_r(g, H) \quad (33)$$

With $H \rightarrow 0$, this direction h_r becomes one of the directions h_0^i . This corresponds to the remanent state

$$h_r = h_0^x \quad (34)$$

If the material is at first magnetized to saturation and then the field is being relaxed then the magnetization directions will be the ones given in Eq. (33). In this case the magnetization direction is uniquely determined by the crystal orientation g and the field H . Hence, in a polycrystal, the magnetization curve from saturation to remanence is mainly determined by the crystallographic texture (Figure 7).

The total magnetization energy of the textured polycrystal is given by the integral

$$\bar{E}(H) = \oint E(g, H) \cdot f(g) \cdot dg \quad (35)$$

with $h = h_r$ according to Eq. (33). The distribution of magnetization directions is given by

$$P^M(y, H) = \int_{y \parallel h_r} f(g) \cdot d\psi \quad (36)$$

$$y = g \cdot h_r(g, H)$$

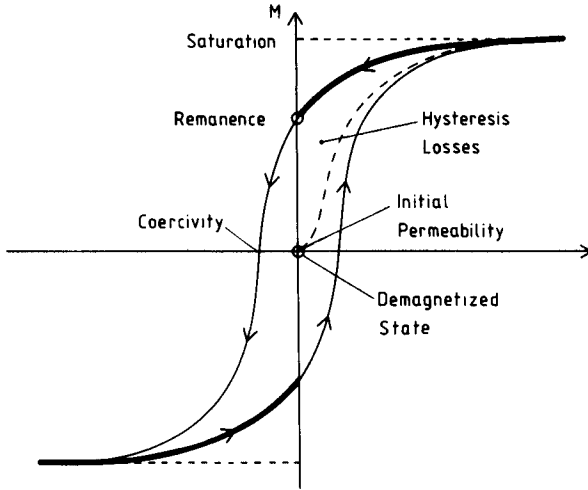


Figure 7 The hysteresis loop (schematic). The part between saturation and remanence is uniquely related to the crystallographic texture.

Hence, in the validity range of this approximation, the magnetization energy $\bar{E}(H)$ and the distribution of magnetization directions $P^M(y, H)$ are completely determined by the crystallographic texture. In the saturation region, the magnetization is in the sample direction y which is parallel to the direction of the magnetic field H . Then Eq. (35) takes on the simpler form

$$\bar{E}(y) = \oint E^K(h) \cdot R_y(h) \cdot dh \tag{37}$$

In other parts of the hysteresis loop, the magnetization may assume different directions h^i according to Eq. (32). The population of these directions will then mainly be determined by other contributions to the total magnetization energy not considered in Eq. (30), e.g. by the stress energy or energies due to inhomogeneities. If these contributions to the total energy are strong enough, then even the directions themselves will be changed. Stresses and structural inhomogeneities are closely related to crystal orientation but they are not uniquely determined by the orientation. Hence, in these cases it is quite difficult to separate the influence of texture from other structural variables. Thus, a general theory of the texture dependence for these parts of the hysteresis loop cannot be given. Nevertheless, also these parts of the hysteresis loop are strongly texture dependent.

The magnetization energy can be determined from torque curve measurements. Thereby the absolute value of the external field $|H|$ is being kept constant. The sample is oriented such that the field direction H is parallel to the sample direction $y = \{\alpha, \beta\}$. The sample usually has the form of a circular disc with its axis fixed perpendicular to the field direction ($\alpha = 90^\circ$) then the torque with respect to the angle β can be measured

$$M(\beta) = \frac{\partial \bar{E}(\beta)}{\partial \beta} \tag{38}$$

MAGNETOSTRICTION

When a ferromagnetic crystal undergoes the magnetic ordering, i.e. the spontaneous magnetization is being developed in the crystal direction h then the crystal lattice deforms according to the strain tensor ε (see e.g. Chikazumi, 1964). This magnetization effect can be described by a fourth-rank tensor

$$\varepsilon_{ij} = \lambda_{ijkl}^0 \cdot h_k \cdot h_l \quad (39)$$

where λ_{ijkl}^0 are the components of the magnetorestriction tensor referred to the crystal coordinate system K_B . In a polycrystalline material, the macroscopic mean value of the strains is to be considered. Strictly speaking, this cannot be done without taking elastic interactions into account. Nevertheless in a first approximation, which is often used, the simple orientation mean value of Eq. (39) can be taken. For this purpose Eq. (39) must be referred to the common sample coordinate system K_A

$$\varepsilon_{ij} = \lambda_{ijkl}(g) \cdot y_k \cdot y_l \quad (40)$$

Thereby the sample direction y of the spontaneous magnetization may depend non-uniquely on g and H as described by Eq. (32). A unique situation exists in the saturated state where the direction of the spontaneous magnetization is the same sample direction y in all crystals. The orientation mean value of Eq. (40) may then be written

$$\bar{\varepsilon}_{ij} = \bar{\lambda}_{ijkl} \cdot y_k \cdot y_l \quad (41)$$

with

$$\bar{\lambda}_{ijkl} = \oint \lambda_{ijkl}(g) \cdot f(g) \cdot dg \quad (42)$$

(This is equivalent to the Reuss-averaging of elastic strains).

Hence, saturation magnetostriction depends only on the crystallographic texture. Furthermore, the tensor components $\lambda_{ijkl}(g)$ can be expressed by the components λ_{mnop}^0 referred to the crystal coordinate system K_B and the components of the transformation matrix

$$\lambda_{ijkl}(g) = \lambda_{mnop}^0 \cdot g_{im} \cdot g_{jn} \cdot g_{ko} \cdot g_{lp} \quad (43)$$

with g_{ij} according to Eq. (2). Hence, Eq. (42) can be written

$$\bar{\lambda}_{ijkl} = \lambda_{mnop}^0 \cdot \bar{T}_{ijkl}^{mnop} \quad (44)$$

where the quantities \bar{T} depend only on the texture

$$\bar{T}_{ijkl}^{mnop} = \oint g_{im} \cdot g_{jn} \cdot g_{ko} \cdot g_{lp} \cdot f(g) \cdot dg \quad (45)$$

and λ_{mnop}^0 are the single crystal constants (Morris, 1969).

INDUCED ANISOTROPY

If the ferromagnetic material is a two-component solid solution the pair interaction energy between A - B -pairs may depend on the angle between the

magnetization direction h and the pair direction h_p

$$E_{AB} = K \cdot \cos^2 \langle h, h_p \rangle \quad (46)$$

Hence, the total energy depends on the number n^i of A - B -pairs in the crystallographically equivalent directions h_p^i (see e.g. Chikazumi, 1964)

$$E(h) = K \sum_i n^i \cdot \cos^2 \langle h, h_p^i \rangle \quad (47)$$

If the A and B -atoms have the possibility to change their positions during magnetic annealing then the quantities n^i may assume different values which is called directional ordering. If the magnetization direction during magnetic annealing is the crystal direction k then $n^i(k)$ minimize Eq. (47)

$$E(k) = K \sum_i n^i(k) \cdot \cos^2 \langle k, h_p^i \rangle = \min \quad (48)$$

with these values of n^i , Eq. (47) takes on the form

$$E(h, k) = K \cdot \sum_i n^i(k) \cdot \cos^2 \langle h, h_p^i \rangle \quad (49)$$

which can be written in the form

$$E(h, k) = P_{ijkl}^0 \cdot h_i \cdot h_j \cdot k_k \cdot k_l \quad (50)$$

Thereby P_{ijkl}^0 are the components of a fourth-rank tensor referred to the crystal coordinate system. In order to consider the orientation mean value of Eq. (50) this tensor has to be referred to the sample coordinate system

$$E(y, z) = P_{ijkl}(g) \cdot y_i \cdot y_j \cdot z_k \cdot z_l \quad (51)$$

where y and z are the sample directions of the magnetization during measurement and magnetic annealing respectively. Eq. (51) can then be averaged

$$\bar{E}(y, z) = \bar{P}_{ijkl} \cdot y_i \cdot y_j \cdot z_k \cdot z_l \quad (52)$$

with

$$\bar{P}_{ijkl} = P_{mnop}^0 \cdot \bar{T}_{ijkl}^{mnop} \quad (53)$$

The texture coefficients \bar{T} are the same as in Eq. (44) i.e. they are defined by Eq. (45).

SERIES REPRESENTATION OF THE TEXTURE

The texture function $f(g)$ defined in Eq. (3) can be represented by a series of harmonic functions which obey the crystal and sample symmetry according to Eq. (4) (Bunge, 1982)

$$f(g) = \sum_{\lambda=0}^L \sum_{\mu=1}^{M(\lambda)} \sum_{\nu=1}^{N(\lambda)} C_{\lambda}^{\mu\nu} \cdot \bar{T}_{\lambda}^{\mu\nu}(g) \quad (54)$$

Thereby (\cdot) represents the sample symmetry and $(:)$ the crystal symmetry and M depends on the crystal symmetry and N on the sample symmetry. The degree L

of the series is, in principle, to be extended to infinity. In practice, however, a finite degree is sufficient e.g. $L = 22$, or 34 are often used.

Similarly the functions P and R defined in Eq. (5,6) can be written

$$P_h(y) = \sum_{\lambda=0}^L \sum_{\nu=1}^{N(\lambda)} F_{\lambda}^{\nu}(h) \cdot \dot{K}_{\lambda}^{\nu}(y) \quad (55)$$

$$R_y(h) = \sum_{\lambda=0}^L \sum_{\mu=1}^{M(\lambda)} H_{\lambda}^{\mu}(y) \cdot \dot{K}_{\lambda}^{\mu}(h) \quad (56)$$

with the coefficients

$$F_{\lambda}^{\nu}(h) = \frac{4\pi}{2\lambda + 1} \sum_{\mu=1}^{M(\lambda)} C_{\lambda}^{\mu\nu} \cdot \dot{K}_{\lambda}^{*\mu}(h) \quad (57)$$

$$H_{\lambda}^{\mu}(y) = \frac{4\pi}{2\lambda + 1} \sum_{\nu=1}^{N(\lambda)} C_{\lambda}^{\mu\nu} \cdot \dot{K}_{\lambda}^{\nu}(y) \quad (58)$$

Thereby $\dot{K}_{\lambda}^{\mu}(h)$ are spherical harmonics of crystal symmetry and $\dot{K}_{\lambda}^{\nu}(y)$ those of sample symmetry.

The magnetic texture defined in Eq. (16) can be developed into a series similar to Eq. (54) with the lower magnetic symmetry replacing crystal symmetry and the sample symmetry being triclinic in the most general case. The relationship, Eq. (18), between magnetic and crystallographic texture can then be expressed in terms of the coefficients of the two series.

If we use the most general representation (without symmetries) for the magnetic texture

$$f^M(g) = \sum_{\lambda=0}^L \sum_{m=-\lambda}^{+\lambda} \sum_{n=-\lambda}^{+\lambda} M C_{\lambda}^{mn} \cdot T_{\lambda}^{mn}(g) \quad (59)$$

then Eq. (18) takes on the form

$$\sum_{m=-\lambda}^{+\lambda} M C_{\lambda}^{mn} \sum_{g^c} T_{\lambda}^{ms}(g^c) = \sum_{\mu=1}^{M(\lambda)} \sum_{\nu=1}^{N(\lambda)} C_{\lambda}^{\mu\nu} \cdot \dot{A}^{s\mu} \cdot \dot{A}^{n\nu} \quad (60)$$

whereby the quantities A are symmetry coefficients of the crystal and sample symmetry respectively as defined by Bunge (1982). The series expansion is an optimal representation for the calculation of orientation mean values Eq. (35, 37, 45). For this purpose, the energy $E(g, H)$ in Eq. (35) must also be written as series expansion

$$E(g, H) = \sum_{\lambda=0}^{L_0} \sum_{\mu=1}^{M(\lambda)} \sum_{\nu=1}^{N(\lambda)} e_{\lambda}^{\mu\nu}(H) \cdot \dot{T}_{\lambda}^{\mu\nu}(g) \quad (61)$$

The degree L_0 may be different from L used in Eq. (54). The integral Eq. (35) can then be expressed in terms of the respective coefficients

$$\bar{E}(H) = \sum_{\lambda=0}^{[L, L_0]} \sum_{\mu=1}^{M(\lambda)} \sum_{\nu=1}^{N(\lambda)} \frac{e_{\lambda}^{\mu\nu} \cdot C_{\lambda}^{\mu\nu}}{2\lambda + 1} \quad (62)$$

where $[L, L_0]$ is the lowest one of the two values L and L_0 . The mean value

Eq. (37) can be obtained by expressing $E^K(h)$ by a series similar to Eq. (56)

$$E^K(h) = \sum_{\lambda=0}^{L_0} \sum_{\mu=1}^{M(\lambda)} e_{\lambda}^{\mu} \cdot \dot{K}_{\lambda}^{\mu}(h) \quad (63)$$

Eq. (37) can then be written

$$\begin{aligned} \bar{E}(y) &= \sum_{\lambda=0}^{[L_0, L_0]} \sum_{\nu=1}^{N(\lambda)} \left[\sum_{\mu=1}^{M(\lambda)} \frac{e_{\lambda}^{\mu\nu} \cdot C_{\lambda}^{\mu\nu}}{2\lambda + 1} \right] \dot{K}_{\lambda}^{\nu}(y) \\ &= \sum_{\lambda=0}^{[L_0, L_0]} \sum_{\nu=1}^{N(\lambda)} \bar{e}_{\lambda}^{\nu} \cdot \dot{K}_{\lambda}^{\nu}(y) \end{aligned} \quad (64)$$

The magnetization energy $E^K(h)$ is satisfactorily represented by $L_0 = 6$ and often even $L_0 = 4$ is considered sufficient. Hence, Eq. (64) contains only very few terms. Especially, it is seen that only the texture coefficients $C_{\lambda}^{\mu\nu}$ up to $\lambda = L_0$ enter the mean value expression. The higher order texture coefficients with $\lambda > L_0$ have no influence. Hence, if only mean values of magnetic properties are considered an abbreviated method of texture determination may be sufficient. The texture coefficients \bar{T} defined in Eq. (45) are needed to average all kinds of fourth-rank tensor properties of textured materials. It can be shown that the product of the four factors g_{ij} in Eq. (45) can be expressed in the form of a finite series terminated at $\lambda = 4$

$$g_{im} \cdot g_{jn} \cdot g_{ko} \cdot g_{lp} = \sum_{\lambda=0}^4 \sum_{\mu=1}^{M(\lambda)} \sum_{\nu=1}^{N(\lambda)} t_{\lambda}^{\mu\nu(mnop)} \cdot \bar{T}_{\lambda}^{\mu\nu}(g) \quad (65)$$

Then Eq. (45) takes on the form

$$\bar{T}_{ijkl}^{mnop} = \sum_{\lambda=0}^4 \sum_{\mu=1}^{M(\lambda)} \sum_{\nu=1}^{N(\lambda)} \frac{t_{\lambda}^{\mu\nu(mnop)} \cdot C_{\lambda}^{\mu\nu}}{2\lambda + 1} \quad (66)$$

where $t_{\lambda}^{\mu\nu}$ are numerical coefficients which have been tabulated by Morris (1969) for crystal symmetries higher than orthorhombic.

Most of the technologically used magnetic materials belong to the cubic or hexagonal crystal symmetry whereby most of the magnetically soft materials are cubic and the important hard materials are hexagonal. Usually, the magnetization energy $E^K(h)$ is written in the cubic case

$$E^K(h) = K_4 \cdot \varphi_4(h) + K_6 \cdot \varphi_6(h) \quad (67)$$

with

$$\varphi_4(h) = (h_1^2 \cdot h_2^2 + h_2^2 \cdot h_3^2 + h_3^2 \cdot h_1^2) \quad (68)$$

$$\varphi_6(h) = h_1^2 \cdot h_2^2 \cdot h_3^2 \quad (69)$$

Hence, the coefficients e_{λ}^{μ} in Eq. (63) have the form (see e.g. Bunge 1982 p. 316 ff)

$$\begin{aligned} e_4^1 &= 0.3094 \cdot \left[K_4 + \frac{K_6}{11} \right] \\ e_6^1 &= 0.012038 \cdot K_6 \end{aligned} \quad (70)$$

In the case of hexagonal symmetry, magnetic energy is often expressed in the form

$$E(\Phi) = K_2 \cdot \sin^2 \Phi + K_4 \cdot \sin^4 \Phi \quad (71)$$

where Φ is the angle between the crystal direction h and the c -axis. In this case the coefficients e_λ^h take the form (e.g. Bunge 1982 p. 316 ff)

$$\begin{aligned} e_2^1 &= -1.5853 \cdot \left[\frac{3}{2} K_2 + \frac{16}{21} K_4 \right] \\ e_4^1 &= 0.2701 \cdot K_4 \end{aligned} \quad (72)$$

If we further assume the material to have the form of a sheet i.e. orthorhombic sample symmetry then Eq. (64) contains the coefficients

$$\begin{aligned} \bar{e}_4^\nu &= \frac{1}{9} e_4^1 \cdot C_4^{1\nu}; & \nu &= 0, 2, 4 \\ \bar{e}_6^\nu &= \frac{1}{13} e_6^1 \cdot C_6^{1\nu}; & \nu &= 0, 2, 4, 6 \end{aligned} \quad (73)$$

in the cubic case and

$$\begin{aligned} \bar{e}_2^\nu &= \frac{1}{3} e_2^1 \cdot C_2^{1\nu}; & \nu &= 0, 2 \\ \bar{e}_4^\nu &= \frac{1}{9} e_4^1 \cdot C_4^{1\nu}; & \nu &= 0, 2, 4 \end{aligned} \quad (74)$$

in the hexagonal case. Hence, only seven or five texture coefficients enter the mean value expression Eq. (64).

Texture determination is usually carried out by pole figure inversion, i.e. the solution of Eq. (7), whereby the texture function $f(g)$ may be expressed in terms of the series expansion coefficients Eq. (54). With the usual degree of expansion ($\lambda = 22$) and cubic orthorhombic symmetry the series consists of 185 terms, of which, however, only seven are needed according to Eq. (64) and Eq. (73). These latter coefficients can also be obtained by much simpler measuring procedures using, for instance, a fixed angle texture analyzer as described by Kopineck (1986) (compare also Bunge, Kopineck and Wagner, this volume).

EXPERIMENTAL EXAMPLES

The approximation of the magnetization energy given in Eq. (67) and its texture average given by Eq. (64) with the coefficients of Eq. (73) applies quite well to the state of saturation. This was shown, for instance, by Szpunar and Ojanen (1975) using torque curve measurements in textured Fe-Si transformer steels. An example of their results is shown in Figure 8 which compares the measured and calculated values.

Although the fourth order approximation of the magnetization energy strictly applies only to the state of saturation, it also provides a quite satisfactory fit to the total hysteresis losses and even to the total core losses as was shown, for instance, by Hutchinson and Swift (1973). In this case, the single crystal parameters K_4 and K_6 , contained in Eq. (67), are to be determined empirically by fitting the core loss curves for strips taken under different angles to the rolling direction from magnetic transformer steel as is shown in Figure 9.

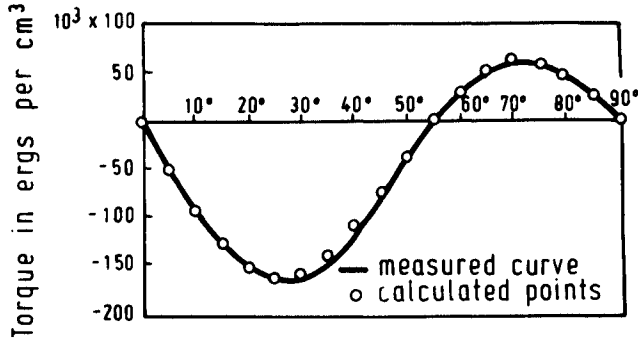


Figure 8 Torque curve of an iron silicon sheet calculated from the texture and measured experimentally. After Szpunar and Ojanen.

MAGNETIC TEXTURE MEASUREMENT

Magnetic anisotropy measurements have often been used as a simple and economic method for texture inspection (see e.g. Wassermann, 1962). It is, however, obvious that this does not provide an alternative to complete X-ray texture analysis. Using Eq. (64) with magnetic energy measurements in different sample directions y , the coefficients \bar{e}_λ^y with $\lambda \leq 6$ can be obtained at the best. These coefficients are related to the texture coefficients $C_\lambda^{h'v}$ according to Eq. (73). Hence, only seven of the total of say 185 texture coefficients can be determined by magnetic methods. With these coefficients substituted in Eq. (54), only a low resolution approximation to the complete texture functions is obtained (see also Bunge, 1982 p. 340 ff and Bunge (to be published)).

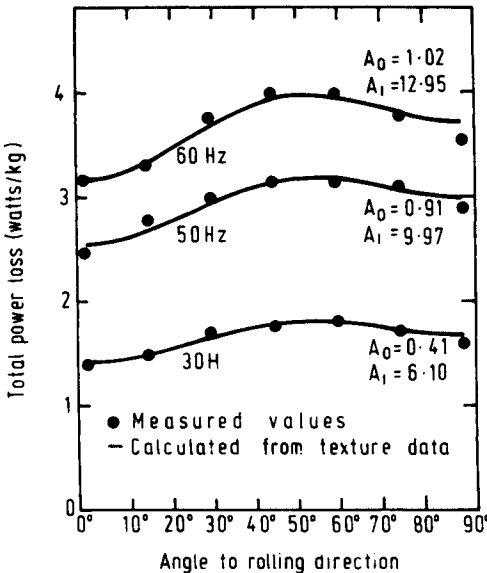


Figure 9 Power losses in Fe-Si transformer steel as a function to rolling direction. After Hutchinson and Swift.

DETERMINATION OF THE MAGNETIC TEXTURE

The crystallographic texture defined in Eq. (3) can be determined from several of its pole figures defined in Eq. (5), which are related to the texture function by Eq. (7). The texture function is thus obtained from pole figures by solving a system of equations (Eq. (7)). This is called "pole figure inversion".

The same procedure can also be applied to the magnetic texture defined in Eq. (16), provided that the lower symmetry (e.g. tetragonal) of the magnetized state can somehow be distinguished from the higher crystallographic symmetric (e.g. the cubic symmetry). This is the case, if neutron diffraction is being used. The neutron scattering factor contains a nuclear and a magnetic part. The first one reveals the crystallographic, the latter one of the magnetic symmetry. The intensity of unpolarized neutrons reflected from the lattice plane (hkl) perpendicular to the crystal direction h can be expressed in terms of the relative intensity factor

$$A = 1 + b \cdot \sin^2 \chi \quad (75)$$

where b is the ratio of magnetic versus nuclear scattering (see e.g. Bacon 1975). This ratio depends on the value of $\sin \Theta/\lambda$, and χ is the angle between the normal direction h to the reflecting lattice plane (hkl) and the magnetization direction.

Pole figures of lattice planes having the same Bragg-angle cannot be measured separately. If we exclude here "occasional" superpositions, then these are the pole figures which are equivalent according to the crystallographic symmetry. Hence, the pole figures measured by neutron diffraction can be expressed in the form

$$P_{\{h\}}^{\text{Neutr}}(y) = \frac{1}{n_h} \sum_{o^c}^N [1 + b_h \cdot \sin^2 \chi_{o^c \cdot h}] \cdot P_{o^c \cdot h}^{\text{Magn}}(y) \quad (76)$$

where o^c are the crystal symmetry operations including those with inversional character, N is the order of the crystal symmetry group and n_h is a normalization factor defined by

$$n_h = \sum_{o^c}^N [1 + b_h \cdot \sin^2 \chi_{o^c \cdot h}] \quad (77)$$

Because of Eq. (18) the first part of the term in brackets in Eq. (76) corresponds to the crystallographic texture. Hence, Eq. (76) can be written

$$P_{\{h\}}^{\text{Neutr}}(y) = \frac{N}{n_h} \cdot P_h^{\text{Cryst}}(y) + \frac{b_h}{n_h} \sum_{o^c}^N \sin^2 \chi_{o^c \cdot h} \cdot P_{o^c \cdot h}^{\text{Magn}}(y) \quad (78)$$

In the case of the completely demagnetized state, the magnetic texture is identical with the crystallographic texture, Eq. (19). Hence, in this case it is

$$P_{\{h\}}^{\text{Neutr}}(y) = \left[\frac{N}{n_h} + \frac{n_h - N}{n_h} \right] \cdot P_h^{\text{Cryst}}(y) \quad (79)$$

The first term in brackets is the part due to nuclear scattering, the second one is that due to magnetic scattering.

In the most general case Eq. (76) can be written

$$P_{(h)}^{\text{Neutr}}(y) = \sum_i q_i \cdot P_{h_i}^{\text{Magn}}(y) \quad (80)$$

where the index i stands for directions which are symmetrically equivalent according to the crystal symmetry o^c but not equivalent according to the magnetic symmetry o^M . The superposition factors q_i have the form

$$q_i = \frac{[1 + b_h \cdot \sin^2 \chi_{o^c \cdot h}]}{\sum_{o^c} [1 + b_h \cdot \sin^2 \chi_{o^c \cdot h}]} \quad (81)$$

with

$$\sum_i q_i = 1 \quad (82)$$

Hence, in order to determine the magnetic texture from neutron diffraction pole figures, the pole figure inversion problem has to be solved for the case of superposed pole figures as was considered theoretically by Bunge (1982). The problem can be solved easily using an iterative method as described by Dahms and Bunge (1988).

If the crystallographic texture is already known, e.g. from X-ray diffraction, then Eq. (80) is to be solved under the conditions Eq. (60) which contain the assumed restriction that the spontaneous magnetization is only in the crystallographically equivalent directions h_0 . Using an iterative method, Eq. (60) can easily be included in the calculation procedure.

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