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TEXTURED IMAGES SEGMENTATION BY MORPHOLOGICAL PARTITION FILTERS

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ABSTRACT

This paper presents a new segmentation procedure for textured images, using morphological partition filters. These filters are characaterized by a controllable selectivity of action with respect to both directivity and spatial frequency. They are thus able to homogenize regions whose structure corresponds to their characteristics, while leaving others untouched. Automatic adjustment of their structuring element is obtained through computation of the image covariograms, during the various stages of the procedure. This segmentation procedure appears to perform well in cases of highly structured textures.

1. INTRODUCTION

A difficult problem in image segmentation concerns the detection of textured regions which differentiate themselves from their surroundings. In fact, a substantial number of computer vision tasks deal with textured images, and their performance crucially depends on the success of the segmentation procedure at hand.

Mathematical morphology, first introduced by G. Matheron [9] and J. Serra [11], in the context of petrography, seems to be a well suited image analysis tool for textured image segmentation.

In order to devise a segmentation procedure based on morphological operators, two issues must be addressed:

(1) the definition of a class of operators which, providing appropriate parameter adjustments, would act selectively on a region containing a particular texture and amplify its "contrast" relative to neighbouring regions.

(2) the definition of a procedure allowing the automatic, context-dependent adjustment of these operators characteristics.

Recently introduced by G. Matheron [9] and J. Serra [12] in the context of binary images, the new class of partition filters seems particularly suited to meet the first requirement. Their action can be regionally selective through an appropriate choice of their structuring element. Moreover, if the structuring element is connected, it tends to unify regions whose structure corresponds to its characteristics. However, in order to apply these filters to the segmentation of textured images, their definition to the domain of grey-level images must be extended and their dual filters must be introduced.

This article presents a segmentation procedure based on the iterative application of partition filters whose structuring elements are automatically adjusted at each step of the process. This procedure has been successfully applied to the segmentation of images composed of natural textures.

2. MORPHOLOGICAL PARTITION FILTERS

The basic concepts of mathematical morphology will not be reviewed here; a complete presentation of the domain can be found in the works of J. Serra [10] and G. Matheron [8]. The reader might also refer to introductions by J. Serra [11], S. Sternberg [14], and R. Haralick [4].

Partition filters consist of a combination of two types of operators, (1): a marking operator whose purpose is to partition the image definition space, and (2): a series of morphological operators which are independent of the marking operator. The application of the marking operator has the effect of atomizing the image into its connected particles, according to an appropriate definition of connectivity (arc connectivity is used here, since it is well suited to current image analysis applications). Once the image has been split up, a filter is used to regroup the particles which, owing to their shapes and spatial distributions, are inclined to merge. The essential feature of such filters is their ability to detect and uniformize regions which exhibit some degree of textural homogeneity while preventing interaction between different regions.

Certain notions such as algebraic opening, connected closing and maximal closing must be introduced prior to the definition of partition filters. These operators are defined and illustrated in the following paragraphs, dealing first with the case of binary images and generalizing them to grey-scale images.

2.1 Connected closing operator: A closing operator h is connected (or preserves connectivity) if the transform of every connected element is a connected element and the transform of the empty set is the empty set. Let P(E) represent the set of all parts of E (image definition space), and C the class of the connected subsets of P(E), C being entirely determined by E and the type of connectivity. A closing is connected if it verifies the predicate:

$$\forall C_i \in C \quad \exists C_j \in C \text{ such that } h(C_i) \subset C_j \qquad (1)$$

Figure 1(a) illustrates the action of a connected closing, the closure of any connected component being a connected component. In constrast, Figure 1(b) shows the action of a non-connected closing, the closure of C being composed of the non-connected set $[C_1, C_2]$.

2.2 Connected algebraic opening: The introduction of partition filters requires the definition of a transformation which allows individual selection of each element C_i of a connected class C. This transformation, called the connected algebraic opening and denoted $\gamma(x)$, is defined as follows:

$$\forall X \in P(E) \quad \gamma_x(X) = \{\bigcup_i C_i;\ C_i \in C \text{ and } x \in C_i \subset X\} \qquad (2)$$

Consequently, $\gamma_x(X)$ is non-empty if and only if element x belongs to X, and designates therefore the connected component of X containing x. Figure 2 shows various applications of this definition.

2.3 Non-generic closing: In order to facilitate the construction of connected closings (expression (1)) it is convenient to consider first the class of non-generic closings g. This type of closing operator verifies the predicate:

$$\forall X \in P(E) \text{ and } \forall x \in g(X) \quad X \cap \gamma_x[g(X)] \neq \phi \qquad (3)$$

It should be noted that every non-generic closing g is connected, while the inverse is not necessarily true. Specifically, Figure 1(a) illustrates a connected closing which does generate new particles, not present initially.

Given two arbitrary closing operators f and f_1, f_1 is a minoring element of f if it verifies:

$$\forall X \in P(E) \quad f_1(X) \subset f(X) \qquad (4)$$

One can show that, among the class of non-generic closings, there is a greater minoring element g_o of f, expressed as:

$$g_o(X) = \bigcup_{x \in X} \gamma_x[f(X)] \qquad (5)$$

Expression (5) suggests a pratical way of determining the action of a connected closing, starting from an arbitrary closing f. Figure 3(a) shows an example of a closing creating new connected components. The application of expression (5) defines then a non-generic closing whose action is illustrated in Figure 3(b).

2.4 Partition closings operator: Once g_o has been defined, it is then possible to define the class Φ_o of connected closings h included in g_o and coinciding with g_o over C:

$$\Phi_o = \{h;\ \forall X \in P(E) \quad h(X) \subset g_o(X) \text{ and } \forall C_i \in C \quad h(C_i) = g_o(C_i)\} \qquad (6)$$

The operator v_o defined as:

$$\forall h \in \Phi_o \quad v_o(h) = \bigcup_{x \in E} g_o(I \cap \gamma_x[h]) \qquad (7)$$

where I designates the identity operator, is then applied on the closings of Φ_o.

It has been shown [12] that Φ_o is stable with respect to v_o i.e. that every application $v_o(h)$ is a connected closing. Finally, if one considers the subset Φ'_o of Φ_o consisting of closings h which remain invariant under transformation v_o:

$$\Phi'_o = \{h \in \Phi_o \ ;\ v_o(h) = h\} \qquad (8)$$

it may be shown that every element of Φ'_o is a non-generic closing.

The role played by the various operators introduced up to this point may be explained in the following way. Given an arbitrary closing f, expression (5) defines, among all of the minoring elements of f, the most active non-generic closing g_o. In addition, expression (6) defines the set Φ_o of all connected closings which, on the one hand, act in an identical manner on each of the connected elements of C taken individually and, on the other hand, are less active than g_o when applied globally to the image.

Expression (7) associates to each closing h of Φ_o a new closing operator $v_o(h)$ whose action on X can be described as follows: on each region of X whose transform is a connected component, $v_o(h)$ acts in the form of the most active non-generic closing g_o. In other words, operator $I \cap \gamma_x[h]$ isolates within the domain E of the image, the elements destined to be merged by the action of h. Thus, applying g_o to this group of elements, results in their merging into a maximal connected component. In this manner, transformation $v_o(h)$ makes it possible, first of all, to isolate in the image domain the regions whose content has a tendency to become uniform, and then to apply a maximal merging to each region individually, thereby avoiding interaction between different regions. Finally, expression (8) defines a subset Φ'_o of connected closings h whose action is equivalent to that of the previously described operators. Due to their characteristics, elements of Φ'_o are called partition closings. Figure 4 shows an example of the application of this type of operator.

In order to exploit these results, these partition closings must be explicitly formulated. In particular, the following theorem, stated and demonstrated by G. Matheron [9], expresses the largest, i.e. the most active of these closings, associated with a particular non-generic closing g_o

<u>Theorem</u>: Given the class Φ'_o corresponding to g_o (expression (8)) its maximal element, i.e. most active, h_M can be expressed as:

$$h_M = \bigcup_{x \in E} T_x \qquad (9)$$

where T_x represent, for each x the largest increasing and idempotent operator which is a minoring element of $\gamma_x[g_o]$ and verifies:

$$\forall X \in P(E) \quad T_x(I \cap T_x)(X) = T_x(X) \tag{10}$$

A definition equivalent to expression (10) is given by:

$\forall X \in P(E)$ and $\forall Y \in P(E)$ such that $X \cap T_x(X) \subset Y \subset X$

$$T_x(X) = T_y(Y) \tag{11}$$

Figure 5 illustrates the action of an operator verifying (10) or (11). It may be noted that every opening or closing operator satisfies these conditions.

The expression of the maximal closing defined in (14) may be obtained by the iterative process:

$$h_I = v_o[g_o]$$
$$\bullet$$
$$\bullet$$
$$h_n = v_o[g_{n-1}] = v_o^{(n)}[g_o] \tag{12}$$

Since the definition space E is finite and each $v_o^{(n)}[g_o]$ is included in $v_o^{(n-1)}[g_o]$, the decreasing sequence (12), reaches for a finite integer N, a limit θ such that:

$$\theta = v_o[\theta] = \bigcup_{x \in E} g_o(I \cap \gamma_x[\theta]) \tag{13}$$

Operator θ therefore belongs to the set Φ'_o, and it has been shown [12] that $\gamma_x[\theta]$ is the largest minoring element verifying (13).

As a result, the following expression allows the iterative generation of h_M:

$$h_M = \bigcup_{x \in E} \gamma_x[\theta] = \lim_{n \to N} \left[\left[\bigcup_{x \in E} g_o(I \cap \gamma_x[g_o] \right]^{(n)} \right] \tag{14}$$

In this expression, Lim[ ] designates the sequential limit of iterations of $v_o^{(n)}[g_o]$.

Expressions (5) and (14) define algorithms necessary to operate a connected closing and a maximal partition closing associated with an arbitrary closing f. It should be noted that expression (14) generally converges in a few iterations which makes the corresponding operators particularly interesting. Figures 6(a) and 6(b) compare the action of h_M and g_o. As appears on these figures the principal effect of h_M is to avoid interactions between the connected components created by g_o, without breaking these components.

The dual operators associated with γ_x, g_o and h_M can be easily defined, assuming the following definition of the dual T^* of a transformation T:

$$T^*(X) = {}^c[T({}^cX)] \tag{15}$$

The operators introduced up to now concerned binary images only. In order to use these concepts in image segmentation, they must now be extended to grey-scale images. Any multilevel function z(x) is equivalent the decreasing family of horizontal sections X_t defined by:

$$X_t(z) = \{x;\ z(x) \geq t\} \tag{16}$$

Conversely z(x) may be expressed in terms of the X_t's

$$z(x) = \sup\{t;\ x \in X_t\} \tag{17}$$

Partition filters are easily generalized to grey-scale images by replacing binary components X by horizontal sections X_t and set operations of intersection and union by minimum and maximum operations respectively.

3. DESCRIPTION OF SEGMENTATION PROCEDURE

Since partition filters, defined by expression (14) are based on a closing operator f, their merging and segmentation effect will be directly influenced by the choice of the structuring element associated with f.

We consider here textured images in which each texture consists of a small set of primitive elements spatially arranged according to particular organization rules. Segmentation of an image involves the detection of abrupt changes, either in the shapes of the primitive elements or in the rules of spatial organization, or in both. The characteristics of the structuring element of the basic closing operator f should therefore be carefully selected. Moreover, in order to use partition filters in non-trivial situations, it is essential that an automatic adjustment procedure be devised for adjusting the parameters of the structuring element, in terms of shape, orientation and size. Once a structuring element has been properly chosen, the segmentation procedure consists of an appropriate sequencing of applications of the transformations defined in the previous section.

3.1 Type of structuring element: The basic closing f considered here consists of the intersection of two closings whose structuring elements are straight-line segments.

3.2 Automatic evaluation of structuring element parameters: The covariogram [11] is an appropriate tool for this task since it accounts for the dispersion of texture primitives. The positions of successive maxima of the covariogram provide an estimate of the minimal spatial periodicity of primitives orientations in the image.

In the case of grey-scale images, a covariogram is established for each horizontal section, which has first been pre-processed by a combination of isotropic openings and closings. This pre-processing prevents non-significant peaks from appearing in the covariograms and thus ease the detection of important structures.

3.2.1 Orientations of the structuring elements: The orientation corresponding to the shortest distance between successive maxima in the covariograms is taken as one of the two orientations defining the structuring elements. This orientation d_1 corresponds to the most finely textured region. The second orientation d_2 is chosen in a similar manner while imposing the constraint:

$$\text{angle}(d_1,d_2) \leq \begin{cases} \pi/3 \ (\text{mod}\pi) & \text{for an hexagonal grid} \\ \pi/4 \ (\text{mod}\pi) & \text{for a square grid} \end{cases} \quad (18)$$

This constraint results from the fact that variations in the orientation associated, with the placement of similar primitives, usually remain within the limits indicated in expression (18).

3.2.2 Lengths of the structuring elements: The length of each linear structuring element is set equal to the shortest distance between maxima in the corresponding covariogram. As indicated previously, this distance usually corresponds to the minimal repetition rate of primitives in this orientation.

3.4 Segmentation procedure: In order to differentiate between two textured regions characterized by different spatial frequencies, it is necessary to apply, first a partition closing whose characteristics correspond to the structure of the first region and whose action leave the second region unchanged, followed by a partition opening whose characteristics match the structures of the second region. In the case of images consisting of two different textures, the complete segmentation procedure can be summarized as follows:

Phase 1: Pre-processing

In order to rub out insignificant details, each horizontal section of the image is treated by an isotropic opening followed by an isotropic closing.

Phase 2: Application of partition filters

A partition closing and its dual partition opening are successively applied to the image. This operation involves computing the characteristics (orientations and lengths) of the structuring elements, as explained previously.

Phase 3: Homogenization of regions

The image resulting from the application of the partition filters (phase 2) contains regions of increased honogeneity, as compared with the original image. This homogeneity however is far from total, due to the fact that natural textures usually present a fair amount of variations in primitive shape as well as spatial distribution. It is therefore essential to eliminate remaining artefacts and refine the homogenization of the regions. Simple isotropic closing and opening operations are generally capable of strenghtening the bimodality of the resulting images.

Phase 4: Partition of the image

The grey-level histogram of the image resulting from phase 3 is established. This histogram being strongly bimodal, a threshold is easily found, which allows the partition of the image.

4. RESULTS AND DISCUSSION

This segmentation technique has been tested on images made up of natural textures taken from the Brodatz album [17]. Textures presenting a high degree of spatial structure were selected, because of their preponderance in industrial scenes. Hexagonal grid digitization was adopted, owing to its advantages in terms of isotropy and number of principal orientations. All textures were histogram-equalized in order to eliminate effects of first order statistical behavior.

Figure 7 presents three textured images, on which the procedure has been applied. The first image contains one type of primitive element only, distributed according to two different spatial arrangements. The second and third images are composed of two natural textures having different primitives and spatial organizations. Figures 8, and 9 present the results of Phases 2, and 4 respectively for each of the three images selected. Figure 8 allows us to observe the action of partition filters. One can notice on example 1 that, under the action of the maximal partition closing, the structure of one of the regions remains unchanged whereas the second region has a tendency to become uniform. Conversely, the maximal partition opening (Figure 8(b), example 1), applied on the result of the maximal closing, acts mainly on the other region, thus enhancing the segmentation effect. Example 2 demonstrates that the maximal partition closing acts on the most finely textured region, while at the same time leaving the second texture untouched, and further, that the maximal partition opening tends to homogenize the macro-textured region. Example 3 is a case where the maximal closing and the maximal opening act on the same region without introducing significant changes into the spatial organization of the second. Figure 9 illustrates the final segmentation results.

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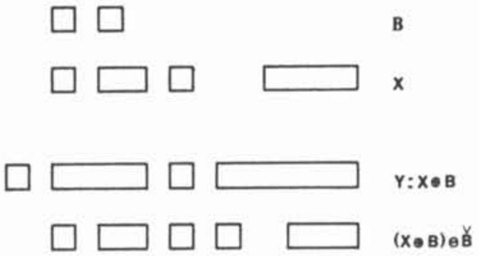


Figure 1(a) - Connected closing operator

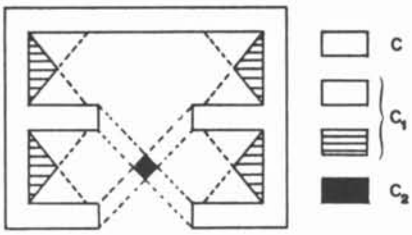


Figure 1(b) - Non-connected closing operator (intersection of two closing by linear segments at ±60°)

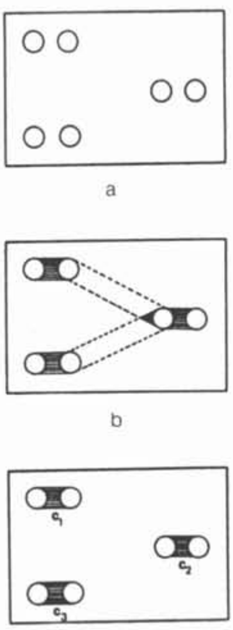


Figure 4 - Partition closing operator
(a) original image
(b) effect of non-generic closing operator (structuring element = union of element of Figure1(b) and horizontal linear segment)
(c) effect of associated partition closing

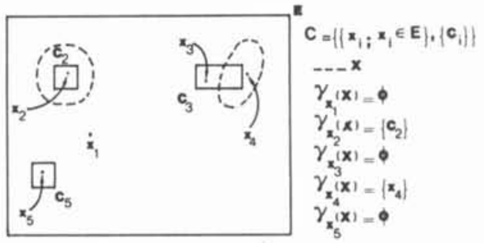


Figure 2 - Connected algebraic opening operator

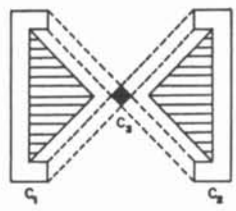


Figure 3(a) - Generic closing operator

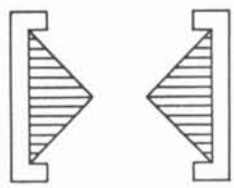


Figure 3(b) - Non-generic closing operator

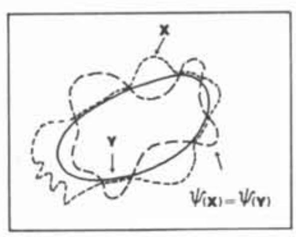


Figure 5 - Operator T verifying expressions (10) and (11)

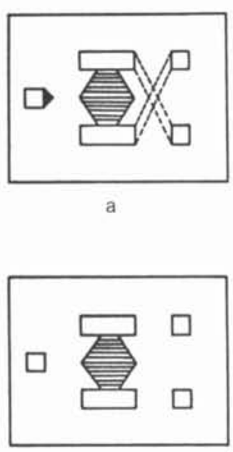
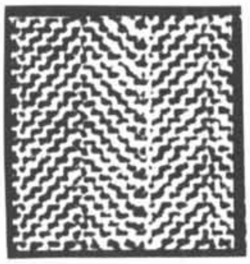


Figure 6 - (a) action of g_0
(b) action of h_M



1

2

3

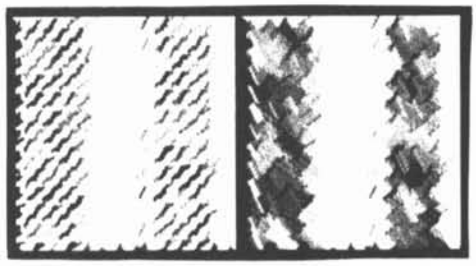
Figure 7 - Original images

1

2

3

Figure 9 - Final segmentation results



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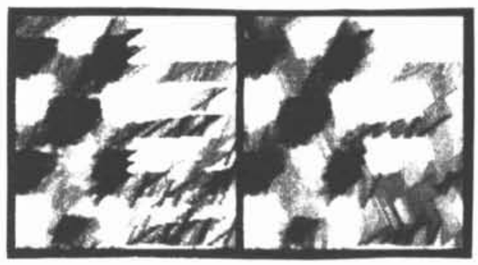

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Figure 8 - Effect of partition filters [a | b]
(a) maximal closing
(b) maximal opening applied on result of (a)

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