

# The 2dF Galaxy Redshift Survey: Spherical Harmonics analysis of fluctuations in the final catalogue

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## ABSTRACT

We present the result of a decomposition of the 2dFGRS galaxy overdensity field into an orthonormal basis of spherical harmonics and spherical Bessel functions. Galaxies are expected to directly follow the bulk motion of the density field on large scales, so the absolute amplitude of the observed large-scale redshift-space distortions caused by this motion is expected to be independent of galaxy properties. By splitting the overdensity field into radial and angular components, we linearly model the observed distortion and obtain the cosmological constraint  $\Omega_m^{0.6} \sigma_8 = 0.46 \pm 0.06$ . The amplitude of the linear redshift-space distortions relative to the galaxy overdensity field is dependent on galaxy properties and, for  $L_*$  galaxies at redshift  $z = 0$ , we measure  $\beta(L_*, 0) = 0.58 \pm 0.08$ , and the amplitude of the overdensity fluctuations  $b(L_*, 0) \sigma_8 = 0.79 \pm 0.03$ , marginalising over the power spectrum shape parameters. Assuming a fixed power spectrum shape consistent with the full Fourier analysis produces very similar parameter constraints.

**Key words:** large-scale structure of Universe, cosmological parameters

## 1 INTRODUCTION

Analysis of galaxy redshift surveys provides a statistical measure of the surviving primordial density perturbations.

These fluctuations have a well known dependency on cosmological parameters (e.g. Eisenstein & Hu 1998), and can therefore be used to constrain cosmological models. The use

of large scale structure as a cosmological probe has acquired an increased importance in the new era of high precision cosmology, which follows high-quality measurements of the cosmic microwave background (CMB) power spectrum (Bennett et al. 2003; Hinshaw et al. 2003). The extra information from galaxy surveys helps to lift many of the degeneracies intrinsic to the CMB data and enhances the scientific potential of both data sets (e.g. Efstathiou et al. 2002; Percival et al. 2002; Spergel et al. 2003; Verde et al. 2003).

In this paper we decompose the large-scale structure density fluctuations observed in the 2dF Galaxy Redshift Survey (2dFGRS; Colless et al. 2001;2003) into an orthonormal basis of spherical harmonics and spherical Bessel functions. In Percival et al. (2001; P01) we decomposed the partially complete 2dFGRS into Fourier modes using the method outlined by Feldman, Kaiser & Peacock (1994). In a companion paper (Cole et al. 2004; C04) we analyse the final catalogue using Fourier modes. In P01 and C04, the Fourier modes were spherically averaged and fitted with model power spectra convolved with the spherically averaged survey window function. Redshift-space distortions destroy the spherical symmetry of the convolved power and potentially distort the recovered power from that expected with a simple spherical convolution. Analysis of mock catalogues presented in P01 and a detailed study presented in C04 showed that, in spite of these complications, cosmological parameter constraints can still be recovered from a basic Fourier analysis.

However, a decomposition into spherical harmonics and spherical Bessel functions rather than Fourier modes distinguishes radial and angular modes, and enables redshift-space distortions to be easily introduced into the analysis method (without the far-field approximation, Kaiser 1987), as well as allowing for the effects of the radial selection function and angular sky coverage (Heavens & Taylor 1995; HT). The down-side is that Spherical Harmonics methods are, in general, more complex than Fourier techniques and are computationally more expensive. This is particularly apparent when only a relatively small fraction of the sky is to be modelled, as the observed modes are then the result of a convolution of the true modes with a wide window function. For nearly all sky surveys (e.g. IRAS surveys), correlations between modes are reduced, and the window is narrower leading to a reduced computational budget.

Consequently, a number of Spherical Harmonics decompositions have been previously performed for the IRAS surveys. The primary focus of much of the earlier work was the measurement of  $\beta(L, z) \equiv \Omega_m(z)^{0.6}/b(L, z)$ , a measure of the increased fluctuation amplitude caused by the linear movement of matter onto density peaks and out from voids (Kaiser 1987). Here  $\Omega_m(z)$  is the matter density and  $b(L, z)$  is a simplified measure of the relevant galaxy bias. See Berlind, Narayanan & Weinberg (2001) for a detailed study of  $\beta(L, z)$  measurements assuming more realistic galaxy bias models.

For the IRAS 1.2-Jy survey, HT and Ballinger, Heavens & Taylor (1995) found  $\beta \sim 1 \pm 0.5$  for fixed and varying power spectrum shape respectively, and similar constraints were also found by Fisher, Scharf & Lahav (1994), Fisher et al. (1995). However, Cole, Fisher & Weinberg (1995) found  $\beta = 0.52 \pm 0.13$  and Fisher & Nusser (1996) found  $\beta = 0.6 \pm 0.2$  for the 1.2-Jy survey using the quadrupole-to-

monopole ratio for the decomposition of the power spectrum into Legendre polynomials. No explanation for the apparent discrepancy between these results has yet been found, although we note that the results are consistent at approximately the 1- $\sigma$  level if the large errors are taken into account for the Spherical Harmonics decompositions.

The IRAS Point Source Catalogue Redshift Survey (PSCz; Saunders et al. 2000) has also been analysed using a Spherical Harmonics decomposition by a number of authors (Tadros et al. 1999; Hamilton, Tegmark & Padmanabhan 2000; Taylor et al. 2001) who found  $\beta \sim 0.4$ . More recently, Tegmark, Hamilton & Xu (2002) presented an analysis using spherical harmonics to decompose the first 100k redshifts released from the 2dFGRS and found  $\beta = 0.49 \pm 0.16$  for the  $b_j$  selected galaxies in this survey, consistent with the  $\xi(\sigma, \pi)$  analyses of Peacock et al. (2001) and Hawkins et al. (2003). The measured  $\beta$  constraints are expected to vary between samples through the dependence on the varying galaxy bias. For example, by analysing the bispectrum of the PSCz survey Feldman et al. (2001) found a smaller large-scale bias than a similar analysis of the 2dFGRS by Verde et al. (2002).

In addition to the linear distortions, random galaxy motions within galaxy groups produce the well known Fingers-of-God effect where structures are elongated along the line-of-sight. These random motions mean that the observed power is a convolution of the underlying power with a narrow window. The observed power therefore depends on the form of this window and the amplitude of the velocity dispersion as a function of scale.

The 2dFGRS and Sloan Digital Sky Survey (SDSS; Abazajian et al. 2004) cover sufficient volume that it is now possible to recover information about the shape of the power spectrum in addition to the redshift-space distortions (P01; C04; Tegmark, Hamilton & Xu 2002; Tegmark et al. 2003a). However, the decreased random errors (cosmic variance) of these new measurements means that systematic uncertainties have become increasingly important. In particular, galaxies are biased tracers of the matter distribution: the relation between the galaxy and mass density fields is probably both nonlinear and stochastic to some extent (e.g. Dekel & Lahav 1999), so that the power spectra of galaxies and mass differ in general. Assuming that the bias tends towards a constant on large scales, then we can write  $P_g(k) = b^2 P_m(k)$ , where subscripts  $m$  and  $g$  denote matter and galaxies respectively. For the 2dFGRS galaxies, although the average bias is close to unity (Lahav et al. 2002; Verde et al. 2002), the bias is dependent on galaxy luminosity (Norberg et al. 2001; 2002a; Zehavi et al. 2002) find a very similar dependence for SDSS galaxies), with  $\langle b(L, z)/b(L_*, z) \rangle = 0.85 + 0.15L/L_*$  where the bias  $b(L, z)$  is assumed to be a simple function of galaxy luminosity and  $L_*$  is defined such that  $M_{b_j} - 5 \log_{10} h = -19.7$  (Norberg et al. 2002b). Because average galaxy luminosity is a function of distance, this bias can distort the shape of the recovered power spectrum (Tegmark et al. 2003a; Percival, Verde & Peacock 2004).

In this paper we decompose the final 2dFGRS catalogue into an orthonormal basis of spherical harmonics and spherical Bessel functions and fit cosmological models to the resulting mode amplitudes. To compress the modes we adopt a modified Karhunen-Loève (KL) data compression method that separates angular and radial modes (Vogeley & Szalay

1996; Tegmark, Taylor & Heavens 1997; Hamilton, Tegmark & Padmanabhan 2000; Tegmark, Hamilton & Xu 2002). We also include a consistent correction for luminosity-dependent bias that includes the effect of this bias on both the measured power and fitted models. We have performed two fits to the recovered modes. First we measured the galaxy power spectrum amplitude,  $b(L_*, 0)\sigma_8$  and the linear infall amplitude,  $\beta(L_*, 0)$  for a fixed power spectrum shape. We then considered fitting a more general selection of cosmological models to these data.

A detailed analysis of the internal consistency of the 2dFGRS catalogue with respect to measuring  $P(k)$  was performed using a Fourier decomposition of the galaxy density field and is presented in C04. This analysis included looking at the effect of changing the calibration, maximum redshift, weighting, region, and galaxy colour range considered. This work is not duplicated using our decomposition technique, and we instead refer the interested reader to that paper. Tests presented in this paper are primarily focused on the analysis method, although we consider the effect of the catalogue calibration in Section 6.

The layout of this paper is as follows. In Section 2 we describe the 2dFGRS catalogue analysed, and in Section 3 we consider mock catalogues used to test our analysis method. A brief overview of the methodology is presented in Section 4. A full description of the Spherical Harmonics method used is provided in Appendix A. The results are presented for both the 2dFGRS and mock catalogues in Sections 5.2 & 5.3. A discussion of various tests performed is given in Section 6. We conclude in Section 7.

## 2 THE 2dFGRS CATALOGUE

In this work, we consider the final 2dFGRS release catalogue. However, the formalism adopted is simplified if we consider a catalogue with a selection function that is separable in radial and angular directions (see Appendix A for details of the formalism). There are two complications in the 2dFGRS catalogue that cause departures from such behaviour (as discussed in Colless et al. 2001;2003).

(i) The photometric calibration of the UKST plates from which the 2dFGRS sample was drawn and the extinction correction have been revised after the initial sample selection. Because revision of the galaxy magnitudes and the angular magnitude limit are required, this means that the survey depth varies across the sky.

(ii) Due to seeing variations between observations, the overall completeness varies with apparent magnitude with a form that depends on the field redshift completeness. This is characterized by a parameter  $\mu$ , with the varying completeness given by  $c_z(m, \mu) = 0.99[1 - \exp(m - \mu)]$  (Colless et al. 2001).

Rather than adapt the formalism, we have chosen to use a reduced version of the 2dFGRS release catalogue with a window function that is separable in radial and angular directions. These issues were also discussed with reference the 100k release catalogue by Tegmark, Hamilton & Xu (2002) whose method also required a sample with window function and weights separable in radial and angular directions. Correcting for these effects is relatively straightforward, if

**Table 1.** Limiting extinction-corrected magnitudes, numbers of galaxies, and assumed radial selection-function parameters for each of the two 2dFGRS regions modelled. The parameters controlling the radial distribution are defined by Eq. 1.

region	$M_{\text{lim}}$	$N_{\text{gal}}$	$z_c$	$b$	$g$
SGP	19.29	84824	0.130	2.21	1.34
NGP	19.17	57932	0.128	2.45	1.24

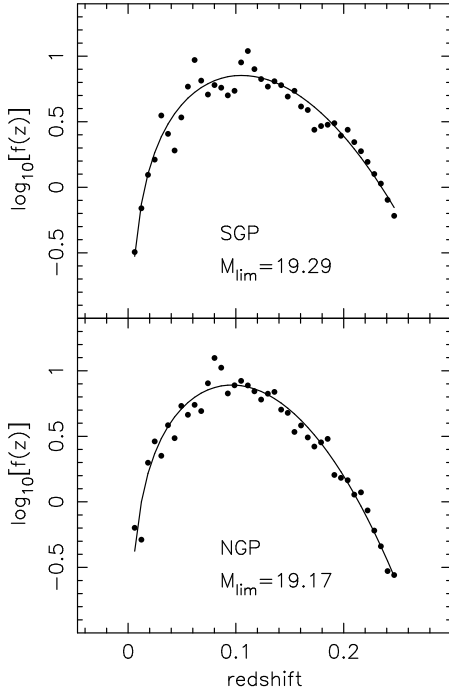
a little painful as we have to remove galaxies with valid redshifts from the analysis. First we need to select a uniform revised magnitude limit at which to cut the catalogue. Galaxies fainter than this limit are removed from the revised catalogue, as are galaxies that were selected using an actual magnitude limit that was brighter than the revised limit. Selecting the revised magnitude limit at which to cut the catalogue is a compromise between covering as large an angular region as possible (resulting in a narrow angular window function), covering as large a weighted volume as possible (reducing cosmic variance), or retaining as many galaxies as possible (reducing shot noise). However, we can model variations in the angular window function and, in Percival et al. (2001), we showed that the 2dFGRS sample is primarily cosmic variance limited. We therefore chose the magnitude limit to maximize the effective volume of the survey.

The random fields, a number of circular 2-degree fields randomly placed in the low extinction regions of the southern APM galaxy survey were excluded from our analysis, in order to focus on two contiguous regions with well-behaved selection functions. These two regions of the survey, one near the north galactic pole (NGP) and another near the south galactic pole (SGP) were analysed separately, and optimization of the magnitude limit was performed for each region independently. The resulting limits are given in Table 1. In order to correct for the magnitude dependent completeness, we removed a randomly selected sample of the bright galaxies in order to provide uniform completeness as a function of magnitude.

The redshift distribution of each sample was matched using a function of the form

$$f(z) \propto z^g \exp \left[ - \left( \frac{z}{z_c} \right)^b \right], \quad (1)$$

where the parameters  $z_c$ ,  $b$  &  $g$  were calculated by fitting to the weighted (Eq. A5) redshift distribution, calculated in 40 bins equally spaced in  $z$ . These resulting parameter values are given in Table 1, and the redshift distributions are compared with the fits in Fig. 1. In addition to the radial and angular distributions of the sample, we also need to match the normalization of the catalogue to the expected distribution. We choose to normalize each catalogue by matching  $\int d\mathbf{r} \bar{n}(\mathbf{r})w(\mathbf{r})$ , where  $\bar{n}(\mathbf{r})$  is the expected galaxy distribution function and  $w(\mathbf{r})$  is the weight applied to each galaxy (Eq. A5), for reasons described in Percival, Verde & Peacock (2004).



**Figure 1.** Redshift distribution of the reduced galaxy catalogues for the two regions considered (solid circles), compared with the best fit redshift distribution for each of the form given by Eq. 1. The magnitude limit adopted for each sample is given in each panel.

### 3 THE MOCK CATALOGUES

As a test of the Spherical Harmonics procedure adopted, we have applied our method to recover parameters from the 22 LCDM03 Hubble Volume mock catalogues available from <http://star-www.dur.ac.uk/~cole/mocks/main.html> (Cole et al. 1998). These catalogues were calculated using an empirically-motivated biasing scheme to place galaxies within  $N$ -body simulations, and were designed to cover the 2dFGRS volume. We have applied the same magnitude and completeness cuts to these data, as applied to the 2dFGRS catalogue (Section 2). In order to allow for slight variations between the redshift distribution of the mocks and the 2dFGRS catalogue, we fit the redshift distribution of the mock catalogues independently from the 2dFGRS data. Because we adopt the magnitude limits used for the 2dFGRS data, the NGP and SGP regions in the mock catalogues have different redshift distributions and these are fitted separately. For simplicity, we assume a single expected redshift distribution for each region for all of the mocks, calculated by fitting to the redshift distribution of the combination of all of the mocks. The number of galaxies in each catalogue is sufficient that the model of  $f(z)$  only changes slightly when considering either catalogues individually, or the combination of all 22 catalogues. This change is sufficiently small that it does not significantly alter either the recovered parameters from the mock catalogues or their distribution.

We use these mock catalogues in a number of ways. By comparing the average recovered parameters and known input parameters of the simulations, we test for systematic problems with the method. In fact, we did not analyse the 2dFGRS data until we had confirmed the validity of the

method through application to these mock catalogues. We test our recovery of the linear redshift-space distortion parameter  $\beta(L_*, z)$  by analysing mocks within which galaxy peculiar velocities were altered (Section 6.7). Additionally, we use the distribution of recovered values to test the confidence intervals that we can place on recovered parameters (Section 5.4).

### 4 METHOD OVERVIEW

The use of Spherical Harmonics to decompose galaxy surveys dates back to Peebles (1973), and is a powerful technique for statistically analysing the distribution of galaxies. The formalism used in this paper is based in part on that developed by HT and described by Tadros et al. (1999). However, there are some key differences and extensions, which warrant the full description given in Appendix A. In this section we outline the procedure for a non-specialist reader.

The galaxy density field was decomposed into an orthonormal basis consisting of spherical Bessel functions and spherical harmonics. In general, we refer to this as a Spherical Harmonics decomposition. As in P01 & C04, we decomposed the density field in terms of proper distance and therefore needed to assign a radial distance to each galaxy. For this, we adopted a flat cosmology with  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$ . The dependence of the recovered power spectrum and  $\beta(L_*, z)$  on this “prior” is weak, and was explored in P01. We assume a constant galaxy clustering (CGC) model, where the amplitude of galaxy clustering is independent of redshift, although it is dependent on galaxy luminosity through the relation of Norberg et al. (2001) given in Eq. A1. This relates the clustering amplitude of galaxies of luminosity  $L$  to that of  $L_*$  galaxies, and by weighting each galaxy by the reciprocal of this relation, we correct for luminosity-dependent bias.

The Spherical Harmonics decomposition of the mean expected distribution of galaxies is then subtracted, calculated using a fit to the radial distribution and an angular mask (this was modelled using a random catalogue in the Fourier analyses of P01 & C04). This converts from a decomposition of the density field to the overdensity field.

In the Fourier based analyses of P01 and C04, we modelled the observed power spectrum. In the analysis presented in this paper we instead model the transformed overdensity field. The expected value of the transform of the overdensity field for any cosmological model is zero by definition. Consequently, apart from a weak dependence on a prior cosmological model hard-wired into the analysis method, the primary dependence on cosmological parameters is encapsulated in the covariance matrix used to determine the likelihood of each model.

The primary difficulty in calculating the covariance matrix for a given cosmological model is correctly accounting for the geometry of the 2dFGRS sample. This results in a significant convolution of the true power, and is performed as a discrete sum over Spherical Harmonic modes in a computationally intensive part of the adopted procedure. To first order, the large-scale redshift-space distortions are linearly dependent on the density field, and we can therefore split the covariance matrix into four components corresponding to the mass-mass, mass-velocity and velocity-velocity power

spectra (cf. Tegmark, Hamilton & Xu 2002; Tegmark et al. 2003a) and the shot noise. This is discussed after Eq. A31 in Appendix A. The velocity component of the covariance matrix is dependent on matter density field rather than the galaxy density field, and we include a correction for the linear evolution of this field. For this we assume that  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$  although the resulting covariance matrix is only weakly dependent on this “prior”.

We include the contribution from the small-scale velocity dispersion of galaxies by undertaking an additional convolution of the radial component of these matrices. We choose to model scales where the small-scale velocity dispersion does not contribute significantly to the overdensity field, and demonstrate this weak dependence in Section 6.5.

The transformation between Fourier modes and Spherical Harmonics is unitary, so each Spherical Harmonic mode corresponds directly to a particular Fourier wavelength. Within our chosen decomposition of the density field, there are 86667 Spherical Harmonic modes with  $0.02 < k < 0.15 h \text{ Mpc}^{-1}$ , and it is impractical to use all of these modes in a likelihood analysis as the inversion of an  $86667 \times 86667$  matrix is slow and may be unstable for a problem such as this. The modes were therefore compressed, leaving 1223 & 1785 combinations of modes for the NGP and SGP respectively. The data compression procedure adopted was designed to remove nearly degenerate modes, which could cause numerical problems, and to optimally reduce the remaining data. The compressed data, and the corresponding covariance matrices are then combined to calculate the likelihood of a given model assuming Gaussian statistics.

Only  $\sim 5\%$  of the computer code used in the PSCz analysis of Tadros et al. (1999) was reused in the current work, as both revision of the method and a significant speed-up of the process were required to model the 2dFGRS. In particular, the geometry of the 2dFGRS sample means that the convolution to correct for the survey window function requires calculation for a larger number of modes than all-sky surveys such as the PSCz, and the method consequently takes longer to run. Because of this revision, the method required thorough testing, both by analysing mock catalogues and by considering the specific tests described in Section 6.

## 5 RESULTS

Results are presented for the 2dFGRS catalogue described in Section 2, and for the mock catalogues described in Section 3. Parameter constraints were derived fitting to modes with  $0.02 < k < 0.15 h \text{ Mpc}^{-1}$ , the range considered in P01. Because the Spherical Harmonics method includes the effects of the small-scale velocity dispersion and uses a non-linear power spectrum, we could in principle extend the fitted  $k$ -range to smaller scales. However, our derivation of the covariance matrix is only based on cosmic variance and shot noise. No allowance is made for systematic offsets caused by our modelling of small-scale effects (velocity dispersion, non-linear power and a possible scale-dependent galaxy bias). Consequently, it is better to avoid regions in  $k$ -space that are significantly affected by these complications, rather than assume that we can model these effects perfectly. Additionally, the number of modes that can be analysed is limited by computation time and the large-scale  $k$ -range selected

includes most of cosmological signal and follows Gaussian statistics.

The Spherical Harmonics method involves a convolution of the window and the model power over a large number of modes (Eq. A22). For a fixed power spectrum shape, the covariance matrix can be written as a linear sum of four components with different dependence on  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$ . It is straightforward to store these components and these parameters can be fitted without having to perform the convolution for each set of parameters. In Section 5.2 we consider a fixed power spectrum shape, and present results fitting  $b(L_*, 0)\sigma_8$  &  $\beta(L_*, 0)$  to the 2dFGRS and mock catalogue data.

In an analysis of the power spectrum shape, separate convolutions are required for each model  $P(k)$ . This would be computationally very expensive, but can be circumvented by discretising the model  $P(k)$  in  $k$  and performing a single convolution for each  $k$ -component. In Section 5.3 we fit to the power spectrum shape, assuming a step-wise  $P(k)$  in this way. First, we describe the set of models to be considered.

### 5.1 Cosmological parameters

A simple model is assumed for galaxy bias, with the galaxy overdensity field assumed to be a multiple (the bias  $b[L, 0]$ ) of the present day mass density field

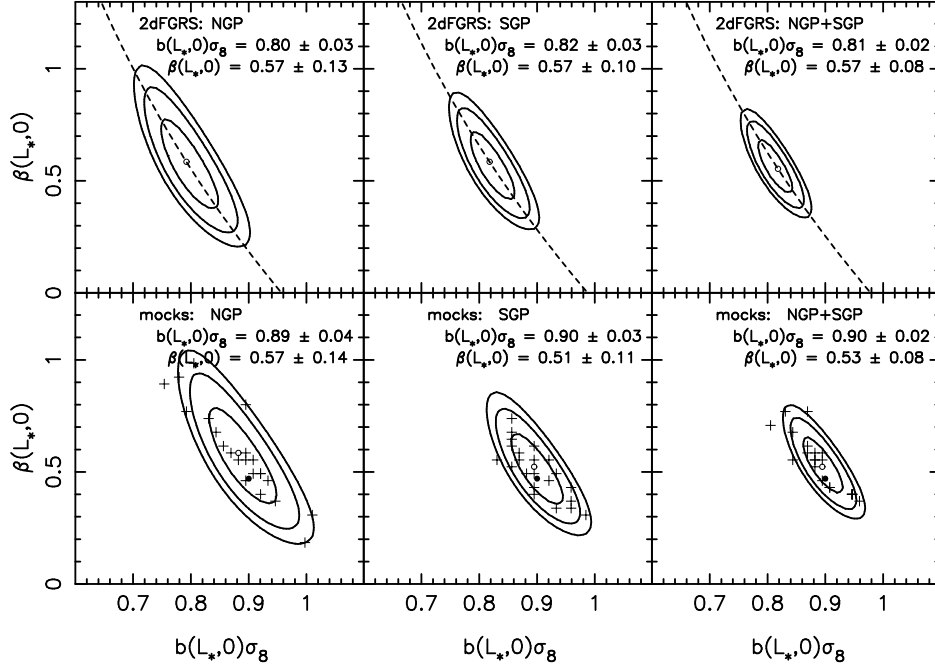
$$\delta(L, \mathbf{r}) = b(L, 0)\delta(\text{mass}, \mathbf{r}), \quad (2)$$

at least for the survey smoothed near our upper wavenumber limit of  $0.15 h \text{ Mpc}^{-1}$ . In the constant galaxy clustering model, the redshift dependence of  $b(L, z)$  is assumed to cancel that of the mass density field so that  $\delta(L, \mathbf{r})$  is independent of redshift. Although galaxy bias has to be more complicated in detail, we may hope that there is a “linear response limit” on large scales: those probed in the analysis presented in this paper. In the stochastic biasing framework proposed by Dekel & Lahav (1999), the simple model corresponds to a dimensionless galaxy-mass correlation coefficient  $r_g = 1$ . Wild et al. (2004) show that the correlation between  $\delta(L, \mathbf{r})$  from different types of galaxies have  $r_g > 0.95$ .

Modelling the expansion of the density field in spherical harmonics and spherical Bessel functions is dependent on the linear redshift-space distortions parameterized by  $\beta(L_*, z) \simeq \Omega_m(z)^{0.6}/b(L_*, z)$ , a function of galaxy luminosity and epoch. The evolution of this parameter is dependent on that of the matter density  $\Omega_m(z)$  and the galaxy bias  $b(L_*, z)$ . These effects are included in the method and are considered in Sections A1 & A4. The recovered expansion is also dependent on the velocity dispersion  $\sigma_{\text{pair}}$  and model assumed for the Fingers-of-God effect (see Section 6.5).

We parameterise the shape of the power spectrum of  $L_*$  galaxies with the Hubble constant  $h$  in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , the scalar spectral index  $n_s$ , and the matter density  $\Omega_m$  through  $\Omega_m h$  and the fraction of matter in baryons  $\Omega_b/\Omega_m$ . The contribution to the matter budget from neutrinos is denoted  $\Omega_\nu$ . The matter power spectrum is normalized using  $\sigma_8$ , the present day rms linear density contrast averaged over spheres of  $8 h^{-1} \text{ Mpc}$  radius.

The shape of the power spectrum to current precision is only weakly dependent on  $h$ , and only sets a strong constraint on a combination of  $\Omega_b/\Omega_m$ ,  $\Omega_m h$ ,  $\Omega_\nu$  and  $n_s$ . In



**Figure 2.** Likelihood contours for the recovered  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  assuming a fixed  $\Lambda$ CDM power spectrum shape. Solid lines in the top row show the recovered contours from the 2dFGRS, while the bottom row gives the average recovered contours from the  $\Lambda$ CDM mock catalogues. Contours correspond to changes in the likelihood from the maximum of  $2\Delta \ln \mathcal{L} = 2.3, 6.0, 9.2$ . These values correspond to the usual two-parameter confidence of 68, 95 and 99 per cent. The open circle marks the ML position, while the solid circle marks the true parameters for the mock catalogues. The crosses give the ML positions for the 22 mock catalogues. Note that on average 57% of the crosses lie within the  $2\Delta \ln \mathcal{L} = 2.3$  contour for the NGP and SGP mock catalogues. The chosen modes are not independent, although they are orthogonal, so we cannot assume that  $\ln \mathcal{L}$  has a  $\chi^2$  distribution. See Section 5.4 for a further discussion of the confidence intervals that we place on recovered parameters. The dashed lines plotted in the upper panels give the locus of models with constant redshift space power spectrum amplitude (see text for details).

this paper, we only consider the very simple model of a constrained flat, scale-invariant adiabatic cosmology with Hubble parameter  $h = 0.72$ , and no significant neutrino contribution  $\Omega_\nu = 0$ . We show that this model is consistent with our analysis, as it is with recent CMB and LSS data sets (e.g. Spergel et al. 2003; Tegmark et al. 2003b). Additionally, we use  $\Omega_b/\Omega_m$  &  $\Omega_m h$  to marginalise over the shape of the power spectrum when considering  $\beta(L_*, 0)$  and  $b(L_*, 0)\sigma_8$ , and marginalise over  $0 < \Omega_b/\Omega_m < 0.4$  and  $0.1 < \Omega_m h < 0.4$ . Given the precision to which the shape of the power spectrum can be constrained, there is an almost perfect degeneracy between  $\Omega_b/\Omega_m$ ,  $\Omega_m h$  and  $n_s$ . For  $n_s \neq 1$ , to first order in  $n_s$ , our best-fit values of  $\Omega_b/\Omega_m$  and  $\Omega_m h$  would change by  $0.46(n_s - 1)$  and  $0.34(1 - n_s)$  respectively.

## 5.2 Results for fixed power spectrum shape

In this Section we fit  $\beta(L_*, 0)$  and  $b(L_*, 0)\sigma_8$  to the data assuming a concordance model power spectrum with  $\Omega_m h = 0.21$ ,  $\Omega_b/\Omega_m = 0.15$ ,  $h = 0.72$  &  $n_s = 1$ , consistent with the recent WMAP results (Spergel et al. 2003), and close to the true parameters of the Hubble volume mocks. Likelihood contours for  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  are presented in Fig. 2 for the 2dFGRS and mock catalogues. The primary degeneracy between these parameters arises because  $b(L_*, 0)\sigma_8$  is a measure of the total power, combining radial and angular modes. Increasing  $\beta(L_*, 0)$  beyond the best-fit

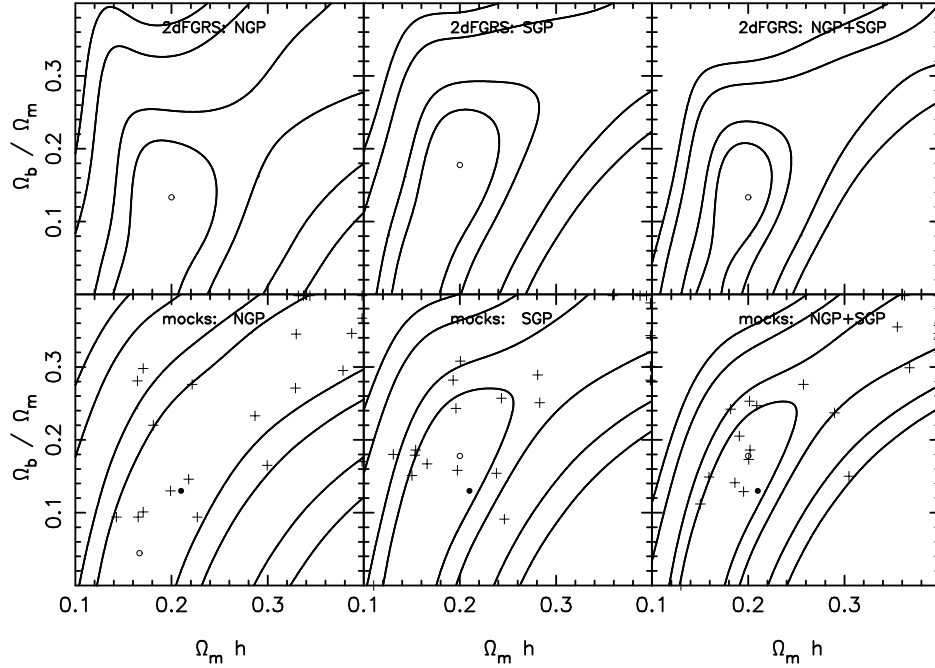
value increases the power of the model radial modes, requiring a decrease in the overall power to approximately fit the data. In order to show that this degeneracy corresponds to models with the same redshift-space power spectrum amplitude, the dashed lines in Fig. 2 show the locus of models with the same redshift-space power spectrum amplitude as the maximum likelihood solution. Here, the redshift-space and real-space power spectra, represented by  $P_s$  and  $P_r$ , are assumed to be related by

$$P_s = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right)P_r. \quad (3)$$

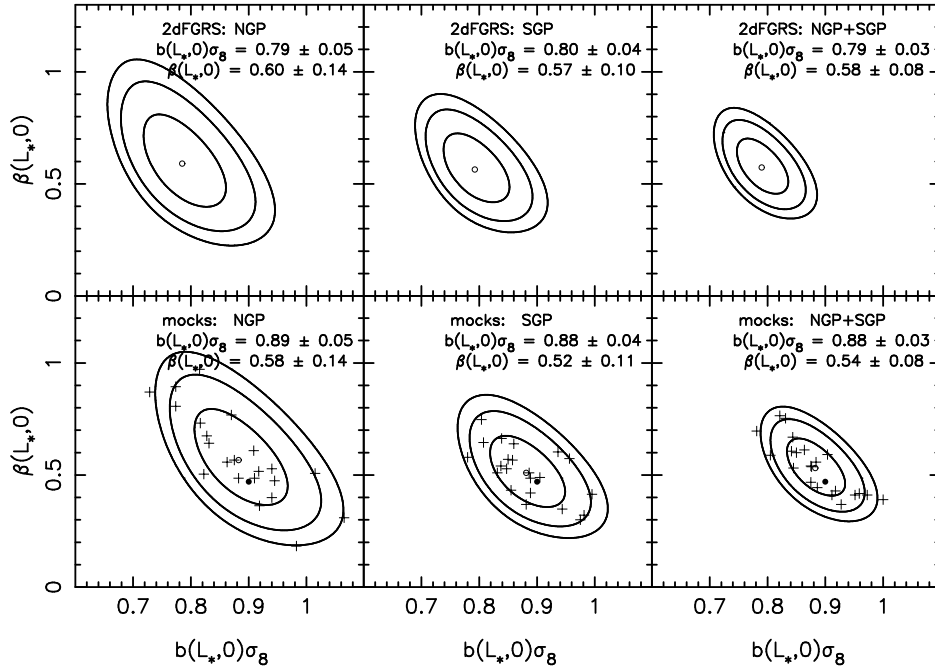
However, we still find tight constraints with  $b(L_*, 0)\sigma_8 = 0.81 \pm 0.02$  and  $\beta(L_*, 0) = 0.57 \pm 0.08$ . In fact, in Section 5.3 we marginalise over a range of model power spectra shapes and show that these constraints are not significantly expanded when the shape of the power spectrum is allowed to vary.

## 5.3 Results without prior on the power spectrum shape

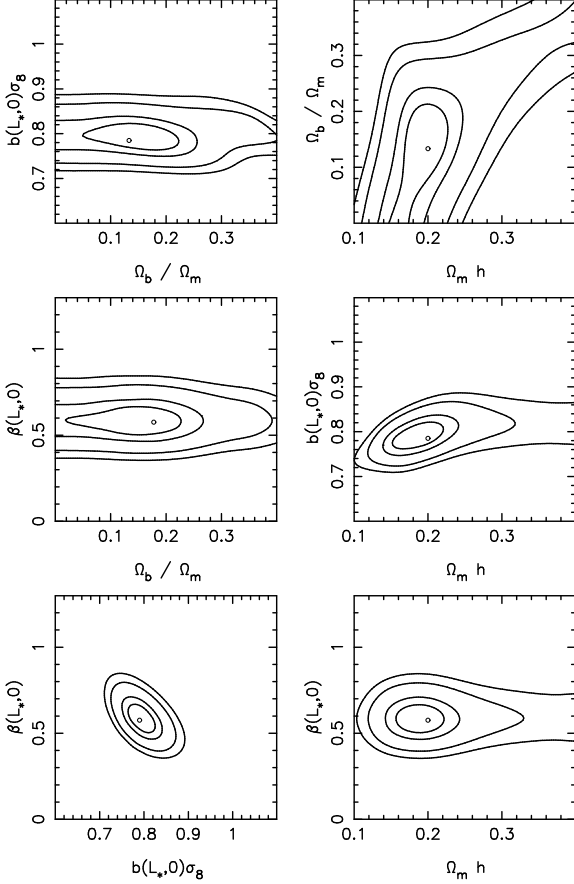
In Fig. 3 we present likelihood contours for  $\Omega_m h$  and  $\Omega_b/\Omega_m$  assuming a  $\Lambda$ CDM power spectrum with fixed  $n_s = 1.0$ , and marginalising over the power spectrum amplitude  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$ . Apart from the implicit dependence via  $\Omega_m h$ , there is virtually no residual sensitivity to  $h$ , so we set it at the Hubble key project value of  $h = 0.72$  (Freedman



**Figure 3.** Likelihood contours for  $\Omega_m h$  and  $\Omega_b/\Omega_m$  assuming a  $\Lambda$ CDM power spectrum with  $h = 0.72$  and  $n_s = 1.0$ . We have marginalised over the power spectrum amplitude and  $\beta(L_*, 0)$ . Solid lines in the top row show the recovered contours from the 2dFGRS, while the bottom row gives the average recovered contours from the  $\Lambda$ CDM mock catalogues. Contours correspond to changes in the likelihood from the maximum of  $2\Delta \ln \mathcal{L} = 1.0, 2.3, 6.0, 9.2$ . In addition to the contours plotted in Fig. 2, we also show the standard one-parameter 68 per cent confidence region to match with figure 5 in P01. The open circle marks the ML position. As in P01, we find a broad degeneracy in the  $(\Omega_m h, \Omega_b/\Omega_m)$  plane, which is weakly lifted with a low baryon fraction favoured for the 2dFGRS data. These parameter constraints are less accurate than those derived in C04 as we use less data, and we limit the number of modes used. ML positions for the 22 mock catalogues are shown by the crosses. It can be seen that a number of the mock catalogues have likelihood surfaces that are not closed, with the ML position being at one edge of the parameter space considered. However, these mocks all follow the general degeneracy between models with the same  $P(k)$  shape.



**Figure 4.** As Fig. 2, but now marginalising over the power spectrum shape as parameterized by  $\Omega_m h$  and  $\Omega_b/\Omega_m$ . As can be seen, allowing for different power spectrum shapes only increases the errors on  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  slightly. The relative interdependence between the power spectrum shape and  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  is considered in more detail in Fig. 5.



**Figure 5.** Contour plots showing changes in the likelihood from the maximum of  $2\Delta \ln \mathcal{L} = 1.0, 2.3, 6.0, 9.2$  for different parameter combinations for the combined likelihood from the 2dFGRS NGP and SGP catalogues, assuming a  $\Lambda$ CDM power spectrum with  $h = 0.72$  and  $n_s = 1.0$ . There are four parameters in total, and in each plot we marginalise over the two other parameters. The primary degeneracy arises between  $\Omega_m h$  and  $\Omega_b/\Omega_m$ , and corresponds to similar power spectrum shapes.  $b(L_*, 0)\sigma_8$  is also degenerate with  $\Omega_m h$ , although  $\beta(L_*, 0)$  is independent of the power spectrum shape.

et al. 2001). Contours are shown for the recovered likelihood calculated using the NGP & SGP catalogues and from the combination of the two. We present the measured likelihood surface from the 2dFGRS catalogue and the average likelihood surface recovered from the mock catalogues. For both the 2dFGRS and the mocks, there is a broad degeneracy between  $\Omega_m h$  and  $\Omega_b/\Omega_m$ , corresponding to models with similar power spectrum shape. This degeneracy is partially lifted by the 2dFGRS data, with a low baryon fraction favoured. Fig. 4 shows a similar plot for  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$ , marginalising over the power spectrum shape (parameterized by  $\Omega_m h$  and  $\Omega_b/\Omega_m$ ). Although this increases the size of the allowed region, the increase is relatively small, and we find  $\beta(L_*, 0) = 0.58 \pm 0.08$ , and  $b(L_*, 0)\sigma_8 = 0.79 \pm 0.03$ .

For the 2dFGRS catalogue, we present likelihood surfaces for all parameter combinations in our simple 4 parameter model in Fig. 5. This plot shows that there is a degeneracy between  $b(L_*, 0)\sigma_8$  and  $\Omega_m h$  (as discussed for example in Lahav et al. 2002). However,  $\beta(L_*, 0)$  appears to be independent of the power spectrum shape.

## 5.4 Confidence intervals for parameters

Although the modes used are uncorrelated because of the Karhunen-Loève data compression (Section A7), they are not independent, and we cannot assume that  $\ln \mathcal{L}$  has a  $\chi^2$  distribution. However, we can still choose to set fixed contours in the likelihood as our confidence limits and simply need to test the amplitude of the contours to be chosen. Luckily, we have 22 mock catalogues from which we can estimate confidence intervals. For  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$ , with a fixed power spectrum, 57% of the data points lie within the  $2\Delta \ln \mathcal{L} = 2.3$  average contour for the NGP and SGP mock catalogues. Marginalising over the power spectrum shape leaves 54% within the contour, 84% with  $2\Delta \ln \mathcal{L} < 6.0$ , and 100% with  $2\Delta \ln \mathcal{L} < 9.2$ . However, for power spectrum shape parameters  $\Omega_m h$  and  $\Omega_b/\Omega_m$ , 77% have  $2\Delta \ln \mathcal{L} < 2.3$ , all but one have  $2\Delta \ln \mathcal{L} < 6.0$ , and this mock has  $2\Delta \ln \mathcal{L} < 9.2$ .

Given the limited number of simulated catalogues, this is in satisfactory agreement. We note that the mocks were drawn from the Hubble Volume simulation (Evrad et al. 2002), and are consequently not completely independent. However, given that the numbers of mocks within the expected confidence intervals are close to those expected for independent Gaussian random variables, we feel justified in using the standard  $\chi^2$  intervals for our quoted parameters.

## 6 TESTS OF THE METHOD

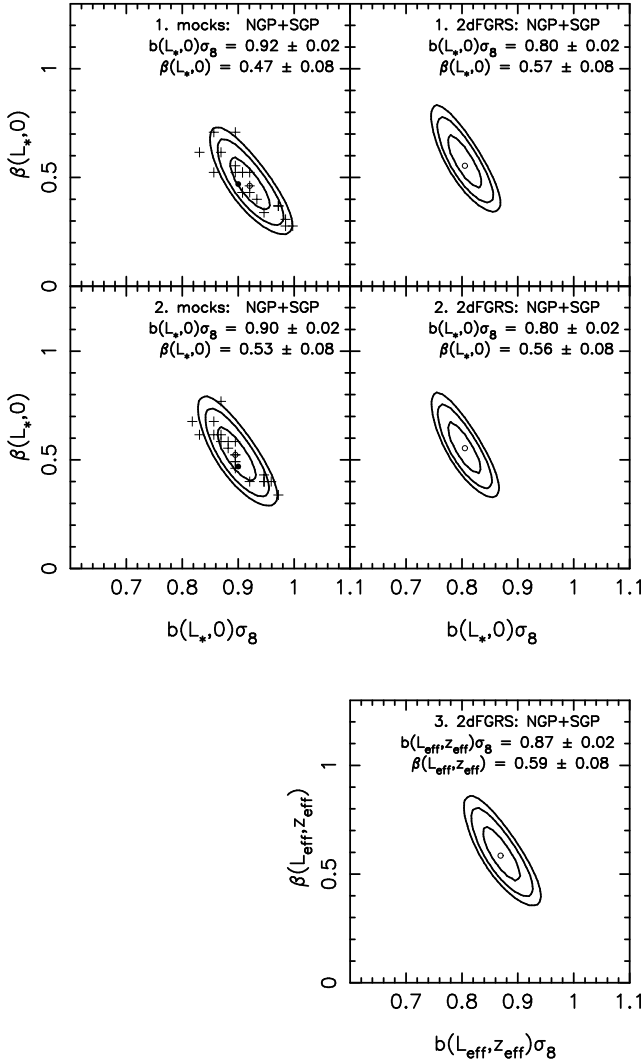
In the Fourier based analysis of P01 and C04, the expected variation in the measured power is only weakly dependent on the cosmological parameters and a fixed covariance matrix could therefore be assumed. In the analysis presented in this paper, the transformed density field is modelled rather than the power, and the likelihood variation due to cosmology is completely modelled using the covariance matrix – indeed, it is the variation of the covariance matrix that alters the likelihood and allows us to estimate the cosmological parameters. Consequently, we need to perform an inversion of this matrix for each cosmological model to be tested (an  $N^3$  operation). For a fixed power spectrum shape, the variation in the inverse covariance matrix with  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  is small and we can use an iterative trick (described in Section A8) to estimate the covariance matrix using an  $N^2$  operation. The covariance matrix obviously varies more significantly when we allow the cosmological parameters to vary more freely and the shape of the power spectrum changes. A full matrix inversion is then required for each model tested. This is computationally intensive and consequently the specific tests presented in this Section are based around recovering  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  for a fixed  $P(k)$  shape.

In Figs. 6 & 7 we present recovered likelihood surfaces calculated with various changes to our method. These plots demonstrate tests of our basic assumptions and of our implementation of the Spherical Harmonics method.

### 6.1 mock catalogues

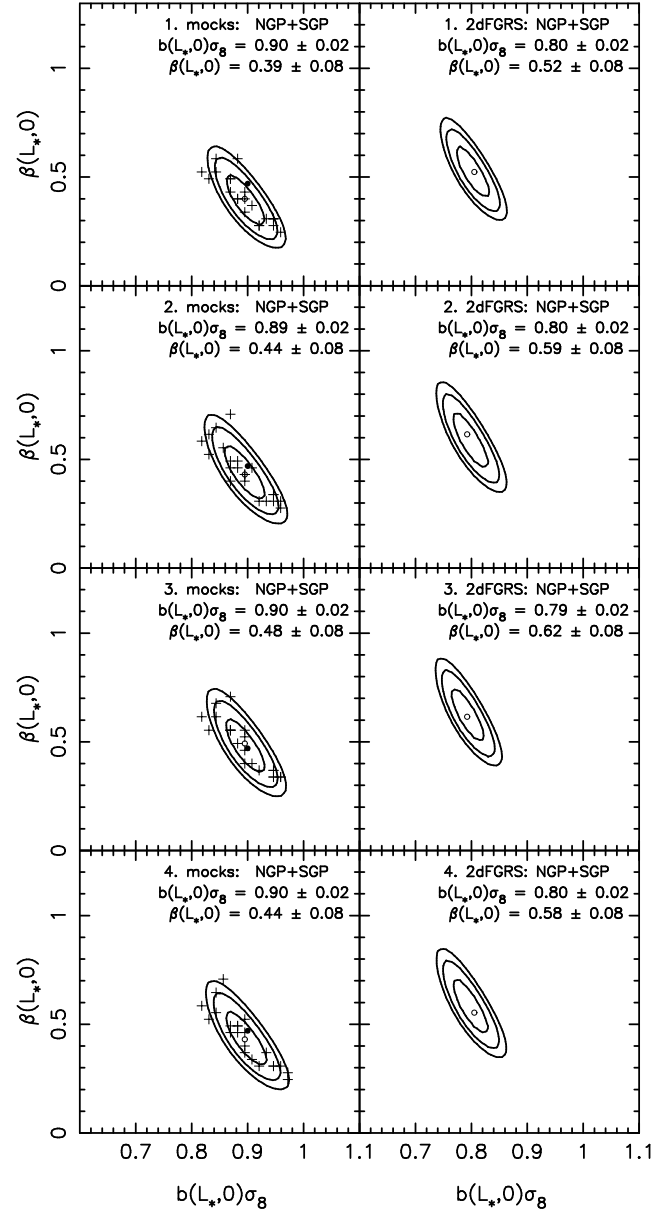
In addition to the 2dFGRS results presented in Figs. 2, 3 & 4, we also plot contours revealing the average likelihood surface determined from the 22 mock catalogues described





**Figure 6.** Likelihood contour plots as in Fig. 2, but now designed to test the Spherical Harmonics method. The different rows correspond to models with: (1.) power spectrum shape corresponding to linear rather than non-linear model. (2.) 1/completeness weighting for galaxies so that the weighted angular mask is uniform over the area of the survey. (3.) No luminosity-bias correction.

in Section 3. The average surface is used so the 2dFGRS and mock contours are directly comparable. For  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  we also give the recovered parameters and errors from the average likelihood surface. These numbers can be compared with the expected values  $b(L_*, 0)\sigma_8 = 0.9$  and  $\beta(L_*, 0) = 0.47$ . Crosses in these plots show the maximum likelihood positions calculated from each of the mock catalogues, while the open circle gives the combined maximum likelihood position, and the solid circles shows the expected values. We see that the recovered value of  $\beta(L_*, 0)$  is slightly higher than expected. However, we will show in Section 6.7 that the recovered value of  $\beta(L_*, 0)$  changes in a consistent way following changes in the peculiar velocities calculated for the galaxies in each mock. Furthermore, we show that there is no evidence for a systematic offset in the recovered  $\beta(L_*, 0)$ , which we would expect to vary with the peculiar



**Figure 7.** Likelihood contour plots as in Fig. 2, but now designed to test the effect of the Fingers-of-God correction applied. The different rows correspond to models with: (1.) no Fingers-of-God correction applied. (2.) an exponential model for the probability distribution caused by the Fingers-of-God effect (Eq. 5) with  $\sigma_{\text{pair}} = 400 \text{ km s}^{-1}$ . (3.) a Gaussian model the probability distribution (Eq. 6) with  $\sigma_{\text{pair}} = 400 \text{ km s}^{-1}$ . (4.) a model with exponential distribution for the correlation function (Eq. 7) with  $\sigma_{\text{pair}} = 400 \text{ km s}^{-1}$ .

velocities. The true value of  $b(L_*, 0)\sigma_8$  is recovered to sufficient precision.

## 6.2 Non-linear power assumption

Although the width of the window function means that the modes are dependent on the real-space power spectrum at  $k > 0.2 h \text{ Mpc}^{-1}$ , this dependence is weak compared with the dependence on the low- $k$ , linear regime (this is shown in Fig. A1). The likelihood calculation used in Eq. A43 relied

on the transformed density field having Gaussian statistics. While this is expected to be true in the linear regime, on the scales of non-linear collapse this assumption must break down. Although the modes must deviate from Gaussianity, we consider the change in shape of the power as a first approximation, and use the fitting formulae of Smith et al. (2003) to determine the model power. In order to test the significance of this, we consider the effect of replacing the non-linear power in the model (Eq. A40) with the linear power. In fact, this has a relatively small effect on the recovered power spectrum amplitude and  $\beta(L_*, 0)$  as shown in Fig. 6.

### 6.3 Completeness weighting

We have tried two angular weighting schemes for the galaxies. The default weights do not have an angular component, and are simply the radial weights of Feldman, Kaiser & Peacock (1994) given by Eq. A5. For comparison we have also tried additionally weighting each galaxy by  $1/(\text{angular completeness})$ , so the weighted galaxy density field at a given  $r$  is independent of angular position (i.e. it is uniform over the survey area). This weighting simplifies the convolution of the model power to correct for the angular geometry of the survey (Eq. A22 & A27), and comparing results from both schemes therefore tests this convolution. The downside of such a weighting is the slight increase in shot noise. Results calculated with this weighting scheme are presented in Fig. 6, and can be compared with the default in Fig. 2: no significant difference is observed between the two schemes.

### 6.4 Luminosity-dependent bias

As described in Section A1, we adopt a constant galaxy clustering (CGC) model for the evolution of the fluctuation amplitudes across the survey, and correct for the expected luminosity dependence of this amplitude by weighting each galaxy by the reciprocal of the expected bias ratio to  $L_*$  galaxies (as suggested by Percival, Verde & Peacock 2004). The expected bias ratio assumed, given by Eq. A1, was calculated from a volume-limited subsample of the 2dFGRS by Norberg et al. (2001). In Fig. 6, we fit models that do not include either this luminosity-dependent bias correction, or the evolution correction for  $\beta(L, z)$ . This likelihood fit measures  $\beta(L_{\text{eff}}, z_{\text{eff}})$ , which is now a function of the effective luminosity  $L_{\text{eff}}$  and effective redshift  $z_{\text{eff}}$  of the survey. For the complete survey, examining the weighted density field gives that  $L_{\text{eff}} = 1.9L_*$  and  $z_{\text{eff}} = 0.17$ . However, we cannot be sure that the Spherical Harmonics modes selected will not change these numbers.

In fact, fitting to the data gives  $\beta(L_{\text{eff}}, z_{\text{eff}}) = 0.59 \pm 0.08$  and  $b(L_{\text{eff}}, z_{\text{eff}})\sigma_8(z_{\text{eff}}) = 0.87 \pm 0.02$ . In order to compare these values with our results that have been corrected for luminosity-dependent bias, we have to consider a number of factors. For the CGC model, the change in the measured power spectrum amplitude should only arise from the galaxy luminosity probed. The effective luminosity of the sample is  $\sim 1.9L_*$ , which gives an expected bias of 1.13 (using Eq. A1). The observed offset in amplitude is 1.08, perhaps indicating that, for the chosen modes,  $L_{\text{eff}} < 1.9L_*$ . Within the CGC model,  $\beta(L_{\text{eff}}, z_{\text{eff}})$  is expected to be related to  $\beta(L_*, 0)$  by

$$\beta(L_{\text{eff}}, z_{\text{eff}}) = \frac{\Omega_m(z_{\text{eff}})^{0.6}}{\Omega_m(0)^{0.6}} D(z_{\text{eff}}) \frac{b(L_*, 0)}{b(L_{\text{eff}}, 0)} \beta(L_*, 0), \quad (4)$$

which gives  $\beta(1.9L_*, 0.17) \sim \beta(L_*, 0)$  for a concordance cosmological model as the different factors approximately cancel. In fact, we measure no significant difference between  $\beta(L_{\text{eff}}, z_{\text{eff}})$  and  $\beta(L_*, 0)$ .

### 6.5 Fingers-of-God correction

In this Section we test the assumed scattering probability that corrects distance errors induced by the peculiar velocities of galaxies inside groups. This probability was used to convolve the model transformed density fields using the matrix presented in Eq. A10. We compare models with exponential and Gaussian forms, and a model that corresponds to an exponential convolution for the correlation function (this corresponds to the model advocated by Ballinger, Peacock & Heavens 1996; Hawkins et al. 2003)

$$p_e(r-y) = \frac{1}{\sqrt{2}\sigma_v} \exp\left[-\frac{\sqrt{2}|r-y|}{\sigma_v}\right], \quad (5)$$

$$p_g(r-y) = \frac{1}{\sqrt{2\pi}\sigma_v} \exp\left[-\frac{(r-y)^2}{2\sigma_v^2}\right], \quad (6)$$

$$p_b(r-y) = \frac{2\sqrt{2}}{\sigma_v} K_0\left[-\frac{\sqrt{2}}{\sigma_v}(r-y)\right]. \quad (7)$$

$\sigma_v$  is the one-dimensional velocity dispersion, related to the commonly used pairwise velocity dispersion by  $\sigma_{\text{pair}} = \sqrt{2}\sigma_v$ .  $K_n$  is an  $n$ th-order modified Bessel function derived as the inverse Fourier transform of the root of a Lorentzian (Taylor et al. 2001).

The Fingers-of-God effect stretches structure along the line-of-sight, whereas large-scale bulk motions tend to foreshorten objects. Although these effects predominantly occur on different scales, there is some overlap, and if the Fingers-of-God effect is not included when modelling the data, the best-fit value of  $\beta(L_*, 0)$  is decreased slightly. In this case, the best-fit model interpolates between the two effects, as demonstrated in Fig. 7, where we present the best-fit  $\beta(L_*, 0)$  with and without including the Fingers-of-God correction.

In the results presented in Fig. 2, we assumed an exponential distribution for the distribution function of random motions with  $\sigma_{\text{pair}} = 350 \text{ km s}^{-1}$  for the 2dFGRS catalogue and  $\sigma_{\text{pair}} = 500 \text{ km s}^{-1}$  for the mock catalogues. We have tried a number of different values of  $0 < \sigma_{\text{pair}} < 500 \text{ km s}^{-1}$  and find only very small variation in the best-fit  $\beta(L_*, 0)$ , as expected because we have chosen modes that peak for  $k < 0.15 h \text{ Mpc}^{-1}$ , where the finger-of-god correction is small. To demonstrate this, in Fig. 7 we present results calculated using Eqns. 5, 6 & 7 with  $\sigma_{\text{pair}} = 400 \text{ km s}^{-1}$  for both the 2dFGRS and mock catalogues. We also compare with the effect of not including any correction for the small-scale velocity dispersion. Little difference is seen in the recovered values of  $\beta(L_*, 0)$ , adding weight to the hypothesis that the Fingers-of-God correction is not important for our determination of  $\beta(L_*, 0)$ .

In the  $\xi(\sigma, \pi)$  analyses of the 2dFGRS presented in Peacock et al. (2001), Madgwick et al. (2003) and Hawkins et al. (2003) a strong degeneracy was revealed between the Fingers-of-God and linear redshift-space distortions. Al-

though such a degeneracy is also present in the results from our Spherical Harmonics analysis, it is weak compared with the  $\xi(\sigma, \pi)$  results. The difference is due to the scales analysed – the correlation function studies estimated the clustering strength on smaller scales where the Fingers-of-God convolution is more important. Because the Fingers-of-God effect has less effect in our analysis, we are less able to constrain its amplitude, and therefore assume a fixed value motivated by the  $\xi(\sigma, \pi)$  analyses, rather than fitting to the data.

## 6.6 2dFGRS catalogue calibration

Outwith this Section we consider the 2dFGRS release catalogue and corresponding calibration. In order to test the dependence of our results on the catalogue calibration, in this Section we report on the analysis of a different version of the catalogue with revised calibration. In the release catalogue (Colless et al. 2003; <http://www.mso.anu.edu.au/2dFGRS/>) the 2dFGRS photographic magnitudes were calibrated using external CCD data from the SDSS Early Data Release and ESO Imaging Survey (EIS) (Colless et al. 2003; Cross et al. 2003). Overlaps between the photographic plates allow this calibration to be propagated to the whole survey. In this section we instead calibrate each plate without the use of external data. The magnitudes in the final released catalogue,  $b_j^{\text{final}}$  and magnitudes,  $b_j^{\text{self}}$ , resulting from this self-calibration are assumed to be related by a linear relation

$$b_j^{\text{self}} = a_{\text{self}} b_j^{\text{final}} + b_{\text{self}}, \quad (8)$$

where the calibration coefficients  $a_{\text{self}}$  and  $b_{\text{self}}$  are allowed to vary from plate to plate. To set the values of these calibration coefficients two constraints are applied. First on each plate we assume that the galaxy luminosity function can be represented by a Schechter function with faint-end slope  $\alpha = 1.2$  and make a maximum likelihood estimate of  $M_*$ . The value of  $M_*$  is sensitive to the difference in  $b_j^{\text{self}}$  and  $b_j^{\text{final}}$  at around  $b_j = 17.5$  and the number of galaxies on each plate is such that the typical random error on  $M_*$  is 0.03 magnitudes. Second we compare the number of galaxies,  $N(z > 0.25)$ , with redshifts greater than  $z = 0.25$  with the number we expect,  $N_{\text{model}}(z > 0.25)$ , based on our standard model of the survey selection function. The value of  $N_{\text{model}}(z > 0.25)$  depends sensitively on the survey magnitude limit and so constrains the difference in  $b_j^{\text{self}}$  and  $b_j^{\text{final}}$  at  $b_j \approx 19.5$ . By demanding that on each plate both  $N(z > 0.25) = N_{\text{model}}(z > 0.25)$  and  $M_* - 5 \log h = 19.73$  we determine  $a_{\text{self}}$  and  $b_{\text{self}}$ . This method of calibrating the catalogue is extreme in that it ignores the CCD calibrating data (apart from setting the overall zero point of  $M_* - 5 \log h = 19.73$ ). A more conservative approach is to combine the external calibration with the internal one and determine  $a_{\text{self}}$  and  $b_{\text{self}}$  by a  $\chi^2$  procedure that takes account of the statistical error on  $M_*$ , the expected variance on  $N(z > 0.25)$  given by mock catalogues and the errors on the calibrating data. Unless the errors on the CCD calibration are artificially inflated this results in a calibration very close to that of final release. Thus we believe that the difference between the results achieved with self-calibrated catalogue and the standard final catalogue represent an up-

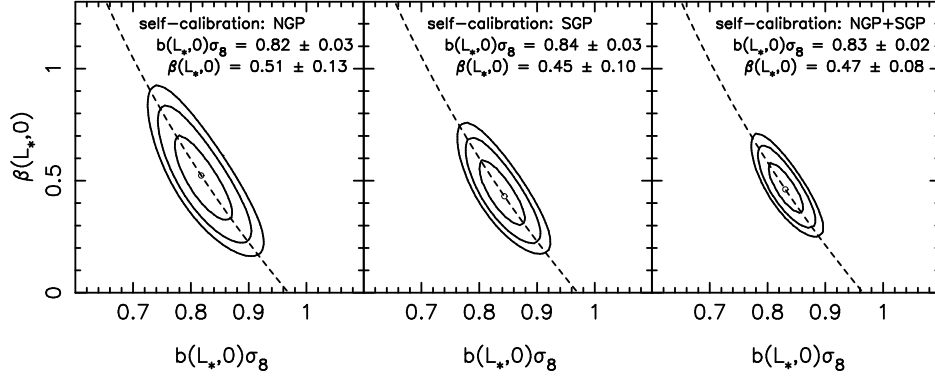
per limit on the effects attributable to uncertainty in the photometric calibration.

For the Spherical Harmonics analysis method, we need to cut the 2dFGRS catalogue so that the radial distribution of galaxies is independent of angular position (this catalogue reduction was described in Section 2). Changing the magnitude limit at which to cut the catalogue changes the angular mask for the reduced sample as angular regions that do not go as faint as the chosen limit are removed. Rather than optimize the magnitude limit at which to cut the self-calibrated catalogue, we instead resample the revised catalogue using the old mask. A magnitude limit was then chosen to fully sample this angular region and give a radial distribution that is independent of angular position. This procedure avoided the computationally expensive recalculation of angular matrices (see Appendix A). However, the radial galaxy distribution and total number of galaxies were different from those in our primary analysis, and a revised radial component of the covariance matrix was required.

Revised parameter constraints on  $\beta(L_*, 0)$  and  $b(L_*, 0)\sigma_8$  are presented in Fig. 8, which can be directly compared with the upper panels in Fig. 2. An incorrect calibration would lead to a resampling of the complete 2dFGRS catalogue (described in Section 2) that would not produce a catalogue with radial galaxy distribution independent of angular position. This would lead to an increase in the amplitude of the observed angular fluctuations. Given that we split the fluctuations into an overall power spectrum and an additional component in the radial direction caused by linear redshift space distortions, an artificial increase in angular clustering would manifest itself as an increase in  $b(L_*, 0)\sigma_8$ , coupled with a decrease in  $\beta(L_*, 0)b(L_*, 0)\sigma_8$ , which controls the absolute amplitude of the linear redshift-space distortions. In fact this is exactly what is observed when comparing results from the release and self-calibration catalogues (Figs. 2 & 8), suggesting that the self-calibration procedure introduces artificial angular distortions into the reduced catalogue. The dashed lines in Fig. 8 show the locus of models with redshift-space power spectrum amplitude (calculated from Eq. 3) at the maximum likelihood (ML) value. Comparing the relative positions of the ML points in Figs. 2 & 8 shows that changing the catalogue calibration moves the maximum likelihood position along this locus, without significantly changing the redshift-space power amplitude. Catalogue calibration and selection represents the most significant potential source of systematic error in our analysis.

## 6.7 Testing $\beta(L_*, 0)$ using mock catalogues

For each galaxy in the mock catalogues, we know the relative contributions to the redshift from the Hubble flow and peculiar velocity. Consequently, we can easily increase or decrease the amplitude of the redshift space distortions to mimic catalogues with different cosmological parameters. In Fig. 9 we plot the recovered power spectrum amplitudes and  $\beta(L_*, 0)$  from catalogues created by increasing or decreasing the peculiar velocity by 50% from the true value. Obviously, this changes both the linear redshift space distortions and the Fingers-of-God, and consequently we fit to these data assuming a revised  $\sigma_{\text{pair}}$ . If we neglected to do this, the average recovered  $\beta(L_*, 0)$  would vary from the true value by less than 50%, as assuming the wrong value of  $\sigma_{\text{pair}}$  has the



**Figure 8.** Likelihood contours for the recovered  $b(L_*, 0)\sigma_8$  and  $\beta(L_*, 0)$  assuming a fixed  $\Lambda$ CDM power spectrum shape as in Fig. 2, but now calculated having revised the calibration of the 2dFGRS catalogue. Details of the revised calibration are presented in Section 6.6.

effect of damping the change in the recovered  $\beta(L_*, 0)$ . However, because the linear redshift space distortions are dominant on the scales being probed in our analysis, we would still see a change in the correct direction. For these catalogues, we find that altering the peculiar velocities results in a consistent change in the recovered  $\beta(L_*, 0)$ , showing that our likelihood test is working well.

In Fig. 9 we also show the recovered parameter constraints from mocks catalogues with no redshift space distortions. Here, we fitted to these data assuming that there was no Fingers-of-God effect, and see that we recover for  $\beta(L_*, 0)$  consistent with 0 for each catalogue.

## 7 SUMMARY AND DISCUSSION

The Spherical Harmonics analysis method of HT and Tadros et al. (1999) has been extended and updated to allow for surveys that cover a relatively small fraction of the sky. Additionally, a consistent approach has been adopted to model luminosity-dependent bias and the evolution of the matter power spectrum. We assume a constant galaxy clustering model for the redshift region  $0 < z < 0.25$  covered by the 2dFGRS survey, in which, although the matter density field does evolve, the galaxy power spectrum is assumed to remain fixed. Galaxy bias is also assumed to be a function of luminosity, and we correct for the effect that this has on the recovered power spectrum.

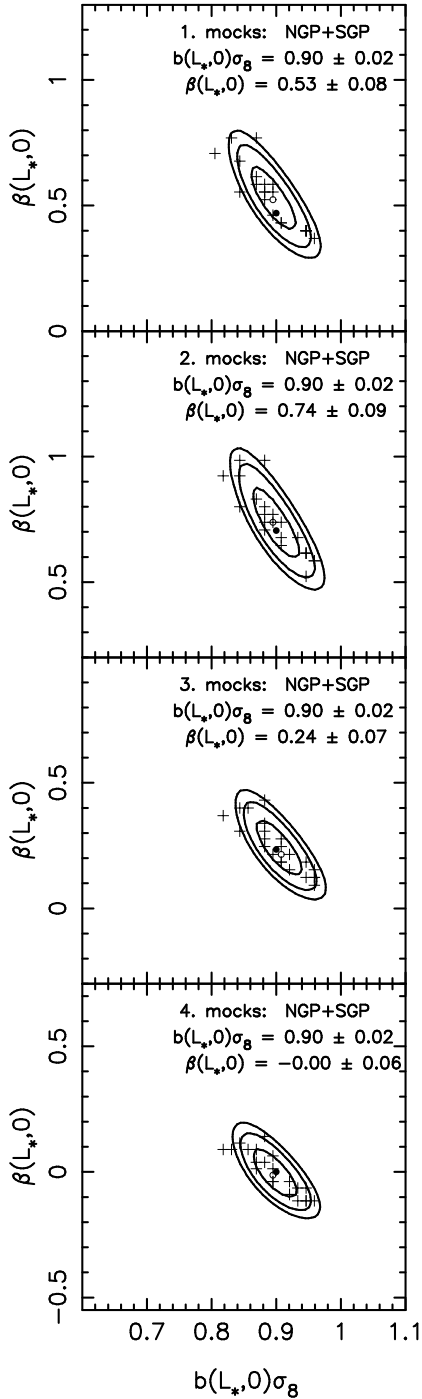
The revised method has been applied to the complete 2dFGRS catalogue, resulting in tight constraints on the amplitude of the linear redshift space distortions. Because the method still requires a survey with selection function separable in radial and angular directions, we have to use a reduced version of the final 2dFGRS catalogue. Additionally, we are forced to compromise on the quantity of data (number of modes) analysed, although we have tried to perform a logical and optimized reduction of the mode number. These considerations mean that we do not obtain the accuracy of the cosmological constraints from the shape of the galaxy power spectrum obtained in our companion Fourier analysis (C04). This reduction in accuracy primarily results from the decrease in the catalogue size. In particular, the analysis is cosmic variance limited and most of the discarded galaxies were luminous and therefore at high redshift where they trace a large volume of the Universe. However, from the

Spherical Harmonics method we do obtain power spectrum shape constraints  $\Omega_b/\Omega_m < 0.21$  as shown in Fig. 3 and, for fixed  $\Omega_b/\Omega_m = 0.17$ , we find  $\Omega_m h = 0.20^{+0.03}_{-0.03}$ , consistent with previous power spectrum analyses from the 2dFGRS and the SDSS.

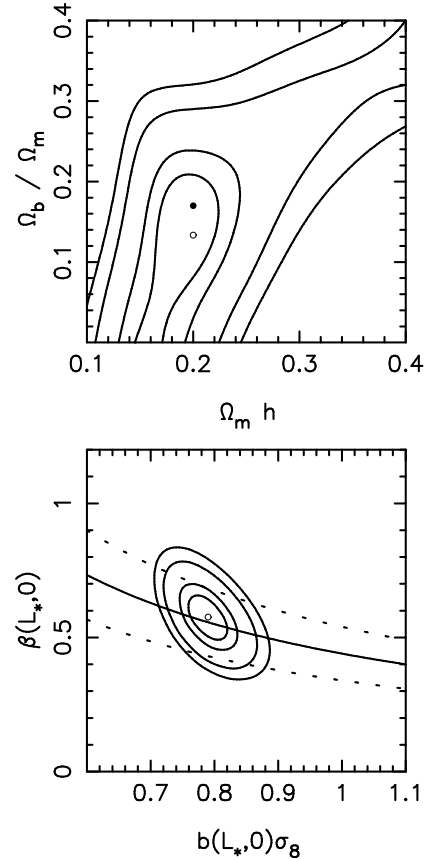
We have also modelled the overdensity distribution in 22  $\Lambda$ CDM mock catalogues, designed to mimic the 2dFGRS. By presenting recovered parameters from these catalogues, we have shown that any systematic biases induced by the analysis method are at a level well below the cosmic variance caused by the size of the survey volume. In particular, it should be emphasized that these mocks include a realistic degree of scale-dependent bias, to reflect the known difference in small-scale clustering between galaxies and the nonlinear CDM distribution (e.g. Jenkins et al. 1998). We have additionally used these catalogues to test the errors placed on recovered parameters and find that assuming a  $\chi^2$  distribution for  $\ln \mathcal{L}$  provides approximately the correct errors. The tests presented, considering the NGP and SGP separately, using the mocks, and varying parts the analysis method were designed to test our Spherical Harmonics formalism and the assumptions that go into this. In particular, we do not test the 2dFGRS sample for internal consistency, for instance splitting by redshift or magnitude limit, although we do find consistent parameter estimates from the northern and southern parts of the survey. A more comprehensive set of tests is presented in C04, using Fourier methods to decompose the density field.

By considering a revised 2dFGRS catalogue calibration, we have examined the effect of small systematic magnitude errors on our analysis. Such errors artificially increase the strength of the angular clustering, leading to an increase in the best-fit  $b(L_*, 0)\sigma_8$  and a corresponding decrease in  $\beta(L_*, 0)$ . We have shown that the revised catalogue tested produces such a change in the recovered parameters, therefore providing evidence in favour of the release calibration. The calibration method and its effect will be further discussed in C04. Here, we simply note that the systematic error in  $\beta(L_*, 0)$  and  $b(L_*, 0)$  from catalogue calibration is of the same order as the random error.

The strength of the Spherical Harmonics method as applied to the 2dFGRS lies in measuring the linear redshift-space distortions, and fitting the real-space power spectrum amplitude. Consequently we are able to measure  $\beta(L_*, 0) =$



**Figure 9.** Likelihood contour plots as in Fig. 2, but now designed to test our recovery of  $\beta(L_*, 0)$  using the mock catalogues. The different panels correspond to: 1. the standard catalogues with peculiar velocities calculated directly from the Hubble Volume simulation. Here the true value of  $\beta(L_*, 0)$  is 0.47. 2. the contribution to the galaxy redshifts from the peculiar velocities has been increased by 50%. Here, we assume  $\sigma_{\text{pair}} = 750 \text{ km s}^{-1}$ . Without this correction,  $\beta(L_*, 0)$  would increase by less than 50%. We expect  $\beta(L_*, 0) = 0.71$ , shown by the solid circle. 3. as 2, but now decreasing the redshift contribution by 50%.  $\sigma_{\text{pair}} = 250 \text{ km s}^{-1}$  is assumed, and we expect  $\beta(L_*, 0) = 0.24$ . 4. recovered parameters from real space catalogues, calculated assuming that  $\sigma_{\text{pair}} = 0$ . Obviously, we expect to recover  $\beta(L_*, 0) = 0.0$ .



**Figure 10.** Likelihood contour plots for the combined NGP + SGP 2dFGRS catalogue as in Fig. 5 compared with best fit parameters from WMAP (Bennett et al. 2003; Spergel et al. 2003). The constraint on the characteristic amplitude of velocity fluctuations from the 1-year WMAP data is  $\sigma_8 \Omega_m^{0.6} = 0.44 \pm 0.10$ , which is shown in the left panel by the thick solid line, with  $1\sigma$  errors given by the dotted lines. In the right panel, the solid circle shows the best-fit parameter values of  $\Omega_m h = 0.20$  &  $\Omega_b / \Omega_m = 0.17$ . As can be seen, the constraints resulting from the 2dFGRS power spectrum shape and the linear distortions are consistent with the WMAP data.

$0.58 \pm 0.08$ , and  $b(L_*, 0)\sigma_8 = 0.79 \pm 0.03$  for  $L_*$  galaxies at  $z = 0$ , marginalising over the power spectrum shape. This result is dependent on the constant galaxy clustering model and on the bias-luminosity relationship derived by Norberg et al. (2001), and covers  $0.02 < k < 0.15 h \text{ Mpc}^{-1}$ . Our measurement of  $\beta(L_*, 0)$  is derived on larger scales than the  $\xi(\sigma, \pi)$  analyses of the 2dFGRS presented in Peacock et al. (2001) and Hawkins et al. (2003), and scale-dependent bias could therefore explain why our result is slightly higher than the numbers obtained in these analyses.

Tegmark, Hamilton & Xu (2002) performed a similar spherical harmonics analysis of the 100k release of the 2dFGRS. As in the analysis presented here, they also required a catalogue that was separable in radial and angular directions, and cut the 100k release catalogue to 66050 galaxies. From these galaxies, they measured  $\beta(L_{\text{eff}}, z_{\text{eff}}) = 0.49 \pm 0.16$ , consistent with our result (see Section 6.4 for a discussion of the conversion between  $\beta(L_{\text{eff}}, z_{\text{eff}})$  and  $\beta(L_*, 0)$ ). Our result not only allows for luminosity-dependent bias

and evolution, it also uses over twice as many galaxies as the Tegmark, Hamilton & Xu (2002) analysis.

On large-scales, galaxies are expected to directly trace the bulk motion of the density field, so the absolute amplitude of the observed redshift-space distortions caused by this motion is expected to be independent of galaxy properties. This assumption has been tested empirically by considering the mean relative velocity of galaxy pairs in different samples (Juszkiewicz et al. 2000; Feldman et al. 2003). Rather than fitting  $\beta(L_*, 0)$ , the relative importance of the linear redshift-space distortions compared with the real-space galaxy power spectrum, we can instead fit the absolute amplitude of these fluctuations. This results in the cosmological constraint  $\Omega_m^{0.6} \sigma_8 = 0.46 \pm 0.06$ .

The relatively high power of  $\sigma_8$  compared to  $\Omega_m$  in this constraint means that an additional constraint on  $\Omega_m$  provides a tight constraint on  $\sigma_8$ . For example, fixing  $\Omega_m = 0.3$  gives  $\sigma_8 = 0.95 \pm 0.12$  ( $\sim 15\%$  error), while fixing  $\sigma_8 = 0.95$  gives  $\Omega_m = 0.3 \pm 0.08$  ( $\sim 26\%$  error). We note that our constraint is approximately  $1\sigma$  higher than a recent combination of weak-lensing measurements that gave  $\sigma_8 \simeq 0.83 \pm 0.04$  for  $\Omega_m = 0.3$  (Refregier 2003). Additionally, combining the weak-lensing constraint with our measurement of  $b(L_*, 0)\sigma_8$  suggests that  $b(L_*, 0) \sim 0.9$  in agreement with the analyses of Lahav et al. (2002) & Verde et al. (2002). Similarly, combining our measurement of  $\beta(L_*, 0)$  with recent constraints on  $\Omega_m$  (such as those derived by Spergel et al. 2003) suggests that  $b(L_*, 0) \sim 0.9$ , and we see that we have a consistent picture of both the amplitude of the real-space power spectrum and linear redshift-space distortions within the concordance  $\Lambda$ CDM model.

A comparison of our results with parameter constraints from WMAP is presented in Fig. 10. In this paper, we do not attempt to perform a full likelihood search for the best-fit cosmological model using the combined 2dFGRS and WMAP data sets. Instead, we simply consider the consistency between the WMAP data and our measurements of the 2dFGRS. In Fig. 10 we plot the WMAP constraint on  $\Omega_m^{0.6} \sigma_8$  as derived in Spergel et al. (2003), compared with our constraints on  $\beta(L_*, 0)$  and  $b(L_*, 0)\sigma_8$ . WMAP obviously tells us nothing about  $b(L_*, 0)$ , so there is a perfect degeneracy between these parameters. However, the constraints are seen to be consistent. In fact our constraint is a significant improvement on the WMAP constraint, primarily because of the uncertainty in the optical depth to the last scattering surface, parameterized by  $\tau$ .

Because  $h = 0.72$  is fixed in the simple cosmological model assumed to parameterise the power spectrum shape, the horizon angle degeneracy for flat cosmological models (Percival et al 2002; Page et al. 2003) is automatically lifted. The position of the first peak in the CMB power spectrum therefore provides a tight constraint on  $\Omega_m$ . In fact, given this simple model, the constraints on  $\Omega_m$  and  $\Omega_b/\Omega_m$  from WMAP are so tight that we chose to plot a point to show them in Fig. 10, rather than a confidence region. However, had we considered a larger set of models in which  $h$  was allowed to vary, then an extra constraint is required to break the horizon angle degeneracy even for the WMAP data (Page et al. 2003). In this paper we provide a new cosmological constraint by measuring the strength of the linear distortions caused by the bulk flow of the density field mapped by the final 2dFGRS catalogue.

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## APPENDIX A: METHOD

The Spherical Harmonics method applied to the 2dFGRS in this paper has a number of significant differences from the formalisms developed for the IRAS surveys (Fisher et al. 1994;1995; HT; Tadros et al. 1999). The revisions are primarily due to the complicated geometry of the 2dFGRS survey (whereas the IRAS surveys nearly covered the whole sky), although we additionally apply a correction for varying galaxy bias, dependent on both galaxy luminosity and redshift. For these reasons we provide a simple, complete description of the formalism in this appendix. Note that throughout we use a single Greek subscript (e.g.  $\nu$ ) to represent a triplet (e.g.  $\ell_\nu, n_\nu, m_\nu$ ), so the spherical harmonic  $Y_\nu(\theta, \phi) \equiv Y_{\ell_\nu, m_\nu}(\theta, \phi)$  and the spherical Bessel functions,  $j_\nu(s) \equiv j_{\ell_\nu}(k_{\ell_\nu, n_\nu} s)$ .  $-\nu$  is defined to represent the triplet  $(\ell_\nu, -m_\nu, n_\nu)$ . We also adopt the following convention for coordinate positions:  $\mathbf{r}$  is the true (or real space) position of a galaxy,  $\mathbf{s}$  is the observed redshift space position given the linear in-fall velocity of the galaxy.  $\mathbf{s}'$  and  $\mathbf{r}'$  correspond to  $\mathbf{s}$  and  $\mathbf{r}$  including the systematic offset in the measured distance caused by the small-scale velocity dispersion of galaxies within larger virialised objects.

### A1 Galaxy bias model

As in Lahav et al. (2002), we adopt a constant galaxy clustering (CGC) model for the evolution of galaxy bias over the range of redshift covered by the 2dFGRS sample used in this analysis ( $0 < z < 0.25$ ) i.e. we assume that the normalization of the galaxy density field is independent of redshift, for any galaxy luminosity  $L$ . We also assume that the relative expected bias  $\hat{r}_b(L)$  of galaxies of luminosity  $L$  relative to that of  $L_*$  galaxies is a function of luminosity

$$\hat{r}_b(L) = \left\langle \frac{b(L, z)}{b(L_*, z)} \right\rangle = 0.85 + 0.15 \frac{L}{L_*}, \quad (\text{A1})$$

and that this ratio is independent of redshift. This dependence is implied by the relative clustering of 2dFGRS galaxies (Norberg et al. 2001).

In the analysis presented in this paper, the galaxy bias is modelled using a very simple linear form with the mean redshift-space density of galaxies of luminosity  $L$  given by

$$\rho(\mathbf{r}') = \bar{\rho}(\mathbf{r}') [1 + b(L, 0)\delta(\text{mass}, \mathbf{r}')] \quad (\text{A2})$$

$$= \bar{\rho}(\mathbf{r}') [1 + \hat{r}_b(L)\delta(L_*, \mathbf{r}')], \quad (\text{A3})$$

where  $\delta(\text{mass}, \mathbf{r}')$  is the present day mass density field, and  $\delta(L_*, \mathbf{r}')$  is the density field of galaxies of luminosity  $L_*$ , which is assumed to be independent of epoch.

The galaxy bias model described above was used to correct the observed galaxy overdensity field, enabling measurement of the shape and amplitude of the power spectrum of  $L_*$  galaxies. Following the CGC model, we do not have to correct the recovered clustering signal for evolution, provided that we wish to measure the galaxy rather than the mass power spectrum. However, because galaxy luminosity varies systematically with redshift, we do need to correct for luminosity-dependent bias, and we do this in a way analogous to the Fourier method presented by Percival, Verde & Peacock (2004), by weighting the contribution of each galaxy to the measured density field by the reciprocal of the expected bias ratio  $\hat{r}_b(L)$  given by Eq. A1.

In the following description of the formalism, we only consider galaxies of luminosity  $L$ . Without loss of generality, this result can be expanded to cover a sample of galaxies with different luminosities by simply summing (or integrating) over the range of luminosities (as in Percival, Verde & Peacock 2004).

### A2 The Spherical Harmonic formalism

Further description of the Spherical Harmonics formalism may be found in Fisher et al. (1994; 1995), HT and Tadros et al. (1999). Expanding the density field of the redshift-space distribution of galaxies of luminosity  $L$  in spherical harmonics and spherical Bessel functions gives

$$\rho_\nu(L, \mathbf{s}') = c_\nu \int d^3s' \frac{\rho(L, \mathbf{s}')}{\hat{r}_b(L)} w(\mathbf{s}') j_\nu(s') Y_\nu^*(\theta, \phi), \quad (\text{A4})$$

where  $w(\mathbf{s}')$  is a weighting function for which we adopt the standard Feldman, Kaiser & Peacock (1994) weight

$$w(\mathbf{s}') = \frac{1}{1 + \bar{\rho}(\mathbf{s}') \langle P(k) \rangle}. \quad (\text{A5})$$

Here,  $\bar{\rho}(\mathbf{s}')$  is the mean galaxy redshift-space density for all galaxies,  $\langle P(k) \rangle$  is an estimate of the power spectrum, and  $\mathbf{s}'$  is the 3D redshift-space position variable. Note that, to simplify the procedure, we do not use luminosity-dependent weights as advocated by Percival, Verde & Peacock (2004).  $c_\nu$  are normalization constants, and  $\rho(\mathbf{s}')$  is the galaxy redshift-space density. For a galaxy survey,  $\rho(\mathbf{s}')$  is a sum of delta functions and the above integral decomposes to a sum over the galaxies.

The inverse transformation is given by

$$\frac{\rho(L, \mathbf{s}')}{\hat{r}_b(L)} w(\mathbf{s}') = \sum_\nu c_\nu \rho_\nu(L, \mathbf{s}') j_\nu(s') Y_\nu(\theta, \phi). \quad (\text{A6})$$

Adopting the set of harmonics with

$$\left. \frac{d}{dr} j_\nu(r) \right|_{r_{\text{max}}} = 0, \quad (\text{A7})$$

(i.e. with no boundary distortions at  $r_{\max} = 706.2 h^{-1}$  Mpc), the normalization of the transform requires  $c_\nu$  to satisfy

$$c_\nu^{-2} = \int dr j_\nu^2(r) r^2. \quad (\text{A8})$$

### A3 Small-scale velocity dispersion correction

We have applied the correction (described by HT) for the effect of the small-scale non-linear peculiar velocity field caused by the random motion of galaxies in groups. Because we are only interested in the large-scale linear power spectrum in this paper, the exact details of this correction are not significant (this is discussed further in Section 6.5). The effect of the velocity field is to smooth the observed overdensity field along the line-of-sight in a way that is equivalent to convolving with a matrix  $S_{\nu\mu}$ :

$$(\delta_{r'})_\nu = \sum_\mu S_{\nu\mu} (\delta_r)_\mu, \quad (\text{A9})$$

where

$$S_{\nu\mu} = c_\nu c_\mu \Delta_{\ell_\nu, \ell_\mu}^K \Delta_{m_\nu, m_\mu}^K \times \iint p(r-y) j_\mu(r) j_\nu(y) r dr y dy. \quad (\text{A10})$$

Here  $\Delta^K$  is the Kronecker delta function, and  $p(r-y)$  is the one-dimensional scattering probability for the velocity dispersion. Models for  $p(r-y)$  are given in Eqns. 5, 6, & 7, and the choice of model is discussed further in Section 6.5. Note that this formalism assumes that the induced dispersion is not a strong function of group mass.

### A4 Modelling the transformed density field

The correction for luminosity-dependent bias given by Eq. A1 is a function of galaxy properties, not the measured galaxy position. The galaxy density multiplied by this bias correction is therefore conserved with respect to a change in coordinates with number conservation implying

$$d^3 s' \frac{\rho(L, \mathbf{s}')}{\hat{r}_b(L)} = d^3 \mathbf{r}' \frac{\rho(L, \mathbf{r}')}{\hat{r}_b(L)}. \quad (\text{A11})$$

The dependence of the redshift distortion term lies in the weighting and spherical Bessel functions and, following HT, we expand to first order in  $\Delta r' \equiv s' - r'$ ,

$$w(\mathbf{s}') j_\nu(s') \simeq w(\mathbf{r}') j_\nu(r') + \Delta r' \frac{d}{dr} [w(\mathbf{r}') j_\nu(r')]. \quad (\text{A12})$$

Using the Poisson equation to relate the gravitational potential with the density field,

$$\Delta r'_{\text{lin}} = \Omega_m(z[r'])^{0.6} \times \sum_\nu \frac{1}{k_\nu^2} c_\nu \delta_\nu(\text{mass}, \mathbf{r}') \frac{dj_\nu(r')}{dr} Y_\nu(\theta, \phi), \quad (\text{A13})$$

where  $\delta_\nu(\text{mass}, r')$  is the transform of the mass over-density field. Because the linear redshift-space distortions are a function of the mass over-density field, they are independent of galaxy luminosity. However, this means that they are expected to grow through the linear growth factor  $D(z)$ , normalized to  $D(0) = 1$ , within the CGC model. This and the

redshift dependence of  $\Omega_m(z)/\Omega_m(0)$  are calculated assuming a concordance model. We can now rewrite these distortions in terms of the transformed density field of galaxies of luminosity  $L_*$

$$\Delta r'_{\text{lin}} = \frac{\Omega_m(z[r'])^{0.6}}{b(L_*, 0)} D(z[r']) \times \sum_\nu \frac{1}{k_\nu^2} c_\nu \delta_\nu(L_*, \mathbf{r}') \frac{dj_\nu(r')}{dr} Y_\nu(\theta, \phi). \quad (\text{A14})$$

Defining

$$\beta(L_*, 0) \equiv \frac{\Omega_m(0)^{0.6}}{b(L_*, 0)}, \quad (\text{A15})$$

reduces this expression to

$$\Delta r'_{\text{lin}} = \beta(L_*, 0) \frac{\Omega_m(z[r'])^{0.6}}{\Omega_m(0)^{0.6}} D(z[r']) \sum_\nu \frac{1}{k_\nu^2} c_\nu \delta_\nu(L_*, \mathbf{r}') \frac{dj_\nu(r')}{dr} Y_\nu(\theta, \phi). \quad (\text{A16})$$

Including a correction for the local group velocity  $\mathbf{v}_{LG}$ , assumed to be  $622 \text{ km s}^{-1}$  towards (B1950) RA =  $162^\circ$ , Dec =  $-27^\circ$  (Lineweaver et al. 1996; Courteau & van den Bergh 1999), gives

$$\Delta r' = \Delta r'_{\text{lin}} - \mathbf{v}_{LG} \cdot \hat{\mathbf{r}}'. \quad (\text{A17})$$

The local group velocity correction has a very minor effect on the results presented in this paper, but was included for completeness.

For galaxies of luminosity  $L$ , transforming the density field gives

$$\rho(L, \mathbf{r}') = \bar{\rho}(L, \mathbf{r}') \times \left[ 1 + \sum_\nu c_\nu \delta_\nu(L, \mathbf{r}') j_\nu(r') Y_\nu(\theta, \phi) \right], \quad (\text{A18})$$

where  $\bar{\rho}(L, \mathbf{r}')$  is the observed mean density of galaxies of luminosity  $L$  in the survey. In fact, the mean number of galaxies as a function of the redshift-space distance  $\bar{\rho}(L, \mathbf{s}')$  is more easily determined than  $\bar{\rho}(L, \mathbf{r}')$ . It would be possible to reformulate the Spherical Harmonics formalism to use  $\bar{\rho}(L, \mathbf{s}')$  by separating the convolution of the window from the linear redshift-space distortion correction. Given the relatively small effect that the coordinate translation  $\mathbf{r}' \rightarrow \mathbf{s}'$  has on  $\bar{\rho}(L, \mathbf{r}')$ , we have instead chosen to use the original HT formalism with  $\bar{\rho}(L, \mathbf{r}') \simeq \bar{\rho}(L, \mathbf{s}')$  as measured from the survey. Converting from  $\delta_\nu(L, \mathbf{r}')$  to consider the fluctuations traced by  $L_*$  galaxies gives

$$\rho(L, \mathbf{r}') = \bar{\rho}(L, \mathbf{r}') \times \left[ 1 + \sum_\nu c_\nu \hat{r}_b(L) \delta_\nu(L_*, \mathbf{r}') j_\nu(r') Y_\nu(\theta, \phi) \right], \quad (\text{A19})$$

and we see that when we combine Eqns. A4, & A19 to determine  $\rho(L, \mathbf{r}')$  as a function of  $\langle \delta_\nu(L_*, \mathbf{r}') \rangle$ , the factors of  $\hat{r}_b(L)$  in both of these Equations will cancel.

Combining Eqns. A4, A9, A12, A17 & A19 gives

$$D_\nu \equiv \rho_\nu(L, \mathbf{r}') - \bar{\rho}_\nu(L, \mathbf{r}') - \rho_\nu(LG, \mathbf{r}') \quad (\text{A20})$$

$$= \sum_\mu (\Phi_{\nu\mu} + \beta(L_*, 0) V_{\nu\mu}) \delta_\mu(L_*, \mathbf{r}') \quad (\text{A21})$$



$$= \sum_{\mu} \sum_{\eta} (\Phi_{\nu\mu} + \beta(L_*, 0)V_{\nu\mu}) S_{\mu\eta} \delta_{\eta}(L_*, \mathbf{r}) \quad (\text{A22})$$

where the mean-field harmonics are defined as

$$\bar{\rho}_{\nu}(L, \mathbf{r}') = c_{\nu} \int d^3r' \frac{\bar{\rho}(L, \mathbf{r}')}{\hat{r}_b(L)} w(\mathbf{r}') j_{\nu}(r') Y_{\nu}^*(\theta, \phi), \quad (\text{A23})$$

the local group contribution is given by

$$\rho_{\nu}(\text{LG}, \mathbf{r}') = c_{\nu} \int d^3r' (\mathbf{v}_{\text{LG}} \cdot \hat{\mathbf{r}}') \frac{\bar{\rho}(L, \mathbf{r}')}{\hat{r}_b(L)} \times \frac{d}{dr'} [w(\mathbf{r}') j_{\nu}(r')] Y_{\nu}^*(\theta, \phi), \quad (\text{A24})$$

and the  $\Phi$  and  $V$  matrices are defined as

$$\Phi_{\nu\mu} = c_{\nu} c_{\mu} \int d^3r' \bar{\rho}(L, \mathbf{r}') w(\mathbf{r}') j_{\nu}(r') j_{\mu}(r') \times Y_{\nu}^*(\theta, \phi) Y_{\mu}(\theta, \phi), \quad (\text{A25})$$

and

$$V_{\nu\mu} = \frac{c_{\nu} c_{\mu}}{k_{\mu}^2} \int d^3r' \frac{\bar{\rho}(L, \mathbf{r}')}{\hat{r}_b(L)} \frac{\Omega_m(z)^{0.6}}{\Omega_m(0)^{0.6}} D(z) \times \frac{d}{dr'} [w(\mathbf{r}') j_{\nu}(r')] \frac{d}{dr'} j_{\mu}(r') Y_{\nu}^*(\theta, \phi) Y_{\mu}(\theta, \phi). \quad (\text{A26})$$

Assuming that the mean observed density field  $\bar{\rho}(L, \mathbf{r}') = \bar{\rho}(L, r') M(\theta, \phi)$  and the weighting  $w(\mathbf{r}') = w(r') w(\theta, \phi)$  can be split into angular and radial components, then the 3D integrals required to calculate the  $\Phi$  and  $V$  matrices have the same angular contribution

$$W_{\nu\mu} = \int d\theta d\phi w(\theta, \phi) Y_{\nu}^*(\theta, \phi) M(\theta, \phi) Y_{\mu}(\theta, \phi), \quad (\text{A27})$$

where  $M(\theta, \phi)$  is the sky mask of the survey. This therefore only needs to be calculated once.

The effect of the survey geometry (matrix  $\Phi_{\nu\mu}$ ) is independent of the luminosity-dependent bias correction: the  $1/\hat{r}_b(L)$  factor in Eq. A4 was designed to cancel the offset in  $\delta(L, \mathbf{r}')$  (Eq. A3). Note that we have included the redshift evolution part of  $\beta(L_*, z)$  in Eq. A26, and in the calculation performed, so that we fit the data with  $\beta(L_*, 0)$ . Ignoring this correction gives a measured  $\beta(L_*, z)$  approximately 10% larger than  $\beta(L_*, 0)$ , because it corresponds to an effective redshift  $\sim 0.17$ .

## A5 Construction of the covariance matrix

In this Section we only work with the real space position, and all overdensities correspond to  $L_*$  galaxies. For simplicity, we therefore define  $\delta_{\mu} \equiv \delta_{\mu}(L_*, \mathbf{r}')$ . We also define

$$\Psi_{\nu\mu} \equiv \sum_{\eta} (\Phi_{\nu\eta} + \beta(L_*, 0)V_{\nu\eta}) S_{\eta\mu}, \quad (\text{A28})$$

so that Eq. A22 becomes

$$D_{\nu} = \sum_{\mu} \Psi_{\nu\mu} \delta_{\mu}. \quad (\text{A29})$$

The real and imaginary parts of  $D_{\nu}$  are given by

$$\text{Re } D_{\nu} = \sum_{\eta} (\text{Re } \Psi_{\nu\eta} \text{Re } \delta_{\eta} - \text{Im } \Psi_{\nu\eta} \text{Im } \delta_{\eta}) \quad (\text{A30})$$

$$\text{Im } D_{\nu} = \sum_{\eta} (\text{Im } \Psi_{\nu\eta} \text{Re } \delta_{\eta} + \text{Re } \Psi_{\nu\eta} \text{Im } \delta_{\eta}). \quad (\text{A31})$$

From Eqns. A28 & A29 it can be seen that, for a single mode, the expected value  $\langle \text{Re } D_{\nu} \text{Re } D_{\mu} \rangle$  or  $\langle \text{Im } D_{\nu} \text{Im } D_{\mu} \rangle$  can be split into three components dependent on  $\beta(L_*, 0)^n$  with  $n = 0, 1, 2$ .

Given the large number of modes within the linear regime, rather than estimating the covariances of all modes, we reduce the problem to considering a number of combinations of the real and imaginary parts of  $D_{\nu}$ . We will discuss how we optimally chose the direction of the component vectors in the space of all modes in Section A7. Suppose the revised mode combinations that we wish to consider are given by

$$\hat{D}_a = \sum_{\nu} E_{a\nu}^r \text{Re } D_{\nu} + \sum_{\nu} E_{a\nu}^i \text{Im } D_{\nu}. \quad (\text{A32})$$

Note that in this Equation  $a$  does not represent a triplet of  $\ell, m$ , &  $n$ , but is instead simply an index of the modes chosen. Using Equations A30 & A31, we can decompose into multiples of the real and imaginary components of  $\delta$

$$\hat{D}_a = \sum_{\eta} (\Upsilon_{a\eta}^r \text{Re } \delta_{\eta} + \Upsilon_{a\eta}^i \text{Im } \delta_{\eta}), \quad (\text{A33})$$

where

$$\Upsilon_{a\eta}^r = \sum_{\nu} (E_{a\nu}^r \text{Re } \Psi_{\nu\eta} + E_{a\nu}^i \text{Im } \Psi_{\nu\eta}) \quad (\text{A34})$$

$$\Upsilon_{a\eta}^i = \sum_{\nu} (E_{a\nu}^i \text{Re } \Psi_{\nu\eta} - E_{a\nu}^r \text{Im } \Psi_{\nu\eta}). \quad (\text{A35})$$

The expected values of  $\langle \hat{D}_a \hat{D}_b \rangle$  are then

$$\langle \hat{D}_a \hat{D}_b \rangle = \sum_{\eta} \sum_{\gamma} \langle (\Upsilon_{a\eta}^r \text{Re } \delta_{\eta} + \Upsilon_{a\eta}^i \text{Im } \delta_{\eta}) \times (\Upsilon_{b\gamma}^r \text{Re } \delta_{\gamma} + \Upsilon_{b\gamma}^i \text{Im } \delta_{\gamma}) \rangle. \quad (\text{A36})$$

Assuming a standard Gaussian density field, the double sum in Eq. A36 can be reduced to a single sum using the following relations

$$\langle \text{Re } \delta_{\nu} \text{Im } \delta_{\mu} \rangle = 0 \quad (\text{A37})$$

$$\langle \text{Re } \delta_{\nu} \text{Re } \delta_{\mu} \rangle = [\Delta_{\nu, \mu}^K + (-1)^{m_{\nu}} \Delta_{\nu, -\mu}^K] \frac{P(k_{\nu})}{2} \quad (\text{A38})$$

$$\langle \text{Im } \delta_{\nu} \text{Im } \delta_{\mu} \rangle = [\Delta_{\nu, \mu}^K - (-1)^{m_{\nu}} \Delta_{\nu, -\mu}^K] \frac{P(k_{\nu})}{2}, \quad (\text{A39})$$

where the  $\Delta_{\nu, -\mu}^K$  terms arise because  $\delta_{\nu}$  obeys the Hermitian relation  $\delta_{\nu}^* = (-1)^{m_{\nu}} \delta_{-\nu}$ . These terms are only important for geometries that lack azimuthal symmetry, such as the 2dFGRS and are less important for the PSCz survey. The dependence on  $P(k)$  follows because the transformation from the Fourier basis to the Spherical Harmonics basis is unitary and the amplitude of the complex variable is unchanged. Using these relations, Eq. A36 reduces to

$$\langle \hat{D}_a \hat{D}_b \rangle = \sum_{\eta} \frac{P(k_{\nu})}{2} [\Upsilon_{a\eta}^r \Upsilon_{b\eta}^r + \Upsilon_{a\eta}^i \Upsilon_{b\eta}^i + (-1)^{m_{\eta}} \Upsilon_{a\eta}^r \Upsilon_{b-\eta}^r - (-1)^{m_{\eta}} \Upsilon_{a\eta}^i \Upsilon_{b-\eta}^i]. \quad (\text{A40})$$

This equation gives the geometrical component of the covariance matrix resulting from the mixing of modes caused by the survey geometry and large-scale redshift-space distortions. Note that, by substituting Eqns. A28, A34 & A35 into

this equation we could split the geometric part of the covariance matrix into 3 components with varying dependence on  $\beta(L_*, 0)$ . This is actually the case in our implementation of the method so we only have to calculate these three components once for any value of  $\beta(L_*, 0)$ .

In addition, there is a shot noise component which can be calculated by the methods of Peebles (1973). This term enters into the above formalism because the density field  $\rho(L, \mathbf{s}')$  in Eq. A4 is actually the sum of a series of delta functions, each at the position of a galaxy. The expected value of  $\langle D_\mu D_\nu \rangle$  therefore includes two terms (as in Appendix A of Feldman, Kaiser & Peacock 1994) corresponding to the convolved power and the shot noise. Allowing  $a$  and  $b$  to represent either real or imaginary parts, the expected value of the noise component for each mode, for a particular galaxy luminosity is

$$\langle aN_\nu bN_\mu \rangle = c_\nu c_\mu \int d^3r \frac{\bar{\rho}(L, \mathbf{r})}{\bar{r}_b^2(L)} w^2(\mathbf{r}) j_\nu(r) j_\mu(r) r^2 \times aY_\nu^*(\theta, \phi) bY_\mu^*(\theta, \phi). \quad (\text{A41})$$

To allow for all galaxy luminosities, we simply integrate over luminosity as in Percival, Verde & Peacock (2004).

Allowing for the combinations of modes defined in Eq. A32,

$$\begin{aligned} \langle \hat{N}_a \hat{N}_b \rangle &= \sum_\nu \sum_\mu (E_{a\nu}^r E_{b\mu}^r \langle \text{Re } N_\nu \text{Re } N_\mu \rangle \\ &\quad + E_{a\nu}^i E_{b\mu}^i \langle \text{Im } N_\nu \text{Im } N_\mu \rangle + E_{a\nu}^r E_{b\mu}^i \langle \text{Re } N_\nu \text{Im } N_\mu \rangle \\ &\quad + E_{a\nu}^i E_{b\mu}^r \langle \text{Im } N_\nu \text{Re } N_\mu \rangle) \end{aligned} \quad (\text{A42})$$

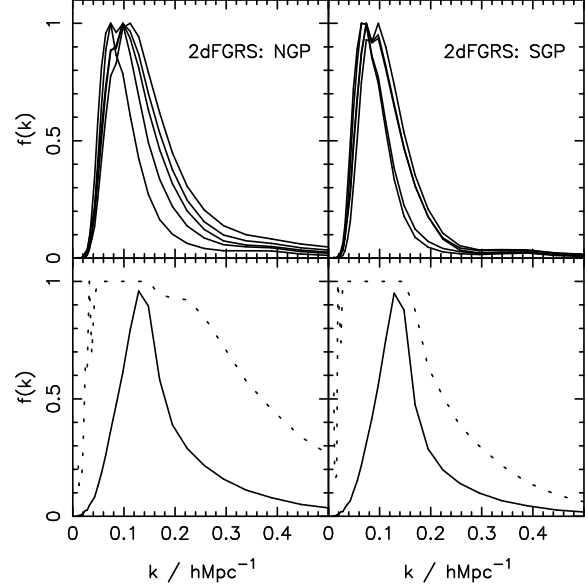
The components of the covariance matrix of the reduced data are  $\langle \hat{D}_a \hat{D}_b \rangle + \langle \hat{N}_a \hat{N}_b \rangle$  as given by Equations A40 & A42.

## A6 Some practical issues

The calculation of the angular part of the mixing matrices,  $W_{\nu\mu}$  (given by Eq. A27) is more CPU intensive than the calculation of the radial components. Because of this, Tadros et al. (1999) utilized Clebsch-Gordan matrices to relate a single transform of the angular mask to the full transition matrix given by Eq. A27. However at the high  $\ell$ -values required for the complex geometry of the 2dFGRS survey it is computationally expensive to calculate these accurately. Because the integral in Eq. A27 can be reduced to a sum over the angular mask, a direct integration proved stable and computationally faster than the more complicated Clebsch-Gordan method. At low  $\ell$ -values both methods agreed to sufficient precision.

In the large-scale regime  $k < 0.15 h \text{Mpc}^{-1}$ , and in the regime where the assumed redshift distribution does not have a significant effect on the recovered power  $k > 0.02 h \text{Mpc}^{-1}$  (see P01), there are 86667 modes with  $m \geq 0$ . This statistically complete set includes real and imaginary modes separately, but only includes modes with  $m \geq 0$  because  $D_\nu$  (Equation A22) obeys the Hermitian relation and positive and negative  $m$ -modes are degenerate. The maximum  $n$  of the modes in this set is 33, and the maximum  $\ell$  is 101.

Obviously we cannot invert a  $86667 \times 86667$  covariance matrix with each mode as a single element for every model



**Figure A1.** Normalized contribution to  $P(k)$  as a function of  $k$  for 5 example modes for the NGP and SGP (top row). In the lower row we present the average (solid line) and maximum (dotted line) of the normalized distribution of  $k$ -contributions, calculated from all modes used.

we wish to test, and we therefore need to reduce the number of modes compared. Another serious consideration is that many of the modes are nearly degenerate. Because we can only calculate the components required with finite precision, nearly degenerate modes often become completely degenerate due to numerical issues and therefore need to be removed from the analysis: covariance matrices with negative eigenvalues are unphysical. Removing degenerate modes is discussed in the context of data compression in the next Section.

There are two convolutions that we need to perform in order to determine the covariance matrix, given by Eqns. A28 & A29. The number of modes summed when numerically performing these convolutions is limited by computational time. The first convolution is given by Equation A28 and results from the small-scale velocity dispersion correction. This convolution is a simple convolution in  $n$  and is relatively narrow in the linear regime that we consider in this paper. In fact we chose to convolve over  $1 \leq n \leq 100$ . The second convolution is given by Equation A40, and is performed for  $\ell \leq 200$ . This is complete for  $k < 0.29 h \text{Mpc}^{-1}$ , and contains  $> 4 \times 10^6$  modes. A limit in  $\ell$  was chosen rather than a limit in  $k$  as the CPU time taken to perform the convolution is dependent on  $\ell_{\text{max}}$ . The  $k$ -distribution of contributions to a few of the chosen modes is presented in Fig. A1. Note that although the convolved set of modes is complete for  $k < 0.29 h \text{Mpc}^{-1}$ , the fall-off to higher  $k$  is very gentle, and most of the signal beyond this limit will still be included in the convolution.

## A7 Data compression

As mentioned in Section A6, there are 86667 Spherical Harmonic modes with  $0.02 < k < 0.15 h \text{Mpc}^{-1}$ , and it is im-

practical to use all of these modes in a likelihood analysis. Consequently, we reject modes for the following reasons

(i) The 2dFGRS regions considered have a relatively small azimuthal angle, so modes that are relatively smooth in this direction will be close to degenerate. We therefore set a limit of  $\ell - |m| > 5$  for the modes analysed. This limit effectively constrains the number of azimuthal wavelengths in the modes used.

(ii) Modes with similar  $\ell$ -values were found to be closely degenerate. Rather than applying a more optimal form of data compression, it was decided to simply sample the range of  $\ell$ -values with  $\Delta\ell = 10$ . This spacing was chosen by examining the number of small eigenvalues in the three components of the covariance matrix as described after Eq. A40.

In addition, we carry out the following steps to remove degenerate modes in the covariance matrix and to compress the data further. These steps are performed first in the angular direction (assuming modes with different  $\ell$  and  $n$  are independent), then on all of the remaining modes.

(i) Even after rejection of near  $\ell$ -values, nearly degenerate combinations of modes remain, which, given the limited numerical resolution achievable, could give negative eigenvalues in the covariance matrix. Because of this, only modes with eigenvalues in the covariance matrix greater than  $10^{-5}$  times the maximum eigenvalue, well above the round-off error, are retained in the three components of the covariance matrix as described after Eq. A40. This step is effectively a principal-component reduction of the covariance matrix eigenvectors.

(ii) Finally, we perform a Karhunen-Loève decomposition of the covariance matrix optimized to constrain  $\beta(L_*, 0)$ . After our angular reduction we retain 2155 & 2172 modes for the NGP and SGP respectively after this step. Following radial compression we are left with 1223 & 1785 modes for the NGP and SGP respectively. The number of modes retained for the NGP is smaller than for the SGP because the smaller angular coverage means that more modes are nearly degenerate.

## A8 Calculating the likelihood

Following the hypothesis that  $\text{Re } \mathbf{D}_\nu$  and  $\text{Im } \mathbf{D}_\nu$  are Gaussian random variables, the likelihood function for the variables of interest can be written

$$\mathcal{L}[\mathbf{D}|\beta(L_*, 0), P(k)] = \frac{1}{(2\pi)^{N/2} |\mathbf{C}|^{1/2}} \exp \left[ -\frac{1}{2} \mathbf{D}^T \mathbf{C}^{-1} \mathbf{D} \right]. \quad (\text{A43})$$

Matrix inversion is an  $N^3$  process, so finding the inverse covariance matrix can be prohibitively slow in order to test a large number of models. However, the KL procedure described in Section A7 means that the covariance matrix is diagonal for a model chosen to be close to the best fit position. To first order, we might be tempted to assume that the covariance matrix is diagonal over the range of models to be tested. However, this can bias the solution depending on the exact form of the matrix. A compromise is to apply the iterative Newton-Raphson method of root-finding to matrix inversion (section 2.2.5 of Press et al. 1992) starting with the diagonal inverse covariance matrix as the first estimate.

Given an estimate of the inverse covariance matrix  $H_0$ , our revised estimate is  $H_1 = 2H_0 - H_0 C H_0$ . Because  $H_0$  is diagonal, the first step of this iterative method only takes of order  $N^2$  operations. This trick allows the likelihood to be quickly calculated for a large number of models, and we use this method in Section 5.2, when we consider a fixed power spectrum shape.

However, over the larger range of models considered in Section 5.3, the covariance matrix changes significantly, and the estimate  $H_1$  is not sufficiently accurate. Instead, a full matrix inversion is performed for each model, so mapping the likelihood hypersurface becomes computationally expensive. A fast method for mapping surfaces which has recently become fashionable in cosmology is the Markov-chain Monte-Carlo technique, where an iterative walk is performed in parameter space seeking local likelihood maxima (e.g. Lewis & Bridle 2002; Verde et al. 2003; Tegmark et al. 2003b). However, we only wish to consider variation of 4 parameters ( $\beta(L_*, z)$ ,  $b(L_*, 0)\sigma_8$ ,  $\Omega_m h$ , &  $\Omega_b/\Omega_m$ ) in a very simple model described in Section 5.1, so it is easy to map the likelihood surface using a grid.