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## The 80 Percent Pension Funding Target, High Assumed Returns, and Generational Inequity

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## **WORKING PAPER SERIES**

### **THE 80 PERCENT PENSION FUNDING TARGET, HIGH ASSUMED RETURNS, AND GENERATIONAL INEQUITY**

Robert M. Costrell

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## **THE 80 PERCENT PENSION FUNDING TARGET, HIGH ASSUMED RETURNS, AND GENERATIONAL INEQUITY**

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Abstract: Generational inequity in pension funding is highly sensitive to the lax policies of 80-percent funding targets and high assumed returns to investment. I develop a simple, powerful relationship between steady-state (SS) inequity in contributions – the percent of extra contributions to fund prior cohorts – and the SS unfunded ratio. I then show how the SS unfunded ratio is governed by x-percent funding targets and the gap between assumed and true returns. The SS degree of inequity is over 60 percent under an 80 percent funding target and over 50 percent with a one point gap between assumed and true returns.

Key words: public pensions, unfunded liabilities, generational inequity

# THE 80 PERCENT PENSION FUNDING TARGET, HIGH ASSUMED RETURNS, AND GENERATIONAL INEQUITY

## I. Introduction

Expenditures on school pensions have doubled in the last decade and now exceed \$1,000 per pupil ([Costrell, 2015a](#)). The rise largely reflects payments on unfunded liabilities. The lack of political will to fully address these liabilities has been abetted by two ideas: (1) full funding is unnecessary for “sustainability” – 80 percent is good enough; and (2) high assumed returns (e.g. 8 percent) are also sustainable, even if returns have fallen short of late. The purpose of this paper is to formally analyze two policies – an “x-percent funding policy” and a “high assumed return policy” (which is often thought of as an “x-percent funding policy” by the backdoor). Using the simple mathematics of ordinary difference equations, I focus on the existence, stability, and characteristics of steady state (SS) – most notably the SS degree of generational inequity.

What is the empirical and policy motivation for formally exploring these questions? To be clear, there are almost no plans that explicitly build a target funded ratio of less than 100 percent into their funding formula, so the motivation here is not to analyze such funding formulas actually in use.<sup>1</sup> Instead, one motivation – a modest one – is to simply show how such a target ratio would play out, were it to be incorporated into an otherwise standard funding formula. More importantly, quite a number of plans that target full funding have fallen well short of meeting that target for many years, and have often used the 80 percent rationale to avoid taking corrective action.<sup>2</sup> The two main reasons for the shortfalls ([Munnell, et. al, 2015](#);

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<sup>1</sup> The exceptions here are the five state systems of Illinois, of which the Teacher Retirement System is the largest. Under Illinois statute, the 2045 target is 90 percent. In addition to the target itself, the method by which that target is built into the funding formula is quite non-standard – unlike the method modeled below. The actuary for Illinois’ funds has repeatedly criticized these non-standard practices, as “Illinois math” ([Teachers’ Retirement System of the State of Illinois, 2016](#) and earlier). There are other states that have built 80-percent triggers into their COLAs.

<sup>2</sup> See the statements of New Jersey and Connecticut policy-makers cited in [American Academy of Actuaries, 2012](#).

[Costrell, 2015b](#)) are (i) the failure to make actuarially required contributions; and (ii) overly optimistic actuarial assumptions -- notably the assumed investment return -- which artificially depress required contributions in the short run. The incentive to rectify such shortfalls depends critically on the debate over whether 80 percent (or some similar figure) is “good enough.” With regard to (i), the “x-percent funding policy” model below may be interpreted as a formal analysis of underfunding “as if” a funded ratio of less than 100 percent had been built into the formula. With regard to (ii), the “high assumed returns” model below is directly pertinent to existing practice. Both policies have strong implications for generational inequity.

Before delving into the analysis, I will first review the background and literature on the “80 percent standard” and the rationale for my formal analytical approach. The new results from that approach will be summarized below, but the main policy implication can be stated here: small departures of the policy variables from sound practice – target funded ratios below 100 percent and high assumed returns -- result in large SS underfunding and generational inequity.

## II. Background and Previous Literature on the “80 Percent” Standard

Continuing public pension funding shortfalls, years after the market crash of 2008, have given circulation to the notion that “full funding” is unnecessary for a “healthy” pension plan. A particularly common form of this idea is the “80 percent” standard, according to which professional opinion allegedly views an 80 percent funded ratio (assets/liabilities) as the threshold for a sound plan (see the Appendix of [American Academy of Actuaries](#), 2012, for examples). The origins of this standard are obscure, and possibly mythical (see [Miller](#), 2012; and reply by [Brainard and Zorn](#), 2012). Indeed, there is now a website, the “[80 Percent Pension Funding Hall of Shame](#)” (Campbell, 2014) and associated [database](#), devoted to exposing those

public officials and pension industry participants who perpetuate this “standard.” That said, the claim does raise questions of the precise sense in which a certain minimum funded ratio indicates a fund’s “health,” or, more specifically, its “sustainability.” Can target ratios below 100 percent be “sustainable,” and, if so, how far below 100 percent, and what are the consequences?

Some observers of the public pension industry have astutely clarified the main issues with the “80 percent standard.” Notably, [Miller \(2012\)](#) makes two key distinctions. First, he points out that it is one thing to hit 80 percent funding at the bottom of the market, while still averaging 100 percent over the cycle by (for example) reaching 125 percent at the peak.<sup>3</sup> It is quite another matter to average well below 100 percent indefinitely. The [American Academy of Actuaries \(2012\)](#) makes a similar (though not identical) distinction between a 100 percent target ratio for actuarial funding (which they recommend) and an 80 percent snapshot at a point in time, which may or may not be on track to reach the 100 percent target.

The second distinction [Miller](#) makes concerns the criteria for plan health: sustainability vs. generational equity.<sup>4</sup> Even if the plan averages below 100 percent indefinitely, it may still be “sustainable” at contribution rates that cover amortization of the unfunded liability. However, this runs afoul of generational equity, with “current taxpayers supporting retirees who didn’t ever work for them” and/or the employees paying down previous cohorts’ unfunded liabilities.

Still, confusion remains in publications of public pension industry spokesmen (let alone the general public and policy-makers). The Government Finance Officers Association ([GFOA](#), 2009) specifies a target funded ratio of “100 percent or more” as the first required practice for “sustainable funding.” As [Miller](#) has clarified, this is actually a requirement for generational

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<sup>3</sup> However, as [Miller](#) points out, funded ratios above 100 percent create a political problem, tempting lawmakers to take pension holidays or enhance benefits.

<sup>4</sup> [American Academy of Actuaries \(2012\)](#) also alludes to this distinction.

equity, rather than sustainability. However, the water was further muddied by spokesmen for the National Association of State Retirement Administrators (NASRA) and the prominent actuarial firm Gabriel, Roeder, Smith & Company (GRS). In a joint statement of these two entities (replying, in part, to [Miller](#)), [Brainard and Zorn \(2012\)](#) reiterate [GFOA's](#) recommendation of 100 percent target funding for sustainability, but then suggest it is not critical to ever reach that target: “many pension plans remain underfunded for decades without causing fiscal stress for the plan sponsor or requiring benefits to be reduced.” For them, the “critical factor” in evaluating plan health is whether or not the required contributions are so high as to create “fiscal stress.” More to the point, their statement seems to ratify an x-percent standard as a rationale for inaction in addressing the sources of underfunding, whether it is failure to meet required contributions or overly optimistic assumed investment return.

### III. Rationale for the Formal Analytical Approach and Summary of Results

This paper brings a simple analytical model to bear on these issues. Although recently developed simulation models incorporate risk and other features, insight into the questions posed here can be obtained by a model stripped to its essentials (e.g., risk-free investment) and solvable from a simple ordinary difference equation. For example, the notion of “sustainability” – a term that is often used without precise definition – is logically defined in such a model as stability of a steady state, at an equilibrium funded ratio and contribution rate.

The first benefit of such a model is to sort out formally who is right and who is wrong in the commentaries discussed above. Miller is right: SS funded ratios below 100 percent are “sustainable,” if generationally inequitable, in a precise mathematical sense. There is a continuum of stable (“sustainable”) steady states. There is no x-percent minimum for the SS

funded ratio in this model, other than the 0-percent criterion of solvency. Low SS funded ratios correspond to high SS contribution rates, exceeding normal cost (the cost of pre-funding each cohort's benefits). The problem with x-percent funding, therefore, is generational inequity, not "sustainability." The model helps us better understand how this result obtains, showing that the system's stability is unrelated to the funding target, and identifying instead the features of an actuarial funding formula that are required to secure stability.

The model also generates additional results not previously articulated. My model generates a simple, powerful relationship between the SS unfunded ratio and a meaningful measure of generational inequity. The model also provides deeper understanding of the SS funded ratio itself. I show that if a system *targets* an x-percent funded ratio in its amortization formula, the SS ratio will be lower yet. That is because contributions at the x-percent target include no amortization, and normal cost alone is insufficient to sustain that target. Conversely, to achieve any given SS funded ratio, one must set a higher target funded ratio for amortization purposes. As a corollary of this result, to merely achieve solvency (0 percent SS ratio) the target ratio must be set at a positive floor. To take that extreme case, if the target ratio is set at that lower bound, the resulting steady state reproduces a pay-go system, under the guise of a pre-funding formula, with amortization payments making up the difference between the pay-go rate and the normal cost of pre-funding benefits. More generally, the math simplifies nicely to allow easy calibration of the relationship between the SS ratio and the funding target, and to demonstrate its sensitivity. Indeed, some proposed targets are alarmingly close to the floor for SS solvency, which also shows how great the inequity can be under proposed targets. Finally, I model the policy of high assumed returns,<sup>5</sup> which inflates the funded ratio. The model shows

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<sup>5</sup> There has been considerable controversy over the assumed rate of return and its use for discounting liabilities. The debate distinguishes between the accounting rate used for reporting liabilities – which should be the risk-free rate,



that even the inflated ratio is underfunded in SS, let alone the true ratio, for reasons that are not always understood. Moreover, the measured SS degree of generational inequity will exceed that based on the measured SS unfunded ratio. The main policy takeaway is that small deviations from the target of full funding or of the assumed from true returns generate large degrees of generational inequity.

#### IV. The Basic Model

Consider a population of public employees (e.g., “teachers”) in a traditional defined benefit plan, such as a "final-average-salary" plan, where the initial annuity equals years of service  $\times$  "multiplier"  $\times$  final average salary. For example, with a multiplier of 2.0 percent, after serving 30 years one may receive 60 percent of final average salary for life, plus any COLAs.

The basic pension funding math can be set out with the following notation:

$W_t$  = payroll in period t, the product of salary and number of teachers

$G = W_t/W_{t-1} = 1 +$  growth rate of payroll

$R = 1 +$  return on investment (assumed to be certain)

$c_t$  = contribution to pension fund, as fraction of payroll, in period t (joint: employer and employee), to be specified further below.

$c^n$  = "normal cost," fraction of payroll to pre-fund pension

$c^p$  = "pay-go cost," fraction of payroll to pay annual pension benefits

$A_t$  = assets in pension fund, at end of period t

$L_t$  = accrued liabilities of the pension fund, at end of period t

$f_t = A_t/L_t$ , funded ratio at end of period t

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rather than the assumed return on investment -- and the rate used for funding purposes. In this paper, my focus is the funding system, so I do not distinguish between the assumed return and the liability discount rate.

Note that  $R$  is taken as constant, as my focus here is not the role of market fluctuations. In addition,  $G$ ,  $c^n$ , and  $c^p$  are taken as constants.<sup>6</sup> In doing so, I am assuming the population is in steady-state (a “mature” population, to use the actuarial term), so that I can focus on the behavior of funding, in and out of steady-state, to determine its “sustainability.”

The timing sequence of the model is this: during period  $t$ , teachers receive total payroll  $W_t$  from taxpayers; teachers and taxpayers jointly contribute  $c_t W_t$  to the fund; retirees receive pension benefits of  $c^p W_t$  from the fund; and the fund earns returns  $R$  on the end-of-previous-period balance,  $A_{t-1}$ . Thus, the fund evolves as:

$$(1) \quad A_t = RA_{t-1} + (c_t - c^p)W_t.$$

The accrued liability at time  $t$  is the present value of accrued benefits to be paid in the future, which, for a mature population, grows in step with payroll, at rate  $G$ . It can also be expressed as the prior liability grown by  $R$  (since the previously accrued benefits are one year closer), plus the current accrual of new benefits -- normal cost -- less the benefits paid out:<sup>7</sup>

$$(2) \quad L_t = GL_{t-1} = RL_{t-1} + (c^n - c^p)W_t.$$

Substituting  $L_{t-1} = L_t/G$  on the RHS and solving, we have:

$$(3) \quad L_t = [G/(R-G)](c^p - c^n)W_t.$$

This steady-state relationship between accrued liabilities and payroll will be quite useful below.

It represents the difference between the present value of all future benefit payments and all future

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<sup>6</sup> As a special case, one may consider a simple two-period model, with overlapping generations (OLG) (Samuelson, 1958; Diamond, 1965). Each generation of public employees works for one period (e.g. 30 years) and then retires (also for 30 years). In this simple model, let  $p$  represent one’s pension, as a fraction of prior salary. Then  $c^n = p/R$ ,  $c^p = p/G$ , and  $L_t = pW_t/R$ , where  $R$  and  $G$  are now understood to be (say) 30-year compound rates. With these expressions, some of the results below simplify, as indicated in corresponding notes.

<sup>7</sup> The precise measurement of normal cost varies with actuarial convention in the multi-period case. That is because the partition of the present value of all future benefits between those that have already accrued and those yet to be accrued depends on conventions regarding when to recognize projected service and wage growth based on current and prior service and wages. To fix ideas, one may consider entry age normal (EAN), the current standard under GASB, but the analysis below pertains to other methods as well.

liability accruals (normal costs).<sup>8</sup> These are, respectively, a fraction  $c^p$  and  $c^n$  of the present value of future payroll,  $[G/(R-G)]W_t$ . Note also that since accrued liabilities are positive, the condition  $R > G$  (the relevant case, as discussed below) implies  $c^p > c^n$ . It is more costly to pay for previous cohorts' pension than to pre-fund one's own, if previous cohorts are not small ( $G$  is not high) and/or the return to investment  $R$  is high.

### A General Result on Generational Inequity in Steady State

The system's dynamic behavior will depend on the funding formula governing contributions,  $c_t$ , but even before specifying that formula, we can derive the steady-state relationship between the contribution rate and the funded ratio. Since  $L_t$  grows at rate  $G$ , so must  $A_t$ , for the funded ratio  $f_t$  to be constant at its steady-state value, call it  $f^*$ . From (1), this means the SS contribution rate,  $c^*$ , must satisfy:

$$(4) \quad G = A_t/A_{t-1} = R + (c^* - c^p)W_t/A_{t-1} = R + (c^* - c^p)W_t/(f^*L_{t-1}) = R + (c^* - c^p)W_t/(f^*L_t/G).$$

Substituting from (3) and solving, we have a fairly general result:

$$(5) \quad c^* = c^n + (c^p - c^n)(1 - f^*).$$

The first term on the RHS, the normal cost, is the cost of prefunding any cohort's future benefits. The second term represents each generation's contribution over and above what is required to pay for its own benefits. More specifically,  $(c^p - c^n)$  represents the *potential* extra burden imposed on the current cohort to fund the benefits of prior cohorts. The fraction of that burden paid in steady-state,  $(1 - f^*)$ , may be considered a measure of generational inequity. At full funding ( $f^* = 1$ ), that burden is zero; at the opposite extreme ( $f^* = 0$ ), the full burden is borne, so

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<sup>8</sup> This follows from the basic identity given in the previous note that the present value of all future benefit payments equals the present value of all benefits yet to be accrued and the present value of all benefits previously accrued, but not yet paid out. The latter term is the accrued liability.

$c^* = c^p$ . That is, we have a strikingly simple result for generational inequity: the extra annual burden borne by each cohort (a flow variable) is the SS unfunded ratio (a ratio of stock variables) times the difference between the pay-go rate and the normal cost.

In the remainder of this paper, I will examine the determinants of the SS funded ratio  $f^*$  (and, hence, the degree of generational inequity, through the result just established), under the two policies in question:  $x$ -percent funding and high assumed returns. First, however, I examine the issue of stability, since steady state is of little interest unless it is stable. Indeed, going back to the debate over these policies, our understanding of the term “sustainable” is that a meaningful steady state exists, and it is stable.<sup>9</sup>

### Stability

The dynamic of the funded ratio,  $f_t = A_t/L_t$ , is found by using (1)-(3):

$$(6) \quad f_t = A_t/L_t = RA_{t-1}/GL_{t-1} + (c_t - c^p)W_t/L_t = (R/G)f_{t-1} + [(R-G)/G](c_t - c^p)/(c^p - c^n).$$

The system evolves from the initial condition  $f_0$  based on the sequence of contribution rates  $c_t$ .

Consider briefly the case of exogenous  $c$ , to see why it must be endogenized, in accord with standard actuarial practice. If  $c_t$  is fixed at  $c$ , (6) is a one-variable linear difference equation, with steady state funded ratio,  $f^* = (c - c^p)/(c^n - c^p)$  and solution  $f_t = (R/G)^t(f_0 - f^*) + f^*$ . The stability condition is  $R < G$ . Specifically, as is well-established in the OLG literature, if  $R < G$ , pay-go ( $c = c^p$ ) is sustainable: contributions cover pension payments with no need to hold any balance in the fund ( $f^* = 0$ ). If the funded ratio is positive to begin with, it will shrink toward

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<sup>9</sup> GFOA (2009) implicitly characterizes a “sustainable” plan as one where “contributions ... expressed as a percentage of active member payroll ... remain approximately level from one year to the next.”

zero. Any higher contribution rate,  $c > c^p$ , is inefficient,<sup>10</sup> and leads to unnecessary asset accumulation ( $f^* > 0$ ).

The problem is that typically  $R > G$ . Indeed, this assumption has guided pension policy for the last few decades.<sup>11</sup> Certainly once the baby boomers entered the workforce, actuaries and economists reading the demography reports knew the baby bust would require moving from pay-go to pre-funding (generally starting in the vicinity of 1980). The actuarially assumed rate of return on public pension funds currently averages 7.7 percent, in the [Public Plans Data](#) of the Boston College Center for Retirement Research,<sup>12</sup> exceeding the average assumed rate of payroll growth of about 3.7 percent.<sup>13</sup> With  $R > G$ , the system is unstable for constant  $c$ . The funded ratio would veer off to plus/minus infinity, as  $f_0 \gtrless f^* = (c^p - c)/(c^p - c^n)$ . Specifically, the normal cost is less than pay-go, but if contributions were simply set to  $c^n$  (so  $f^* = 1$ ), the system would collapse for any starting balance  $f_0 < 1$ .

## V. Target Funded Ratio < 100 Percent

### Endogenizing the Contribution Rate: Stability

Contributions need to be endogenous to address the stability problem when  $R > G$ . Standard actuarial funding formulas do so through contributions that annually amortize some portion of unfunded liabilities, after paying normal cost. In this way, contributions are adjusted

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<sup>10</sup> Conversely to the case discussed above,  $R < G$  means  $c^p < c^n$ . It is cheaper to pay for previous cohorts' pension than to pre-fund one's own, if the previous cohort is small ( $G$  is high) and/or the return to investment  $R$  is low. Of course, this literally requires growth at rate  $G$  forever, so that no "last generation" gets stuck holding the bag.

<sup>11</sup> It is also a standard result from growth theory, although Piketty (2014) created something of a stir by finding this result in the historical record. See Mankiw (2015).

<sup>12</sup> This is the unweighted FY13 estimate, as reported for 145 of the 150 state and local plans covered; the asset-weighted estimate is almost identical. It is also equal to the national average for state and local pensions, from the Census of Governments, as reported by the [Boston College Retirement Center](#) (accessed January 8, 2016). The assumed rate has been reduced in recent years for a number of plans, but the national average from the Census of Government has not dropped much, from 7.98 percent in 2003 to 7.64 percent in 2013.

<sup>13</sup> This is both the weighted and unweighted FY13 estimate, as reported for 42 of the 150 plans covered.

over time, with the intention of steering the funded ratio back toward a stable target. Consider the following stylized model of actuarially required contributions, as a fraction of payroll:

$$(7) \quad c_t = c^n + s(f^\circ L_{t-1} - A_{t-1})/W_t$$

The second term is the amortization payment. Normally it is a fraction,  $s$ , of the difference between liabilities and assets (as most recently measured), but I have slightly generalized it here to allow for an  $x$ -percent target funded ratio  $f^\circ$ .

There are two potential rationales for this formulation (even though there are no plans that formally embed a target ratio below 100 percent in this fashion<sup>14</sup>). First, if plans were to formalize a target below 100 percent within a standard formula, this would seem to be the natural way to do it. (As I will show, however, for this to work,  $f^\circ$  would have to be set higher than the true target.) A second rationale pertains to an observed practice, where the funded ratio persists at less than 100 percent due to shortfalls in contributions, but authorities take no corrective action, falling back on the “80 percent” rule for a rationale. In that case, (7) may be thought of as an implicit funding formula, modeling contributions “as if”  $f^\circ < 1$  were embedded in it.

Using (3), we can write (7) as:<sup>15</sup>

$$(8) \quad c_t = c^n + s(f^\circ - f_{t-1})(L_{t-1}/GW_{t-1}) = c^n + s(f^\circ - f_{t-1})(c^p - c^n)/(R-G).$$

Substituting (8) into (6) and simplifying, we have

$$(9) \quad f_t = (R/G)f_{t-1} + [(R-G)/G][s(f^\circ - f_{t-1})/(R-G) - 1] = [(R-s)/G]f_{t-1} + [sf^\circ - (R - G)]/G$$

Thus, we have again characterized the system with a simple linear difference equation.

The stability condition is now  $(R-s)/G < 1$ , which can be met by setting the speed of amortization  $s > R - G$ . In this way, amortization payments will exceed the growth-adjusted interest on the

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<sup>14</sup> The Illinois plans that formalize a 90 percent target do so in a non-standard formula that works backward from a fixed (but distant) target year. According to the actuaries for the Illinois state plans (who dub the state’s funding formula as “Illinois math”), the funding method effectively sets a lower target than the standard method.

<sup>15</sup> In the 2-period case this simplifies to  $c_t = c^n + s(f^\circ - f_{t-1})c^n/G$ .

(target) unfunded liability. Conversely, to prevent oscillation, one would not want to over-adjust by setting  $s > R$ , which would more than pay off the (target) unfunded liability in one period.<sup>16</sup> Thus, the economically meaningful range is  $s \in (R-G, R]$ .<sup>17</sup> Note that the stability condition is independent of the target funded ratio. This contrasts with the suggestion that an 80-percent funded ratio (or some other figure) is tied to the “sustainability” of the system.<sup>18</sup>

### Steady State

The steady-state funded ratio, derived from (9), is:

$$(10) \quad f^* = [sf^\circ - (R - G)]/[s - (R - G)].$$

If the target is full-funding ( $f^\circ=1$ ), then the steady-state will indeed be full funding ( $f^*=1$ ).

However, if the target is for less than full-funding, the steady-state ratio will be lower yet.<sup>19</sup> That

is, if the target is set to 80 percent funding, the actual steady state will be lower than 80 percent.

The reason is that once we reach 80 percent, amortization will be zero, so contributions will simply cover normal cost ( $c_t = c^n$ ), which is not enough to sustain steady state unless we are at full funding. Indeed, the *meaning* of contributing normal cost is that each cohort has prefunded

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<sup>16</sup> Consider the borderline case  $s = R$ . If the target is full funding, then, as (9) shows, for any funded ratio  $f_{t-1}$ , contributions over the next period brings  $f_t$  back to unity. More generally, for  $f^\circ < 1$ , the steady state, given below, will also be attained in one period. For the other borderline case,  $s = R - G$ ,  $f_t$  will either remain stationary at any arbitrary level (if  $f^\circ = 1$ ) or will decline indefinitely (if  $f^\circ < 1$ ); as shown below, there is no well-defined steady state.

<sup>17</sup> The two conditions can be written as:  $R(f^\circ L_{t-1} - A_{t-1}) \geq (c_t - c^n)W_t > (R-G)(f^\circ L_{t-1} - A_{t-1})$ . The first term is principal plus interest on the (target) UAL, the middle term is the amortization payment (from (7)), and the last term is the growth-adjusted interest on the (target) unfunded liability. One does not need to pay full interest for the funded ratio to be stable, since payroll growth alone will erode the unfunded ratio.

<sup>18</sup> The behavior of the funded ratio, (9), is also independent of the relationship between  $c^p$  and  $c^n$ , unlike the more general formulation given in (6). That is, the conventional actuarial formula for contributions, slightly generalized in (8), eliminates the terms in  $c^p$  and  $c^n$  from the funded formula dynamic, since  $(c_t - c^p)$  in (6) is now proportional to  $(c^p - c^n)$ . Specifically, this follows from the fact that normal cost, net of pay-go cost appears on both the liability accrual (2) and asset accrual (1) under contribution formula (7).

<sup>19</sup> From (10),  $f^*[s - (R - G)] = [sf^\circ - (R - G)] < f^\circ[s - (R - G)]$  for  $f^\circ < 1$ , since  $R > G$ , so  $f^* < f^\circ$ .

its benefits, so that  $f = 1$ . If  $f_{t-1} = f^\circ < 1$ , so that  $c_t = c^n$  (by (8)), the funded ratio will fall<sup>20</sup> (and contributions will rise) until  $f$  approaches the steady state given in (10),  $f^* < f^\circ < 1$ .

Conversely, policy-makers may still achieve a "sustainable" funded ratio less than one, but to do so one must set the target funded ratio for amortization purposes ( $f^\circ$ ) between the true target and unity. That is, if the true target is 80 percent, the amortization target must be set higher than 80 percent.

As a corollary, if the true target is zero percent (i.e. barely solvent), the target ratio for amortization must be set somewhat greater than zero. That is, there is a positive floor for the amortization target ratio, below which insolvency will ensue. Specifically, (10) shows that  $f^* = 0$  for  $f^\circ = f^\circ_{\min} \equiv (R-G)/s \in (0, 1)$ . Thus, there is a sense in which one could say that a minimum  $x$ -percent target ratio is necessary. I will calibrate suggested estimates of that floor below to see how it compares with the 80 percent standard.

Figure 1 illustrates the relationship between the target funded ratio,  $f^\circ$ , and the steady state  $f^*$ , for a given value of  $s$ . The relationship lies below the 45-degree line. This can be read from the target  $f^\circ$  to the consequent steady state, which is lower, or from the steady state one seeks to the required target for amortization, which is higher. The horizontal intercept is  $f^\circ_{\min}$ .

[Figure 1 approximately here]

As shown earlier, in (5), the SS contribution rate  $c^*$  will include a fraction of the extra burden to fund the benefits of prior cohorts,  $(c^p - c^n)$ , and that fraction is the SS unfunded ratio,  $(1-f^*)$ . The model shows how that fraction is related to the target funded ratio  $f^\circ$ . Specifically, using (10) in (5), the steady-state contribution rate is:

$$(11) \quad c^* = c^n + (c^p - c^n)(1 - f^*) = c^n + (c^p - c^n)s(1 - f^\circ)/[s - (R - G)] > c^n + (c^p - c^n)(1 - f^\circ)$$

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<sup>20</sup> It is readily shown, using (6) and  $c_t = c^n$  that for  $f_{t-1} = f^\circ < 1$ ,  $f_t < f_{t-1}$ .



As the target funded ratio drops from 100 percent to, say, 80 percent, our measure of generational inequity rises from zero to something greater than 20 percent (indeed, much greater, as shown below) of the extra burden ( $c^p - c^n$ ), since  $(1-f^*) > (1 - f^\circ)$ . In the limiting case, as the target funded ratio reaches  $f^\circ_{\min} \equiv (R - G)/s$ , so  $f^* = 0$ , the full extra burden is born:  $c^* = c^p$ . The form that extra burden takes is the amortization payment. That is, if we set the target funded ratio as low as possible, consistent with steady-state solvency, the amortization payments suffice to top up the normal costs to the pay-go rate. The only difference between this system and the simple pay-go system is that with  $R > G$ , the system with a constant contribution rate is unstable, while in this system, with  $s > (R - G)$ , the amortization payments will adjust outside of steady-state to provide stable convergence.

### Standard N-Period Amortization and Calibration of Magnitudes

To get more specific (and to calibrate magnitudes), we specify the amortization rate,  $s$ , using a standard actuarial formula. The most common formula is “level percent” of payroll. Amortization payments are set to grow in step with payroll, at rate  $G$  and to pay off the unfunded liability in  $N$  years (usually 30).<sup>21</sup> Under the “open” version – commonly used – the  $N$ -year horizon is renewed every year, so the UAL is never fully paid off, but, as shown below, the funded ratio asymptotically approaches one, or the target ratio. Under this formula,<sup>22</sup>

$$(12) \quad s = (R-G)/[1 - (G/R)^N].$$

Note that this formula will always satisfy the stability condition  $s \in (R-G, R]$ , as  $s = R$  for  $N = 1$  and  $s \rightarrow (R - G)$  as  $N \rightarrow \infty$ .

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<sup>21</sup> “Level dollar” amortization is the special case with  $G = 1$ .

<sup>22</sup> This assumes that the amortization series starting in period  $t$  is set to amortize the UAL of period  $t-1$ , with interest accrued in period  $t$ , i.e. the present value of the amortization series starting in period  $t$  is  $R \cdot UAL_{t-1}$ . The derivation is a simple application of the sum of a geometric series.

Substituting (12) into (9), and simplifying, we have

$$(13) \quad f_t = \{[G - R(G/R)^N]/G[1 - (G/R)^N]\}f_{t-1} + [(R-G)(f^\circ - 1 + (G/R)^N)/G[1 - (G/R)^N]].$$

Solving (13) for the steady state,  $f_t = f_{t-1} = f^*$  (or substituting (12) directly into (10)), we have a very simple solution, which can be readily calibrated:

$$(14) \quad f^* = 1 - (1 - f^\circ)(R/G)^N.$$

With the mean actuarial assumptions reported above,  $R = 1.077$  and  $G = 1.037$ , and  $N = 30$ , if the target funded ratio were set at 80 percent, the actual steady state would be quite a bit lower: 37.8 percent. Conversely, to achieve a steady state of 80 percent, the target ratio for amortization purposes would have to be much closer to one: 93.6 percent.

At the other extreme, the minimum target to avoid SS insolvency is  $f^\circ_{\min} = [1 - (G/R)^N] = 67.9$  percent. Stated differently, a pay-go system can be mimicked with an actuarial funding formula that sets amortization based on 67.9 percent target funding. Interestingly, some commentators now state that funding at about 70 percent is good enough.<sup>23</sup> If that were formalized in the amortization formula, the result would be near-insolvency.

Table 1 provides a more expansive picture of the SS funded ratios as a function of the target funded ratio and  $(R/G)$ . The bold red row represents  $(R/G)$  under the mean actuarial assumptions above,  $1.077/1.037 = 1.039$ . At this value of  $(R/G)$ , every percentage point reduction in the target ratio reduces the SS funded ratio by over 3 points. That is the slope in Figure 1. Clearly, the SS funded ratio is quite sensitive to variation in the target ratio.

[Table 1 approximately here]

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<sup>23</sup> Dean Baker co-director of the Center for Economic Policy Research, as reported in [Politico](#) and unnamed “investment analysts” cited in the [Baltimore Sun](#). (“[80 Percent Pension Funding Hall of Shame](#)” database)

We can now calibrate the degree of generational inequity and the same result holds: it is very sensitive to the target funded ratio. The steady-state contribution rate, given by (5), can be related directly to the target funded ratio, for given R and G:

$$(15) \quad c^* = c^n + (c^p - c^n)(1 - f^*) = c^n + (c^p - c^n)(1 - f^\circ)(R/G)^N.$$

Just as the SS funded ratio varies by over 3 points for every point in the target ratio (for R/G = 1.039), so too for the SS unfunded ratio,  $(1 - f^*)$ , and, hence, the share of the extra burden to fund the benefits of prior cohorts. For  $f^\circ = 80$  percent, the share of the extra burden rises not to 20 percent, but to 62.2 percent. At the target ratio of 70 percent, bandied about by some, the SS result would be nearly complete generational inequity – virtually pay-go.

To be sure, those who invoke an x-percent target no doubt have in mind something like an SS funded ratio (instead of a target ratio as modeled here). But one may well doubt if they have a firm idea of how to operationalize that target. The model here suggests how one might think of plans' contribution behavior “as if” a target funded ratio were embedded in the amortization formula, with the ensuing actual funded ratio. To illustrate, consider the national average, that employer contributions covered only 85.3 percent of ARC in 2013.<sup>24</sup> The ARC rate for these plans averaged 17.6 percent, of which 8.0 percent was the employer normal cost and 9.6 percent amortization. From these data one can infer that the average plan paid 73.3 percent of the amortization. The same dataset gives  $f = 74.1$  percent. This is “as if” the target funded ratio  $f^\circ = 93.1\%$ <sup>25</sup> which implies  $f^* = 78.4\%$ , by (14). Thus (perhaps coincidentally), these data are consistent with the idea that the average plan is acting as if it were content with 80

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<sup>24</sup> This is for those plans in Boston College's Public Plans database whose members are in Social Security (for their ARCs to be comparable with each other). The figures given are all weighted averages for about 100 plans.

<sup>25</sup> The 73.3 percent figure is calculated as  $(0.853 \times 0.176 - 0.080)/0.096$ , without rounding. The ratio of amortization “as if” the target ratio were  $f^\circ$  to the full-funding amortization is  $(f^\circ - f_{t-1})/(1 - f_{t-1})$ . Setting this to 0.733, with  $f_{t-1} = 0.741$  implies  $f^\circ = 0.931$ .

percent funding and, implicitly uses the corresponding target ratio to pare back the amortization payments. Of course, this average masks a wide variation between those plans aiming at full-funding and those which further reduce contributions, consistent with a much lower steady state.

## VI. Assumed Return > R

The previous analysis helps understand the properties of a system that either builds underfunding into its amortization formula or acts “as if” it has done so by shortchanging the contributions required to reach full funding, with the rationale that “80 percent is good enough.” In many plans, however, the main cause of underfunding is that the fund’s assumed return has exceeded the market return, in recent years. If that gap is temporary and the fund’s performance is expected to revert to longer term historical norms on which the assumed return is purportedly based, then the “80 percent” rationale is simply a statement that, while short today, we are still on track to full funding. However, if we are now in an environment where prospective returns are lower than previously assumed, and the assumed rate is maintained or only minimally reduced, then we have a situation analogous to that modeled above, where underfunding is built into the system, through the back door of the assumed return, and is effectively rationalized with the 80-percent standard. In this section I formally examine a system where the assumed return exceeds the true return, and provide new insights into the resulting steady state.

We begin with some additions to the preceding notation:

$$R' = 1 + \text{assumed return on investment} > R = 1 + \text{true return}$$

$$c^m = \text{measured normal cost} < c^n = \text{true normal cost (measured at } R)$$

$$L'_t = \text{measured liabilities, at end of period } t < L_t = \text{true liabilities (measured at } R)$$

$$f'_t = \text{measured funded ratio at end of period } t, A_t/L'_t > f_t = \text{true funded ratio}$$

The dynamic of assets is unchanged from equation (1), since their evolution rests on actual returns,  $R$ . Equations (2) - (3) continue to represent the dynamic of true liabilities, but for measured liabilities, these analogous expressions hold:

$$(16) \quad L'_t = GL'_{t-1} = R'L'_{t-1} + (c^m - c^p)W_t$$

$$(17) \quad L'_t = [G/(R'-G)](c^p - c^m)W_t.$$

Similarly, equation (6) still holds for the dynamic of the true funded ratio, but using (1), (16)-(17), we find the following dynamic for the measured funded ratio:

$$(18) \quad f'_t = (R/G)f'_{t-1} + [(R'-G)/G](c_t - c^p)/(c^p - c^m).$$

Consider the actuarially-determined contribution rate with amortization speed  $s'$ :

$$(19) \quad c_t = c^m + s'(L'_{t-1} - A_{t-1})/W_t = c^m + s'(1 - f'_{t-1})(c^p - c^m)/(R'-G),$$

where the target is full funding, and we have used (17). The immediate impact on contributions of assuming  $R'$  instead of  $R$  is complex, but includes a reduction in measured normal cost and unfunded ratio; it will be instructive to contrast these with the steady-state impact, below.

Substituting into (18), we have the dynamic for the measured funded ratio:

$$(20) \quad f'_t = [(R - s')/G]f'_{t-1} + [s' + G - R']/G.$$

The stability condition,  $s' > R - G$ , is analogous to that of the target funding model.<sup>26</sup>

Turning to the steady state, it would not be surprising to find that the true funded ratio is less than one, since it is below the measured ratio, which is targeted for full funding. It may, however, be surprising that not even the measured ratio -- inflated by the higher discount rate for liabilities -- reaches one in steady state. Solving (20), we have:

$$(21) \quad f'^* = [s' + G - R']/[s' + G - R] < 1 \text{ for } R' > R.$$

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<sup>26</sup> Once again, this dynamic does not depend on normal cost (true or measured,  $c^n$  or  $c^m$ ) or the cost of benefits ( $c^p$ ). The reason is the same: normal cost -- however measured -- net of pay-go cost appears on both the liability accrual and asset accrual, so it vanishes from the dynamic of the measured funded ratio.

The reason  $f^* < 1$  may be different from what intuition suggests. Suppose  $f_{t-1} = 1$ . Since we are at full-funding, as measured using  $R'$ , amortization payments cease and  $c_t = c^m$  (by (19)). One might think part (or all) of the problem is that  $c^m$  low-balls true normal cost, so the liabilities that accrue are not fully funded. But that is not the problem. *Measured* liabilities accrue at  $c^m$ , so if contributions also include normal cost as measured, this is not the reason the measured ratio falls below 1. As (18) shows, when  $c_t = c^m$ , the terms in  $c^m$  drop out, no matter how normal cost is measured – even if it were measured accurately. The problem is not in the accruals of measured liabilities or assets (i.e., contributions), but rather the fact that liabilities are rolled forward at  $R'$ , while assets (net of contributions) only accumulate at rate  $R$ .<sup>27</sup> Thus, (20) shows that if  $f_{t-1} = 1$ ,  $f_t$  drops to  $1 - (R'-R)/G$ , and will continue dropping until reaching the steady-state in (21).

To calibrate the system, consider the amortization formula (12), with assumed return  $R'$ :

$$(22) \quad s' = (R'-G)/[1 - (G/R')^N].$$

Substituting in (20), we have:<sup>28</sup>

$$(23) \quad f_t = \{[R - R(G/R')^N - (R'-G)]/G[1-(G/R')^N]\}f_{t-1} + (R'-G)(G/R')^N/G[1-(G/R')^N] \text{ and}$$

$$(24) \quad f^* = (R'-G)(G/R')^N/\{(R-G)(G/R')^N + (R'-R)\} \\ = (R'-G)/[(R'-R)(R'/G)^N + (R-G)] < 1 \text{ for } R' > R > G.$$

Consider first the benchmark case discussed above:  $R' = 1.077$ ,  $G = 1.037$ ,  $N = 30$ . If we suppose that the true return is 0.5 percentage points lower,  $R = 1.072$ , the steady-state value of the measured funded ratio,  $f^* = 79.1$  percent. Thus, if one accepts the 80 percent standard as “good enough,” one might be willing to maintain an assumed rate of return that is half a

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<sup>27</sup> Our emphasis here is on the impact of high assumed returns on the growth of measured liabilities, rather than the impact of low market returns on the growth of assets. That is, unlike some previous literature ([Munnell, et al., 2015](#); [Costrell, 2015b](#)), to assess the impact of  $R' > R$ , the counterfactual here is “what if the assumed return had been reduced to the true return,” rather than “what if the market had performed up to assumed return?”

<sup>28</sup> The coefficient on  $f_{t-1} < 1$ , so that stability condition still holds. For small  $N$  (e.g.  $N = 1$ ), the coefficient goes negative, so the system becomes oscillatory, but that seems unlikely under conventional choices of  $N$ .

percentage point too high. However, the steady state is very sensitive to the assumed return. If the true return is a full point lower,  $f^* = 65.4$  percent and if it is two points lower (i.e. 5.7 percent true return),  $f^*$  falls below 50 percent.

Table 2 illustrates the sensitivity, by depicting  $f^*$  for various values of  $R'/G$  and  $R'/R$ . The values of  $R'/R$  vary from 1.000 to 1.030, with increments of 0.005, which correspond approximately to 0.5 percent increments on the spread between  $R'$  and  $R$ . Looking across any given row<sup>29</sup> one can see the sensitivity of  $f^*$  to the assumed return. The relationship is not linear ( $f^*$  only approaches zero asymptotically), but  $f^*$  rapidly falls below 50 percent.

As we have seen above, the steady-state unfunded ratio corresponds directly to the degree of generational inequity, but for our *measured* magnitudes with  $R' > R$ , there is a twist. Write the steady-state contribution rate by substituting (22) into (19):

$$(25) \quad c^* = c^n + (c^p - c^n)(1 - f^*)/[1 - (G/R')^N].$$

The potential extra burden of funding prior cohorts is measured as  $(c^p - c^n)$  and the share of that actually borne is  $(1 - f^*)/[1 - (G/R')^N]$ . That is, unlike the true burden (given in (5)), each cohort carries a share of the measured burden that *exceeds* the measured unfunded ratio. For example, in our benchmark case ( $R' = 1.077$ ,  $G = 1.037$ ), if  $R'$  exceeds  $R$  by half point -- so the steady-state measured funded ratio is about 80 percent -- each cohort carries not 20 percent, but about 30 percent of the (measured) extra burden. At a full point spread, that rises to about 50 percent, and at a two point spread that goes to about 75 percent.

Finally, it is worth pointing out that even though the initial impact of a policy of high assumed returns is undoubtedly to reduce contributions (both normal cost and amortization), in steady-state, it raises contributions over what they would be if  $R' = R$ . We can see this in (5),

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<sup>29</sup> The benchmark case of  $R' = 1.077$  and  $G = 1.037$  corresponds approximately to the row with  $R'/G = 1.04$ .

which still holds here. With  $R' = R$ ,  $f^* = 1$  and  $c^* = c^n$ , but with  $R' > R$ ,  $f^* < 1$  and  $c^* > c^n$ .<sup>30</sup>

This is no surprise – it is certainly understood that a policy of high assumed returns is a policy of “pay later,” i.e. generational inequity. Our contribution here is to shed further light on the mechanism by which this occurs, and to show the order of magnitudes involved. The mechanism is the steady-state amortization payments generated by the failure of the system to reach full funding by the *inflated* ratio (let alone the true ratio) and that failure owes to the inflated growth of measured liabilities (rather than the deflated measure of normal cost). The magnitudes of the generational inequity generated by small deviations of assumed from true returns (0.5 to 2.0 percentage point) may be surprising: 30 – 75 percent of the measured extra burden of funding prior cohorts.

## VII. Conclusion

The “80 percent standard” – mythical though its origins may be -- raises the possibility that steady states may be sustainable with less than full funding. This is true. Indeed, steady state funding ratios as low as zero may be sustainable, in a mathematical sense, if the funding formula includes an amortization payment that adjusts outside of steady state (to provide stability) and generates sufficient contributions in steady-state to help fund the benefits of prior cohorts as well as the current cohort. I have shown here that the extra contributions for prior cohorts – the generational inequity – takes a simple form in steady state: it is the steady-state unfunded ratio times the difference between the pay-go rate and the normal cost.

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<sup>30</sup> Since (5) and (25) both hold in steady-state for  $c^*$ , together they define the complex relationship between the measured and true unfunded ratios,  $(1-f^*)$  and  $(1-f)$ . Equivalently, we can use (3) and (17) to write  $f_t/f_t = L_t/L_t = [(R-G)/(R'-G)][(c^p - c^m)/(c^p - c^n)]$ . Although, the steady-state value of the measured funded ratio does not depend on the gaps among  $c^p$ ,  $c^n$ , and  $c^m$ , these do matter for the true funded ratio.



I have also examined the determinants of the SS unfunded ratio under an “x-percent” amortization formula (either formal or informal) and a policy of high assumed returns. Under an x-percent amortization formula, the SS unfunded ratio (and, hence, the degree of generational inequity), will exceed the target unfunded ratio. Thus, to achieve a 20 percent SS unfunded ratio will require a target funded ratio for amortization of more than 80 percent; I calibrate it at about 94 percent under mean actuarial assumptions. Conversely, a target ratio of 80 percent for amortization would result in an SS funded ratio of under 40 percent, and a 70 percent target – a figure bandied about by some analysts – would lead to near-insolvency. This would impose upon each cohort nearly the full extra burden of funding prior cohorts, mimicking the pay-go system that actuarial funding was originally designed to replace.

Under a policy of high assumed returns, with a target funded ratio of 100 percent, the SS funded ratio – *even as inflated by the high discount rate* – will fall short.<sup>31</sup> I calibrate, under mean actuarial assumptions, that if the goal is a 20 percent SS unfunded ratio as measured, the true return can run half a percentage point below assumptions. However, the degree of generational inequity (as measured) will not be 20 percent, but 30 percent. If the spread between assumed and actual returns reaches 2 percentage points, the extra burden borne by each cohort can easily reach 75 percent.

More generally, the main *policy* takeaway of my analysis is that the SS degree of generational inequity is highly sensitive to the policies considered. The “80 percent” mantra lends a spurious complacency regarding departures from sound finance: small deviations from the 100 percent funding target or of the assumed return from true returns generate large degrees of generational inequity.

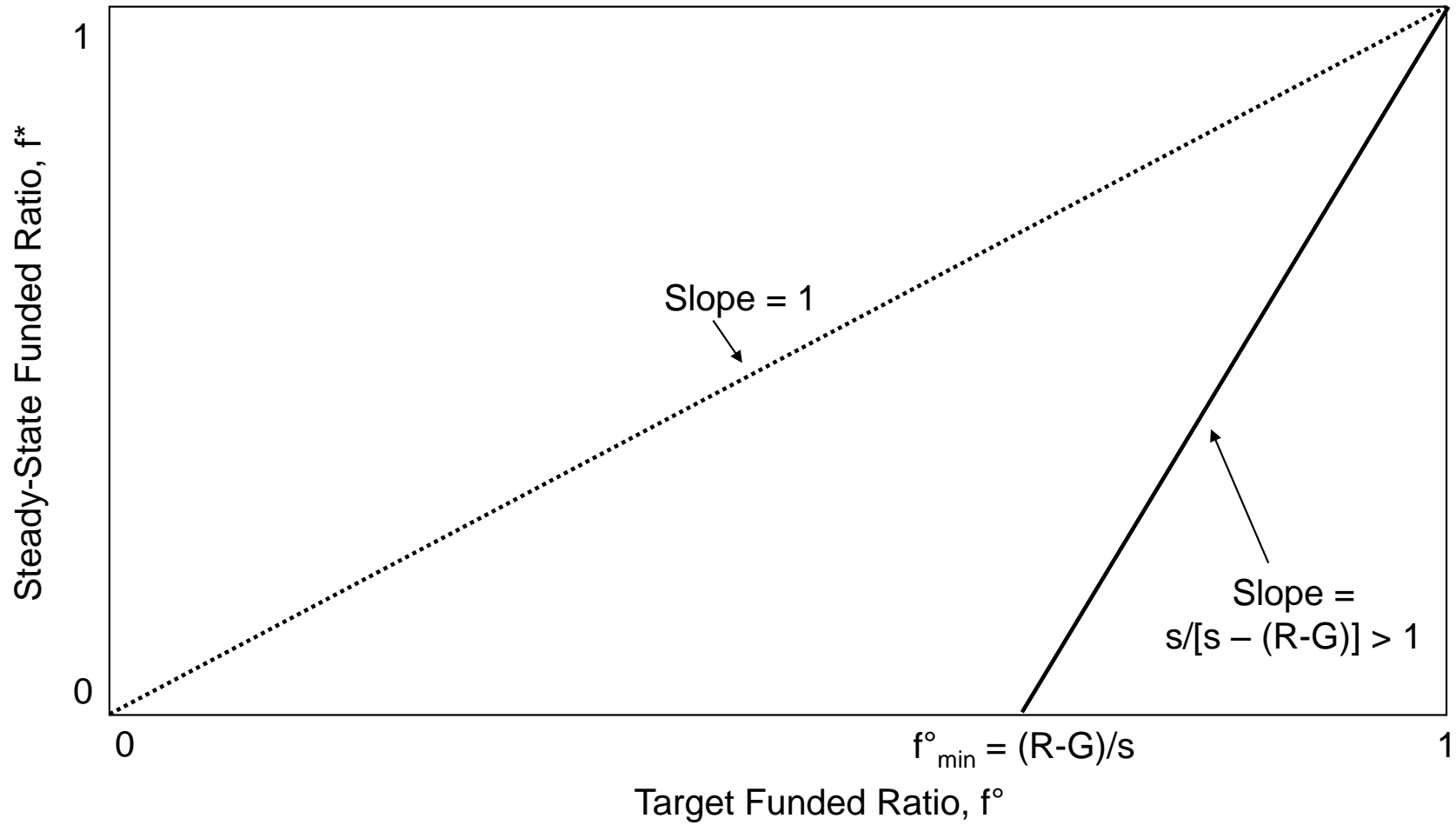
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<sup>31</sup> The reasons for this result, not always understood, are independent of the deflated measure of normal cost.

## **References**

- American Academy of Actuaries. 2012. "[The 80% Pension Funding Standard Myth.](#)"
- Boston College Center for Retirement Research. "[Public Plans Data.](#)"
- Brainard, Keith, and Paul Zorn. 2012. "[The 80-percent threshold: Its source as a healthy or minimum funding level for public pension plans.](#)" National Association of State Retirement Administrators and Gabriel, Roeder, Smith and Company.
- Campbell, Mary Pat. 2014. "[The 80 Percent Pension Funding Hall of Shame](#)" and [database.](#)
- Costrell, Robert M. 2015a, "[School Pension Costs Have Doubled Over the Last Decade, Now Top \\$1,000 Per Pupil Nationally.](#)" [teacherpensions.org.](#)
- Costrell, Robert M. 2015b. "[Assessing the Impact of Investment Shortfalls on Unfunded Pension Liabilities: The Allure of Neat, but Faulty Counterfactuals.](#)" Economics Research Network, Public Policy Center Research Paper Series, U of Arkansas Department of Education Reform, vol. 1, No. 4.
- Diamond, Peter A. 1965. "National Debt in a Neoclassical Growth Model." *The American Economic Review*, 55(5), Part 1 (Dec., 1965), pp. 1126-1150.
- Government Finance Officers Association. 2009. "[Sustainable Funding Practices of Defined Benefit Pension Plans.](#)"
- Mankiw, N. Gregory. 2015. "[Yes,  \$r > g\$ . So What?](#)" *American Economic Review: Papers & Proceedings*, 105(5), pp. 43-47.
- Miller, Girard. 2012. "[Pension Puffery.](#)" *Governing* (January 5).
- Munnell, Alicia H., Jean-Pierre Aubry, and Mark Cafarelli, 2015. "[How Did State/Local Plans Become Underfunded?](#)" Center for Retirement Research at Boston College, Number 42, January 2015.
- Piketty, Thomas. 2014. *Capital in the Twenty-first Century*. Cambridge, MA: Belknap Press of the Harvard University Press.
- Samuelson, Paul A. 1958. "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money." *The Journal of Political Economy*, 66(6), Dec., 1958, pp. 467-482.
- Teachers' Retirement System of the State of Illinois. 2016. "[Actuarial Valuation Report of the June 30, 2015 Actuarial Valuation of Pension Benefits.](#)"

Figure 1  
 Steady-State vs. Target Funded Ratio  
 equation (10),  $0 < R - G < s \leq R$ ; equation (14),  $s = (R-G)/[1-(G/R)^N]$



**Table 1**  
**Steady State Funded Ratios Under x-percent Funding Targets**  
 (equation (14); N = 30)

		<u>target funded ratio</u>						
		<b>0.70</b>	<b>0.75</b>	<b>0.80</b>	<b>0.85</b>	<b>0.90</b>	<b>0.95</b>	<b>1.00</b>
<b>1.02</b>		0.46	0.55	0.64	0.73	0.82	0.91	1.00
<b>1.03</b>		0.27	0.39	0.51	0.64	0.76	0.88	1.00
<b>1.039</b>		<b>0.07</b>	<b>0.22</b>	<b>0.38</b>	<b>0.53</b>	<b>0.69</b>	<b>0.84</b>	<b>1.00</b>
<b>1.05</b>	<b>R/G</b>			0.14	0.35	0.57	0.78	1.00
<b>1.06</b>					0.14	0.43	0.71	1.00
<b>1.07</b>						0.24	0.62	1.00
<b>1.08</b>							0.50	1.00

**Table 2**  
**Steady State Measured Funded Ratios Under High Assumed Returns**  
 (equation (24); N = 30)

		<b><u>assumed/true rate of return, R'/R</u></b>						
		<b>1.000</b>	<b>1.005</b>	<b>1.010</b>	<b>1.015</b>	<b>1.020</b>	<b>1.025</b>	<b>1.030</b>
<b><u>R'/G</u></b>	<b>1.02</b>	1.00	0.83	0.71	0.62	0.55	0.50	0.45
	<b>1.03</b>	1.00	0.80	0.67	0.58	0.51	0.46	0.41
	<b>1.04</b>	1.00	0.78	0.63	0.54	0.47	0.41	0.37
	<b>1.05</b>	1.00	0.74	0.59	0.49	0.42	0.37	0.33
	<b>1.06</b>	1.00	0.71	0.55	0.45	0.38	0.33	0.29
	<b>1.07</b>	1.00	0.67	0.50	0.40	0.34	0.29	0.25
	<b>1.08</b>	1.00	0.62	0.45	0.36	0.29	0.25	0.22