

THE α -REGULAR CLASSES OF THE GENERALIZED SYMMETRIC GROUP

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(Received 2 April, 1975)

Introduction. The α -regular classes of any finite group G are important since they are those classes on which the projective characters of G with factor set α take non-zero value, and thus a knowledge of the α -regular classes gives the number of irreducible projective representations of G with factor set α (see [4]). Here we look at the particular case of the generalized symmetric group $C_m \text{ wr } S_l$. The analogous problem of constructing the irreducible projective representations of $C_m \text{ wr } S_l$ has been dealt with in [6] by generalizing Clifford's theory of inducing from normal subgroups, but unfortunately, it is not in general possible to determine the irreducible projective characters (and hence the α -regular classes) by this method.

Necessary definitions of factor sets and properties of α -regular elements are given in §1, but a knowledge of the theory of projective representations of finite groups is assumed (see [4], [2]). In §2, we give a brief description of the group $C_m \text{ wr } S_l$ as well as the most important results of [1]. The α -regular classes of $C_m \text{ wr } S_l$ with respect to $(-1, 1, 1)$, $(-1, -1, -1)$, $(1, -1, 1)$ are determined in §3 and §4, and we tabulate these results, together with the α -regular classes of the remaining factor sets in Theorem 5.2.

In all that follows, G is a finite group, \mathbf{C} the complex field, and \mathbf{C}^* the group of non-zero elements of \mathbf{C} .

1. Factor sets.

DEFINITION 1.1. A mapping $\alpha: G \times G \rightarrow \mathbf{C}^*$ is called a *factor set* of G if

$$\alpha(x, y)\alpha(xy, z) = \alpha(x, yz)\alpha(y, z) \quad \text{for all } x, y, z \in G, \text{ and}$$

$$\alpha(1_G, 1_G) = 1,$$

where 1_G is the identity of G .

DEFINITION 1.2. Let α be a factor set of G . We define $\alpha': G \times G \rightarrow \mathbf{C}^*$ by

$$\alpha'(x, y) = \alpha(x, y)\alpha(y, x)^{-1} \quad \text{for all } x, y \in G.$$

DEFINITION 1.3. An element $a \in G$ is α -regular if $\alpha'(a, x) = 1$ for all $x \in C_G(a)$, the centralizer of a in G .

LEMMA 1.4. If $a \in G$ is α -regular, so is every conjugate of a in G . Thus the property of being α -regular is a class function on G .

Proof. See [4].

LEMMA 1.5. Let T be a projective representation of G with factor set α , and let χ_T be the character of T . If $\chi_T(a) \neq 0$, then a is α -regular.

Proof. Let $x \in C_G(a)$. Then $T(x)T(a) = \alpha'(a, x)T(a)T(x)$, and so $\chi_T(a)(1 - \alpha'(a, x)) = 0$, which gives the result.

LEMMA 1.6. *Let $a, b, c \in G$ be such that $b, c \in C_G(a)$. Then $\alpha'(a, bc) = \alpha'(a, b)\alpha'(a, c)$.*

Proof. By repeated applications of 1.1 we have

$$\begin{aligned} \alpha'(a, bc) &= \alpha(a, bc)\alpha(bc, a)^{-1} = \alpha(a, bc)\alpha(b, c)\alpha(b, c)^{-1}\alpha(bc, a)^{-1} \\ &= \alpha(a, b)\alpha(ab, c)\alpha(b, ca)^{-1}\alpha(c, a)^{-1} = \alpha(a, b)\alpha(a, c)\alpha(b, a)^{-1}\alpha(c, a)^{-1} \end{aligned}$$

DEFINITION 1.7. Let α be a factor set of G , and assume there exists some integer n such that $\alpha(x, y)^n = 1$ for all $x, y \in G$. The smallest value of n such that this holds is called the *order* of α . If no such n exists, α is said to be of infinite order.

LEMMA 1.8. *Let α be a factor set of G of finite order n and let s be any integer such that $(n, s) = 1$. If $a \in G$ is such that a^s is α -regular, then a is also α -regular.*

Proof. Let $x \in C_G(a)$. Then $x \in C_G(a^s)$ and hence $\alpha'(a, x)^s = \alpha'(a^s, x) = 1$ by Lemma 1.6. However, as α is of order n , we must have $\alpha'(a, x)^n = 1$, and hence $\alpha'(a, x) = 1$.

2. The generalized symmetric group. $C_m \text{ wr } S_l$ is the wreath product of the cyclic group C_m of order m with the symmetric group S_l on l symbols (see e.g. [3]). Here, however, it is more convenient to think of the group in other terms. It has a presentation

$$\begin{aligned} C_m \text{ wr } S_l = \langle r_i (i = 1, \dots, l-1), w_j (j = 1, \dots, l) \mid r_i^2 = (r_i r_{i+1})^3 = (r_i r_j)^2 = 1, (|j-i| \geq 2), \\ w_j^m = 1, w_i w_j = w_j w_i, r_i w_i = w_{i+1} r_i, r_i w_j = w_j r_i, j \neq i, i+1 \rangle \end{aligned}$$

(see [6]). It is called the generalized symmetric group because we may think of r_i as the transposition $(i, i+1)$ and w_j as the mapping $j \rightarrow \xi j$, where ξ is some primitive m th root of 1. Thus $C_m \text{ wr } S_l$ permutes the letters $\{1, \dots, l\}$ as well as multiplying any number of them by some power of ξ .

Let

$$\sigma = \begin{pmatrix} 1 & 2 & \dots & l \\ \xi^{k_1} b_1 & \xi^{k_2} b_2 & & \xi^{k_l} b_l \end{pmatrix} \in C_m \text{ wr } S_l,$$

where $\{b_1, \dots, b_l\} = \{1, \dots, l\}$, and the k_i are positive integers. We define $\Phi: C_m \text{ wr } S_l \rightarrow S_l$ by

$$\Phi(\sigma) = \begin{pmatrix} 1 & 2 & \dots & l \\ b_1 & b_2 & & b_l \end{pmatrix}.$$

Then Φ is a homomorphism.

DEFINITION 2.1. $\sigma \in C_m \text{ wr } S_l$ is *even* if $\Phi(\sigma)$ is an even element of S_l and *odd* otherwise. In terms of the generators $\{r_i, w_j\}$ given above, σ is even if and only if the number of r_i appearing in any expression for σ is even.

Any element $\sigma \in C_m$ wr S_t may be written down uniquely (up to reordering) as a product of disjoint cycles $\sigma = \theta_1 \dots \theta_t$ for some t , where

$$\theta_i = \begin{pmatrix} b_{i_1} & b_{i_2} & \dots & b_{i_{i_1}} \\ \xi^{k_{i_1}} b_{i_1} & \xi^{k_{i_2}} b_{i_2} & & \xi^{k_{i_{i_1}}} b_{i_{i_1}} \end{pmatrix}, \quad i = 1, \dots, t. \tag{1}$$

DEFINITION 2.2. Let θ_i be any cycle of the above form. We define

$$\text{diag } \theta_i = \begin{pmatrix} b_{i_1} \\ \xi b_{i_1} \end{pmatrix} \begin{pmatrix} b_{i_2} \\ \xi b_{i_2} \end{pmatrix} \dots \begin{pmatrix} b_{i_{i_1}} \\ \xi b_{i_{i_1}} \end{pmatrix} = w_{b_{i_1}} w_{b_{i_2}} \dots w_{b_{i_{i_1}}}$$

We note that $\text{diag } \theta_i$ is always an even element of C_m wr S_t .

DEFINITION 2.3. Let $\sigma = \theta_1 \dots \theta_t \in C_m$ wr S_t , where $\{\theta_i \mid i = 1, \dots, t\}$ are the disjoint cycles of σ . For any such θ_i we define $f(\theta_i) = \sum_{j=1}^{i_1} k_{i_j}$ (as in (1)).

Let a_{pq} be the number of cycles θ_i of σ of length q such that $f(\theta_i) \equiv p \pmod{m}$, for $1 \leq p \leq m, 1 \leq q \leq l$. The $m \times l$ matrix (a_{pq}) is called the *type* $\text{Ty}(\sigma)$ of σ .

THEOREM 2.4. $\sigma, \sigma_1 \in C_m$ wr S_t are conjugate if and only if $\text{Ty}(\sigma) = \text{Ty}(\sigma_1)$.

Proof. See [3].

In [1], the following results are proved.

THEOREM 2.5. Each projective representation T of C_m wr S_l may be generated (in the sense of [6]) by a set of matrices $\{R_1, \dots, R_{l-1}, V_1, \dots, V_l\}$, where $R_i = T(r_i)$, and $V_j = T(w_j)$, in which case the corresponding factor set α of T is of order 2 or 1. Furthermore the matrices $\{R_i, V_j\}$ satisfy the following relations: $R_i^2 = I, (R_i R_{i+1})^3 = I, (R_i R_j)^2 = \gamma I, |i-j| \geq 2, V_i^m = I, V_i V_j = \mu V_j V_i, j \neq i, R_i V_i = V_{i+1} R_i, R_i V_j = \lambda V_j R_i, j \neq i, i+1$, where

$$\gamma^2 = \lambda^{(2,m)} = \mu^{(2,m)} = 1.$$

The Schur Multiplier of C_m wr S_l is then given by

$$H^2(C_m \text{ wr } S_l, \mathbf{C}^*) = \begin{cases} C_2 = \{(\gamma)\} & \text{if } m \text{ is odd, } l \geq 4, \\ \{1\} & \text{if } m \text{ is odd, } l < 4, \\ C_2^3 = \{(\gamma, \lambda, \mu)\}, & \text{if } m \text{ is even, } l \geq 4, \\ C_2^2 = \{(\lambda, \mu)\} & \text{if } m \text{ is even, } l = 3, \\ C_2 = \{(\mu)\} & \text{if } m \text{ is even, } l = 2, \\ \{1\} & \text{if } m \text{ is even, } l = 1. \end{cases}$$

For simplicity of notation, we will always use $\{(\gamma, \lambda, \mu)\}$ to denote the multiplier, with the convention that γ, λ , or $\mu = 1$ for certain values of m, l (given by the above result).

The relations between the $\{R_i, V_j\}$ given in 2.5 imply the following result which is expressed in terms of the factor set α of the projective representation T of C_m wr S_l generated by $\{R_i, V_j\}$.

- LEMMA 2.6. (i) $\alpha'(r_i, r_j) = \gamma, \quad |i-j| \geq 2$
 (ii) $\alpha'(r_i, w_j) = \lambda, \quad j \neq i, i+1$
 (iii) $\alpha'(w_i, w_j) = \mu, \quad j \neq i.$

The proof is easy and is omitted.

Before proceeding to determine the α regular classes of C_m wr S_l for all (γ, λ, μ) , we describe a class of matrices which will be used to construct matrices $\{R_i, V_j\}$ satisfying 2.5.

LEMMA 2.7. *Let k be any positive integer. There exist matrices $\{N_1, \dots, N_{2k+1}\}$ of degree 2^k satisfying*

- (i) $N_j^2 = I, \quad j = 1, \dots, 2k+1,$
- (ii) $N_j N_h = -N_h N_j \quad j \neq h,$
- (iii) $N_1 N_2 \dots N_{2k+1} = (i)^k I, \quad (i = \sqrt{-1}),$
- (iv) *No other product of distinct matrices $N_{j_1} \dots N_{j_t} = \zeta I$, for any $\zeta \in \mathbf{C}^*$ (apart from a reordering of (iii)).*
- (v) $N_{j_1} \dots N_{j_t}$ *has non-zero trace if and only if $N_{j_1} \dots N_{j_t} = \zeta I$, for some $\zeta \in \mathbf{C}^*$.*

Proof. Let M_1, \dots, M_{2k+1} be defined as in [7, p. 198]. Put $N_j = M_{2k+2-j}$ if j is odd, and $N_j = (i)M_{2k+2-j}$ if j is even.

3. In the case $(\gamma, \lambda, \mu) = (1, 1, 1)$, the projective representations of C_m wr S_l are linear (ordinary) representations, and hence all classes are α -regular. Next we consider the α -regular classes of C_m wr S_l , when α is the factor set of the projective representation T generated by matrices $\{R_i, V_j\}$ for $(\gamma, \lambda, \mu) = (-1, 1, 1)$. (Henceforth, we will write $\alpha \in (-1, 1, 1)$). Let $\{N_1, \dots, N_{2k+1}\}$ be the matrices defined in Lemma 2.7, where $k = [\frac{l}{2}]$ (integer part). Putting $R_i = (1/\sqrt{2})(N_i - N_{i+1}), \quad i = 1, \dots, l-1, \quad V_j = I, \quad j = 1, \dots, l,$ we see that $\{R_i, V_j\}$ satisfy the conditions of 2.5 for $(-1, 1, 1)$.

(i) Let $\sigma = \theta_1 \dots \theta_t$ be even, where $\{\theta_i \mid i = 1, \dots, t\}$ are the disjoint cycles of σ . Assume further that all θ_i are even cycles. Thus, there exists an odd integer p such that

$$\sigma^p = w_{i_1}^{a_1} \dots w_{i_r}^{a_r}, \quad (a_i \in \mathbf{Z}_+)$$

If T is the projective representation of C_m wr S_l generated by the above matrices, (see (6)), then

$$T(\sigma^p) = I, \text{ and hence } \sigma^p \text{ is } \alpha\text{-regular by 1.5.}$$

Thus σ is α -regular by Lemma 1.8. If θ_1 is an odd cycle,

$$\begin{aligned} \alpha'(\theta_1, \sigma) &= \alpha'(\theta_1, \theta_1)\alpha'(\theta_1, \theta_2, \dots, \theta_t) \text{ (by Lemma 1.6)} \\ &= -1 \text{ (by Lemma 2.6)} \end{aligned}$$

and as $\theta_1 \in C_{C_m \text{ wr } S_l}(\sigma)$, σ is not α -regular.

(ii) Now assume $\sigma = \theta_1 \dots \theta_t$ is odd, and let $\text{Ty}(\theta_1) = \text{Ty}(\theta_2)$. By Lemma 1.4, we may assume, without loss of generality that

$$\theta_1 = \begin{pmatrix} a_1 & \dots & a_q \\ a_2 & \dots & \xi^p a_1 \end{pmatrix} \text{ and } \theta_2 = \begin{pmatrix} b_1 & \dots & b_q \\ b_2 & \dots & \xi^p b_1 \end{pmatrix},$$

where $\{a_i, b_j\} \subseteq \{1, \dots, l\}$. Define

$$\theta_{\dagger} = \begin{pmatrix} a_1 & b_1 & a_2 & \dots & a_q & b_q \\ b_1 & a_2 & b_2 & \dots & b_q & \zeta^p a_1 \end{pmatrix}$$

Then $\theta_{\dagger}^2 = \theta_1 \theta_2$, and thus $\theta_{\dagger} \in C_{C_m \text{ wr } S_l}(\sigma)$. Further,

$$\begin{aligned} \alpha'(\theta_{\dagger}, \sigma) &= \alpha'(\theta_{\dagger}, \theta_1 \theta_2) \alpha'(\theta_{\dagger}, \theta_3 \dots, \theta_l) \text{ (by Lemma 1.6)} \\ &= -1 \text{ (by Lemma 2.6)} \end{aligned}$$

and hence σ is not α -regular. If all θ_i are of different type, $C_{C_m \text{ wr } S_l}(\sigma)$ consists of elements of the form

$$\prod_{i=1}^l (\theta_i)^{u_i} (\text{diag } \theta_i)^{v_i}, \text{ where } u_i, v_i \in \mathbf{Z}_+$$

However, $\text{diag } \theta_i$ is α -regular by (i), and thus $\alpha'(\text{diag } \theta_i, \sigma) = 1$. Finally $\alpha'(\theta_i, \sigma) = 1$ by Lemmas 1.6 and 2.6 (henceforth these references will be omitted), and thus σ is α -regular.

4. We now consider the remaining factor sets. By Theorem 2.5, $m \equiv 0 \pmod{2}$, and in this case, we can make the following definition.

DEFINITION 4.1. $\sigma \in C_m \text{ wr } S_l$ of type (a_{pq}) is *positive* if $\sum_q \sum_{p \text{ odd}} a_{pq} \equiv 0 \pmod{2}$, and *negative* otherwise.

In terms of the generators of $C_m \text{ wr } S_l$ given in §2, we see that σ is positive if and only if the number of w_j appearing in any expression for σ is even.

In the following, v_i will always denote a positive cycle, and τ_j a negative cycle.

4.2. $\alpha \in (-1, -1, -1)$. This case is only briefly sketched, since it is a straightforward generalization of the particular case $m = 2$ given in [5]. If $k = \lfloor \frac{1}{2}l \rfloor$, we define $\{N_1 \dots, N_{2k+1}\}$ as in Lemma 2.7, and put

$$R_i = (1/\sqrt{2})(N_i - N_{i+1}) \quad i = 1, \dots, l-1, \text{ and } V_j = (-1)^j N_j, \quad j = 1, \dots, l.$$

Then $\{R_i, V_j\}$ generate a projective representation T of $C_m \text{ wr } S_l$, whose factor set $\alpha \in (-1, -1, -1)$. By using an argument similar to that used by Schur in [7], it is easy to show that $\chi_r(\sigma) \neq 0$ if and only if either

- (i) $\sigma = v_1 \dots v_r \tau_1 \dots \tau_s$, where all v_i are even and all τ_j are odd, or (only when l is odd)
- (ii) $\sigma = \tau_1 \dots \tau_s$

(see [5]). All σ of the above form are α -regular by Lemma 1.5. We can however, by using the argument given in [5], prove that these are the only α -regular elements (details omitted).

4.3. $\alpha \in (1, -1, 1)$. Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, and put $R_j = A$, $j = 1, \dots, l-1$ $V_j = (-1)^j B$, $j = 1, \dots, l$. The projective representation T generated by $\{R_i, V_j\}$ has factor set $\alpha \in (1, -1, 1)$ (see Theorem 2.5).

(i) Let $\sigma = v_1 \dots v_r \tau_1 \dots \tau_s$, where σ is even, and s is even. $T(\sigma) = \pm I$, and thus all elements of this form are α -regular by 1.5.

(ii) Assume $\sigma = v_1 \dots v_r \tau_1 \dots \tau_s$ (σ even, s odd) is α -regular. Then $T(\sigma) = \pm B$ (see [6]). If v_1 is odd, $\alpha'(v_1, \sigma) = -1$, and similarly if τ_1 is odd, $\alpha'(\tau_1, \sigma) = -1$. Thus all cycles must be even. If $\text{Ty}(v_1) = \text{Ty}(v_2)$, we define v_{\dagger} as in §3(ii), and $\alpha'(v_{\dagger}, \sigma) = -1$. Similarly if $\text{Ty}(\tau_1) = \text{Ty}(\tau_2)$, $\alpha'(\tau_{\dagger}, \sigma) = -1$, and thus all cycles must be of different type. However, in this case, $C_{C_m \text{ wr } S_1}(\sigma)$ consists of elements ϕ of the form

$$\phi = \prod_{i=1}^r (\text{diag } v_i)^{a_i} (v_i)^{b_i} \prod_{j=1}^s (\text{diag } \tau_j)^{c_j} (\tau_j)^{d_j}$$

(see [3]), and as all v_i, τ_j are even cycles, $T(\phi) = \pm I$ or $\pm B$. Thus $\alpha'(\phi, \sigma) = 1$.

(iii) Assume $\sigma = v_1 \dots v_r \tau_1 \dots \tau_s$ (σ odd, s even) is α -regular. If v_1 is even, v_1 is α -regular by (i) and thus $\alpha'(\text{diag } v_1, v_1) = 1$. $\alpha'(\text{diag } v_1, v_2 \dots v_r \tau_1 \dots \tau_s) = -1$, and hence $\alpha'(\text{diag } v_1, \sigma) = -1$. If τ_1 exists, even or odd, then $\alpha'(\tau_1, \sigma) = -1$, and thus we must have $\sigma = v_1 \dots v_r$, all v_i odd. $C_{C_m \text{ wr } S_1}(\sigma)$ consists of elements ϕ of the form

$$\phi = \prod_{i=1}^r v_i^{a_i} (\text{diag } v_i)^{b_i} \theta,$$

where θ is conjugate to an element permuting the sets of symbols in cycles with similar type as they stand (see [3]), and as each cycle is of even length, θ is itself an even, positive element of $C_m \text{ wr } S_1$. Thus $T(\phi) = \pm A$ or $\pm I$, and as $T(\sigma) = \pm A$, $\alpha'(\sigma, \phi) = 1$.

(iv) Assume $\sigma = v_1 \dots v_r \tau_1 \dots \tau_s$ (σ odd, s odd) is α -regular. By a similar argument to (iii), we can show that σ must be of the form $\sigma = \tau_1 \dots \tau_s$, with all τ_i odd. As above, $C_{C_m \text{ wr } S_1}(\sigma)$ consists of elements ϕ of the form

$$\phi = \prod_{i=1}^s (\tau_i)^{a_i} (\text{diag } \tau_i)^{b_i} \theta,$$

where θ is again even and positive. Thus $T(\phi) = \pm I$ or $\pm BA$, according to whether $\sum_{i=1}^s a_i$ is even or odd. However, $T(\sigma) = \pm BA$, and thus, $\alpha'(\phi, \sigma) = 1$.

5. The α -regular classes. We now tabulate the α -regular classes of $C_m \text{ wr } S_1$ for all factor sets (γ, λ, μ) , the results in the cases $(-1, 1, -1)$, $(-1, -1, 1)$, $(1, 1, -1)$, $(1, -1, -1)$ being given without proof. The techniques used in these cases are, however, the same as those used in §3 and §4. In the actual computations, the following result was used repeatedly.

LEMMA 5.1. *If $\sigma \in C_m \text{ wr } S_1$ is (γ, λ, μ) -regular then it is $(\gamma_1, \lambda_1, \mu_1)$ -regular if and only if it is $(\gamma\gamma_1, \lambda\lambda_1, \mu\mu_1)$ -regular.*

THEOREM 5.2. *The α -regular elements σ of $C_m \text{ wr } S_1$ are the following.*

- (a) $\alpha \in (1, 1, 1)$. All classes are α -regular.
- (b) $\alpha \in (-1, 1, 1)$. $\sigma = \theta_1 \dots \theta_t$, where the θ_i are the disjoint cycles of σ , and either
 - (i) σ is even and all θ_i are even, or
 - (ii) σ is odd and all θ_i are of different type.

Henceforth m is even, and $\sigma = v_1 \dots v_r \tau_1 \dots \tau_s$, where $\{v_i\}$ are the disjoint positive cycles, and $\{\tau_i\}$ are the disjoint negative cycles of σ .

- (c) $\alpha \in (-1, -1, -1)$. *Either*
 (i) all v_i are even and all τ_j are odd, or
 (ii) $\sigma = \tau_1 \dots \tau_s$ (only when l is odd).
- (d) $\alpha \in (1, -1, 1)$. *Either*
 (i) σ is even and s is even, or
 (ii) σ is even, s is odd and all cycles are even and of different type, or
 (iii) σ is odd and all cycles are odd and positive, or
 (iv) σ is odd and all cycles odd and negative.
- (e) $\alpha \in (-1, 1, -1)$. *Either*
 (i) σ is even, s is even, all v_i are even and all τ_j odd, or
 (ii) σ is even, s is odd and all cycles are even, negative and of different type, or
 (iii) σ is odd, s is odd and all cycles are negative.
- (f) $\alpha \in (-1, -1, 1)$. *Either*
 (i) σ is even, s is even and all cycles are even, or
 (ii) σ is even, s is odd and all cycles of different type, or
 (iii) σ is odd and all cycles are odd, positive and of different type, or
 (iv) σ is odd, s is odd and all cycles are odd, negative and of different type.
- (g) $\alpha \in (1, 1, -1)$. *Either*
 (i) $\sigma = \tau_1 \dots \tau_s$, s is odd and all τ_i are of different type, or
 (ii) $\sigma = v_1 \dots v_r$ and all v_i are even.
- (h) $\alpha \in (1, -1, -1)$. *Either*
 (i) σ is even and all cycles are even and positive, or
 (ii) σ is even, s is odd and all cycles are even and negative, or
 (iii) σ is odd, s is even and all cycles are negative and of different type, or
 (iv) σ is odd, s is odd, all v_i are even, all τ_j are odd and all cycles are of different type.

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