

# The ABA Problem in Multicore Data Structures with Collaborating Operations

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**Abstract**—An increasing number of modern real-time systems and the nowadays ubiquitous multicore architectures demand the application of programming techniques for reliable and efficient concurrent synchronization. Some recently developed Compare-And-Swap (CAS) based nonblocking techniques hold the promise of delivering practical and safer concurrency. The ABA<sup>2</sup> problem is a fundamental problem to many CAS-based designs. Its significance has increased with the suggested use of CAS as a core atomic primitive for the implementation of portable lock-free algorithms. The ABA problem’s occurrence is due to the intricate and complex interactions of the application’s concurrent operations and, if not remedied, ABA can significantly corrupt the semantics of a nonblocking algorithm. The current state of the art leaves the elimination of the ABA hazards to the ingenuity of the software designer. In this work we provide the first systematic and detailed analysis of the ABA problem in lock-free Descriptor-based designs. We study the semantics of Descriptor-based lock-free data structures and propose a classification of their operations that helps us better understand the ABA problem and subsequently derive an effective ABA prevention scheme. We supplement our analysis with a statistical model of the probability for an ABA event in a concurrent system. Our ABA prevention approach outperforms by a large factor the use of the alternative CAS-based ABA prevention schemes. It offers speeds comparable to the use of the architecture-specific CAS2 instruction used for version counting. We demonstrate our ABA prevention scheme by integrating it into an advanced nonblocking data structure, a lock-free dynamically resizable array.

**Index Terms**—nonblocking synchronization, collaborating threads, multicore computing

## I. INTRODUCTION

The modern ubiquitous multi-core architectures demand the design of programming libraries and tools that allow fast and reliable concurrency. In addition, providing safe and efficient concurrent synchronization is of critical importance to the engineering of many modern real-time systems. Lock-free programming techniques [15] have been demonstrated to be effective in delivering performance gains and preventing some hazards, typically associated with the application of mutual exclusion, such as deadlock, livelock, and priority inversion [8], [3]. As explained by Herlihy [15], a concurrent object is *nonblocking* if it guarantees that *some* process in the system will make progress in a *finite* number of steps. An object that guarantees that *each* process will make progress in a

*finite* number of steps is defined as *wait-free* [15]. Lock-free algorithms exploit a set of portable atomic primitives such as the word-size Compare-and-Swap (CAS) instruction [9]. The design of nonblocking data structures poses significant challenges and their development and optimization is a current topic of research [8], [15]. To ease the implementation complexity, some approaches suggest the application of uncommon hardware primitives such as Load-Link/Store-Conditional (LL/SC) or a Double-CAS (DCAS) [5]. Most commonly, to provide a portable and practical nonblocking design, developers rely solely on the use of widely available hardware primitives such as the single-word CAS. The ABA problem [4] is a fundamental problem to many CAS-based nonblocking designs and its occurrence can seriously corrupt the semantics of a nonblocking algorithm [9], [21], [3]. While of a simple nature and derived from the application of a basic hardware primitive, the ABA problem’s occurrence is due to the intricate and complex interactions of the application’s concurrent operations. The importance of the ABA problem has been reiterated in the recent years with the application of CAS for the development of nonblocking programming techniques.

Avoiding the hazards of ABA imposes an extra challenge for a lock-free algorithm’s design and implementation. To the best of our knowledge, the literature does not offer an explicit and detailed analysis of the ABA problem, its relation to the most commonly applied nonblocking programming techniques (such as the use of Descriptors [3]) and correctness guarantees, and the possibilities for its avoidance. Thus, at the present moment of time, eliminating the hazards of ABA in a nonblocking algorithm is left to the ingenuity of the software designer. In this work we study in details and define the conditions that lead to ABA in a nonblocking Descriptor-based design. Based on our analysis, we define a generic and practical technique, called the  $\lambda\delta$  approach, for ABA avoidance for a lock-free Descriptor-based linearizable design (Section IV). We demonstrate the application of our approach by incorporating it in a complex and advanced nonblocking data structure, a lock-free dynamically resizable array (vector) [3]. The ISO C++ Standard Template Library [25] vector offers a combination of dynamic memory management and constant-time random access. We survey the literature for other known ABA prevention techniques (usually described

<sup>2</sup>ABA is not an acronym and is a shortcut for stating that a value at a shared location can change from A to B and then back to A

as a part of a nonblocking algorithm’s implementation) and study in detail three known solutions to the ABA problem (Sections II-A and II-C). Our work discusses the assumed and desired semantics of a nonblocking container (Section III). In addition, we present the first statistical model describing the practical probability for an ABA event (Section II-D). Our performance evaluation (Section V) establishes that the single-word CAS-based  $\lambda\delta$  approach is fast, efficient, and practical. The  $\lambda\delta$  approach outperforms by a large factor the application of garbage collection for the safe management of each shared location (discussed in Section II-C) and offers speed of execution comparable to the direct application of the architecture-specific CAS2 instruction used for reference counting.

## II. THE ABA PROBLEM

The Compare-And-Swap (CAS) atomic primitive (commonly known as Compare and Exchange, `CMPXCHG`, on the Intel *x86* and *Itanium* architectures [16]) is a CPU instruction that allows a processor to atomically test and modify a single-word memory location. CAS requires three arguments: a memory location ( $L_i$ ), an old value ( $A_i$ ), and a new value ( $B_i$ ). The instruction atomically exchanges the value stored at  $L_i$  with  $B_i$ , provided that  $L_i$ ’s current value equals  $A_i$ . The result indicates whether the exchange was performed. For the majority of implementations the return value is the value last read from  $L_i$  (that is  $B_i$  if the exchange succeeded). Some CAS variants, often called Compare-And-Set, have a return value of type boolean. The hardware architecture ensures the atomicity of the operation by applying a fine-grained hardware lock such as a cache or a bus lock (as is the case for IA-32 [16]). The application of a CAS-controlled speculative manipulation of a shared location ( $L_i$ ) is a fundamental programming technique in the engineering of nonblocking algorithms [15] (an example is shown in Algorithm 1).

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### Algorithm 1 CAS-based speculative manipulation of $L_i$

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1: repeat
2:   value_type  $A_i = \hat{L}_i$ 
3:   value_type  $B_i = \text{fComputeB}$ 
4: until CAS( $L_i, A_i, B_i$ ) ==  $B_i$ 

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In our pseudocode we use the symbols  $\hat{\cdot}$ ,  $\&$ , and  $\cdot$  to indicate pointer dereferencing, obtaining an object’s address, and integrated pointer dereferencing and field access. When the value stored at  $L_i$  is the target value of a CAS-based speculative manipulation, we call  $L_i$  and  $\hat{L}_i$  *control location* and *control value*, respectively. We indicate the control value’s type with the string `value_type`. The size of `value_type` must be equal or less than the maximum number of bits that a hardware CAS instruction can exchange atomically (typically the size of a single memory word). In the most common cases, `value_type` is either an integer or a pointer value. In the latter case, the implementor might reserve two extra bits per each control value and use them for implementation-specific value marking [8]. This is possible if we assume that the pointer values stored at  $L_i$  are aligned and the two low-order bits

have been cleared during the initialization. In Algorithm 1, the function `fComputeB` yields the new value,  $B_i$ , to be stored at  $L_i$ .

**Definition 1.** *The ABA problem is a false positive execution of a CAS-based speculation on a shared location  $L_i$ .*

As illustrated in Table I, ABA can occur if a process  $P_1$  is interrupted at any time after it has read the old value ( $A_i$ ) and before it attempts to execute the CAS instruction from Algorithm 1. An interrupting process ( $P_k$ ) might change the value at  $L_i$  to  $B_i$ . Afterwards, either  $P_k$  or any other process  $P_j \neq P_1$  can eventually store  $A_i$  back to  $L_i$ . When  $P_1$  resumes, its CAS loop succeeds (false positive execution) despite the fact that  $L_i$ ’s value has been meanwhile manipulated.

**Definition 2.** *A nonblocking algorithm is ABA-free when its semantics cannot be corrupted by the occurrence of ABA.*

ABA-freedom is achieved when: *a)* occurrence of ABA is harmless to the algorithm’s semantics or *b)* ABA is avoided. The former scenario is uncommon and strictly specific to the algorithm’s semantics. The latter scenario is the general case and in this work we focus on providing details of how to eliminate ABA.

### A. Known ABA Avoidance Techniques I

A general strategy for ABA avoidance is based on the fundamental guarantee that no process  $P_j$  ( $P_j \neq P_1$ ) can possibly store  $A_i$  again at location  $L_i$  (Step 3, Table I). One way to satisfy such a guarantee is to require all values stored in a given control location to be *unique*. To enforce this uniqueness invariant we can place a constraint on the user and request each value stored at  $L_i$  to be used only once (*Known Solution 1*). Enforcing this constraint can be facilitated if a programming language’s type system supports uniqueness typing [27] that forbids the use of more than a single reference to an object. We are not familiar with any programming language or library that implements uniqueness typing in a concurrent environment. For a large majority of concurrent algorithms, enforcing uniqueness typing would not be a suitable solution since their applications imply the usage of a value or reference more than once.

An alternative approach to satisfying the uniqueness invariant is to apply a *version tag* attached to each value (or the use of an `AtomicStampedReference` in Java [15]). The usage of version tags is the most commonly cited solution for ABA avoidance [9]. The approach is effective, when it is possible to apply, but suffers from a significant flaw: a single-word CAS is insufficient for the atomic update of a word-sized control value and a word-sized version tag. An effective application of a version tag [6] requires the hardware architecture to support a more complex atomic primitive that allows the atomic update of two memory locations, such as CAS2 (compare-and-swap two co-located words) or DCAS (compare-and-swap two memory locations). The availability of such atomic primitives might lead to much simpler, elegant, and efficient concurrent designs (in contrast to a CAS-based design). It is not desirable

to suggest a CAS2/DCAS-based ABA solution for a CAS-based algorithm, unless the implementor explores the optimization possibilities of the algorithm upon the availability of CAS2/DCAS. A proposed hardware implementation (entirely built into a present cache coherence protocol) of an innovative Alert-On-Update (AOU) instruction [24] has been suggested by Spear et al. to eliminate the CAS deficiency of allowing ABA. Some suggested approaches [22] split a version counter into two half-words (*Known Solution 2*): a half-word used to store the control value and a half-word used as a version tag. Such techniques lead to severe limitations on the addressable memory space and the number of possible writes into the shared location. To guarantee the uniqueness invariant of a control value of pointer type in a concurrent system with dynamic memory usage, we face an extra challenge: even if we write a pointer value no more than once in a given control location, the memory allocator might reuse the address of an already freed object ( $A_i$ ) and pose an ABA hazard. To prevent this scenario, all control values of pointer type must be guarded by a concurrent nonblocking garbage collection scheme such as Hazard Pointers [21] (that uses a list of hazard pointers per thread) or Herlihy et al.’s Pass The Buck algorithm [14] (that utilizes a dedicated thread to periodically reclaim unguarded objects). While enhancing the safety of a concurrent algorithm (when needed), the application of a complementary garbage collection mechanism might come at a significant performance cost (Section V).

### B. The Descriptor Object

Linearizability is an important correctness condition for concurrent objects [15]. A concurrent operation is linearizable if it appears to execute instantaneously in a given point of time  $\tau_{lin}$  between the time  $\tau_{inv}$  of its invocation and the time  $\tau_{end}$  of its completion. The literature often refers to  $\tau_{lin}$  as a *linearization point*. The consistency model implied by the linearizability requirement is stronger than the widely applied Lamport’s sequential consistency model [17]. According to Lamport’s definition, sequential consistency requires that the results of a concurrent execution are equivalent to the results yielded by *some* sequential execution (given the fact that the operations performed by each individual processor appear in the sequential history in the order as defined by the program). The implementations of many nonblocking data structures require the update of *two or more memory locations* in a linearizable fashion [3], [8]. The engineering of such operations (e.g. `push_back` and `resize` in a dynamically resizable array) is particularly challenging in a CAS-based design. A common programming technique applied to guarantee the linearizability requirements for such operations is the use of a *Descriptor Object* ( $\delta$  object) [3], [8]. The pseudocode in Algorithm 2 shows the generalized two-step execution of a Descriptor Object. Our definition of a Descriptor Object requires the Descriptor to store three types of information:

- (1) Global data describing the state of the shared container ( $v\delta$ ), e.g. the `size` of a dynamically resizable array [3].

- (2) A record of a pending operation on a given memory location. We call such a record requesting an update at a shared location  $L_i$  from an old value, `old_val`, to a new value, `new_val`, a *Write Descriptor* ( $\omega\delta$ ). The shorthand notation we use is  $\omega\delta @ L_i : \text{old\_val} \rightarrow \text{new\_val}$ . The fields in the Write Descriptor Object store the target location as well as the old and the new values.
- (3) A boolean value indicating whether  $\omega\delta$  contains a pending write operation that needs to be completed.

The use of a Descriptor allows an interrupting thread to help the interrupted thread complete an operation rather than wait for its completion. As shown in Algorithm 2, the technique is used to implement, using only two CAS instructions, a linearizable update of two memory locations: 1. a reference to a Descriptor Object (data type pointer to  $\delta$  stored in a location  $L_\delta$ ) and 2. an element of type `value_type` stored in  $L_i$ . In Step 1, Algorithm 2, we perform a CAS-based speculation of a shared location  $L_\delta$  that contains a reference to a Descriptor Object. The CAS-based speculation routine’s purpose is to replace an existing Descriptor Object with a new one. Step 1 executes in the following fashion:

1. we read the value of the current  $\delta$  reference stored in  $L_\delta$  (line 3),
2. if the current  $\delta$  object contains a pending operation, we need to help its completion (lines 4-5),
3. we record the current value,  $A_i$ , in location  $L_i$  (line 6) and compute the new value,  $B_i$ , to be stored in  $L_i$  (line 7),
4. a new  $\omega\delta$  object is allocated on the heap, initialized (by calling  $f_{\omega\delta}$ ), and its fields `Target`, `OldValue`, and `NewValue` are set (lines 8-11),
5. any state carrying data stored in a Descriptor Object must be computed (by calling  $f_{v\delta}$ ). Such data might be a shared element or a container’s size (line 12),
6. a new Descriptor Object is initialized containing the new Write Descriptor and the new descriptor’s data. The new descriptor’s *pending operation* flag (`WDpending`) is set to `true` (lines 13-14),
7. we attempt a swap of the old Descriptor Object with the new one (line 15). Should the CAS fail, we know that there is another process that has interrupted us and meanwhile succeeded to modify  $L_\delta$  and progress. We need to go back at the beginning of the loop and repeat all the steps. Should the CAS succeed, we proceed with Step 2 and perform the update at  $L_i$ .

The size of a Descriptor Object is larger than a memory word. Thus, we need to store and manipulate a Descriptor Object through a reference. Since the control value of Step 1 stores a pointer to a Descriptor Object, to prevent ABA at  $L_\delta$ , all references to descriptors must be memory managed by a safe nonblocking garbage collection scheme. We use the prefix  $\mu$  for all variables that require safe memory management. In Step 2 we execute the Write Descriptor, `WD`, in order to update the value at  $L_i$ . Any interrupting thread (after the completion of Step 1) detects the pending flag of  $\omega\delta$  and, should the flag’s value be still positive, it proceeds to executing the requested

update  $\omega\delta @ L_i : A_i \rightarrow B_i$ . There is no need to execute a CAS-based loop and the call to a single CAS is sufficient for the completion of  $\omega\delta$ . Should the CAS from Step 2 succeed, we have completed the two-step execution of the Descriptor Object. Should it fail, we know that there is an interrupting thread that has completed it already. A false positive execution of the CAS operation from Step 2 can lead to a *spurious write* of  $B_i$  into  $L_i$ , violate the operation’s linearizability guarantee, and corrupt the semantics of a nonblocking algorithm. In the following sections (Sections II-C, IV) we discuss a number of possible techniques that help us avoid ABA in this scenario.

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**Algorithm 2** Two-step execution of a  $\delta$  object

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1: Step 1: place a new descriptor in  $L_\delta$ 
2: repeat
3:    $\delta \mu\text{OldDesc} = \sim L_\delta$ 
4:   if  $\mu\text{OldDesc.WDpending} == \text{true}$  then
5:     execute  $\mu\text{OldDesc.WD}$ 
6:   value_type  $A_i = \sim L_i$ 
7:   value_type  $B_i = \text{fComputeB}$ 
8:    $\omega\delta \text{WD} = f_{\omega\delta}()$ 
9:    $\text{WD.Target} = L_i$ 
10:   $\text{WD.OldElement} = A_i$ 
11:   $\text{WD.NewElement} = B_i$ 
12:   $v\delta \text{DescData} = f_{v\delta}()$ 
13:   $\delta \mu\text{NewDesc} = f_\delta(\text{DescData}, \text{WD})$ 
14:   $\mu\text{NewDesc.WDpending} = \text{true}$ 
15: until  $\text{CAS}(L_\delta, \mu\text{OldDesc}, \mu\text{NewDesc}) == \mu\text{NewDesc}$ 
16:
17: Step 2: execute the write descriptor
18: if  $\mu\text{NewDesc.WDpending}$  then
19:    $\text{CAS}(\text{WD.Target}, \text{WD.OldElement}, \text{WD.NewElement}) == \text{WD.NewElement}$ 
20:    $\mu\text{NewDesc.WDpending} = \text{false}$ 

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### C. Known ABA Avoidance Techniques II

A known approach for avoiding a false positive execution of the Write Descriptor from Algorithm 2 is the application of *value semantics* for all values of type `value_type` (*Known Solution 3*). As discussed in [13] and [3], an ABA avoidance scheme based on value semantics relies on:

- a. *Extra level of indirection*: all values are stored in shared memory indirectly through pointers. Each write of a given value  $v_i$  to a shared location  $L_i$  needs to allocate on the heap a new reference to  $v_i$  ( $\eta_{v_i}$ ), store  $\eta_{v_i}$  into  $L_i$ , and finally safely delete the pointer value removed from  $L_i$ .
- b. *Nonblocking garbage collection (GC)*: all references stored in shared memory (such as  $\eta_{v_i}$ ) need to be safely managed by a nonblocking garbage collection scheme (e.g. Hazard Pointers, Pass The Buck).

As reflected in our performance test results (Section V), the usage of both an extra level of indirection as well as the heavy reliance on a nonblocking GC scheme for managing the Descriptor Objects *and* the references to `value_type` objects is very expensive with respect to the space and time complexity of a nonblocking algorithm. However, the use of value semantics is the **only known approach** for ABA avoidance in the execution of a Descriptor Object. In Section IV we present a 3-step execution approach that helps us eliminate ABA, avoid the need for an extra level of indirection, and reduce the usage of the computationally expensive GC scheme.

### D. Statistical Analysis of ABA

In this section we develop a statistical model of the probability for an ABA event. We need the following sequence of events for ABA to occur.

- S1.  $P_a$  succeeds at performing a write at  $L_\delta$ . ( $P_a$  can only be a push operation for an ABA to be possible).
- S2.  $P_a$  is interrupted.
- S3.  $P_b$  reads  $L_\delta$ . ( $P_b$  must be a tail operation to pose an ABA hazard).
- S4.  $P_b$  is interrupted.
- S5.  $P_a$  resumes and updates  $L_i$  from its  $A$  to  $B$ .
- S6.  $P_k$  (any thread different than  $P_b$ ) stores  $A$  to  $L_i$ .  $P_k$  could be a write operation only and the value of the write is  $A$ . However, we can have the same effect if  $P_k$  does not execute a write but instead we have an immediate sequence of:
  - S6A.  $P_k$  executes a pop
  - S6B.  $P_k$  or any thread different than  $P_b$  executes a push with a value  $A$ .
- S7.  $P_b$  resumes and runs into ABA.

This sequence of events involves several very low probability events:

- E1. Processor  $P_a$  needs to be interrupted right after updating the descriptor  $L_\delta$ , but before completing the push operation to change  $L_i$ .
- E2. Similarly,  $P_b$  should be interrupted right after reading  $L_\delta$ , but before it executes the descriptor stored at  $L_\delta$  or has the chance to complete its own tail operation.
- E3. E2 should happen before  $P_a$  resumes.
- E4. The write operation of E1 and the read operation of E2 should be at the same descriptor location.
- E5. When  $P_b$  resumes,  $L_i$  should be first updated by  $P_a$ , and then updated back to its original value.

An interrupt is a low probability event by itself, however what makes these events highly unlikely is that two interrupts should come between the small interval of updating a descriptor, and accessing (first a write and a read) to the global address. Note that E1 and E2 are similar events, and E3 essentially ties these events together to the same miniscule time interval. Thus if the probability of E1 is  $x$ , probability of both happening essentially at the same time is  $x^2$ .

Moreover, we need these events to write to and read from the same descriptor variable (while there is only one global Descriptor Object in the vector’s design [3], we might have several Descriptor Objects that are read by different processes and pending). Let  $D$  be the size of the descriptor set, and  $p_w(d)$  and  $p_r(d)$  be the probability of  $P_a$  writing on  $d$ , and reading from descriptor  $L - d$ , respectively. Then this probability will be

$$\sum_{d=1}^D p_w(d)p_r(d)$$

Assuming all descriptor events are equally likely to be accessed, this probability will be  $\sum_{d=1}^D \frac{1}{D^2} = \frac{1}{D}$ .

E5 requires  $P_a$  to resume before  $P_b$ , and then another processor to restore the original value to  $L_i$ . The probability of this event is inversely proportional to the size of the alphabet for  $L_i$ , and the size of the active set that can be used for writes. This means that  $L_i$  should take values from a finite, small-sized alphabet for an ABA to have a practical probability to occur, and values should be repeated. Note that the repetition

requirement also excludes algorithms where the stack is used to identifiers of tasks that are ready to execute, which is a common usage in many graph algorithms and kernels, such as breadth-first or depth-first search, finding strongly connected components, etc. Even when the same value stored multiple times on the stack in these algorithms, they happen with long time gaps. For simplicity if we assume a processor can write to any of  $N$  positions with equal likelihood, and any of the  $K$  values with equally likelihood, the probability for a write to restore the original value would be  $\frac{1}{NK}$ .

At the time of this work, we did not have statistics for quantifying this probability. But to give the reader an idea about how small this probability is, we will provide the following argument. If the probability of an interrupt is  $10^{-4}$ , that is an interrupt occurs at every  $10^4$  instructions. Probability of this interrupt coming between an descriptor update and write will be orders of magnitude smaller, say  $10^{-8}$ . Two of these interrupts happening back to back would be  $10^{-16}$ . If there are  $D = 100$  descriptor positions,  $K = 100$  different values for  $L_i$ ,  $N = 100$  is the size of the active set then the probability of this event will be  $10^{-22}$ . This means if we have a computer that executes  $10^{15}$  instructions per second, we expect an ABA event to occur at every  $10^7$  seconds, which corresponds to once in every 4 months.

### III. DESCRIPTOR-BASED OPERATIONS CLASSIFICATION

The practical implementation of a hand-crafted lock-free container is notoriously difficult. Recent research into the design of lock-free data structures includes linked-lists ([11], [20]), double-ended queues ([19], [26]), stacks [13], hash tables ([20], [23]), binary search trees [7], and a dynamically resizable array [3]. For example, a shared vector’s random access, data locality, and dynamic memory management pose serious challenges for its nonblocking implementation [3]. The use of a Descriptor Object provides the programming technique for the implementation of some of the complex nonblocking operations in a shared container, such as the `push_back`, `pop_back`, and `reserve` operations in a shared vector [3]. The use and execution of a Write Descriptor guarantees the linearizable update of two or more memory locations. Here, to better understand the interactions among these operations and the cause of ABA, we classify the operations in a nonblocking Descriptor-based design.

**Definition 3.** *An operation whose success depends on the creation and execution of a Write Descriptor is called an  $\omega\delta$ -executing operation.*

The operation `push_back` of a shared vector [3] is an example of an  $\omega\delta$ -executing operation. Such  $\omega\delta$ -executing operations have *lock-free* semantics and the progress of an individual operation is subject to the contention on the shared location  $L_i$  (under heavy contention, the body of the CAS-based loop from Step 1, Algorithm 2 might need to be re-executed). For a shared vector, operations such as `pop_back` do not need to execute a Write Descriptor Object [3]. Their

progress is dependent on the state of the global data stored in the Descriptor Object, such as the size of a container.

**Definition 4.** *An operation whose success depends on the state of the  $v\delta$  data stored in the Descriptor Object is a  $\delta$ -modifying operation.*

A  $\delta$ -modifying operation, such as `pop_back`, needs only update the shared global data (the `size` of type  $v\delta$ ) in the Descriptor Object (thus `pop_back` seeks an atomic update of only one memory location:  $L_\delta$ ). Since an  $\omega\delta$ -executing operation by definition always performs an exchange of the entire Descriptor Object, every  $\omega\delta$ -executing operation is also  $\delta$ -modifying. The semantics of a  $\delta$ -modifying operation are *lock-free* and the progress of an individual operation is determined by the interrupts by other  $\delta$ -modifying operations. An  $\omega\delta$ -executing operation is also  $\delta$ -modifying but as is the case with `pop_back`, not all  $\delta$ -modifying operations are  $\omega\delta$ -executing. Certain operations, such as the random access `read` and `write` in a vector [3], do not need to access the Descriptor Object and progress regardless of the state of the descriptor. Such operations are non- $\delta$ -modifying and have *wait-free* semantics (thus no delay if there is contention at  $L_\delta$ ).

**Definition 5.** *An operation whose success does not depend on the state of the Descriptor Object is a non- $\delta$ -modifying operation.*

#### A. Concurrent Operations

Similarly to a number of fundamental studies in nonblocking design [15], [8], we assume the following premises: each processor can execute a number of operations. This establishes a *history* of invocations and responses and defines a *real-time order* between them. An operation  $O_1$  is said to precede an operation  $O_2$  if  $O_2$ ’s invocation occurs after  $O_1$ ’s response. Operations that do not have real-time ordering are defined as *concurrent*. A *sequential history* is one where all invocations have immediate responses. A *linearizable history* is one where: *a.* all invocations and responses can be reordered so that they are equivalent to a sequential history, *b.* the yielded sequential history must correspond to the semantic requirements of the sequential definition of the object, and *c.* in case a given response precedes an invocation in the concurrent execution, then it must precede it in the derived sequential history. It is the last requirement that differentiates the consistency model implied by the definition of linearizability with Lamport’s sequential consistency model and makes linearizability stricter. When two  $\delta$ -modifying operations ( $O_{\delta_1}$  and  $O_{\delta_2}$ ) are concurrent [15], according to Algorithm 2,  $O_{\delta_1}$  precedes  $O_{\delta_2}$  in the linearization history if and only if  $O_{\delta_1}$  completes Step 1, Algorithm 2 prior to  $O_{\delta_2}$ .

**Definition 6.** *We refer to the instant of successful execution of the global Descriptor exchange at  $L_\delta$  (line 15, Algorithm 2) as  $\tau_\delta$ .*

**Definition 7.** *A point in the execution of a  $\delta$  object that determines the order of an  $\omega\delta$ -executing operation acting on*

location  $L_i$  relative to other writer operations acting on the same location  $L_i$ , is referred to as the  $\lambda\delta$ -point ( $\tau_{\lambda\delta}$ ) of a Write Descriptor.

The order of execution of the  $\lambda\delta$ -points of two concurrent  $\omega\delta$ -executing operations determines their order in the linearization history. The  $\lambda\delta$ -point does not necessarily need to coincide with the operation's linearization point,  $\tau_{lin}$ . The core rule for a linearizable operation is that it must appear to execute in a single instant of time with respect to other concurrent operations. The linearization point need not correspond to a single fixed instruction in the body of the operation's implementation and can vary depending on the interrupts the operation experiences. In contrast, the  $\lambda\delta$ -point of an  $\omega\delta$  object corresponds to a single instruction in the object's implementation. In the pseudo code in Algorithm 2  $\tau_{\lambda\delta} \equiv \tau_{\delta}$ .

Let us designate the point of time when a certain  $\delta$ -modifying operation reads the state of the Descriptor Object by  $\tau_{read\delta}$ , and the instants when a thread reads a value from and writes a value into a location  $L_i$  by  $\tau_{access_i}$  and  $\tau_{write_i}$ , respectively. Table II demonstrates the occurrence of ABA in the execution of a  $\delta$  object with two concurrent  $\delta$ -modifying operations ( $O_{\delta_1}$  and  $O_{\delta_2}$ ) and a concurrent write,  $O_i$ , to  $L_i$ . We assume that the  $\delta$  object's implementation follows Algorithm 2. The placement of the  $\lambda\delta$ -point plays a critical role for achieving ABA safety in the implementation of an  $\omega\delta$ -executing operation. As shown in Table II, at time  $\tau_{wd}$  when  $O_{\delta_2}$  executes the write descriptor,  $O_{\delta_2}$  has no way of knowing whether  $O_{\delta_1}$  has completed its update at  $L_i$  or not. Since  $O_{\delta_1}$ 's  $\lambda\delta$ -point  $\equiv \tau_{\delta}$ , the only way to know about the status of  $O_{\delta_1}$  is to read  $L_i$ . Using a single-word CAS operation prevents  $O_{\delta_2}$  from atomically checking the status of  $L_i$  and executing the update at  $L_i$ .

**Definition 8.** A concurrent execution of one or more non- $\omega\delta$ -executing  $\delta$ -modifying operations with one  $\omega\delta$ -executing operation,  $O_{\delta_1}$ , performing an update at location  $L_i$  is ABA-free if  $O_{\delta_1}$ 's  $\lambda\delta$ -point  $\equiv \tau_{access_i}$ . We refer to an  $\omega\delta$ -executing operation where its  $\lambda\delta$ -point  $\equiv \tau_{access_i}$  as a  $\lambda\delta$ -modifying operation.

Assume that in Table II the  $O_{\delta_1}$ 's  $\lambda\delta$ -point  $\equiv \tau_{access_i}$ . As shown in Table II, the ABA problem in this scenario occurs when there is a hazard of a spurious execution of  $O_{\delta_1}$ 's Write Descriptor. Having a  $\lambda\delta$ -modifying implementation of  $O_{\delta_1}$  allows any non- $\omega\delta$ -executing  $\delta$ -modifying operation such as  $O_{\delta_2}$  to check  $O_{\delta_1}$ 's progress while attempting the atomic update at  $L_i$  requested by  $O_{\delta_1}$ 's Write Descriptor. Our 3-step descriptor execution approach, discussed in Section IV, offers a solution based on Definition 8. In an implementation with two or more concurrent  $\omega\delta$ -executing operations, each  $\omega\delta$ -executing operation must be  $\lambda\delta$ -modifying in order to eliminate the hazard of a spurious execution of an  $\omega\delta$  that has been picked up by a collaborating operation. To effectively avoid the ABA hazard at  $L_i$  during a Descriptor-based linearizable update of  $L_\delta$  and  $L_i$  (see Algorithm 2), we generalize two

fundamental strategies:

- (a) Guarantee that a Write Descriptor created by  $O_{\delta_1}$ , or any other  $\omega\delta$ -executing operation, succeeds at most once. We refer to such a  $\delta$  object as a *once-execute-descriptor*. Definition 8 offers the condition leading to a solution of this type. In our example in Table II, a *once-execute-descriptor* strategy would cause the attempt to re-execute the write descriptor by  $O_{\delta_1}$  (Step 7, Table II) or by any other operation to fail. Our 3-step  $\delta$  execution approach presented in Section IV is one possible way of implementing a *once-execute-descriptor*.
- (b) Guarantee that no concurrent interleaving of operations can lead to a write of a value posing ABA hazard (such as  $B_i$  in Table II) at  $L_i$ . Relying on a methodology that employs unique values, such as *Known Solution 1*, as well as the application of Semantically Enhanced Containers [2] are approaches of this type. Requiring uniqueness typing for ABA prevention is an overkill. The guarantee we need is that no thread can restore an old value  $A_i$  in a shared location  $L_i$  while there is an alive  $\omega\delta$  object in the system requesting  $\omega\delta @ L_i : A_i \rightarrow \text{any\_value}_i$ . Modern mainstream programming languages do not provide the tools to express and enforce such a concurrent and dynamic correctness condition.

#### IV. ABA-FREE EXECUTION OF THE DESCRIPTOR OBJECT

In Algorithm 3 we suggest a design strategy for the implementation of a  $\lambda\delta$ -modifying operation. Our approach is based on a 3-step execution of the  $\delta$  object. While similar to Algorithm 2, the approach shown in Algorithm 3 differs by executing a fundamental additional step: in Step 1 we store a pointer to the new descriptor in  $L_i$  prior to the attempt to store it in  $L_\delta$  in Step 2. Since all  $\delta$  objects are memory managed, we are guaranteed that no other thread would attempt a write of the value  $\mu\text{NewDesc}$  in  $L_i$  or any other shared memory location. The operation is  $\lambda\delta$ -modifying because, after the new descriptor is placed in  $L_i$ , any interrupting writer thread accessing  $L_i$  is aware of the Write Descriptor stored at  $L_i$  and is required to complete the remaining two steps in the execution of the Write Descriptor. However, should the CAS execution in Step 2 (line 26) fail, we have to unroll the changes at  $L_i$  performed in Step 1 by restoring  $L_i$ 's old value preserved in  $\text{WD.OldElement}$  (line 20) and retry the execution of the routine (line 21). To implement Algorithm 3, we have to be able to distinguish between objects of type `value_type` and  $\delta$ . A possible solution is to require that all `value_type` variables are pointers and all pointer values stored in  $L_i$  are aligned with the two low-order bits cleared during their initialization. That way, we can use the two low-order bits for designating the type of the pointer values. Subsequently, every read must check the type of the pointer obtained from a shared memory location prior to manipulating it. Once an operation succeeds at completing Step 1, Algorithm 3, location  $L_i$  contains a pointer to a  $\delta$  object that includes both:  $L_i$ 's previous value of type `value_type` and a write descriptor WD that provides a record for the steps necessary for the

operation’s completion. Any non- $\delta$ -modifying operation, such as a random access read in a shared vector, can obtain the value of  $L_i$  (of type `value_type`) by accessing `WD.OldElement` (thus going through a *temporary* indirection) and ignore the Descriptor Object. Upon the success of Step 3, Algorithm 3, the temporary level of indirection is eliminated. Such an approach would preserve the wait-free execution of a non- $\delta$ -modifying operation. The  $\omega\delta$  data type needs to be amended to include a field `TempElement` (line 9, Algorithm 3) that records the value of the temporary  $\delta$  pointer stored in  $L_i$ . The cost of the  $\lambda\delta$  operation is 3 CAS executions to achieve the linearizable update of two shared memory locations ( $L_i$  and  $L_\delta$ ). We stress that in Algorithm 3, our assumption is that the competing operations both attempt to store an element at a location  $L_i$ . It is possible to think of a scenario where only the operation that succeeds first stores its update at  $L_i$  and then any interrupted operation would have to change its update location to  $L_{i+1}$  or any subsequent location [3]. In this case, we suggest that in Step 1, we perform an additional step: we would have to check whether  $L_i$  contains a pending Descriptor object and if it does, help it complete. That way, we would prevent concurrent interleaving that might possibly skip the removal of the temporary level of indirection at  $L_i$  and cause an extra use of the costly lock-free garbage collection scheme. Such an implementation is similar to the execution of Harris et al.’s *MCAS* algorithm [12]. In any scenario, just like our  $\lambda\delta$ -modifying approach, for an *MCAS* update of  $L_\delta$  and  $L_i$ , the cost of Harris et al.’s *MCAS* is at least 3 executions of the single-word CAS instruction. Harris et al.’s work on *MCAS* [12] brings forward a significant contribution in the design of lock-free algorithms, however, it lacks any analysis of the hazards of ABA and the way the authors manage to avoid it.

## V. PERFORMANCE EVALUATION

To evaluate the performance of the ABA-free programming techniques discussed in this work, we incorporated the presented ABA elimination approaches in the implementation of a nonblocking dynamically resizable array [3]. Our test results indicate that the  $\lambda\delta$  approach offers ABA prevention with performance comparable to the use of the platform-specific CAS2 instruction to implement version counting. This finding is of particular value to the engineering of some embedded real-time systems where the hardware does not support complex atomic primitives such as CAS2 [18]. We ran performance tests on an Intel IA-32 SMP machine with two 1.83GHz processor cores with 512 MB shared memory and 2 MB L2 shared cache running the MAC 10.5.6 operating system. In our performance analysis we compare:

- (1)  *$\lambda\delta$  approach*: the implementation of a vector with a  $\lambda\delta$ -modifying `push_back` and a  $\delta$ -modifying `pop_back`. In this scenario the cost of `push_back` is 3 single-word CAS operations and `pop_back`’s cost is one single-word CAS instruction. Table VII offers an overview of the shared vector’s operations’ relative cost in terms of number and type of atomic instructions invoked per operation.

---

### Algorithm 3 Implementing a $\lambda\delta$ -modifying operation through a three-step execution of a $\delta$ object

---

```

1: Step 1: place a new descriptor in  $L_i$ 
2: value_type  $B_i = f_{\text{ComputeB}}$ 
3: value_type  $A_i$ 
4:  $\omega\delta$  WD =  $f_{\omega\delta}()$ 
5: WD.Target =  $L_i$ 
6: WD.NewElement =  $B_i$ 
7:  $v\delta$  DescData =  $f_{v\delta}()$ 
8:  $\delta$   $\mu\text{NewDesc} = f_\delta(\text{DescData}, \text{WD})$ 
9: WD.TempElement =  $\&\mu\text{NewDesc}$ 
10:  $\mu\text{NewDesc.WDpending} = \text{true}$ 
11: repeat
12:    $A_i = \hat{L}_i$ 
13:   WD.OldElement =  $A_i$ 
14: until CAS( $L_i, A_i, \mu\text{NewDesc}$ ) ==  $\mu\text{NewDesc}$ 
15:
16: Step 2: place the new descriptor in  $L_\delta$ 
17: bool unroll = false
18: repeat
19:   if unroll then
20:     CAS(WD.Target,  $\mu\text{NewDesc}$ , WD.OldElement)
21:   goto 3
22:    $\delta$   $\mu\text{OldDesc} = \hat{L}_\delta$ 
23:   if  $\mu\text{OldDesc.WDpending} == \text{true}$  then
24:     execute  $\mu\text{OldDesc.WD}$ 
25:     unroll = true
26: until CAS( $L_\delta, \mu\text{OldDesc}, \mu\text{NewDesc}$ ) ==  $\mu\text{NewDesc}$ 
27:
28: Step 3: execute the Write Descriptor
29: if  $\mu\text{NewDesc.WDpending}$  then
30:   CAS(WD.Target, WD.TempElement, WD.NewElement) == WD.NewElement
31:    $\mu\text{NewDesc.WDpending} = \text{false}$ 

```

---

- (2) *All-GC approach*: the application of Known Solution 3 (Section II-C), namely the use of an extra level of indirection and memory management for each element. Because of its performance and availability, we have chosen to implement and apply Herlihy et al.’s Pass The Buck algorithm [14]. In addition, we use Pass The Buck to protect the Descriptor Objects for all of the tested approaches.
- (3) *CAS2-based approach*: the application of CAS2 for maintaining a reference counter for each element. A CAS2-based version counting implementation is easy to apply to almost any pre-existent CAS-based algorithm. While a CAS2-based solution is not portable, we believe that the approach is applicable for a large number of modern architectures. For this reason, it is included in our performance evaluation. In the performance tests, we apply CAS2 (and version counting) for updates at the shared memory locations at  $L_i$  and a single-word CAS to update the Descriptor Object at  $L_\delta$ .

Similarly to the evaluation of other lock-free algorithms [7], we designed our experiments by generating a workload of the various operations. We varied the number of threads, starting from 1 and exponentially increased their number to 64. Each thread executed 500,000 lock-free operations on the shared container. We measured the execution time (in seconds) that all threads needed to complete. Each iteration of every thread executed an operation with a certain probability (`push_back` (+), `pop_back` (-), random access write (w), random access read (r)). We show the performance graph for a distribution of +:40%, -:40%, w:10%, r:10% on

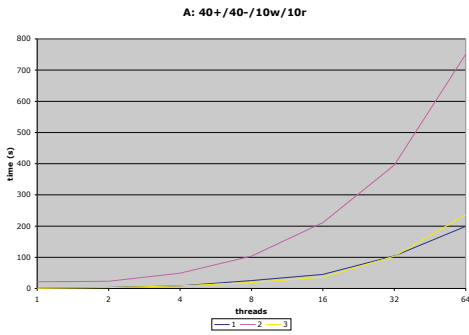


Fig. 1. Performance Results A

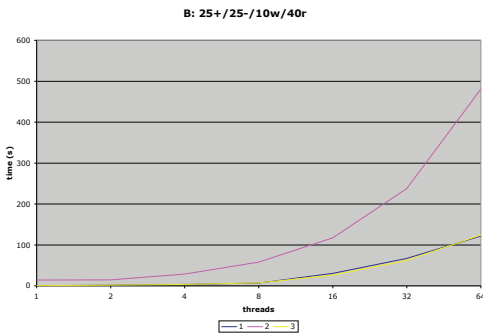


Fig. 2. Performance Results B

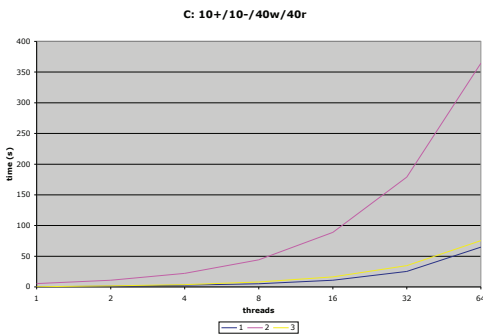


Fig. 3. Performance Results C

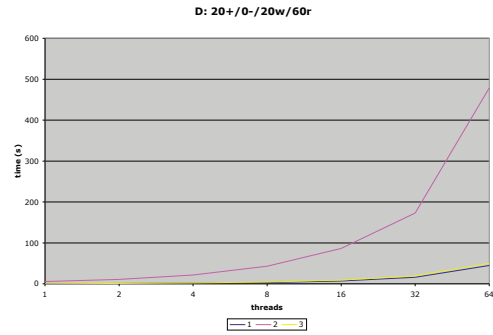


Fig. 4. Performance Results D

Figure 1. Figure 2 demonstrates the performance results with less contention at the vector's tail,  $+25\%$ ,  $-25\%$ ,  $w:10\%$ ,  $r:40\%$ . Figure 3 illustrates the test results with a distribution containing predominantly random access read and write operations,  $+10\%$ ,  $-10\%$ ,  $w:40\%$ ,  $r:40\%$ . Figure 4 reflects our performance evaluation on a vector's use with mostly random access read operations:  $+20\%$ ,  $-0\%$ ,  $w:20\%$ ,  $r:60\%$ , a scenario often referred to as the most common real-world use of a shared container [7]. The number of threads is plotted along the  $x$ -axis, while the time needed to complete all operations is shown along the  $y$ -axis. According to the performance results, compared to the *All-GC* approach, the  $\lambda\delta$  approach delivers consistent performance gains in all possible operation mixes by a large factor, a factor of at least 3.5 in the cases with less contention at the tail and a factor of 10 or more when there is a high concentration of tail operations. These observations come as a confirmation to our expectations that introducing an extra level of indirection and the necessity to memory manage each individual element with PTB (or an alternative memory management scheme) to avoid ABA comes with a pricy performance overhead. The  $\lambda\delta$  approach offers an alternative by introducing the notion of a  $\lambda\delta$ -point and enforces it through a 3-step execution of the  $\delta$  object. The application of version counting based on the architecture-specific CAS2 operation is the most commonly cited approach for ABA prevention in the literature. Our performance evaluation shows that the  $\lambda\delta$  approach delivers performance comparable to the use of CAS2-based version counting. CAS2 is a complex atomic primitive and its application comes with a higher cost when compared to the application of atomic write or a single-word CAS. In the performance tests we executed, we notice that in the scenarios where random access write is invoked more frequently (Figures 3 and 4), the performance of the CAS2 version counting approach suffers a performance penalty and runs slower than the  $\lambda\delta$  approach by about 12% to 20%. According to our performance evaluation, the  $\lambda\delta$  approach is a systematic, effective, portable, and generic solution for ABA avoidance for Descriptor-based nonblocking designs. The  $\lambda\delta$



scheme does not induce a performance penalty when compared to the architecture-specific application of CAS2-based version counting and offers a considerable performance gain when compared to the use of *All-GC*.

## VI. CONCLUSION AND FUTURE WORK

In this work we studied the ABA problem and the conditions leading to its occurrence in a Descriptor-based lock-free linearizable design. We offered a systematic and generic solution, called the  $\lambda\delta$  approach, that outperforms by a significant factor the use of garbage collection for the safe management of each shared location and offers speed of execution comparable to the application of the architecture-specific CAS2 instruction used for version counting. Having a practical alternative to the application of the architecture-specific CAS2 is of particular importance to the design of some modern embedded systems [18]. We defined a condition for ABA-free synchronization that allows us to reason about the ABA safety of a lock-free algorithm. We presented a practical, generic, and portable implementation of the  $\lambda\delta$  approach and evaluated it by integrating the  $\lambda\delta$  technique into a nonblocking shared vector. The literature *does not offer a detailed analysis of the ABA problem* and the general techniques for its avoidance in a lock-free linearizable design. At the present moment of time, the challenges of eliminating ABA are left to the ingenuity of the software designer. The goal of our work is to deliver a guide for ABA comprehension and prevention in Descriptor-based lock-free linearizable algorithms. For the practical application of Descriptor-based nonblocking techniques in real-time systems, it is important to study the service-time bounds of the operations within the context of the Descriptor's CAS-based retry loop. Anderson et al. [1] present a fundamental approach for such formal timing analysis. In our future work we plan to utilize a model-checker [10] to express the  $\lambda\delta$  condition as well as apply Anderson et al.'s [1] approach to derive the timing guarantees for our ABA prevention approach.

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## VII. APPENDIX: TABLES

<i>Step</i>	<i>Action</i>
Step 1	$P_1$ reads $A_i$ from $L_i$
Step 2	$P_k$ interrupts $P_1$ ; $P_k$ stores the value $B_i$ into $L_i$
Step 3	$P_j$ stores the value $A_i$ into $L_i$
Step 4	$P_1$ resumes; $P_1$ executes a false positive CAS

TABLE I  
ABA AT  $L_i$

<i>Step</i>	<i>Action</i>
Step 1	$O_{\delta_1}: \tau_{read_\delta}$
Step 2	$O_{\delta_1}: \tau_{access_i}$
Step 3	$O_{\delta_1}: \tau_\delta$
Step 4	$O_{\delta_2}: \tau_{read_\delta}$
Step 5	$O_{\delta_1}: \tau_{wd}$
Step 6	$O_i: \tau_{write_i}$
Step 7	$O_{\delta_2}: \tau_{wd}$

TABLE II  
ABA OCCURRENCE IN THE EXECUTION OF A DESCRIPTOR OBJECT

ABA prevention approach / operation	push_back	pop_back	read_j	write_j
1. $\lambda\delta$ approach	3 CAS	1 CAS	atomic read	atomic write
2. All-GC approach	2 CAS + GC	1 CAS + GC	atomic read	atomic write + GC
3. CAS2-based approach	1 CAS2 + 1 CAS	1 CAS	atomic read	1 CAS2

TABLE III  
A SHARED VECTOR'S OPERATIONS COST (BEST CASE SCENARIO)

operation	<i>push</i>	<i>pop</i>	<i>read</i>	<i>write</i>
<i>push</i>	<i>ABA free</i>	<i>ABA</i>	<i>ABA free</i>	<i>ABA</i>
<i>pop</i>	<i>ABA free</i>	<i>ABA</i>	<i>ABA free</i>	<i>ABA free</i>
<i>read</i>	<i>ABA free</i>	<i>ABA free</i>	<i>ABA free</i>	<i>ABA free</i>
<i>write</i>	<i>ABA</i>	<i>ABA free</i>	<i>ABA free</i>	<i>ABA</i>

TABLE IV  
ABA-FREE AND ABA-PRONE INTERLEAVING OF TWO  
CONCURRENT OPERATIONS