Mon. Not. R. astr. Soc. (1974) **169**, 447–470.

THE ACCRETION MODEL OF DWARF NOVAE WITH APPLICATION TO Z CHAMAELEONTIS

G. T. Bath, W. D. Evans, J. Papaloizou and J. E. Pringle

(Received 1974 August 12)

SUMMARY

The processes that arise as a result of mass transfer between the red and blue components in dwarf nova type binary systems are examined. It is shown that the outbursts themselves may be accounted for by unstable mass transfer by the red component. The consequent behaviour of the impact hot spot and of the accretion disc surrounding the blue component is shown in the steady state approximation to be in detailed agreement with the observed behaviour of the eclipsing system Z Cha, during both the quiescent normal state, and the outburst state. The cause of such a red component instability is considered together with effects due to time dependent evolution of accretion discs. Problems associated with a nuclear burning instability as an alternative explanation of dwarf nova outbursts are examined. In particular it is shown that the disc behaviour observed in Z Cha is difficult to achieve through such a mechanism. It is noted that many types of variable and unstable stars may involve similar processes to those discussed in this paper.

1. INTRODUCTION

In the past two decades our knowledge of dwarf novae, and the related nova-like variables and old novae, has increased considerably. The model proposed by Kraft (1962) of an interacting binary system in which the red component fills its Roche lobe and transfers mass on to an accreting blue component seems to be valid for all these classes of active stars. Crawford & Kraft (1956) first suggested such a model for the nova-like variable AE Aqr and later Kraft (1962, 1963) showed conclusively that such a model is consistent with observations of a large number of dwarf novae and old novae. The position has been well summarized in review articles by Kraft (1963), Mumford (1967) and Smak (1971a).

There is, as yet, no consensus of opinion as to the cause of the nova and dwarf nova outbursts. Two models have been developed: one assigns the outbursts to unstable nuclear burning on the surface of the blue component and the other to phases of unstable mass transfer from the red component. In this paper we restrict our attention for the most part to dwarf novae and develop the second approach along the lines suggested by Bath (1973); that is, that the observed properties of the outbursts can be accounted for by quasiperiodic unstable mass outflow from the red star and the consequent sudden liberation of accretion energy in the disc surrounding the blue component. (This is essentially the same as alternative (b) for the outbursts of the dwarf nova U Geminorum put forward by Smak (1971b) and more generally Smak (1971a).) We note that all models of dwarf novae require mass transfer from the secondary, and theoretical studies of such a flow, discussed in Section 5, demonstrate that it can display the required quasiperiodic instabilities. We indicate in Section 7 our reservations with regard to the present state of theories of nuclear burning on the surface of white dwarfs.

We examine in detail the observations of the dwarf nova Z Chamaeleontis by Warner (1974a). These observations are of great importance in that for the first time we can examine a system in which the dominant light source during outburst is eclipsed, enabling us to obtain a clear indication of its size and structure. We show how these observations of Z Cha, both during its normal or 'quiescent' state and during outburst can be satisfactorily accounted for by accretion alone. From Warner's observations of Z Cha it is clear that during outburst the disc surrounding the primary increases dramatically in brightness (and possibly size). This is a natural consequence of a sudden onset of enhanced mass transfer. Warner, however, suggests that this increase in brightness is instead due to a strong shock, generated by explosive nuclear burning on the white dwarf, propagating outwards through the disc. We examine this alternative in Section 7 and conclude that it is not clear that such a shock can propagate in a centrifugally supported disc.

We show on the other hand that for the expected quiescent and outburst mass transfer rates through an accretion disc both the quantitative values of the disc luminosity, and the qualitative luminosity distribution across the disc are in agreement with the observed behaviour of Z Cha. We demonstrate further that the impact 'hot spot' situated at the outer edge of the disc contributes a luminosity to the system which is in agreement with such an accretion model.

In the next section, Section 2, we outline in more detail the framework of the model we adopt for Z Cha. In Section 3 we discuss the theoretical consequences of such a model and compare these with the observed behaviour in Section 4. In Section 5 we outline why instabilities are to be expected in the mass transfer rate and what form they take. In Section 6 we consider the dynamical effects of variable mass transfer on the accretion disc around the primary. In Section 7 we outline our reservations about some aspects of other models, in particular with regard to explosive nuclear burning, shock propagation in an accretion disc and observed transient optical periodicities. In Section 7 we draw conclusions and discuss the implications of this model for other dwarf novae and the classical novae. We stress again that although we discuss Z Cha in particular, we consider much of what is said to be relevant to other dwarf novae and nova-like variables.

2. WORKING MODEL

The dwarf nova, Z Cha, was found by Mumford (1969) to be an eclipsing binary with a period of 107 min. Warner's recent observations (1974a) show that it has deep primary eclipses during both quiescence and outburst. A shoulder is clearly visible prior to primary eclipse in the 'normal' state. During the 'outburst' state the intensity and phase of the shoulder is variable. At quiescence the time between first and second contact indicates that the eclipsed component has dimensions of $\sim 10^9$ cm. During outburst the primary eclipse remains centred at the same phase, but changes to a broader V-shaped eclipse. This can be interpreted (Warner 1974a) as the eclipse of a bright disc, radius $1\text{–}2 \times 10^{10}$ cm centred on the primary, viewed at an angle of inclination $80° \lesssim i \lesssim 90°$.

As a working model we take the following parameters for the system. We assume the primary to have mass $M_1 = 1\, M_\odot$ and radius $R_1 = 10^9$ cm. We take the secondary to have a mass $M_2 = 0{\cdot}20\, M_\odot$. With a binary period of 107 min, this requires a binary separation, $A = 5{\cdot}5 \times 10^{10}$ cm. We have chosen this mass so that the secondary fills its Roche lobe if it is on the main sequence. For this reason our system differs from that adopted by Warner, having a somewhat larger mass ratio, q. It will, however, be apparent that our general model and conclusions are relatively independent of the adopted value of q.

We assume that the dynamics of the mass transfer process during both outburst and quiescent states can be described by the following model. Mass outflow from the secondary through the Lagrangian point forms a stream which is initially in free-fall towards the primary. Because of its angular momentum relative to the primary this stream cannot be accreted directly by the primary. Instead it must form an accretion disc, within which, as a consequence of differential rotation and some form of viscosity, angular momentum is transported outwards and mass flows inwards (Prendergast & Burbidge 1968; Lynden-Bell 1969). For simplicity we shall assume that both during outburst and during the quiescent state, a steady state exists in the disc, the only difference being that during outburst the mass transfer rate is greatly increased. During quiescence the steady approximation is probably valid and we discuss the consequence of time-dependent disc behaviour during outburst later (Section 6).

A small proportion of the accretion energy is released where the stream of transferred material is incident upon the accretion disc around the primary. Following Smak (1971a), Warner & Nather (1971) we associate this with the position of the hot spot which gives rise to the shoulder in the binary light curve. The precise position of this hot spot depends on the detailed gas dynamics in the binary system (see Section 3(c)) and for the sake of argument we take its distance from the primary in Z Cha to be equal to the outer radius of the disc, R_d, at $1 \cdot 4 \times 10^{10}$ cm. It will become apparent in what way this assumption affects any of our conclusions as the development of our model proceeds.

3. THEORETICAL CONSIDERATIONS

We now examine the optical appearance of the accretion disc and the hot spot from a theoretical point of view. In particular we address ourselves to the question of why the hot spot should appear so prominent in a number of systems, including Z Cha, whereas most of the accretion energy is liberated near the surface of the primary where the gravitational potential well is deepest. We examine the difference in optical appearance of the disc between the quiescent and outburst states, emphasizing the importance of bolometric corrections. We also consider the apparent luminosity distribution across the disc and the resulting shapes of primary eclipse for a system like Z Cha.

(a) *The integrated optical luminosity of the accretion disc*

As we mentioned above, we take the disc to be in a steady state with a steady mass flux, F, accreting through it on to the primary. For the parameters we consider, the accretion disc will be optically thick and radiation pressure is not important. In this case we may assume that each element of the disc radiates like a blackbody.* These assumptions completely determine the radial temperature dependence of the disc. The power per unit area radiated by the disc on one side is given as a function of R by (Shakura & Sunyaev 1973).

* In fact we should really take into account the fact that emission from a collisionally ionized gas drops by many orders of magnitude below the temperature of $\sim 10^4$ K at which hydrogen recombines. Thus, during quiescence, the outer parts of the disc may have an electron temperature of $\sim 10^4$ K, with the dominant emission process being due to line

$$P(R) = \frac{3FGM_1}{8\pi R^3}\left[1-\left(\frac{R_1}{R}\right)^{1/2}\right].$$

The temperature in the disc as a function of radius is then given by

$$T(R) = T_* x^{-3/4}(1-x^{-1/2})^{1/4}$$

where $x = R/R_1$ (for the disc in Z Cha we take x to lie between 1 and 14), and

$$T_* = 4\cdot1 \times 10^4 \, F_{16}^{1/4} \, K$$

where $F_{16} = F/10^{16}$ g s^{-1}. We note the maximum temperature in the disc occurs at $x = 49/36$ and is $T_{max} = 0\cdot488 \; T_*$. The total luminosity from the disc is given (with an error of order R_1/R_s) by

$$L_d = \frac{FGM_1}{2R_1}$$

$$= 6\cdot7 \times 10^{32} \, F_{16} \text{ erg s}^{-1}$$

for the parameters we have chosen for Z Cha.

A comparable amount of luminosity is generated in the boundary layer between the accretion disc and the primary (see Lynden-Bell & Pringle 1974). Lynden-Bell & Pringle estimate that the temperature of this boundary layer is at least $2\cdot40 \; T_*$ and could be greater if the radiation emitted there is not thermalized. For the model parameters we consider, radiation from the boundary layer is emitted predominantly in the far ultraviolet (or possibly soft X-ray) region of the spectrum and does not contribute significantly to the optical light of the system (except possibly by a reflection effect off the accretion disc and/or the secondary).

We may now calculate the fraction of radiant energy from the disc, ϵ_d, emitted redward of 3500 Å (i.e. 'in the optical'). The proportion $\epsilon_1(T)$ of radiant energy emitted redward of 3500 Å was obtained from tables of the Planck function for the temperature range corresponding to different radii in the disc. The efficiency of the whole disc, ϵ_d, was then calculated by integrating numerically the following expression:

$$\epsilon_d = \frac{\int_{R_1}^{R_d} P(R)\, \epsilon_1(T(R))\, 2\pi \, RdR}{\int_{R_1}^{R_d} P(R)\, 2\pi \, RdR}.$$

This is shown as a function of F_{16} in Table I. We then determine the optical luminosity of the disc as

$$L_d^{opt} = \epsilon_d L_d$$

which is also shown in Table I. We note that similar calculations must be carried out to obtain the bolometric correction for observations in, for example, the U, B or V bands. The efficiency, ϵ_d, decreases with increasing mass flux since for higher accretion rates T_* increases and so a greater fraction of the radiation is emitted at wavelengths shorter than 3500 Å.

emission, and with the ionization levels adjusting accordingly (see Pringle & Rees 1972). For simplicity we can ignore this for the present without jeopardizing the accuracy to which we are working. We note that this may lead us to slightly overestimate the visual luminosity of the disc, and that this outer part of the disc can give rise to (broad) emission lines in the spectrum.

TABLE I

Log F_{16}	T^* (K)	L_d (erg s^{-1})	ϵ_d	L_d^{opt} (erg s^{-1})	T_s (K)	L_s (erg s^{-1})	ϵ_s	L_s^{opt} (erg s^{-1})	0·5 cos 80° $\times L_d^{opt}$ (erg s^{-1})
−2	$1·3\times10^4$	$5·5\times10^{30}$	0·95	$5·2\times10^{30}$	$8·4\times10^3$	$4·8\times10^{29}$	0·73	$3·5\times10^{29}$	$4·5\times10^{29}$
−1	$2·3\times10^4$	$5·5\times10^{31}$	0·76	$4·2\times10^{31}$	$1·4\times10^4$	$4·8\times10^{30}$	0·37	$1·8\times10^{30}$	$3·6\times10^{30}$
0	$4·1\times10^4$	$5·5\times10^{32}$	0·45	$2·5\times10^{32}$	$2·3\times10^4$	$4·8\times10^{31}$	0·14	$6·8\times10^{30}$	$2·2\times10^{31}$
1	$7·2\times10^4$	$5·5\times10^{33}$	0·18	$1·0\times10^{33}$	$3·9\times10^4$	$4·8\times10^{32}$	0·040	$1·9\times10^{31}$	$8·7\times10^{31}$
2	$1·3\times10^5$	$5·5\times10^{34}$	0·056	$3·1\times10^{33}$	$6·5\times10^4$	$4·8\times10^{33}$	0·010	$4·9\times10^{31}$	$2·7\times10^{32}$
3	$2·3\times10^5$	$5·5\times10^{35}$	0·014	$7·4\times10^{33}$	$1·1\times10^5$	$4·8\times10^{34}$	0·0023	$1·1\times10^{32}$	$6·4\times10^{32}$
4	$4·1\times10^5$	$5·5\times10^{36}$	0·0027	$1·5\times10^{34}$	$1·8\times10^5$	$4·8\times10^{35}$	0·00056	$2·7\times10^{32}$	$1·3\times10^{33}$

From Table I it is clear that the total *optical* emission by the disc is never greater than $\sim 10^{34}$ erg s^{-1} for accretion on to a white dwarf at mass transfer rates $\lesssim 10^{20}$ g s^{-1}. Only by increasing the disc thickness dramatically, leading to absorption of radiation at wavelengths shorter than ~ 3500 Å, could significantly larger optical luminosities be achieved via a reflection effect.

(b) *The surface brightness distribution in the disc*

We now investigate the optical surface brightness across the accretion disc. We consider two extreme cases:

(i) All the radiation from the disc is emitted in the optical and the apparent luminosity is proportional to the mass transfer rate F; and

(ii) Only the Rayleigh–Jeans part of the Planck spectrum is observed and the apparent luminosity is proportional to $F^{1/4}$.

That is, we assume that the optical luminosity is proportional to either T^4 or T. We estimate that for $F_{16} \gtrsim 10^2$ the disc will radiate roughly as case (ii) since the temperatures in the disc are then $\gtrsim 2 \times 10^4$ K. At $F_{16} \lesssim 10^{-1}$ the disc will radiate approximately as case (i). At fluxes between these a more detailed calculation is necessary to determine the precise brightness profile. We ignore any intrinsic luminosity contribution of the primary and also obscuration by the primary of the central region of the disc which occurs in systems with large angles of inclination.

We simulate the eclipse by drawing an opaque rectangular strip across the disc. To determine the rate of change of luminosity during such an eclipse, we have evaluated the luminosity $I(x)$ arising from an infinitesimal strip perpendicular to a diameter of the disc ($I(x)$ has dimensions of energy per unit time per unit length). If the disc has radius R_d and is centred at the origin of a set of Cartesian axes then the luminosity from a strip x = constant is proportional to:

$$I(x) = \int_0^{\sqrt{R_d^2 - x^2}} R^{-3n} \left(1 - \left(\frac{R_1}{R}\right)^{1/2}\right)^n dy$$

where $R^2 = x^2 + y^2$, and, for case (i) $n = 1$ and for case (ii) $n = \frac{1}{4}$. The function $I(x)$ for each case is shown in Fig. 1. These graphs give a clear indication of the different eclipse shapes to be expected in each case. In case (i) the light is concentrated towards the centre of the disc. This would give rise to the sharp square eclipses of the kind which are seen in Z Cha during its quiescent state. In case (ii) the light distribution is much flatter and the disc appears to have an almost uniform brightness distribution. If the eclipsing object is about the same size as the emitting disc, this would give rise to V-shaped eclipses similar to those observed in Z Cha during outburst. We return to a detailed comparison later (Section 4).

(c) *The hot spot*

Gas flow in a binary system is a complex subject which we shall not attempt to treat in any quantitive way here. There are however a few qualitative remarks which seem worth making about steady mass transfer flow in which an accretion disc is involved. Near the primary the gas dynamics is unaffected by the presence of the other star and a steady circular accretion disc can be set up. Within the disc there is a net flow of angular momentum outwards (allowing a mass flux inwards). Thus at the outer edge of the disc, there must be, as well as an inward mass flux, an outward flux of angular momentum. The stream of material from the Lagrangian point provides an inward mass flux and also an excess contribution of angular

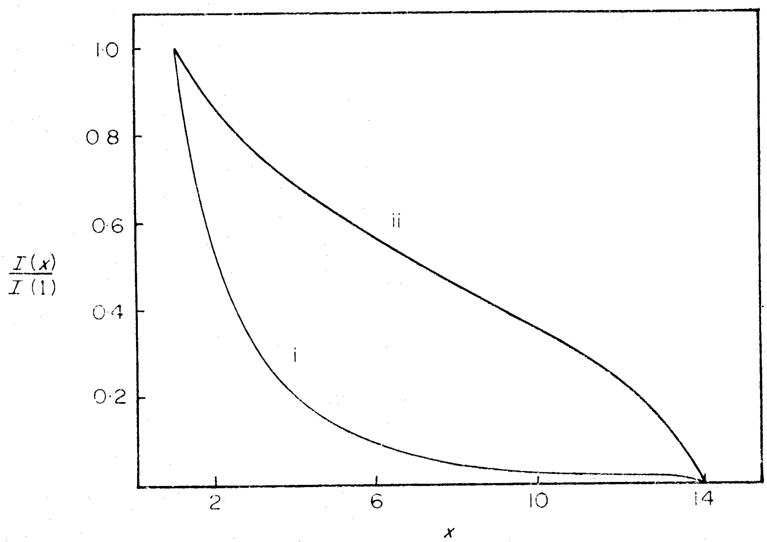

FIG. 1. *Brightness distribution across the disc. I(x) is the luminosity emitted in an infinitesimal strip perpendicular to a diameter of the disc, at a distance, x, from the centre. Curve i shows the luminosity distribution for the case in which all the energy is radiated in the optical regime. Curve ii shows the extreme limit in which all the optical luminosity is radiated in the Rayleigh–Jeans part of the Planck spectrum.*

momentum with respect to the primary accreting star. The collision of this gas stream with the outer edge of the disc produces dissipation of kinetic energy through shock waves and radiation by the hot spot. To provide the necessary associated outward flux of angular momentum there must be an outward flow of material from the hot spot region. We estimate (*cf.* Prendergast & Burbidge 1968) that about a half of the incident stream must be spun out from the accretion disc. It seems that this ejected material will be in the form of a spray or a splash rather than in an ordered stream and where this material ends up is by no means clear. Most of it will probably return to the secondary, but it is possible that some (or all) of it could be lost from the system entirely. We note that the effect that mass transfer has on the binary period and on the rotation of the secondary depends critically on what happens to this spray of material, or more importantly to the angular momentum that it carries. It is therefore not clear how observed binary orbital period changes and mass transfer rates are linked.

The position of the hot spot in the binary system is even more uncertain. The stream of material from the Lagrangian point may be affected by the spray of material ejected from the hot spot region*—thus the usual assumption that in a steady state the stream follows the path of a test particle may be invalid. In addition any slight non-corotation of the outer layers of the secondary with the orbital period would change the initial direction and consequent path of the stream. The distance of the hot spot from the primary depends on how deep this incoming stream can penetrate the accretion disc. That is, it depends, amongst other properties, on the surface density of the disc, which for a given mass flux F is inversely proportional to the effective kinematic viscosity in the disc. It is well known that

* We note also that the presence of such a large amount of material, in or near the plane of the binary system almost certainly gives rise to the observed low energy cut offs in the spectra of those binary X-ray sources which are observed to display eclipses and therefore have $i \sim 90°$.

the effective viscosity is the most uncertain parameter in all theories of accretion discs.

It is therefore not possible to deduce the position of the hot spot from a general energy or angular momentum principle. The most satisfactory approach so far to this problem has been the full two-dimensional hydrodynamic treatment of mass-transfer flow by Prendergast & Taam (1974).

Bearing in mind these problems we crudely estimate, on the basis of a calculation of free particle orbits and the observed phase of the shoulder in the quiescent state, the radius of the hot spot, R_s, to be $\sim 1{\cdot}4 \times 10^{10}$ cm. The total luminosity of the hot spot may be written

$$L_s = f\frac{FGM_1}{2R_s}$$

where f is an 'uncertainty factor'. If $f = 1$, then this luminosity represents the power released by a mass flux F falling from infinite radius into a circular orbit about the primary at radius R_s. There are a number of factors which influence the exact value of f relevant in any system.

(i) The incoming stream does not fall from infinity but only from the Lagrangian point. This has the effect of reducing f, especially for large R_s.

(ii) F is the accretion rate on to the primary. As we remarked above, the actual mass flux in the stream must be greater than this, possibly by a factor ~ 2.

(iii) The viscous torques which transport angular momentum outwards in the disc must also transport energy outwards. Thus energy, as well as angular momentum, must be lost from the disc at the outer edge.

For the purpose of calculations we assume that f is of order unity. We therefore put the energy emitted in the hot spot equal to $FGM_1/(2R_s)$.

We assume that there is sufficient material in the radiating region for the radiation to be thermalized. We can then determine the temperature of the emitting region for a given spot surface area. We consider the case where the hot spot radiates as a two-dimensional strip along the outside rim of the disc. We refer to this as the completely anisotropic case as the spot then contributes maximally to the shoulder in the light curve. It is of course possible that the geometry is different. For instance if the spot radiates vertically as a two-dimensional strip on the surface of the disc, no shoulder will be observed. We assume the semi-thickness of the hot spot in the direction perpendicular to the plane of the disc, h, to be given by

$$h = \frac{c_s}{v_c} R_s$$

where c_s is the isothermal sound speed of the matter in the spot and v_c the circular velocity. The size of the radiating region is therefore a function of its temperature. We shall define the area of the spot to be $Q(2h)^2$. We then find for the temperature of the spot, T_s

$$T_s = 3{\cdot}62 \times 10^4 F_{16}^{1/5} Q^{-1/5} \left(\frac{R_s}{1{\cdot}4 \times 10^{10}}\right)^{-4/5} \left(\frac{M_1}{M_\odot}\right)^{2/5} \text{K}.$$

For a constant azimuthal extent of the spot of 20°, as suggested by the size of the shoulder in Warner's observations,

$$Q = 8{\cdot}66 \left(\frac{M_1}{M_\odot}\right)^{1/3} \left(\frac{R_s}{1{\cdot}4 \times 10^{10}}\right)^{-1/9} F_{16}^{-1/9}.$$

In Table I we give values of T_s for various values of F_{16}. In addition we note that

$$h = 3{\cdot}50 \times 10^8 F_{16}^{1/10} Q^{-1/10} \left(\frac{R_s}{1{\cdot}4 \times 10^{10}}\right)^{11/10} \left(\frac{M_1}{M_\odot}\right)^{-3/10} \text{cm.}$$

As before, from the temperature of the spot we have determined the fraction of the total energy radiated longward of 3500 Å, ϵ_s, and hence the luminosity radiated by the spot in the optical for the various values of F_{16}. These are given in Table I. Strictly these luminosities correspond to the observed luminosity in the shoulder of dwarf novae light curves only if the radiation is emitted completely anisotropically. We note that h, T_s and hence L_s^{opt} depend quite sensitively on the uncertain parameter R_s.

4. COMPARISON WITH OBSERVATIONS

In the system Z Cha we may estimate the relative brightness of the spot, disc and secondary from the light curves of Warner. We may then compare the changes in each component with the theoretical changes expected from the considerations of the previous section. We examine in particular the observations at the quiescent state of 1972 December 9 and during outburst of 1973 January 9. We assume that the secondary red component contributes all the radiation at primary minimum in the quiescent phase. The clear, flat-bottomed eclipses would support such a contention. We further assume that all the primary light arises from the disc. Then we estimate from the height of the shoulder and depth of primary eclipse that in the quiescent phase the contribution to the flux as seen at Earth is

$$I_d \propto 900$$
$$I_s \propto 700$$
$$I_2 \propto 400$$

where I_d, I_2 and I_s are the observed relative brightness of the disc, secondary and spot, respectively. If we make the assumption that the secondary does not change in brightness then we estimate that during outburst

$$I_d \propto 16\,800$$
$$I_s \propto 2800$$

These estimates are rather approximate since the light curves themselves are not smooth.

We now compare these changes with those expected for an accretion model. We examine both the relative changes in brightness and the expected absolute luminosities of the disc and spot if the red component is indeed a main sequence star of mass 0·2 $M_\odot$. This latter assumption necessarily determines the apparent luminosity of all components from the observations and in particular requires that in the quiescent state they must all contribute $\sim 10^{31}$ erg s^{-1}.

The most important correction to the theoretical disc luminosities, L_d^{opt} given in Table I arises from the fact that the system has an angle of inclination i, which is close to 90°, i.e. we are seeing the disc almost edge on. This introduces a factor $\cos i$ into the apparent luminosity of the disc. Further since only half the light emerges from each surface of the disc, the apparent brightness is

$$I_d = 0{\cdot}5 \cos i L_d^{\text{opt}},$$

where πI_d is the luminosity of the disc radiated in the direction of an observer on Earth. Values of I_d for $i = 80°$ are given in Table I. In comparing these values with the observed behaviour of Z Cha we must use some caution. Though we have corrected the luminosity for aspect we have not corrected for limb darkening. We crudely estimate that during quiescence this reduces the luminosity by a factor 0·5, but only by about 0·8 during outburst as wavelengths longward of 3500 Å shift further into the Rayleigh–Jeans part of the spectrum. Values of I_d given in Table I should be adjusted accordingly. In the following discussion we shall assume the factors given above, though it should be emphasized that they are a minor correction, and unimportant with regard to the general conclusions we draw.

(a) *Quiescent state*

We assume a mass flux during quiescence of $\log F_{16} = 0$. From Table I we see that this leads to a total luminosity in the disc of $\sim 2{\cdot}5 \times 10^{32}$ erg s^{-1}. For $i = 80°$ this corresponds to an apparent luminosity in the direction of the observer of $I_d \sim 1{\cdot}1 \times 10^{31}$ erg s^{-1}, taking account of limb darkening as outlined above. We may now deduce from the observed ratio of I_d/I_2 that $I_2 \sim 4{\cdot}9 \times 10^{30}$ erg s^{-1}, or, since the secondary is radiating approximately spherically symmetrically, a luminosity for the secondary of $L_2^{opt} \sim 2{\cdot}0 \times 10^{31}$ erg s^{-1}. This is exactly the sort of luminosity one would anticipate for a main sequence star of mass $\sim 0{\cdot}2\ M_\odot$. It is evident that for $i \sim 80°$ and $F \sim 10^{16}$ g s^{-1} the observed luminosity of the disc is comparable with a main sequence star of mass $\sim 0{\cdot}2\ M_\odot$. Thus we would expect primary eclipses of about the observed depth. The values we have deduced above confirm this in detail.

The luminosity of the spot given in Table I suggests that $I_s \sim 6{\cdot}8 \times 10^{30}$ erg s^{-1} corresponding to the spot being face on to the observer. Thus $I_s \sim I_d$ and both $\sim I_2$ as deduced from the assumption that it is a main sequence star. We return to discuss the spot luminosity in more detail later.

We may also deduce the absolute magnitude of system, including the contribution from the secondary, spot and disc. The mean 'spherical' luminosity of the system, $\bar{L}$ (i.e. the luminosity of a star of the same apparent brightness) is given by

$$\bar{L} = 4(I_2 + I_d + 0{\cdot}25\ I_s)$$

where the factor 0·25 arises since the spot is observed for only half the orbit, and then radiates approximately as a sine wave with respect to the observer. We deduce that in the quiescent phase $\bar{L} \sim 7{\cdot}0 \times 10^{31}$ erg s^{-1} corresponding to an absolute magnitude of +9·1. This agrees with the sort of absolute magnitudes which have been deduced for dwarf novae at quiescence. However, we return to this point later, since it is clear that the absolute magnitude is a strong function of the inclination of the disc with respect to the observer in any system. This evidently has important consequences with regard to selection effects in the observation of dwarf novae with varying inclination.

(b) *Outburst state*

From theoretical studies of the red component, discussed in the next section, we conclude that dynamical instabilities will increase the mass transfer rate during outburst to a limiting F of 10^{18}–10^{19} g s^{-1}. Only if heating by the disc becomes a dominant energy source in the red component envelope can F be increased significantly beyond this. For a mass transfer rate of $\log F_{16} = 2$ the total optical luminosity of the disc is $\sim 3{\cdot}1 \times 10^{33}$ erg s^{-1} and the luminosity radiated in the direction of the observer is $I_d \sim 2{\cdot}2 \times 10^{32}$ erg s^{-1}, again taking into account limb darkening as discussed above. Thus we would deduce a theoretical increase in the disc luminosity of ~ 20. This compares with the observed increase of ~ 19. Clearly the luminosity changes predicted by an accretion model are in satisfactory agreement with the observed behaviour of Z Cha.

The absolute magnitude of the system may be deduced as before. The equivalent spherical luminosity of the system is $\bar{L} \sim 1{\cdot}0 \times 10^{33}$ erg s^{-1} corresponding to an absolute magnitude of $+6{\cdot}2$ and an absolute magnitude change of $\Delta M \sim 2{\cdot}9$. The apparent visual magnitude of Z Cha in its quiescent state is $\sim 15{\cdot}3$ and Warner estimates an apparent visual magnitude on January 9 of $\sim 12{\cdot}8$. Thus the observed change in visual magnitude is $\Delta M_V \sim 2{\cdot}5$. Considering the crudeness of both estimates the agreement is completely satisfactory.

(c) *Eclipse shape during quiescence and during outburst*

We examine the luminosity distribution expected for primary eclipse on the basis of the considerations of Section 3(b). It is clear that even ignoring any contribution by the primary itself, the optical luminosity is emitted in the main close to the accreting star in the quiescent phase. For a mass transfer rate of $\log F_{16} \sim 0$ most of the radiation is emitted at temperature $< 2 \times 10^4$ K and bolometric corrections in the centre of the disc are small. The luminosity distribution will be close to that of curve (i) in Fig. 1. During outburst the bolometric corrections for the centre of the disc become large (see values of ϵ_d in Table I) and a distribution similar to that of curve (ii) would be expected. These changes are clearly in agree-

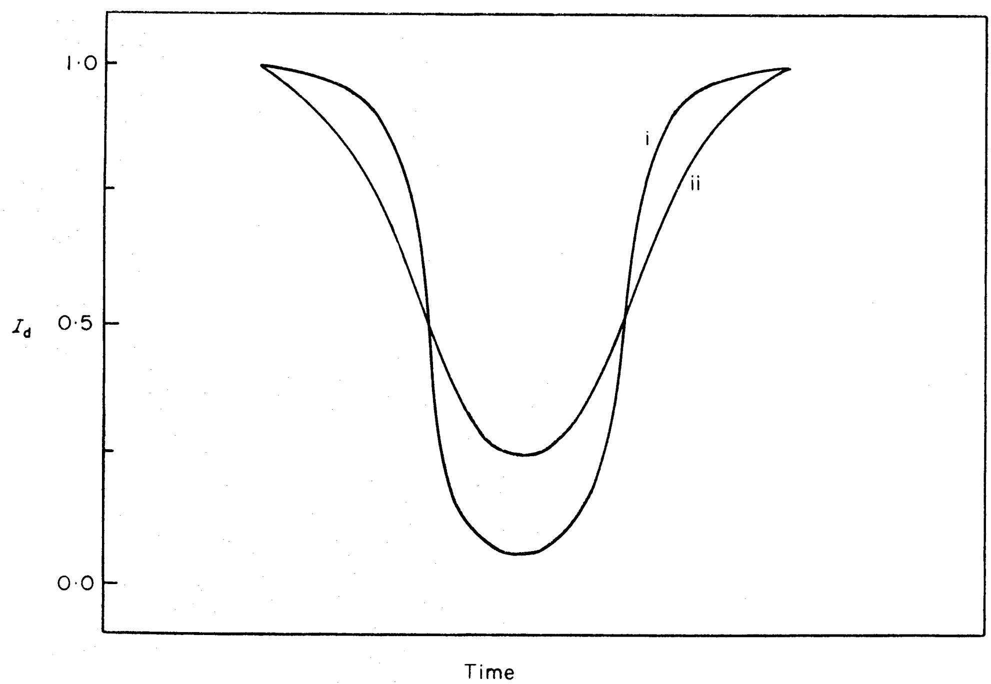

FIG. 2. *Eclipse curve of the disc. The variation in optical luminosity of the disc, I_d, is shown as a function of time for the two extreme cases of all the radiation (i) and only that on the Rayleigh–Jeans limit (ii) being visible in the optical regime. A straight edge eclipse by a rectangular strip of width $1{\cdot}6 \times 10^{10}$ cm is assumed. It can be seen that although the eclipse of the disc of diameter $2{\cdot}8 \times 10^{10}$ cm is always partial, only ~ 6 per cent of the optical luminosity of the disc remains at eclipse minimum in case (i) and ~ 25 per cent in case (ii). To compare these with the observations of Warner (1974a), Fig. 11, the luminosity of the secondary must be added, resulting in an observed eclipse depth ~ 30 per cent in both cases.*

ment with the changes in primary eclipse shape observed in Z Cha. In Fig. 2 we have plotted theoretical steady-state disc eclipse curves for the two extremes, i.e. observing all the radiation from the disc and only that in the Rayleigh–Jeans part of the spectrum, corresponding to the brightness distributions (i) and (ii) in Fig. 1. We have assumed a straight edge eclipse by a rectangular strip of width $1{\cdot}6 \times 10^{10}$ cm. This corresponds approximately to the eclipsing width of the secondary for an inclination of 80°. Comparison of these two extreme cases with the observed behaviour of Z Cha, shown in Fig. 11 of Warner (1974a), clearly demonstrates a close correspondence between the behaviour of the theoretical disc structure and that observed in Z Cha.

(d) *Behaviour of the hot spot*

We now consider the behaviour of the hot spot according to our simple model of Section 3(c). For the completely anisotropic case the aspect correction for the spot is $\sin i$. For angles of inclination relevant to Z Cha this is ~ 1. Therefore the ratio of the intensity of the disc to the spot contribution is:

$$\frac{I_{\mathrm{d}}}{I_{\mathrm{s}}} = \frac{0{\cdot}5 \cos i L_{\mathrm{d}}^{\mathrm{opt}}}{L_{\mathrm{s}}^{\mathrm{opt}}}.$$

In Table II we give values for this ratio for different values of F_{16} and for angles of inclination between 77·5° and 87·5°.

TABLE II

			$I_{\mathrm{d}}/I_{\mathrm{s}}$		
$\log F_{16}$	77·5°	80·0°	82·5°	85·0°	87·5°
−2	1·6	1·3	1·0	0·65	0·32
−1	2·5	2·0	1·5	1·0	0·5
0	4·0	3·2	2·4	1·6	0·8
1	5·7	4·6	3·5	2·3	1·2
2	6·9	5·5	4·1	2·8	1·4
3	7·2	5·8	4·4	2·9	1·5
4	6·0	4·8	3·6	2·4	1·2

If all the spot radiation is emitted anisotropically and we ignore limb-darkening effects then we may compare these relative intensities with those deduced from the behaviour of Z Cha. We see immediately that for this case the hot spot always contributes a significant fraction of the observed light of the system for values of $\log F_{16} \lesssim 0$ and $i \gtrsim 80°$. With increasing mass flux the contribution of the spot decreases initially until $\log F_{16} = 3$. Beyond this the relative contribution of the spot increases. Thus a rather different value of the spot contribution relative to the disc contribution would be expected between quiescence and outburst even for a steady state model of the disc whose radius remains constant during outburst, and an exceedingly simple model of the spot whose radiation remains completely anisotropic at all times. The changes of the ratio $I_{\mathrm{d}}/I_{\mathrm{s}}$ with different F_{16} and i in Table II are simply caused by the complex behaviour of the relevant bolometric corrections for the disc and spot. Of course the uncertainty of the geometry of the hot spot, which may change when the properties of the disc are varying, makes it impossible to give a simple definitive relation between the mass transfer rate and the appearance of the shoulder in the light curve. We regard the eclipse shape and the total brightness of the disc as better indicators of the dominant outburst properties. Nevertheless, comparison of the observed intensity of the shoulder, when it is present, with the simple completely anisotropic theory is of interest.

From the deduced relative intensities of the components of Z Cha we find that the observed ratio I_d/I_s during quiescence is $\sim 1{\cdot}3$ and at outburst ~ 6. We see from Table II that these values are in reasonable agreement with the theoretical predictions for an accretion model with $90° \gtrsim i \gtrsim 80°$ in which the mass transfer rate increases from 10^{16} to 10^{18} g s^{-1} during outburst. For our earlier model with $i = 80°$, we deduce ratios of $I_d/I_s \sim 1{\cdot}6$ and $\sim 4{\cdot}4$ at minimum and maximum mass flux respectively, taking account of limb darkening as before. Considering both the crudeness of the spot model and the approximate nature of the adopted values of the observed intensity contributions from the disc and spot the agreement is satisfactory. We stress however that geometry effects will be important, and note that on January 8 the observations of Warner show no shoulder at the phase expected for a simple hot spot model, i.e. prior to eclipse, but a shoulder *following* eclipse. Similar changes in the position and intensity of the shoulder are not infrequent amongst dwarf novae and nova-like systems (Smak 1971a). Complicated effects of spot geometry, disc expansion, and non-steady disc transport must be invoked to account for such behaviour. However, the important conclusion is emphasized that the *maximum* height of the shoulder can be accounted for on the basis of a thermal completely anisotropic hot spot model in Z Chamaeleontis.

5. MASS TRANSFER INSTABILITIES

The variation in the mass transfer rate, F, to which we have ascribed the changes in disc structure during outburst can be caused by the onset of a state of dynamical instability in the red component envelope. Only in the immediate vicinity of the Lagrangian point is the gravity low enough for the stability to be significantly affected by the presence of the companion. This must be taken into account in any interpretation of the behaviour of the spherical simulations of Bath (1972). Recent work (Bath & Papaloizou, in preparation), using a variational approach to the stability of models with a gravity field given by the Roche potentials gives the following results. There is an approximate correspondence between the two methods regarding the existence of stability and e-folding times, though the tendency of more efficient convection to *stabilize* models is more pronounced in Roche potentials. The behaviour of gravity in the vicinity of the Lagrangian point coupled with standard boundary conditions results in a similar mathematical criterion for stability to that obtained for spherically symmetric models with the pressure held constant on an initial spherical surface.

However, the results also indicate that the instability will probably be confined to a relatively small outflow cone angle. The value depends on the variation of gravity in the vicinity of the Lagrangian point, and the ionization energy available to outflowing material. This results in a natural limit to the growth of the mass-loss rates and luminosities which are, as a consequence, considerably less than the spherical situation. We estimate that the maximum outflow rate resulting from dynamical instabilities will be between 10^{18} and 10^{19} g s^{-1} as a result of this confinement. If self-excited heating of the red component by the accretion disc is triggered by an initial dynamical instability, then greater values of F may result.

A spherically symmetric hydrodynamic code has been developed to simulate the complete time-dependent development of dynamical mass loss instabilities. We assume this is applicable to the mass transfer situation if the cone angle restriction is taken into account by simply reducing the surface area accordingly. The results then indicate that for main sequence red dwarf models the total mass loss per outburst is 10^{22}–10^{23} g on a time scale of hours to days. Because of the restriction of the mass loss to the immediate vicinity of the Lagrangian point the total luminosity change by the secondary during the phase of unstable outflow is negligible compared with other luminosity sources in the system. The outbursts are followed by a phase in which the surface shrinks inside the Roche lobe, readjusting to thermal equilibrium. The outbursts are found to repeat with a period in the region 2–500 days, depending on the initial model. The outburst period is determined by the thermal relaxation times of the envelope, that is, the energy deficit in the envelope at the end of each outburst and the difference in luminosity between the core and surface of the model. This work, together with a detailed discussion of the instability mechanism will be presented elsewhere. In this discussion we only wish to emphasize that the behaviour we have hypothesized as occurring in the red component, that is quasiperiodic dynamical outburst states, is to be expected. The sort of properties which are demanded by the accretion disc behaviour we have examined in this paper are just those which are found from stability studies of the red component.

6. TIME DEPENDENT BEHAVIOUR OF AN ACCRETION DISC

So far we have assumed in our consideration of the accretion disc that the mass flux F through the disc is independent of time, having different values during quiescence and during outburst. A number of authors (Lüst 1952; Lynden-Bell 1962, thesis; Lynden-Bell & Pringle 1974; Lightman 1974; Pringle 1974, thesis) have considered the time-dependent evolution of accretion discs. The details of time evolution depend critically on the viscosity about which little is known, but nevertheless it is possible to make a few general comments.

If the time scale on which the outburst occurs is much less than the time scale on which viscosity acts in the disc, then immediately after mass transfer we may envisage the accretion ' disc ' as consisting of a thin ring of material encircling the primary. Viscosity acts to spread the ring out, with the inner radius of the ring decreasing and the outer one *increasing* (see the Green's Function solution of Lynden-Bell & Pringle 1974). Eventually the ring spreads out completely and the disc settles down to a more or less steady state, but not before most of the matter transferred during outburst has been accreted by the primary. In this situation the luminosity of the disc rises rapidly to a peak of $(\Delta M)\, GM_1/R_1\tau$ where ΔM is the amount of mass transferred during outburst, and $\tau \sim R_s^2/\nu$ is the time scale on which the viscosity, ν, acts in the disc. After the peak the luminosity declines slowly, roughly as an inverse power of time (the exact power depending on the viscosity). An example of such an outburst light curve is shown in Fig. 3, and we note that the shape, especially the slow decline, is characteristic of the outburst light curves of many dwarf novae (Glasby 1970).

During the quiescent state, when we assume the mass transfer rate to be more or less constant, we expect the steady disc approximation to be valid. When the outburst lasts much longer than the viscous time scale τ, that is when the outburst duration is much longer than the decline time scale of the outburst light curve, we

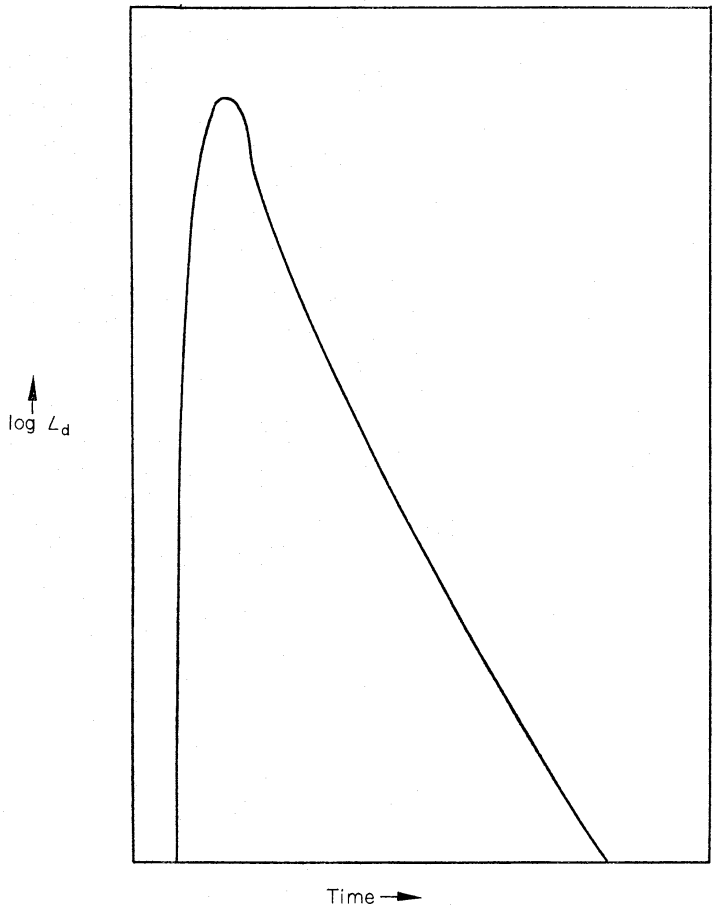

FIG. 3. *Schematic light curve for a time-dependent disc. The quantitative values of time and log L_d are dependent on the disc viscosity and mass, amongst other parameters.*

again expect the steady disc approximation to be valid during the outburst with F appropriately increased—we stress however that the *size* of the disc will probably depend on F. Any structure in the outburst light curve with a time scale longer than τ (for example the standstills of the Z Cam variables) represents a similar structure in the mass transfer rate.

We conclude that the general features of dwarf novae outburst light curves (in particular the sharp rise to maxima and more gradual decline) are well accounted for by the theory of time-dependent accretion discs.

7. NUCLEAR BURNING MODELS OF NOVAE AND DWARF NOVAE

In our accretion model for dwarf novae there remains the question of what becomes of the nuclear energy available through the burning of accreted hydrogen: this energy is about 50 times larger per gram than the accretion energy. Various models have been proposed (Saslaw 1968; Rose 1968; Starrfield *et al.* 1972; Starrfield, Sparks & Truran 1974, in press) suggesting that it is just this source of energy which produces novae, and possibly dwarf novae outbursts. We examine here some of the problems associated with unstable nuclear burning as an explanation of the behaviour of Z Cha, and of novae and dwarf novae in general.

It is clear that, if the energy during outburst is supposed to arise through nuclear burning on the *surface* of the white dwarf, some means must be found to channel most of this energy into the disc as a whole. Warner (1974a) has suggested that the increase in luminosity of the disc at maximum light in Z Cha is due to phases of unstable nuclear burning on the surface giving rise to an outgoing shock through the disc. Warner deduces a mass for the disc of $\sim 10^{-4}\, M_\odot$ by associating the outburst life time with the thermal cooling time of the disc. The accretion contribution to the luminosity during outburst is assumed negligible.

To test this model we have performed numerical calculations of shock waves propagating through such a disc. We consider the propagation of axially symmetric waves through an initially simple disc structure in which the vertically averaged density ρ, pressure P and temperature T are independent of the cylindrical co-ordinate r. We assume that the cooling time is long, so that the shock propagates adiabatically. We also assume that transport of angular momentum occurs sufficiently slowly so that each mass element retains its angular momentum over the time scale considered.

The equations we use are, in Lagrangian coordinates

$$\frac{\partial^2 r}{\partial t^2} = -2\pi r \frac{\partial(P+q)}{\partial m} - \frac{GM_1}{r^2} + \frac{h^2}{r^3} \tag{1}$$

$$\frac{1}{\rho} = 2\pi r \frac{\partial r}{\partial m} \tag{2}$$

$$h^2 = GM_1 r_0 + 2\pi r_0^4 \frac{\partial P_0}{\partial m} \tag{3}$$

$$\frac{\partial P}{\partial t} = \frac{\gamma P}{\rho}\frac{\partial \rho}{\partial t} + \frac{(\gamma-1)\, q}{\rho}\frac{\partial \rho}{\partial t}. \tag{4}$$

Here, h represents the angular momentum per unit mass, P_0 the initial pressure and r_0 the initial radius out to mass per unit vertical thickness m. Terms of the type $\partial P_0/\partial m$ are zero in our constant pressure model. This assumption of P_0 constant is unlikely to be important since centrifugal and gravitational terms will dominate the structure of the initial disc. The artificial viscosity q, of Richtmeyer & Morton (1967) is used. The value of γ is taken as 5/3 throughout. We have considered both the propagation of initial velocity distributions and the effect of the impact of the central object on the inner edge of the disc. All calculations were performed with a $1\, M_\odot$ central object of radius 10^9 cm. T was taken as 10^5 K.

Because of the dominance of centrifugal terms over pressure terms in this problem, shock propagation is radically different when compared with that in a medium with no angular momentum. The reasons for this emerge if one considers the linearized equation for sound waves derived from equations (1)–(4). If we put $r = r_0(1+\xi(r_0)\, e^{i\sigma t})$ we obtain

$$\sigma^2 \xi = \frac{GM_1 \xi}{r_0^3} - \frac{1}{\rho_0 r_0}\frac{\partial}{\partial r_0}\left(\frac{\gamma P_0}{r_0}\frac{\partial r_0^2 \xi}{\partial r_0}\right) + \frac{4\xi}{\rho_0 r_0}\frac{\partial P_0}{\partial r_0}.$$

It can be seen from this that the term $(GM_1/r_0^3)\,\xi$ acts to give a minimum frequency for wave propagation given by $\omega_K^2 \sim GM_1/R_1^3$. It follows from this that any impact on the inside of the disc must be delivered with a characteristic time scale $\tau < \omega_K^{-1} \sim 3$ s. If this is not so, very little wave propagation can result; all the energy goes into causing a secular hydrostatic re-adjustment right at the centre of the disc. Very little energy is communicated to more than twice the initial radius.

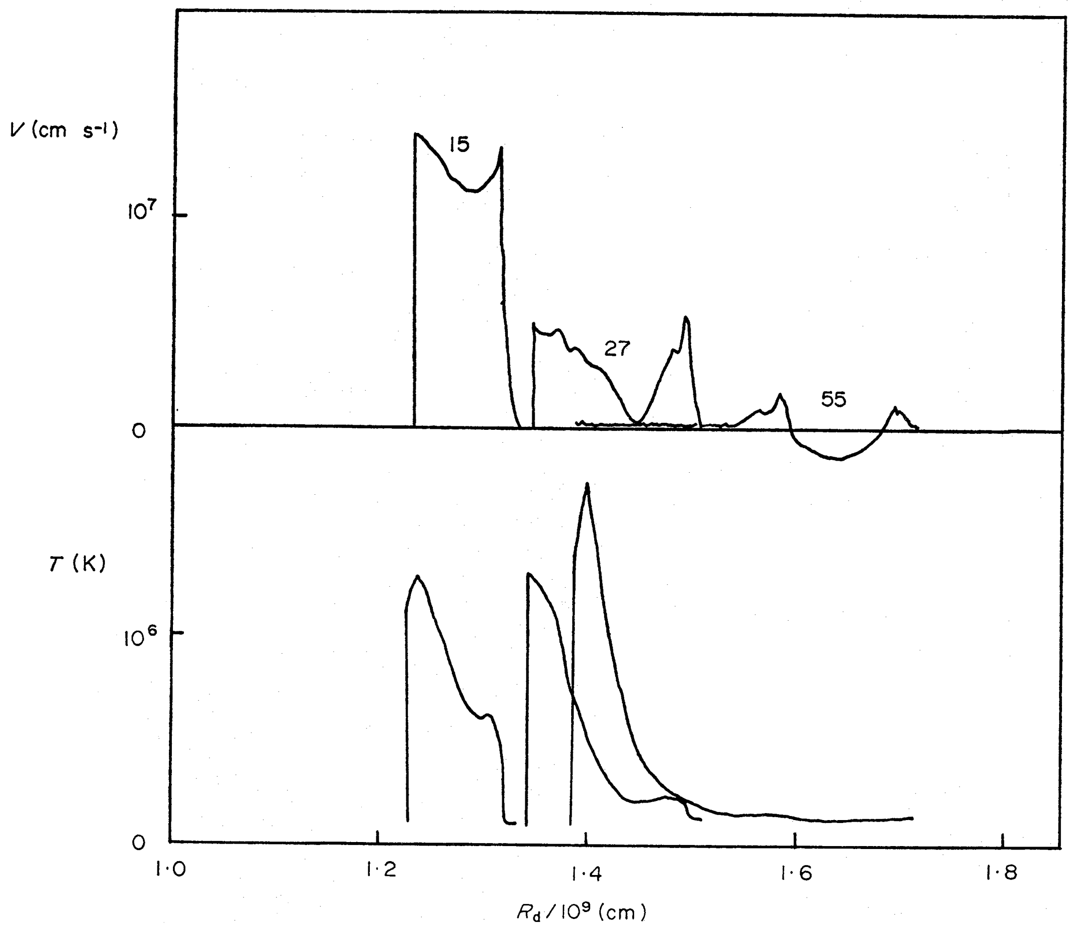

FIG. 4.

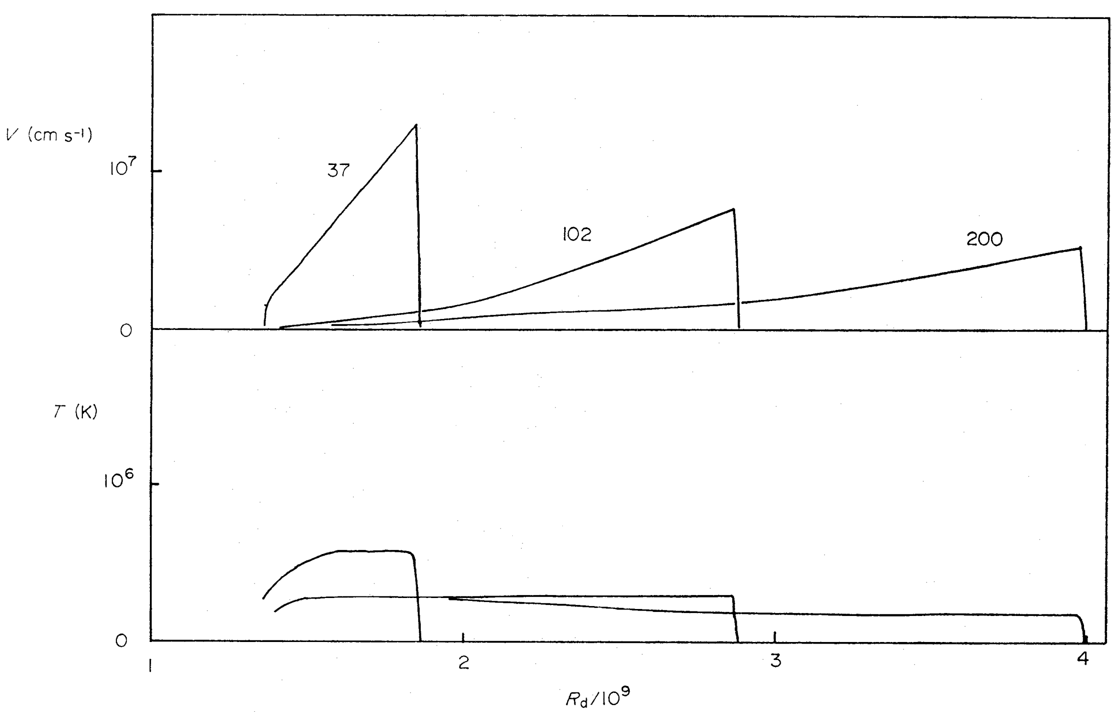

FIG. 5.

FIGS 4 and 5. *The propagation of shocks due to a sudden expansion at the inner boundary of the disc. The numbers above each set of solutions refer to the time, in seconds, after the initial impulse. Fig. 4, in which centrifugal terms are included, exhibits rapid dissipation of shock energy in the inner part of the disc. Fig. 5 shows a standard shock propagating out through a static, non-rotating disc, uniformly heating the material. Note that the ordinate scale is contracted in the latter case.*

Furthermore, even if a velocity distribution can be initiated that is appropriate to a shock wave, it is found that because of the slow propagation of long wavelength disturbances ($\lambda \gtrsim 10^8$ cm), and the effect of the spreading of wave packets, several persistent shock waves form near the origin. These rapidly dissipate their energy and the system again hydrostatically adjusts, very little energy being propagated outwards. It is readily verified that because of these serious constraints on wave propagation, significant energy can only be propagated a reasonable distance into the disc if the white dwarf expands a large fraction of its radius with the escape velocity. Under these unreasonable conditions, the disc is of course unlikely to survive as such to have energy propagated through it.

To illustrate the above points we have set up a hydrodynamic code to solve the equations (1)–(4). The results of several calculations are shown graphically in Figs 4–7. Figs 4 and 5 compare the results of an inside push on a disc with and without gravitational and centrifugal forces, respectively. The inner boundary was moved according to the equation

$$\frac{\partial r}{\partial t} = 8.10^7 \, e^{-t/10}(1 - e^{-t/10})$$

where t is the time in seconds and r is in centimetres. The white dwarf thus increases its radius by 30 per cent in 20 s. It is seen that in the case of a centrifugally supported disc (Fig. 4) very little kinetic energy is communicated to the outer regions of the disc. The energy rapidly dissipates. Very strong heating is produced in the centre of the disc and virtually nothing is felt beyond 1.6×10^9 cm. In the second case with no rotation (Fig. 5) it is seen that a standard shock propagates outwards uniformly heating the disc. The shock reached out to 5×10^9 cm, raising the temperature by about 60 per cent fairly uniformly before the calculation was stopped. Figs 6 and 7 make the comparison between the same media as before, but follow the propagation of a distribution of velocity which was a half sine wave of amplitude 4×10^7 cm s^{-1} and wavelength 7.5×10^8 cm. Again, in the second case a shock propagates through the disc uniformly heating it, while in the first centrifugally supported case the kinetic energy is rapidly dissipated in several shock waves near the origin. Large temperature increases were found near the origin, with virtually no heating beyond 2.3×10^9 cm. It would thus appear from these results that hitting the disc from inside by the primary would either disrupt the disc completely or heat it strongly in the centre. Shock waves uniformly heating it out to large radii are unlikely to be produced. It is also to be noted that because of the small propagation distance, and the radical structural alterations by the shock, these results are unlikely to be affected by taking a different density distribution in the disc. We note that the recent paper of Sanders & Prendergast (1974) illustrates a similar retarding effect of rotation on shock propagation.

A second possibility for uniformly heating the disc is to have a thermal pulse, rather than a shock propagating through it. However, it will be readily seen that such a mechanism also runs into serious difficulty due to heat losses vertically through the disc. A thermal pulse is unlikely to get more than several scale heights through the disc. The scale height near the centre is $\sim 10^7$ cm.

The initial conditions which are required in order to produce explosive nucleosynthesis on the surface of white dwarfs also raise problems. In the spherically symmetric nova models of Starrfield the outer envelope of unburnt hydrogen must contain a considerable enhancement of carbon, nitrogen and oxygen and must

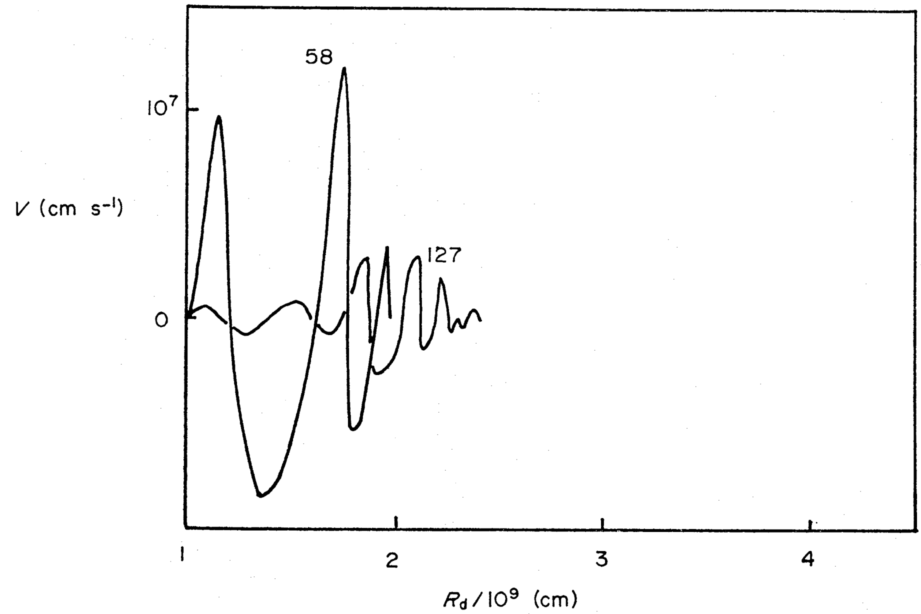

FIG. 6.

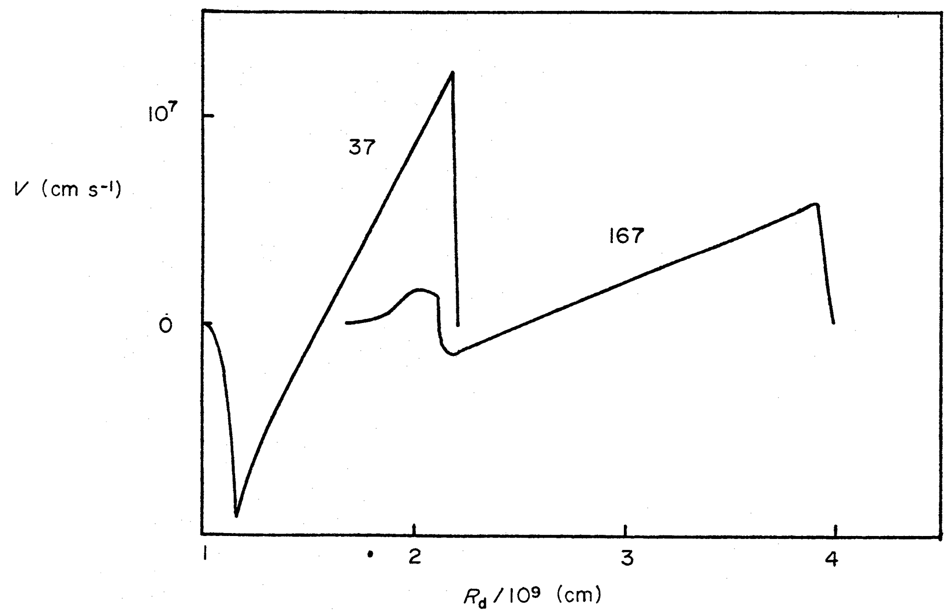

FIG. 7.

FIGS 6 and 7. *As in Figs 4 and 5, but with an initial half sine wave distribution of velocity.*

have a mass of $\sim 10^{-3}\ M_\odot$ to produce the required scale of explosion. However, we have examined the thermal stability of plane-stratified envelopes from a simple analytical point of view and find that such spherically symmetric models become thermally unstable for envelope masses much less than this. Furthermore the models of Kippenhahn, Thomas & Weigert (1968) suggest that very different non-explosive behaviour can occur when burning instabilities set in.

In all Starrfield's nova models the luminosity change occurs as a consequence of a spherically symmetric expansion of the photosphere. For dwarf novae this is not supported by the observations of colour changes, or the changes in eclipse shape in Z Cha. In a typical dwarf nova the burning process must occur every 30 days or so. No such behaviour has been predicted in any model published so far. Even if such behaviour were found, we stress again that nuclear burning energy is released at the surface of the white dwarf and would then need to be propagated outwards through the disc. We find such propagation behaviour is difficult (if not impossible) to achieve.

The usual assumption of spherical symmetry in models of surface nuclear burning may be in error for these systems. It is not clear how the concentration of accreted material in an equatorial region, where the accretion disc interacts with the stellar surface (or on the magnetic poles if the dwarf has a strong ($\gtrsim 10^6$ G) magnetic field—Bath, Evans & Pringle 1974b), will affect the conditions required for nuclear burning. In addition Bath *et al.* (1974b) noted that the time scale on which accreted angular momentum spins up a white dwarf is $\sim 10^7$ yr, after $\sim 0{\cdot}1$ $M_\odot$ has been transferred. Thus the primary may be expected to be rotating close to break up. It has yet to be shown that it will not have a strongly differentially rotating envelope. Conditions for nuclear burning require a high effective gravity which is difficult to achieve on the surface of a rapidly rotating white dwarf. Nuclear burning may in these conditions be quite different to that predicted by spherical models.

Additional evidence for the nuclear burning hypothesis has been claimed (Warner & Robinson 1972) from the discovery of transient, low-amplitude periodicities, visible during outburst with periods in the range 17–35 s. These are interpreted as g-mode pulsations of the white dwarf envelope. The main evidence put forward for this interpretation is that in *one* dwarf nova, Z Cam (Robinson 1973) the period increased with decreasing brightness of the system. Warner & Brickhill (1974) have examined such period changes by extrapolating the results of Osaki & Hansen (1973), relevant to cooling white dwarfs, to high luminosity. Such a comparison is not valid. The pulsation period of a hot white dwarf is essentially determined by the core temperature. To comply with the observed period changes the core temperature must change by a factor ~ 2. This requires a total energy of $\sim 10^{48}$ erg—clearly out of the question for a dwarf nova. If g-mode pulsations are the cause of the observed periodicities, they must be a surface phenomenon and the results of Osaki & Hansen are not applicable. The suggestion that the periodicities in dwarf novae are caused by inhomogeneities in the accretion disc (Bath 1973; Bath, Evans & Papaloizou 1974a) seems to us a more viable alternative, and similar behaviour in the accretion discs in X-ray sources has already been predicted (Syunyaev 1972) and observed (Schreier *et al.* 1971; Oda *et al.* 1972).

8. DISCUSSION AND CONCLUSIONS

We have demonstrated that the properties of the dwarf nova Z Cha during both quiescence and outburst can be satisfactorily explained in terms of variable mass transfer and the liberation of accretion energy alone, and we emphasize that this model is equally applicable to other dwarf novae.*

Although the values of L_d^{opt} in Table I were calculated specifically for Z Cha, they will be approximetely correct for other dwarf novae whose primaries consist of white dwarfs. From Table I we note that as F increases, the value of L_d^{opt} tends roughly to a limit of $\sim 10^{34}$ erg s^{-1}. Thus, unless significant ultraviolet radiation is

* We note that variable mass transfer has been suggested to explain the 35-day period of the X-ray source Hercules X-1 in which the accreting component is a neutron star (Arons & Bahcall 1972; Pringle 1973; McCray 1973).

converted to optical by a reflection effect, either off the secondary or off gas streams, we expect the absolute visual magnitude during outburst to be $\gtrsim +3.5$.* The actual value of the observed absolute visual magnitude depends on the maximum mass transfer rate during outburst and on the angle of inclination of the system. Since the disc is the dominant light source during outburst, eclipsing systems in which the disc is seen almost edge on will appear underluminous. They are therefore less likely to be detected than those systems with moderate or small values of i. The absolute visual magnitude of the system during quiescence, and hence the magnitude change during outburst, depends on which part of the system—the red secondary component, the disc and/or hot spot, or the intrinsic luminosity of the blue primary component—provides the dominant contribution. Dwarf novae are observed to have binary periods in the range $9 \gtrsim P \gtrsim 1$ hr (Mumford 1967). For main sequence secondaries which fill their Roche lobes, this implies $M_2 \lesssim 2\,M_\odot$. For $M_2 > 2\,M_\odot$, however, the luminosity from the secondary will dominate the optical radiation from the disc ($\sim 10^{34}$ erg s^{-1}). Thus dwarf nova type systems with periods $> 0.5^{\rm d}$ may well occur, but would be hard to detect because of the dominant luminosity of the secondary. For example, in Algol-type systems, the intrinsic luminosity of the two stellar components will swamp any accretion energy produced by the mass transfer process, and quasiperiodic unstable mass transfer of the type discussed here would not be readily detectable photometrically.

We note that apart from the system Z Cha where the shape of primary eclipse indicates a component of size $\sim 10^9$ cm, the evidence for the presence of a white dwarf in all dwarf novae systems is not good. The evidence is two-fold. First, medium dispersion spectra of some dwarf novae during quiescence reveal a blue continuum with broad, shallow lines usually indicative of a white dwarf. As we have seen, however, most of the light from the system during quiescence can come from the hot spot and from the accretion disc around the primary and can display similar spectral characteristics to those of a white dwarf. Schvartzman (1971) has also noted that accretion can give rise to spectra resembling that of a bright white dwarf. Second, it is argued that white dwarfs are needed to explain the outbursts in terms of explosive nuclear burning. We have demonstrated that variable mass transfer can satisfactorily account for these outbursts and stress that instability of the mass transfer flow from the red star is independent of the size of the primary companion. Such mass transfer would however be harder to detect in systems in which the primary or accreting component is larger than a white dwarf, for two reasons: (i) a larger star of comparable mass is more luminous and would tend to dominate the accretion energy, and (ii) the total amount of accretion energy available is inversely proportional to the radius of the primary.

In such systems in which the primary is of comparable brightness to or even dominates the accretion disc we expect none the less the emission line spectra of the disc to be visible. The relative strengths of the two spectra would vary in an irregular or quasiregular fashion. Because the primary is much more luminous than a white dwarf, the magnitude variations caused by variable mass transfer should be correspondingly less. We note that symbiotic variables, for example, fit this description quite well and note in particular the behaviour of AR Pav (Thackeray & Hutchings 1974). Smak (1971b) and Hall (1969, 1971) have also pointed out that the light curves, spectra and irregular variations of certain close

* This compares favourably with the observational estimates of a minimum M_V of $+3$ during outburst for dwarf nova eruptions in general (Warner 1974b).

binaries containing bright, normal primary components can be accounted for in terms of disc variations. Smak points out that this is a partial argument against any mechanism which requires white dwarfs in dwarf novae to produce similar disc variations.

The most convincing evidence for the presence of a white dwarf in some dwarf novae is the comparatively short time scale (17–35 s) of the optical periodicities observed during the outburst state (Warner & Robinson 1972). If this is a gravitational time scale, corresponding to $(G\rho)^{-1/2}$, we deduce a density $\sim 4 \times 10^4$ g cm^{-3}. This implies that the object responsible is much more compact than a main sequence star whatever the mechanism responsible. If interpreted in terms of inhomogeneities in the accretion disc then, as has been pointed out (Bath 1973; Bath *et al.* 1974a), the periods, luminosities and lifetimes of the pulses may be accounted for in terms of hot spots near the inner edge of a disc accreting on to a white dwarf. However, short period pulsations have not been detected as yet in a number of dwarf novae (e.g. U Gem, SS Cyg) and it cannot be concluded that all systems necessarily contain white dwarfs.

In Section 7 we noted that for hydrogen falling on to a white dwarf about 50 times as much energy is available to it from nuclear burning than from accretion, and have drawn attention to some of the problems associated with present models of unstable surface nuclear burning. Even if this energy does *eventually* emerge, there is at least one possibility which is not ruled out observationally. The mass transfer rates we have considered amount to an average of $\sim 10^{-9} M_\odot$ yr^{-1} for dwarf novae. If every dwarf nova burnt its accreted nuclear fuel in the form of a classical nova outburst, the period between such outbursts would be at least 10^4–10^5 yr. In 20 yr of observation it is unlikely that any of the ~ 100 dwarf novae known should undergo such an explosion. It seems plausible that the nova explosion itself could preclude mass transfer of the kind we have envisaged for a period following the explosion. This is not, of course, to say that all novae were dwarf novae prior to the explosion, or that novae are necessarily caused by nuclear burning instabilities. We note that the slow nova RR Tel was known to be a variable star with a photographic magnitude range ~ 3 and an approximate period of 1 yr prior to outburst (Mayall 1949). It may also be possible to account for the changes in spectroscopic appearance of RR Tel, and of the somewhat similar system He 2-177 (Webster 1966; Carlson & Henize 1974) in terms of variable mass transfer.

We have emphasized that the bolometric corrections are of importance in interpreting observations of dwarf novae. Thus during quiescence, and especially during outburst, most of the radiation is emitted at frequencies in the ultraviolet. For this reason we stress the importance of observing these systems photometrically and spectroscopically with satellite based ultraviolet detectors.* We have also noted that during outburst the high temperatures close to the primary could give rise to copious soft X-ray (~ 0.3 keV) emission. Dwarf novae are sufficiently close (~ 100 pc) that interstellar absorption at these energies is relatively unimportant. Absorption by material in the system itself may be important especially for systems with $i \sim 90°$.

Mass transfer rates from the red component in excess of $\sim 10^{19}$ g s^{-1} can be self-excited by absorption of accretion radiation. This can explain the 'super-maxima' observed in some systems (e.g. VW Hydri). In the extreme, runaway

* D. W. Sciama has suggested that these objects may be a significant, and perhaps dominant, contributor to ionizing the interstellar medium (private communication).

self-excitation of the red component could produce nova and recurrent nova outbursts. Davidsen & Ostriker (1974) have postulated continuous self-excitation to account for the properties of Cyg X-3, which they point out resembles dwarf novae in several respects.

9. SUMMARY

In the eclipsing system Z Cha we have shown that the changes in binary light curves in quiescent and outburst states can be explained in terms of modulated mass transfer by the red star and accretion on to the primary. In addition we have demonstrated that such a scheme is compatible with the behaviour of dwarf novae in general. The required instabilities in mass flow from the red component can be understood on theoretical grounds and the subsequent behaviour of the system naturally gives rise to outbursts of the observed magnitude and form.

We have pointed out some of the problems associated with the proposed alternative nuclear burning model. In particular we have shown that if nuclear energy is released explosively on the white dwarf surface then it is difficult to propagate the energy out into the disc, as is required to fit the observations.

We stress that many classes of variable and unstable stars may involve mechanisms similar to those discussed here.

ACKNOWLEDGMENTS

We thank Professor B. Warner for communicating results prior to publication. G. T. Bath acknowledges financial support from Merton College, Oxford. W. D. Evans, J. Papaloizou and J. E. Pringle acknowledge financial support from the Science Research Council. We thank a referee for comments which led to clarification of the text and more accurate numerical values.

G. T. Bath, W. D. Evans and J. Papaloizou:
Department of Astrophysics, Oxford

J. E. Pringle:
Institute of Astronomy, Cambridge

Received in original form 1974 *May* 6

REFERENCES

Arons, J. & Bahcall, J. N., 1972. *Astrophys. Lett.*, **11**, 191.
Bath, G. T., 1972. *Astrophys. J.*, **173**, 121.
Bath, G. T., 1973. *Nature Phys. Sci.*, **246**, 84.
Bath, G. T., Evans, W. D. & Papaloizou, J., 1974a. *Mon. Not. R. astr. Soc.*, **167**, 7P.
Bath, G. T., Evans, W. D. & Pringle, J. E., 1974b. *Mon. Not. R. astr. Soc.*, **166**, 113.
Carlson, E. D. & Henize, K. G., 1974. *Astrophys. J. Lett.*, **188**, L47.
Crawford, J. A. & Kraft, R. P., 1956. *Astrophys. J.*, **123**, 44.
Davidsen, A. & Ostriker, J., 1974. *Astrophys. J.*, **189**, 331.
Glasby, J. S., 1970. *The dwarf novae*, Constable, London.
Hall, D. S., 1969. *Bull. Am. astr. Soc.*, **1**, 345.
Hall, D. S., 1971. *IAU Colloquium*, **15**, 217.
Kippenhahn, R., Thomas, H. G. & Weigert, A., 1968. *Z. Astrophys.*, **69**, 265.
Kraft, R. P., 1962. *Astrophys. J.*, **135**, 408.
Kraft, R. P., 1963. *Adv. Astr. Astrophys.*, **2**, 43.

Lightman, A. P., 1974. *Astrophys. J.*, in press.
Lüst, R., 1952. *Naturf.*, **7a**, 87.
Lynden-Bell, D., 1962. Unpublished thesis, University of Cambridge.
Lynden-Bell, D., 1969. *Nature*, **233**, 690.
Lynden-Bell, D. & Pringle, J. E., 1974. *Mon. Not. R. astr. Soc.*, **168**, 603.
Mayall, M. W., 1949. *Harv. Obs. Bull.*, 919.
McCray, R., 1973. *Nature Phys. Sci.*, **243**, 94.
Mumford, G. S., 1967. *Publ. astr. Soc. Pacific*, **79**, 287.
Mumford, G. S., 1969. *Sky Telesc.*, **37**, 288.
Oda, M., Wada, M., Matsuoka, M., Miyamoto, S., Muranaka, N. & Ogawara, Y., 1972. *Astrophys. J. Lett.*, **172**, L13.
Osaki, Y. & Hansen, C. J., 1973. *Astrophys. J.*, **185**, 277.
Prendergast, K. H. & Burbidge, G. R., 1968. *Astrophys. J. Lett.*, **151**, L83.
Prendergast, K. H. & Taam, R. E., 1974. *Astrophys. J.*, **189**, 125.
Pringle, J. E., 1973. *Nature Phys. Sci.*, **243**, 90.
Pringle, J. E., 1974. Unpublished thesis, University of Cambridge.
Pringle, J. E. & Rees, M. J., 1972. *Astr. Astrophys.*, **21**, 1.
Richtmeyer, R. D. & Morton, K. W., 1967. *Difference methods for initial value problems*, 2nd ed., N.Y. Interscience.
Robinson, E. L., 1973. *Astrophys. J.*, **186**, 121.
Rose, W. K., 1968. *Astrophys. J.*, **152**, 245.
Sanders, R. H. & Prendergast, K. H., 1974. *Astrophys. J.*, **188**, 489.
Saslaw, W. C., 1968. *Mon. Not. R. astr. Soc.*, **138**, 337.
Schreier, E., Gursky, H., Kellogg, E., Tananbaum, H. & Giacconi, R., 1971. *Astrophys. J. Lett.*, **170**, L21.
Schvartzman, V. F., 1971. *Astr. Zh.*, **48**, 479 (1971, *Sov. Astr-Astr. J.*, **15**, 377).
Shakura, N. J. & Syunyaev, R. A., 1973. *Astr. Astrophys.*, **24**, 337.
Smak, J., 1971a. *IAU Colloquium*, **15**, 248.
Smak, J., 1971b. *Acta Astronomica*, **21**, 15.
Starrfield, S., Sparks, W. M., Truran, J. W. & Kutter, G. S., 1972. *Astrophys. J.*, **176**, 169.
Starrfield, S., Sparks, W. M. & Truran, J. W., 1974. in press.
Syunyaev, R. A., 1972. *Astr. Zh.*, **49**, 1153 (1973, *Sov. Astr.-Astr. J.*, **16**, 941).
Thackeray, A. D. & Hutchings, J. B., 1974. *Mon. Not. R. astr. Soc.*, **167**, 319.
Warner, B., 1974a. *Mon. Not. R. astr. Soc.*, **168**, 235.
Warner, B., 1974b. *Mon. Not. astr. Soc. Sth Afr.*, **33**, 21.
Warner, B. & Brickhill, A. J., 1974. *Mon. Not. R. astr. Soc.*, **166**, 673.
Warner, B. & Nather, R. E., 1971. *Mon. Not. R. astr. Soc.*, **152**, 219.
Warner, B. & Robinson, E. L., 1972. *Nature Phys. Sci.*, **239**, 2.
Webster, L., 1966. *Publ. astr. Soc. Pacific*, **78**, 136.