THE AERODYNAMICS OF SUPERSONIC BIPLANES*

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1. Introduction. The drag of supersonic wings increases rapidly with increasing thickness. This has led to some speculation about the potentialities of supersonic biplanes, which might afford structural strength and rigidity by virtue of their external structure and hence permit the use of thinner airfoil profiles than would be possible in a monoplane. This brings to mind the possibilities, recognized for several years, of actually reducing the drag of wings by providing the proper wave interactions between the upper and lower wings of a biplane arrangement. That this can be done in the two-dimensional case, i.e., in a biplane of infinite span, was proved in 1935 by Busemann (Ref. 1), who showed that the drag (excluding viscous drag) can be made equal to zero for a biplane at zero lift.

Clearly, it is of interest to study the aerodynamics of finite-span biplanes at supersonic speeds, and especially to estimate the effects of the wing tips on the drag of a finite "Busemann biplane." In this paper we shall report briefly on an investigation (Ref. 8) of the aerodynamics of biplanes having rectangular wings of identical planform. To simplify the work, we shall use here the small-perturbation linear theory, in which all shock and expansion waves are replaced by Mach waves inclined at the free-stream Mach angle. Busemann, to be sure, did not make this approximation in his two-dimensional biplane studies; nevertheless, it should be permissible for the slender airfoils that are of greatest practical interest.

In the linearized theory, the Busemann biplane arrangement becomes the one shown in Fig. 1, i.e., the top and bottom surfaces are flat, the leading-edge Mach wave of either wing intersects the other wing at mid-chord, and the airfoil slopes are related by the formulas, for $x > c/2$,

$$Y_1'(x) = -Y_2'(x - c/2), \qquad Y_2'(x) = -Y_1'(x - c/2).$$

The typical case is then simply that of two isosceles triangles pointing at each other.

In this investigation, the Busemann relationship between gap, chord, and Mach angles shown in Fig. 1 will always be assumed, but it will not be necessary to specify the shape of the profile in deriving some general results. It will be shown that the velocity potential, including all interaction effects, can be calculated by means of integrations involving the wing surface slopes only. The general results will be applied to the numerical calculation of the wave drag, at zero lift, of the typical Busemann arrangement having triangular wing sections.

2. Formulas for source distributions. The equation satisfied by the disturbance velocity potential ϕ in the linearized theory is

$$\beta^2\phi_{xx} - \phi_{yy} - \phi_{zz} = 0; \qquad \beta^2 = M^2 - 1. \tag{1}$$

where subscripts denote partial differentiation with respect to the rectangular Cartesian

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coordinates x, y, z. Here M denotes the free-stream Mach number, and the coordinate x is taken in the direction of the undisturbed stream. It has been assumed in deriving Eq. (1) that ϕ_x, ϕ_y, and ϕ_z are small compared to the stream speed, U. A consistent approximate formula for the pressure coefficient is

$$C_p = 2(p - p_0)/\rho_0 U^2 = -2\phi_x/U, \tag{2}$$

where p_0, ρ_0 are the pressure and density of the undisturbed stream.

An elementary solution of Eq. (1) is the so-called supersonic source, $\phi(x, y, z) = [(x - \xi)^2 - \beta^2(y - \eta)^2 - \beta^2(z - \zeta)^2]^{-1/2}$, provided that the value zero is taken outside

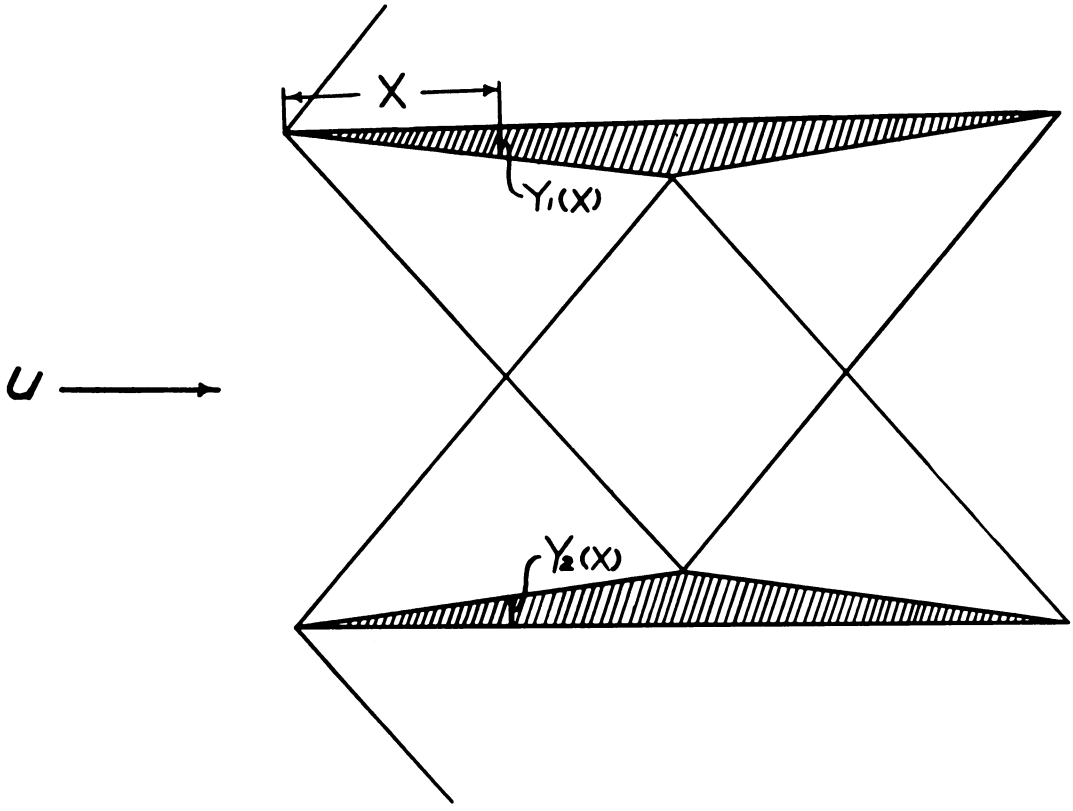

FIG. 1. The Busemann biplane arrangement.

of the Mach cone that originates at the point ξ, η, ζ. For brevity, we shall adopt the following notation:

$$\mu(z) \equiv [(x - \xi)^2 - \beta^2(y - \eta)^2 - \beta^2 z^2]^{-1/2}.$$

It is well known (Refs. 2, 3) that a continuous distribution q of these singularities over a surface parallel to the flow yields a solution satisfying Eq. (1) and the boundary condition $\partial\phi/\partial n = \pi q$ on the surface. Moreover, Evvard (Ref. 4) has shown how a distribution of these sources over a fictitious diaphragm at a wing tip can be used to account for the interaction of upper and lower surfaces of a monoplane wing.

We shall adopt Evvard's scheme here for the calculation of tip effects for both upper and lower wings, placing a diaphragm at each wing tip and introducing the conditions that these diaphragms are stream surfaces of the flow. The potential at points on the top (T) and bottom (B) surfaces of the upper (u) wing is given by

$$\phi_{uT}(x, y) = -\int_S q_{uT}\mu(0)\, dS, \tag{3}$$

$$\phi_{uB}(x, y) = -\int_S q_{uB}\mu(0)\, dS - \int_{S'} q_{lT}\mu(c)\, dS, \tag{4}$$

and there are analogous formulas for the lower (l) wing. The areas of integration S, on the wing under consideration, and S' on the other wing, are shown in Fig. 2.

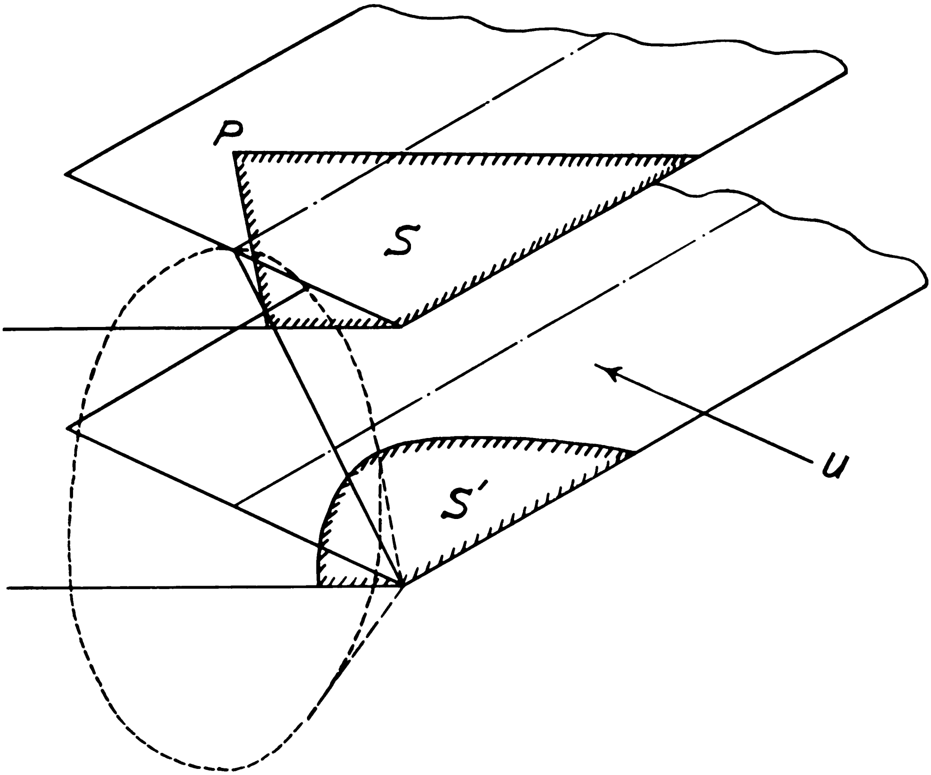

FIG. 2. Diagram showing areas of integration, S of wing considered, and S' of other wing, including portions of diaphragms.

Now the integrations over portions of S and S' can be simplified immediately by use of monoplane results. First of all, it is clear that, in all areas unaffected by biplane interaction, the wing-surface boundary condition requires that $q = U\sigma/\pi$ where σ is the slope of the wing profile in the x direction. Moreover, Evvard has shown, that for monoplanes—and therefore for biplane regions unaffected by interwing interaction—the integration over the diaphragm can be replaced by another integration over part of the wing. For any point forward of mid-chord, i.e., $x < a$, there can be no biplane interaction, hence it is convenient to write the relatively simple expressions for these points before going on to treat the interacting regions.

$x < a$: *no biplane interaction*: Here monoplane results are applicable. For both upper and lower wings, we have (cf. Ref. 4)

$$\phi_T(x, y) = -\frac{U}{\pi}\int_{S_I} \sigma_T\mu(0)\, dS - \frac{U}{2\pi}\int_{S_{I_0}} (\sigma_B - \sigma_T)\mu(0)\, dS, \tag{5}$$

$$\phi_B(x, y) = -\frac{U}{\pi}\int_{S_I} \sigma_B\mu(0)\, dS - \frac{U}{2\pi}\int_{S_{I_0}} (\sigma_T - \sigma_B)\mu(0)\, dS. \tag{6}$$

$x > a$: We consider now a point on the *upper wing, top surface*. If the point lies forward of the Mach line from the tip mid-chord (outside of area N in Fig. 3), there is again

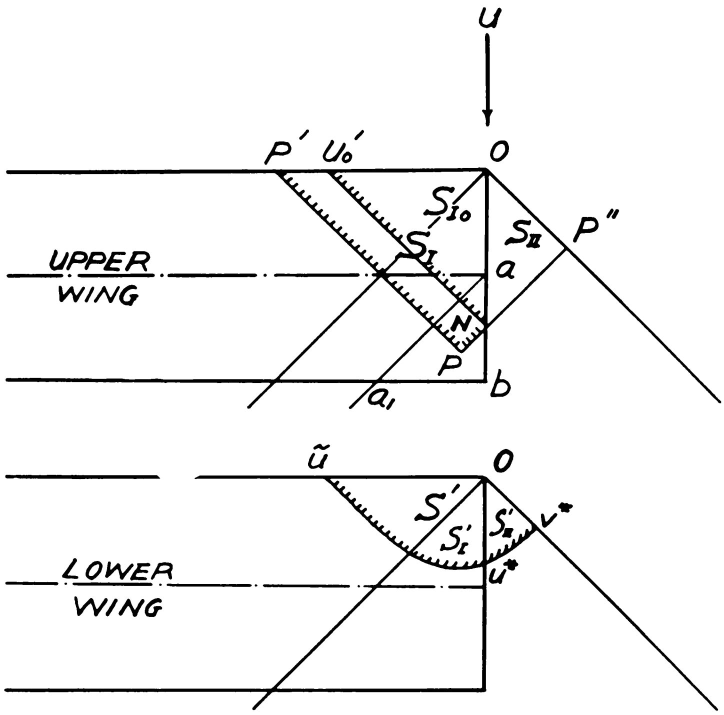

Fig. 3. Diagram defining notation used in calculations for the upper wing.
$aa'b$—N, $PP'Ou_0$—S_I, u_0OP''—S_{II},
$u_0u_0'O$—S_{I_0}, $Ou^*\tilde{u}$—S_I', Ou^*v^*—S_{II}'.

no biplane interference and Eqs. (5) and (6) apply. For a point in N, however, there exists an effect of the lower wing, transmitted through the interaction region of the tip diaphragm. We can write

$$\phi_{uT}(x, y) = -\frac{U}{\pi}\int_{S_I} \sigma_{uT}\mu(0)\, dS - \frac{U}{\pi}\int_{S_{II}} \lambda_u\mu(0)\, dS. \tag{7}$$

Here, and in subsequent formulas, we denote by $\lambda(\xi, \eta)$ the slopes of the tip diaphragms of upper and lower wings. Thus, for any point on the top of the upper-wing diaphragm, q_{uT} is equal to $U\lambda_u/\pi$, and this value has been used in Eq. (7). In regions unaffected by biplane interaction (e.g. for $\xi < a$), $\lambda(\xi, \eta)$ is the same as for a monoplane, and Evvard's results will be used for such regions. In interacting regions λ is still unknown, of course; its determination constitutes the main problem of this investigation. We shall postpone this to the next section, after writing an analogous formula for points on the bottom surface of the upper wing.

All points of the *bottom surface* of the *upper wing*, for which $x > a$ are affected by biplane interaction. Let S'_I and S'_{II} denote the areas of the lower wing and its diaphragm that affect the point (x, y). The wing-surface boundary condition is

$$q_{uB}(x, y) + \frac{U}{\pi^2} \frac{\partial}{\partial c} \left[\int_{S'_I} \sigma_{lT} \mu(c) \, dS + \int_{S'_{II}} \lambda_l \mu(c) \, dS \right] = \frac{U}{\pi} \sigma_{uB}(x, y). \tag{8}$$

This is an explicit formula for $q_{uB}(x, y)$, involving only known quantities. It may be noted that in the region S'_{II}, q_{lT} has been put equal to $U\lambda_l/\pi$. Moreover, here λ_l is a monoplane value unaffected by biplane interference, and is therefore known from Evvard's work. We now have

$$\begin{aligned} \phi_{uB}(x, y) = &- \int_{S_I + S_{II}} q_{uB} \mu(0) \, dS \\ &- \frac{U}{\pi} \int_{S'_I} \sigma_{lT} \mu(c) \, dS - \frac{U}{\pi} \int_{S'_{II}} \lambda_l \mu(c) \, dS, \end{aligned} \tag{9}$$

where q_{uB} in S_I is known from Eq. (8) and λ_l in S'_{II} is known from monoplane theory. Again the calculation of the diaphragm source distribution, q_{uB} in S_{II}, is postponed to the next section.

For the *lower wing* there are formulas exactly analogous to Eqs. (7), (8) and (9), which will not be written out here.

3. Calculation of diaphragm distributions. The conditions that insure that the tip diaphragms will be stream surfaces are the conditions of equal slope and equal pressure on top and bottom. Since, as Evvard has pointed out (Ref. 5), the diaphragms of a rectangular wing tip are not vortex sheets, equal pressures imply equal values of ϕ, the perturbation velocity potential. We have, then, in region S_{II},

$$\frac{\partial \phi_T}{\partial z} = \frac{\partial \phi_B}{\partial z} \qquad \text{and} \qquad \phi_T = \phi_B \, . \tag{10}$$

The first of these equations leads to

$$\begin{aligned} \frac{U}{\pi} \lambda_u(x, y) &= q_{uT}(x, y) \\ &= -q_{uB}(x, y) - \frac{U}{\pi^2} \frac{\partial}{\partial c} \left[\int_{S'_I} \sigma_{lT} \mu(c) \, dS + \int_{S'_{II}} \lambda_l \mu(c) \, dS \right]. \end{aligned} \tag{11}$$

The second Eq. (10) states that, in S_{II},

$$\begin{aligned} &-\frac{U}{\pi} \int_{S_I} \sigma_{uT} \mu(0) \, dS - \frac{U}{\pi} \int_{S_{II}} \lambda_u \mu(0) \, dS \\ &\quad = - \int_{S_I + S_{II}} q_{uB} \mu(0) \, dS - \frac{U}{\pi} \int_{S'_I} \sigma_{lT} \mu(c) \, dS - \frac{U}{\pi} \int_{S'_{II}} \lambda_l \mu(c) \, dS, \end{aligned} \tag{12}$$

where q_{uB} in S_I and S_{II} is given by Eqs. (8) and (11), respectively. We have now an integral equation for the diaphragm slope λ_u : for points x, y in S_{II} ,

$$2 \int_{S_{II}} \lambda_u \mu(0)\, dS = \int_{S_I} (\sigma_{uB} - \sigma_{uT})\mu(0)\, dS$$

$$+ \int_{S'_I} \sigma_{lT}\mu(c)\, dS + \int_{S'_{II}} \lambda_l \mu(c)\, dS \tag{13}$$

$$- \frac{1}{\pi} \int_{S_I + S_{II}} \mu(0) \frac{\partial}{\partial c} \left[\int_{S'_I} \sigma_{lT}\mu(c)\, dS^* + \int_{S'_{II}} \lambda_l \mu(c)\, dS^* \right] dS.$$

There is an analogous equation for λ_l , which will not be written out.

Eq. (13) is to be satisfied for all points x, y on the upper-wing diaphragm. For some areas, there is no biplane interaction, i.e., S'_I and S'_{II} vanish, so that the second and third integrals on the right side of Eq. (13) disappear. It is clear that for these points the third integral vanishes as well, since S_I and S_{II} do not contain any points ξ, η affected by interaction. Consequently, for non-interacting points x, y, Eq. (13) reduces to Evvard's integral equation for the diaphragm slope of a monoplane (Ref. 4).

4. Solution of the integral equation. Eq. (13) can be written in the form

$$\int_{S_{II}} \lambda_u \mu(0)\, dS = F_1(x, y) \tag{14}$$

for points x, y in S_{II} , where $2F_1(x, y)$ denotes the entire right-hand side of Eq. (13), and involves only known functions. We now introduce the new coordinates u, v, measured along the two families of Mach lines on the wing in question:

$$u = \frac{M}{2\beta}(\xi + \beta\eta), \qquad v = \frac{M}{2\beta}(\xi - \beta\eta),$$

$$\xi = \frac{\beta}{M}(u + v), \qquad \eta = \frac{1}{M}(u - v),$$

$$J = \frac{\partial(\xi, \eta)}{\partial(u, v)} = -\frac{2\beta}{M^2}, \tag{15}$$

$$\mu(z) \equiv \{(x - \xi)^2 - \beta^2(y - \eta)^2 - \beta^2 z^2\}^{-1/2}$$

$$= \frac{M}{2\beta}\{(u_1 - u)(v_1 - v) - M^2 z^2/4\}^{-1/2}.$$

Our integral equation now takes the form

$$\int_0^{u_1} \frac{du}{(u_1 - u)^{1/2}} \int_u^{v_1} \frac{\lambda(u, v)\, dv}{(v_1 - v)^{1/2}} = F(u_1 , v_1) \tag{16}$$

for points u_1 , v_1 in S_{II} .

The solution can now be found by means of the following process:

$$\int_0^{u'} \frac{F(u_1, v_1)\,du_1}{(u' - u_1)^{1/2}} = \int_0^{u'} \frac{du_1}{(u' - u_1)^{1/2}} \int_0^{u_1} \frac{du}{(u_1 - u)^{1/2}} \int_u^{v_1} \frac{\lambda(u, v)\,dv}{(v_1 - v)^{1/2}}$$

$$= \int_0^{u'} \frac{du_1}{(u' - u_1)^{1/2}} \int_0^{u_1} \frac{H(u, v_1)\,du}{(u_1 - u)^{1/2}}, \tag{17}$$

say,

$$= \int_0^{u'} H(u, v_1)\,du \int_u^{u'} \frac{du_1}{(u' - u_1)^{1/2}(u_1 - u)^{1/2}} = \pi \int_0^{u'} H(u, v_1)\,du.$$

Differentiating this result with respect to u', we have

$$\frac{\partial}{\partial u'} \int_0^{u'} \frac{F(u_1, v_1)\,du_1}{(u' - u_1)^{1/2}} = \pi \int_{u_1}^{v_1} \frac{\lambda(u', v)\,dv}{(v_1 - v)^{1/2}}. \tag{18}$$

We now multiply both sides of Eq. (18) by $(v' - v_1)^{-1/2}$, integrate with respect to v_1, and exchange order of integration in a manner similar to that just employed. The result is

$$\int_{u'}^{v'} \frac{dv_1}{(v' - v_1)^{1/2}} \left(\frac{\partial}{\partial u'} \int_0^{u'} \frac{F(u_1, v_1)\,du_1}{(u' - u_1)^{1/2}} \right) = \pi^2 \int_{u'}^{v'} \lambda(u', v)\,dv \tag{19}$$

which implies (dropping the primes)

$$\lambda(u, v) = \frac{1}{\pi^2} \frac{\partial}{\partial v} \int_u^v \left(\frac{\partial}{\partial u} \int_0^u \frac{F(u_1, v_1)\,du_1}{(u - u_1)^{1/2}} \right) \frac{dv_1}{(v - v_1)^{1/2}}. \tag{20}$$

This solution can be used to calculate the slopes λ_u in regions of interaction. This completes Eq. (7) for ϕ_{uT}, and, by use of Eq. (11), also completes Eq. (9) for ϕ_{uB}. Eq. (20) constitutes a generalization of Evvard's expression for the tip-diaphragm slope, to which, in fact, it immediately reduces when u, v lie in a region free of biplane interaction.

5. Calculation of the potential. Although the biplane problem is now completely solved in principle, the straightforward calculation of ϕ, especially for regions of biplane interference, by substitution in Eqs. (7) and (9), is extremely tedious. Fortunately, as will now be shown, it is possible to eliminate entirely the integration involving λ_u in these two formulas.

In both Eqs. (7) and (9), the term involving λ_u is

$$\int_{S_{II}} \lambda_u \mu(0)\,dS = -\frac{1}{M} \int_0^{v_1} \frac{du}{(u_1 - u)^{1/2}} \int_u^{v_1} \frac{\lambda_u(u, v)\,dv}{(v_1 - v)^{1/2}}, \tag{21}$$

where now u_1, v_1 lie in region S_I.

We return to Eq. (13), which holds for points in S_{II}, and write it in the form

$$\int_0^{u_1} \frac{du}{(u_1 - u)^{1/2}} G(u, v_1) = \frac{M\pi}{2U} \phi'(u_1, v_1), \qquad (u_1 \leqq v_1), \tag{22}$$

where

$$G(u, v_1) = \int_u^{v_1} \frac{\lambda_u(u, v)\,dv}{(v_1 - v)^{1/2}} - \frac{1}{2} \int_{-u}^{u} \frac{(\sigma_{uB} - \sigma_{uT})\,dv}{(v_1 - v)^{1/2}}$$
$$- \frac{1}{2U} \int_{-u}^{v_1} \frac{dv}{(v_1 - v)^{1/2}} \frac{\partial}{\partial c} \phi'(u, v) \tag{23}$$

and

$$\phi'(u_1, v_1) = -\frac{U}{\pi}\left\{\int_{S'_I} \sigma_{lT}\mu(c)\, dS + \int_{S'_{II}} \lambda_l\mu(c)\, dS\right\}. \tag{24}$$

Actually, ϕ' is the potential contributed at u_1, v_1 by the lower wing.

The solution of Eq. (22) can be written down immediately (Ref. 6); viz.,

$$G(u, v_1) = \frac{M}{2U}\frac{\partial}{\partial u}\int_0^u \frac{\phi'(u', v_1)\, du'}{(u - u')^{1/2}}, \qquad (u \leqq v_1). \tag{25}$$

Since Eq. (22) is correct only for points u_1, v_1 in S_{II}—i.e. for $u_1 \leqq v_1$—we must restrict u in Eq. (25) as indicated.

Now for points outside of the interaction region, i.e., for $u' \leqq M^2c^2/4v_1$, the interaction potential $\phi'(u', v_1)$ is zero. Thus $G(u, v_1)$ is also zero for $u < M^2c^2/4v_1$.

We can now consider an integral involving $G(u, v_1)$; i.e.,

$$I(u_1, v_1, \kappa) \equiv \int_0^\kappa \frac{du}{(u_1 - u)^{1/2}} G(u, v_1) = \int_{M^2c^2/4v_1}^\kappa \frac{du}{(u_1 - u)^{1/2}} G(u, v_1),$$

where $\kappa \leqq u_1$.

If $\kappa \leqq v_1$ also, $G(u, v_1)$ can be taken from Eq. (25):

$$\begin{aligned} I(u_1, v_1, \kappa) &= \frac{M}{2U}\int_{M^2c^2/4v_1}^\kappa \frac{du}{(u_1 - u)^{1/2}}\left\{\frac{\partial}{\partial u}\int_{M^2c^2/4v_1}^u \frac{\phi'(u', v_1)\, du'}{(u - u')^{1/2}}\right\} \\ &= \frac{M}{2U}\int_{M^2c^2/4v_1}^\kappa \phi'(u', v_1)\left(\frac{\kappa - u'}{u_1 - \kappa}\right)^{1/2}\left[\frac{1}{\kappa - u'} - \frac{1}{u_1 - u'}\right] du' \end{aligned} \tag{26}$$

after some manipulation. Recalling the meaning of $G(u, v_1)$, (Eq. (23)), we can write Eq. (26) as

$$\begin{aligned} &\int_0^\kappa \frac{du}{(u_1 - u)^{1/2}}\int_u^{v_1} \frac{\lambda_u(u, v)\, dv}{(v_1 - v)^{1/2}} \\ &\quad = \frac{M}{2U}\int_{M^2c^2/4v_1}^\kappa \phi'(u', v_1)\left(\frac{\kappa - u'}{u_1 - \kappa}\right)^{1/2}\left[\frac{1}{\kappa - u'} - \frac{1}{u_1 - u'}\right] du' \\ &\qquad + \frac{1}{2}\int_0^\kappa \frac{du}{(u_1 - u)^{1/2}}\left\{\int_{-u}^u \frac{(\sigma_{uB} - \sigma_{uT})\, dv}{(v_1 - v)^{1/2}} + \frac{1}{U}\int_{-u}^{v_1} \frac{dv}{(v_1 - v)^{1/2}}\frac{\partial}{\partial c}\phi'(u, v)\right\}. \end{aligned} \tag{27}$$

Since the only restrictions on Eq. (27) are $\kappa \leqq u_1$, and $\kappa \leqq v_1$, it is exactly the result we need for Eq. (21), in which $\kappa = v_1 \leqq u_1$.

We are now prepared to write complete expressions for the potential on top and bottom surfaces of the upper wing, by substitution in Eqs. (7) and (9). Let S_{I_0} be the portion of S_I for which $u \leqq v_1$, as indicated in Fig. 3; then

$$\begin{aligned} \phi_{uT}(x, y) = &-\frac{U}{\pi}\int_{S_I} \sigma_{uT}\mu(0)\, dS - \frac{U}{2\pi}\int_{S_{I_0}} (\sigma_{uB} - \sigma_{uT})\mu(0)\, dS \\ &- \frac{1}{2\pi}\int_{S_{I_0}+S_{II}} \mu(0)\frac{\partial}{\partial c}\phi'(u, v)\, dS \\ &+ \frac{1}{2\pi}\int_{M^2c^2/4v_1}^{v_1} \phi'(u', v_1)\left(\frac{v_1 - u'}{u_1 - v_1}\right)^{1/2}\left[\frac{1}{v_1 - u'} - \frac{1}{u_1 - u'}\right] du', \end{aligned} \tag{28}$$

$$\begin{aligned}\phi_{uB}(x, y) = &-\frac{U}{\pi}\int_{S_I} \sigma_{uB}\mu(0)\,dS - \frac{1}{\pi}\int_{S_I+S_{II}} \mu(0)\,\frac{\partial}{\partial c}\,\phi'(u, v)\,dS \\ &+\frac{U}{2\pi}\int_{S_{I_0}} (\sigma_{uB} - \sigma_{uT})\mu(0)\,dS + \frac{1}{2\pi}\int_{S_{I_0}+S_{II}} \mu(0)\,\frac{\partial}{\partial c}\,\phi'(u, v)\,dS \\ &-\frac{1}{2\pi}\int_{M^2c^2/4v_1}^{v_1} \phi'(u', v_1)\left(\frac{v_1 - u'}{u_1 - v_1}\right)^{1/2}\left[\frac{1}{v_1 - u'} - \frac{1}{u_1 - u'}\right] du' \\ &+\phi'(x, y).\end{aligned} \tag{29}$$

Formulas (28) and (29) permit the calculation of the potential, and consequently the pressure distribution, on the biplane. It is seen that, whereas we have succeeded in eliminating the integrals involving λ_u, for the upper wing, we are left with integrals involving λ_l, to be taken over certain interaction-free areas. In fact, if interplane interaction of a higher order were encountered, such as an area of the lower wing influenced by interacting regions of the upper wing, it would always be possible to eliminate the λ integral expressing the last stage of tip interaction.

5. Application of results: The wave drag of a finite Busemann biplane at zero lift. The general results obtained here have been applied to one typical practical case, to date.

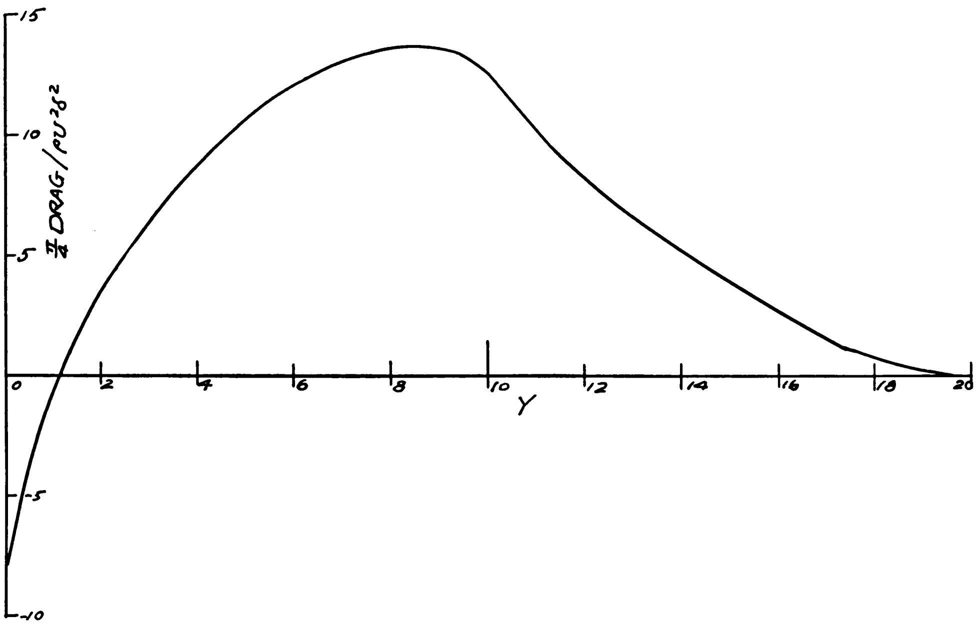

FIG. 4. Spanwise distribution of wave drag near the tip of a rectangular supersonic biplane wing at $M = \sqrt{2}$. The distance y is measured from the tip, and the chord is equal to 20.

This is the case of the finite Busemann biplane, having isosceles-triangular profiles, at zero lift, at a Mach number of $\sqrt{2}$. The computations have been carried out on a computing machine, and integrations have been done by planimeter. These are only preliminary results of an investigation that is still in progress.

These numerical results are shown in Fig. 4, where spanwise distribution of wave drag is plotted. The average drag coefficient of the biplane has been computed from Fig. 4 and found to be

$$C_D = \frac{\text{Total wave drag}}{\rho_0 U^2 S} = 0.823\delta^2/A. \tag{30}$$

where δ denotes the leading-edge angle of the profiles, S the area of one wing, and A the aspect ratio of one wing. It is interesting to compare the wave drag coefficient of a rectangular wing of double-wedge profile, which is

$$C_D = 4\delta^2 \tag{31}$$

For the monoplane, δ denotes the half-angle of the wedge. As would be expected, the ratio of biplane to monoplane wave drags diminishes with increasing aspect ratio.

It is not difficult to show that the force coefficients calculated according to this theory for any Mach number M_1, can be extended to any other value of M by means of the following similarity rule

$$C_D(M) = C_D(M_1)\frac{1 - M_1^2}{1 - M^2} \tag{32}$$

The same correction would apply to the lift coefficient, $C_L(M)$. It is to be understood here that the coefficients $C_D(M)$ and $C_L(M)$ do not apply to the same biplane as $C_D(M_1)$ and $C_L(M_1)$ but to a new configuration proportioned as in Fig. 1 at the Mach number M.

In particular, the result of the present numerical work can be written

$$C_D = 0.823\delta^2/(A\beta^2). \tag{33}$$

References

1. A. Busemann, *Aerodynamischer Auftrieb bei Ueberschallgeschwindigkeit.* Luftfahrtforschung **12**, 210-220, 1935.
2. Jacques Hadamard, *Lectures on Cauchy's problem in linear partial differential equations*, Yale University Press, 1928.
3. Max A. Heaslet and Harvard Lomax, *The use of source-sink and doublet distributions extended to the solution of arbitrary boundary value problems in supersonic flow*, N.A.C.A. Tech. Note No. 1515, 1948.
4. John C. Evvard, *Distribution of wave drag and lift in the vicinity of wing tips at supersonic speeds*, N.A.C.A. Tech. Note No. 1382, 1947.
5. John C. Evvard and L. R. Turner, *Theoretical lift distribution and upwash velocities for thin wings at supersonic speeds*, N.A.C.A. Tech. Note No. 1484, 1947.
6. Maxime Bocher, *An introduction to the study of integral equations*, Cambridge University Press, 1909.
7. M. J. Lighthill, *The supersonic theory of wings of finite span*, Rep. & Memo. No. 2001, British Aero. Res. Comm., 1944.
8. H. S. Tan, *The wave drag of a supersonic biplane of finite span*, A thesis submitted in partial fulfillment of the requirements for the degree Ph.D., Cornell University, 1949.