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THE AGGREGATE IMPLICATIONS OF MACHINE REPLACEMENT: THEORY AND EVIDENCE

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ABSTRACT

This paper studies an economy in which producers must incur resource costs to replace depreciated machines. The process of costly replacement and depreciation creates endogenous fluctuations in productivity, employment and output of a single producer. We also explore the spillover effects of machine replacement by multiple, independent producers. The implications of our model are generally consistent with observed monthly output and productivity fluctuations in automobile plants and with monthly variations in employment and production in the manufacturing sector.

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I. Motivation

This paper investigates the aggregate implications of a non-convexity in technology: the firm's choice of technique. In particular, we study a machine replacement problem in which a firm must decide whether or not to install a new machine or continue to produce with an older, depreciated machine. We first characterize the solution to this problem for a single agent and then study the spillover effects of machine replacement on other aspects of economic activity. The paper concludes with some empirical evidence on machine replacement by automobile producers and its implications for monthly fluctuations in production, employment and productivity in manufacturing. For the most part, our analysis concerns seasonal fluctuations though the work is suggestive for business cycles as well.[1]

In general, the point of introducing non-convexities into macroeconomic models is two-fold.[2] First, in order to induce the large fluctuations in economic activity observed in the data, macroeconomists often study stochastic models in which shocks to the environment induce variations in output and employment through intertemporal substitution effects. Non-convex economies present an alternative in that endogenous fluctuations may emerge in these environments.[3] Second, models with non-convexities may exacerbate the influence of shocks so that more of the variation in economic activity is explained within the model. In particular, small variations in exogenous variables may generate large responses in endogenous variables.

Here we consider the aggregate implications of the decision by a firm regarding the replacement of its machine: i.e. the machine replacement problem. In general, consider a firm for which the productivity of capital falls over time due to depreciation.[4] At any point in time, the firm can replace its capital with a new machine that is of current vintage.[5] We view this as a discrete decision, replace or not, and one that uses the firm's resources.[6] In particular, our specification highlights the lumpy nature of the investment process stemming from a non-convexity in the adjustment process. Machine replacement naturally creates endogenous fluctuations in output which are positively correlated with productivity so that exogenous productivity shocks, as in a seasonal version of Kydland-Prescott [1982] for example, are not necessary to generate this positive correlation.[7]

Section II presents our analysis of the Robinson Crusoe problem for this environment. Here we focus on the predictions of this model for employment, output and productivity. Section III considers the effects of shocks on the timing of machine replacement. We find that machine replacement is more likely at the end of economic downturns since the resource costs of replacing machines is less than if the machines are replaced in other times. This result highlights the potential link between seasonal fluctuations generated by machine replacement and the stage of the business cycle. Section IV embeds this choice problem into a multi-sector general equilibrium model to illustrate the spillover effects of machine replacement on other sectors. Section V discusses timing considerations when there are multiple producers solving the machine replacement problem. In this section, we provide conditions under which firms will have an incentive to synchronize machine replacement so that this discrete decision is not smoothed by aggregation.

Finally, empirical evidence on the importance of machine replacement and other discrete activities is provided in Section VI. In particular, we illustrate the importance of machine replacement by looking at plant level data for some U.S. automobile manufacturers for 1978-85. Among other things, we find that for these plants the variance of production exceeds that of sales due to the discrete decisions in the production process and that, as implied by our model, machine replacement is more likely to arise during periods of low activity. Further, we find that machine replacement (or retooling) is synchronized across producers in both the inter-war years and in the 1978-85 period. We also relate the empirical findings of Beaulieu-Miron [1990] on the seasonal patterns of production throughout manufacturing to the spillover implications of our model.

II. Machine Replacement for Robinson Crusoe

We begin our analysis of the machine replacement problem (MRP) by considering the dynamic choice problem of a single producer, Robinson Crusoe (RC). This agent lives forever; consuming and producing in each period of life. Period t utility is given by $u(c_t) - g(n_t)$ where c_t is period t consumption and n_t is period t labor supply. Assume that $u(\cdot)$ is continuously differentiable, strictly increasing and strictly concave and that $g(\cdot)$ is continuously differentiable, strictly increasing and strictly convex. Further, we normalize utility so that $u(0)=g(0)=0$. RC is endowed with a unit of leisure time in each period so $n_t \leq 1$ for all t. RC discounts the future at rate $\beta \epsilon (0,1]$.

In each period, output, y_t, is produced from labor according to a linear technology of $y_t=\theta_t n_t$, where θ_t indexes the current state of technology. Output cannot be stored so that $c_t=y_t$. We discuss the implications of allowing storage below.

The key to the specification of technology is the determination of θ_t. In each period, RC chooses whether or not to replace his machine. If RC chooses to replace the machine in period t, then $\theta_{t+1}=\hat{\theta}$. If RC chooses not to replace the machine, then $\theta_{t+1}=\rho\theta_t$ where $\rho \epsilon (0,1)$. Let z_t index this choice where $z_t=0$ means that RC did not replace the machine in period t and $z_t=1$ implies that the machine was replaced. If $z_t=1$, then RC incurs a lump sum cost of k units of labor time in period t.[8]

This specification reflects the importance of capital depreciation, given by ρ. The process of replacement may include in it the production of the capital good and its installation at a resource cost of k. Under that interpretation, the one period lag in the replacement process contains both a time to build component and a time delay due to installation. The resource cost to replace capital would then include the labor needed to produce and install the lumpy capital good.

We also have assumed that in the event a machine is replaced, its productivity is independent of time -- there is no technological advance in this model. One could augment the model to allow the productivity of a new machine to grow with time. That is, suppose that $\hat{\theta}_t=\eta\hat{\theta}_{t-1}$ where $\eta>1$ is the rate of technological progress and $\hat{\theta}_t$ is the productivity of a new period t machine. Further, one might argue that as the productivity of the new machines increase, the cost of installation might increase as well due to higher costs of producing and installing (including worker training) the new machines. Analyzing the machine replacement problem in this growth environment is worth further consideration.

Finally, we have assumed that once the new machine is installed, the old one is removed from production entirely. This is appropriate if one thinks literally of replacement as the substitution of one machine for another. Alternatively, one could consider a model in which there is a resource cost from adding a new plant to the production process so that plants with different vintages of capital could be in use simultaneously.[9] Labor would then be allocated across plants in an optimal fashion implying that the opening of a new plant would raise productivity in all plants due to the substitution of labor resources from old to new plants. In addition, if there is a fixed cost of production each period, over time the older plants would be closed.

The cost of replacing the machine is modeled as a lump sum labor requirement. This specification implies that, at the margin, producing more output is costlier when a machine is being replaced and is a simple way to introduce a congestion effect into the technology. Our point here is to model the phenomenon that a firm replacing its machine must incur an increase in the cost of producing output so that production is lower during periods of machine replacement.[10]

Given this structure, the optimization problem of RC is

$$(1)\quad \max_{\{n_t\},\{z_t\}} \sum_{t=0}^{\infty} \beta^t [u(c_t) - g(n_t + z_t k)]$$

subject to:

$$(1.a)\quad c_t = n_t \theta_t \quad , \; z_t \in \{0,1\} \quad \text{and}$$

$$(1.b)\quad \theta_t = \begin{cases} \rho\theta_{t-1} & \text{if } z_{t-1} = 0 \\ \hat{\theta} & \text{if } z_{t-1} = 1. \end{cases}$$

To analyze this problem, note that the single state variable is the state of technology in the previous period. Denote the productivity of this period's machine by θ.[11] If RC innovates, then his utility from this period onward is given by

$$(2)\quad V^I(\theta) = W^I(\theta) + \beta V(\hat{\theta}) \qquad \text{where}$$

$$(3)\quad W^I(\theta) = \max_n u(n\theta) - g(n+k)$$

So the value of innovating is given by the current utility from producing with a machine of productivity θ given that the firm is devoting k units of time to the replacement process. This current utility is given by $W^I(\theta)$. Let $n^I(\theta)$ be the optimal value of labor input when RC innovates and the state of productivity is θ. Once a machine is replaced, then RC's capital in the following period has productivity of $\hat{\theta}$. The value of that machine is given by $V(\hat{\theta})$. The function $V(\cdot)$ is defined below.

If RC does not innovate, then his utility is given by

$$(4) \qquad V^N(\theta) = W^N(\theta) + \beta V(\rho\theta) \qquad \text{where}$$

$$(5) \qquad W^N(\theta) = \max_n \; u(n\theta) - g(n)$$

Here $W^N(\theta)$ is the utility from producing with a machine of productivity θ when the machine is not being replaced. Let $n^N(\theta)$ be the optimal value of labor input in the optimization problem given by (5). We assume $u'(0)>g'(0)$ so that $n^N(\theta)>0$ for $\theta>0$.[12]

Finally, $V(\theta) = \max\{V^I(\theta), V^N(\theta)\}$. Note that both $V^I(\theta)$ and $V^N(\theta)$ are strictly increasing functions of θ since $W^j(\theta)$ is increasing in θ for j=I,N. Therefore $V(\theta)$ is also strictly increasing in θ.

So, in each period, RC decides whether to replace or not by comparing the value of replacement with the value of continuing with the depreciated machine in the following period. We now consider some properties of the solution to this problem.

Lemma 1: If $cu'(c)$ is an increasing function of c, then an increase in θ increases current utility more when the machine is not being replaced than when it is being replaced.

Proof: From (3) and (5), we know that:

$$dW^I(\theta)/d\theta = n^I(\theta)u'(n^I(\theta)\theta) \qquad \text{and}$$

$$dW^N(\theta)/d\theta = n^N(\theta)u'(n^N(\theta)\theta).$$

Since $g(\cdot)$ is strictly convex and $k>0$, $n^I(\theta)<n^N(\theta)\ \forall\ \theta$. Therefore, when $cu'(c)$ is an increasing function of c, $W^N(\theta)$ increases more with an increase in θ than does $W^I(\theta)$. QED.

The point of this lemma is simply that the increase in current welfare from an increase in productivity is higher when the machine is not being replaced. This is a direct consequence of the congestion effect associated with the labor cost, k, of replacing a machine. Since RC produces less when a machine is being replaced, the gain from an increase in θ is lower. The assumption that $cu'(c)$ is an increasing function of c is a restriction on the curvature of $u(\cdot)$ needed to ensure that income effects do not dominate substitution effects.[13] We maintain this assumption throughout the analysis.

<u>Lemma 2</u>: $dV^N(\theta)/d\theta > dV^I(\theta)/d\theta$ for all θ.

<u>Proof</u>: From (2), an increase in θ increases $V^I(\theta)$ due to the resulting change in $W^I(\theta)$ which is positive. From (4), the change in $V^N(\theta)$ comes from the increase in $W^N(\theta)$ and the increase in $V(\rho\theta)$ since the machine is not being replaced. Since $V(\cdot)$ is a non-decreasing function of θ and, from Lemma 1, $dW^I(\theta)/d\theta < dW^N(\theta)/d\theta$, $V^N(\theta)$ increases more due to an increase in θ than does $V^I(\theta)$QED.

Lemma 2 implies that, as a function of θ, $V^N(\theta)$ is steeper than $V^I(\theta)$ for all values of θ. This is an important property in terms of characterizing the value function $V(\theta)$ and hence the decision of RC on whether or not to replace the machine.

The solution to Robinson Crusoe's optimization problem is given by,[14]

<u>Proposition 1</u>: If k is sufficiently close to 0 and ρ is sufficiently close to 1, then there exists a critical level of θ, $\theta^* \epsilon (0,\hat{\theta})$, such that RC replaces the machine iff $\theta \leq \theta^*$.

Proof: We first show that $\theta^*>0$. Assume instead that $\theta^*=0$ which implies $V^N(0)\geq V^I(0)$. $V^N(0)=0$ while $V^I(0)\geq\beta W^N(\hat{\theta}) - g(k)$. At $k=0$, $V^I(0)>0$ since $W^N(\theta)>0$ for all $\theta>0$. Thus at $k=0$, $V^I(0)>V^N(0)=0$ and, by continuity, this holds for k close to 0.

Suppose instead that $\theta^*=\hat{\theta}$, i.e. $V^I(\hat{\theta})\geq V^N(\hat{\theta})$. Here $V^I(\hat{\theta})= (1+\beta)W^I(\hat{\theta}) + \beta^2 W^I(\hat{\theta})/(1-\beta)$ and $V^N(\hat{\theta})=W^N(\hat{\theta}) + \beta W^I(\rho\hat{\theta}) + \beta^2 W^I(\hat{\theta})/(1-\beta)$. At $\rho=1$, since $W^I(\hat{\theta})<W^N(\hat{\theta})$, $V^I(\hat{\theta})<V^N(\hat{\theta})$ and this holds by continuity of $W^I(\theta)$ for ρ near 1.

From these results and Lemma 2, there exists a unique $\theta^*\epsilon(0,\hat{\theta})$ such that $V^N(\theta^*/\rho)\geq V^I(\theta^*/\rho)$ and $V^I(\theta^*)\geq V^N(\theta^*)$, with at least one strict inequality. Uniqueness is guaranteed by Lemma 2. Thus Robinson Crusoe will have an incentive to replace at $\theta=\theta^*$ but not for higher values of θ. Further, once $\theta=\theta^*$, the machine will be replaced. QED.

Figure 1 illustrates this result. The two value functions are drawn as continuous functions of θ which is appropriate for sufficiently short time periods. Following a period of machine replacement, the technology parameter, θ_t, will decrease at a rate determined by ρ until $\theta_t\leq\theta^*$. Ignoring integer problems, the number of periods between replacement is given by the T^* solving $\theta^*=\rho^{T^*}\hat{\theta}$. Then machine replacement will occur and θ will be increased to $\hat{\theta}$.

From $W^N(\theta)$, as given in (5), during the period between machine replacements, the level of employment will fall since $n^N(\theta)$ is increasing in θ. In fact, in the period of replacement, employment in the production of the consumption good will be at its lowest level both because θ is at its minimum and because RC must devote an additional k units of time to replacing the machine. The strict convexity of g(·) implies that machine replacement creates a congestion effect leading to a further reduction in employment.

In some cases, the replacement of machines requires that a plant shut down its operations. This could be modeled by assuming that k=1 so that an entire period's endowment of labor time is used in the replacement of machines.

This model generates a positive correlation between employment and labor productivity. In contrast to Kydland and Prescott [1982], these fluctuations are not driven by exogenous technological change. Instead the productivity variations arise quite naturally through the process of replacing

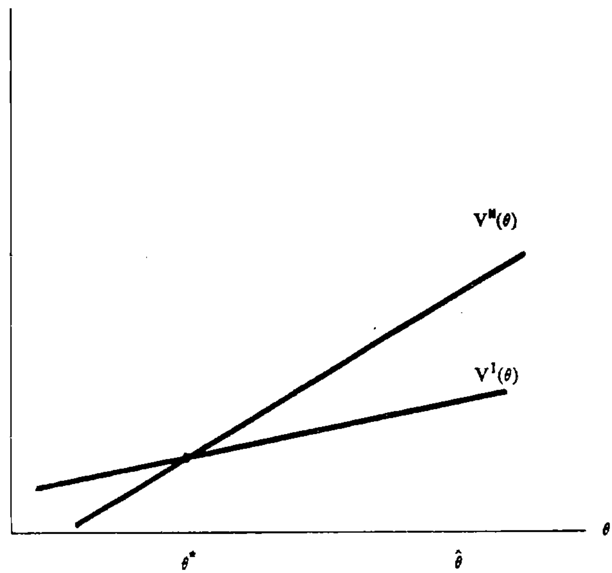


Figure 1

machines.[15] The implied frequency of ouput and productivity fluctuations is dictated by the parameters of the model. In practice, in the auto industry (for example) machine replacement occurs on an annual cycle with resulting seasonal fluctuations (see Section VI for further discussion).

With regard to employment and output fluctuations at the time of replacement, we find that

Lemma 3: $n^N(\theta^*/\rho) > n^I(\theta^*)$ and that $n^N(\theta^*/\rho) < n^I(\theta^*) + k$

Proof: Since cu'(c) is assumed to be an increasing function of c, we know that $n^N(\theta)$ is an increasing function of θ. From (3), $n^I(\theta)$ is decreasing in k as $g(\cdot)$ is strictly convex. Hence, $n^N(\theta^*/\rho) > n^N(\theta^*) > n^I(\theta^*)$ for k>0.

From the first order conditions determining $n^N(\theta^*/\rho)$ and $n^I(\theta^*)$, this implies that $g'(n^N(\theta^*/\rho)) < g'(n^I(\theta^*)+k)$ so that $n^N(\theta^*/\rho) < n^I(\theta^*)+k$. QED.

Using the condition determining employment in the period just prior to replacement ($n^N(\theta^*/\rho)$) and employment of the consumption good during the period of replacement ($n^I(\theta^*)$), Lemma 3 implies that employment in producing the final good is lower in the period of replacement than in the period just prior to replacement ($n^N(\theta^*/\rho) > n^I(\theta^*)$) while total employment in both the production of consumption and investment goods (including replacement) is higher in the period of replacement than in the period just before replacement ($n^N(\theta^*/\rho) < n^I(\theta^*)+k$). In this way, there is an increase in total employment in the economy during replacement and a substitution of employment from consumption to investment goods.

In the model, consumption fluctuates along with output since goods are not storable. This assumption simplifies the analysis since we do not have to be concerned with two state variables. While we have not formally characterized the solution to this problem, we conjecture that the qualitative results of our analysis would carry through. RC would allow capital to become increasingly less productive until a point was reached where the machine would be replaced. Clearly, there would be some consumption smoothing if the depreciation rate on inventories is less than one. Therefore, production smoothing would not be observed in this economy so that the variance of production would exceed that of sales. Further, the positive correlation between labor input and productivity found for the model without inventories would carry over to this setting.

While we have stressed the effects of machine replacement on productivity, replacement for changes in variety are probably important as well, particularly for the automobile sector studied in Section VI. In fact, it is relatively straightforward to reinterpret the model from this perspective. Simply view θ_t as a measure of the current variety of the product and let $\hat{\theta}$ be the desired variety at every point in time. If the machine is not replaced in period t-1, then the period t machine produces goods which do not match consumer's tastes as closely as did the period t-1 machine. Thus, holding employment fixed, the consumption flow from output falls over time reflecting the mismatch between tastes and produced variety. At a cost of k units of labor, the firm can alter variety to produce the type of product most desired by consumers, as in the automobile industry's new model year. This non-convex cost of adjustment thus generates a "product cycle". Note though that productivity is constant in this economy in contrast to the model described earlier. Presumably both of these affects of machine replacement are at work within the automobile sector.

Given this structure, we now consider extensions of this model in three important directions. First, we allow for shocks in the model to understand the relationship between the timing of the machine replacement and aggregate economic activity. Second, we evaluate the implications of the machine replacement problem for other sectors of the economy; i.e. we look at the spillover effects associated with this process and the timing of machine replacement when there are multiple producers. Finally, we look at the interaction of multiple producers solving the machine replacement problem.

III. Machine Replacement and Shocks

We introduce exogenous fluctuations in this economy by incorporating taste and technology shocks into the single agent problem. The point is to understand how the decision on machine replacement at the firm level is influenced by the state of the aggregate economy, represented by these shocks. Here we emphasize the implications of aggregate fluctuations on the individual's choice problem.

In particular, let period t utility be given by $\alpha_t u(c_t) - g(n_t)$ and let period t production be given by $y_t = \theta_t \lambda_t n_t$, where α_t and λ_t are iid shocks to tastes and technology, respectively.[16] Incorporation of these sources of fluctuations alters the specification of the problem as follows. Denote α as the current period realization of the taste shock, and λ the current period realization of the technology shock. If RC innovates, then his utility from this period onward is given by:

(6) $V^I(\theta) = W^I(\theta) + \beta EV(\hat{\theta})$ where

(7) $W^I(\theta) = \max_n \alpha u(\theta\lambda n) - g(n+k)$

and E is the expectational operator. Similarly, if RC does not innovate then his utility is given by:

(9) $V^N(\theta) = W^N(\theta) + \beta EV(\rho\theta)$ where

(10) $W^N(\theta) = \max_n \alpha u(\theta\lambda n) - g(n)$

As before, $V(\theta) = \max \{V^I(\theta), V^N(\theta)\}$. Under this specification, the analogues of Lemmas 1 and 2 and Proposition 1 hold.

The issue of interest is how the critical level of θ is affected by the realizations α and λ. At the optimum, we have:

$$dV^I(\theta)/d\alpha = u(\theta\lambda n^I(\theta)) \quad \text{and}$$

$$dV^N(\theta)/d\alpha = u(\theta\lambda n^N(\theta)).$$

As shown in Lemma 1, $n^I(\theta) < n^N(\theta)$. Thus $dV^I(\theta)/d\alpha < dV^N(\theta)/d\alpha$.

This result is illustrated in Figure 2. Higher realizations of α shift both the V^I and V^N schedules upward. However, the shift in the V^N schedule is proportionately larger than the shift in the V^I schedule. Since machine replacement implies some loss of current production, machine replacement is most likely to occur during periods of low marginal utility. So θ^* is a decreasing function of α.

Now consider technology shocks. At the optimum, we have:

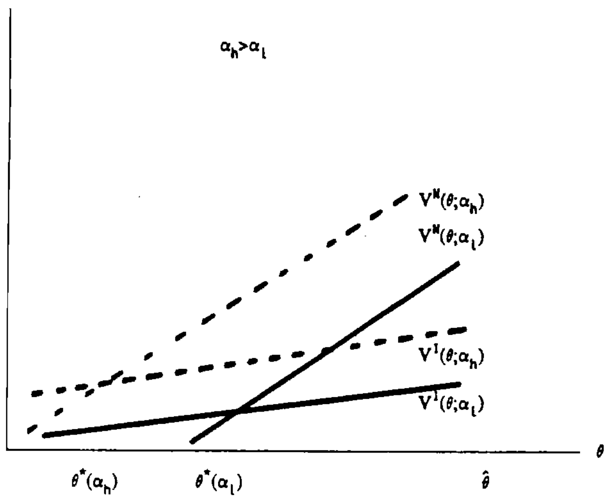

$\alpha_h > \alpha_l$
$V^N(\theta;\alpha_h)$
$V^N(\theta;\alpha_l)$
$V^I(\theta;\alpha_h)$
$V^I(\theta;\alpha_l)$
θ
$\theta^*(\alpha_h)$
$\theta^*(\alpha_l)$
$\hat{\theta}$

Figure 2

$$dV^{I}(\theta)/d\lambda = \alpha u'(\theta\lambda n^{I}(\theta))\theta n^{I}(\theta) \quad \text{and}$$

$$dV^{N}(\theta)/d\lambda = \alpha u'(\theta\lambda n^{N}(\theta))\theta n^{N}(\theta).$$

Given that substitution effects dominate, then, since $n^{I}(\theta) < n^{N}(\theta)$, $dV^{I}(\theta)/d\lambda < dV^{N}(\theta)/d\lambda$. This implies that the higher is λ, the less likely RC will innovate in the current period. The intuition for this result is similar to that for taste shocks. High realizations of λ indicate periods of high productivity and since machine replacement essentially requires some down time, this indicates that machine replacement will be more likely in low productivity periods. Therefore, θ^{*} is also a decreasing function of λ.

Taken together, we find a tendency for the timing of machine replacement to be linked with current demand and cost conditions. Our analysis of iid shocks reveals that machine replacement is most likely in periods of low realized demand and/or low realized productivity. This suggests an interesting covariance between output and productivity fluctuations endogenously induced by machine replacement and output and productivity fluctuations exogenously generated by demand and cost shocks. This covariance implies a potential link between seasonal fluctuations endogenously generated by machine replacement and the stage of the business cycle. Viewed from this perspective, the machine replacement process acts as a potentially important propagation mechanism for adverse business cycle shocks.[17] This covariance involves testable implications regarding the timing of machine replacement. We return to this point in Section VI.

The assumption of iid shocks is quite important in these results. The general principle that emerges is that replacement should occur during downturns when the opportunity cost of labor is small <u>but</u> the new machine should be in operation during good times. For the iid case, the second effect is independent of the current state, (α,λ), so that only the opportunity cost effect was operative. Suppose though, that α followed a deterministic pattern. In that case, it is easy to see that replacement would occur at the end of the downturn when the opportunity cost of labor was low <u>and</u> the new machines would be operative at the start of the period of high α.

Suppose, more generally, that α followed a simple Markov process in which α is either α_h or α_l in each period and π is the transition probability from each of the two states. In that case, there will be two critical values of θ^*, $\theta^*(\alpha_h)$ and $\theta^*(\alpha_l)$ such that machine replacement occurs iff the current state of the machine is below the appropriate value of θ^*. For π close to 0 and β close to 1, $\theta^*(\alpha_h) > \theta^*(\alpha_l)$ so that machine replacement is more likely to occur in high α periods. This is the opposite of our finding in the case of iid shocks. The intuition is that the iid shock case emphasizes the effects of current α on the cost of replacement. For the Markov model with low π, the effect of α on the future stream of utility from consumption is more important. In contrast, for π near 1, replacement is more likely to occur during periods of low α. In this case, the costs of replacement are low and the gains higher when α is low.

IV. Decentralized Solution with Demand Linkages

In this section of the paper, we consider the spillover effects from machine replacement by a single producer. These spillover effects are important because not all production activities are best described by the machine replacement problem and yet, as discussed further in our presentation of empirical evidence in Section VI, Beaulieu-Miron [1990] find that production in the entire manufacturing sector displays similar monthly variations as does the automobile sector.

To capture these features of the data, we consider a multi-sector economy in which demand linkages create spillover effects from the sector undergoing machine replacement to other sectors. In general, these spillovers may arise due to either factor or final demand linkages. We begin by discussing final demand linkages and discuss factor demand linkages at the end of this section.

Suppose there is a single producer that sells good 1 and consumes good 2.[18] Denote by $u(y_t) - g(n_t)$ the payoff to the monopolist in period t where y_t is the level of consumption of the good produced in sector 2 and n_t is the level of work in period t. The function $u(\cdot)$ is assumed to be increasing and strictly concave while the disutility of work, $g(\cdot)$, is a strictly increasing and strictly convex function of n_t. The monopolist lives forever and discounts future utility at rate β. Assume for now that the good produced by the monopolist can not be held in inventory.

The technology for producing good 1 is similar to that studied in the previous section of this paper. The production function for period t is given by $q_t=\theta_t n_t$ where θ_t equals $\hat{\theta}$ if the machine was replaced last period ($z_{t-1}=1$) and equals $\rho\theta_{t-1}$ otherwise, where $\rho\epsilon(0,1)$ and represents the rate of depreciation of the technology. As before, machine replacement requires the use of k units of leisure time by the monopolist.

Good 2 is produced by a large group of price taking agents who live for only a single period and only consume good 1.[19] Good 2 is not inventoriable and can be thought of as a service. Denote by y_t the level of output of good 2 in period t. For simplicity, assume there is a single, competitive sector 2 producer. The producer's preferences are given by $v(q_t^d) - h(y_t)$. So q_t^d is the period t consumption of good 1 by the competitive agent. Assume that $v(\cdot)$ is strictly increasing and strictly concave and $h(\cdot)$ is strictly increasing and strictly convex.

In each period, the monopolist chooses a price of good 1 in terms of good 2, p_t, which the sector 2 agent takes as given in deciding upon his production and consumption plan. Using the budget constraint for this agent, he chooses y_t to maximize $v(y_t/p_t) - h(y_t)$. The first-order condition is $(1/p_t)v'(y_t/p_t)=h'(y_t)$. Implicitly this yields $y^*(p_t)$ as the supply function for sector 2 output. If the function $v(\cdot)$ has the property that $cv'(c)$ is an increasing function of c, then output will be a decreasing function of p.[20] Using the individual's budget constraint, $q^*(p_t)=y^*(p_t)/p_t$ so that demand for good 1 is a decreasing function of p_t.

The monopolist solves

$$\max_{(z_t,p_t)} \sum_{t=0}^{\infty} \beta^t (u(y^*(p_t)) - g(q^*(p_t)/\theta_t + z_t k))$$

subject to:

$$\theta_t = \begin{cases} \rho\theta_{t-1} & \text{if } z_{t-1}=0 \\ \hat{\theta} & \text{if } z_{t-1}=1. \end{cases}$$

In this objective function, $y^*(p_t)$ is the supply of the competitive firm and $q^*(p_t)$ is the monopolist's output. Since the monopolist meets demand forthcoming at the announced price, the monopolist must supply $q^*(p_t)/\theta_t$ units of time to the production of goods and also allocates k units of time to machine replacement in the event $z_t=1$.

Since goods can not be held in inventory, p_t is determined from a static optimization problem for the monopolist given θ_t. Of course, the choice of z_t is the consequence of dynamic optimization.

The first-order condition for the choice of p_t is

$$(11) \qquad u'(y^*(p_t))p_t(1-1/\xi(p_t)) = g'(q^*(p_t)/\theta_t + z_t k)/\theta_t \text{ where}$$

$$\xi(p) = -q^{*\prime}(p)p/q^*(p)$$

and is therefore the elasticity of demand. The term $p_t(1-1/\xi(p_t))$ is marginal revenue, $MR(p_t)$.

Proposition 2: If marginal revenue is an increasing function of the price, then p_t rises between periods of machine replacement.

Proof: Between periods of machine replacement, θ_t falls and $z_t=0$. This implies that the right side of the first order condition increases for a given value of p_t. To maintain equality, the left side must be increased. Since marginal revenue increases in p_t, $y^*(p_t)$ is a decreasing function of p_t and $u(\cdot)$ is strictly concave, p_t must increase to maintain (11).

QED.

Between periods of machine replacement, marginal cost increases since θ_t falls. As long as marginal revenue is increasing in price (decreasing in quantity), this increase in marginal cost will imply that prices will rise over time. Since output of the competitive sector is a decreasing function of the price set by the monopolist, as θ_t rises output of the competitive good and demand for the monopoly good will both fall. In this way, both sectors of the economy move together.[21]

To analyze the monopolist's decision on replacement, define

$$W^{I}(\theta) = \max_{p}\ u(y^{*}(p)) - g(q^{*}(p)/\theta + k) \qquad \text{and}$$

$$W^{N}(\theta) = \max_{p}\ u(y^{*}(p)) - g(q^{*}(p)/\theta).$$

In the replacement period, price is set higher, for a given value of θ, than would be the case if the machine was not being replaced. This is because of the congestion effect, created by the need to devote k units of time to the replacement process in the period before a new machine is active, increases the marginal cost of production.

These functions are the maximized value of period utility for the monopolist producing with a productivity θ machine in the case of machine replacement ($W^{I}(\theta)$) and no machine replacement ($W^{N}(\theta)$). Using these functions, we can state the value functions for the monopolist's problem

$$V^{I}(\theta) = W^{I}(\theta) + \beta V(\hat{\theta}) \qquad \text{and}$$

$$V^{N}(\theta) = W^{N}(\theta) + \beta V(\rho\theta)$$

where $V(\theta) = \max\ (V^{I}(\theta), V^{N}(\theta))$.

Using these relations, the solution to the monopolist's choice of machine replacement is given by

Proposition 3 If $\xi(\cdot)$ is non-increasing in p, ρ is close to 1 and k is close to 0, then there will exist a critical θ, θ^{*}, such that the monopolist will replace the machine iff $\theta \leq \theta^{*}$.

Proof: As in the previous section, we need to verify that $V^{N}(\theta)$ is a steeper function than $V^{I}(\theta)$. It is sufficient that $W^{N}(\theta)$ be steeper than $W^{I}(\theta)$ since θ appears in the $V^{N}(\theta)$ function in a positive way through $V(\rho\theta)$. Taking derivatives of the $W^{N}(\theta)$ and $W^{I}(\theta)$ function yields

$$\frac{dW^{i}(\theta)}{d\theta} = g'(q^{*}(p^{i}(\theta))/\theta + zk)\ q^{*}(p^{i}(\theta))/\theta^{2} \qquad \text{for } i = I,N.$$

Here $z = 1$ for $i=I$ and $z=0$ for $i=N$. Further $q^*(p^i(\theta))$ is the demand for the good at price p^i, $i=I,N$, when θ is the current level of productivity.

Using the first-order condition from the monopolist's choice of the price, this derivative is

$$\frac{dW^i}{d\theta} = \frac{u'(y^*(p^i(\theta)))MR(p^i(\theta))q^*(p^i(\theta))}{\theta} = \frac{u'(y^*(p^i(\theta))[1-(1/\xi(p^i(\theta)))]y^*(p^i(\theta))}{\theta}$$

for $i = I,N$.

From the monopolist's first-order condition, (11), it is easy to see that $p^I(\theta)>p^N(\theta)$ for all θ. This is again the affect of k on the marginal disutility of work. By assumption, we know that cu'(c) is an increasing function of c. Using this and the fact that $y^*(p)$ is a decreasing function, $u'(y^*(p^N(\theta))y^*(p^N(\theta)) > u'(y^*(p^I(\theta))y^*(p^I(\theta))$. Hence if $\xi(p)$ is a non-increasing function of p, $W^N(\theta)$ is steeper than $W^I(\theta)$. Hence, $V^N(\theta)$ is steeper than $V^I(\theta)$.

From this result and assuming that k is close to 0 and ρ near 1, the value functions cross once and only once as in Proposition 1. Let θ^* be the value of θ at which they cross. Then the monopolist will replace the machine for $\theta\leq\theta^*$ and will not replace the machine otherwise. QED.

This result corresponds to that for the Robinson Crusoe economy. As in the previous section, the slopes of value functions for the two options have a particular order so that, for low k and high ρ, the solution is interior. That is, the monopolist always has an incentive to replace the machine but this replacement does not occur every period.

Thus we see that the machine replacement by the monopolist spills over to other activities in the economy through final demand linkages. Machine replacement creates congestion effects and thus higher marginal costs of production for the monopolist. This, in turn, induces competitive firms to reduce their output as well. In the period following replacement, there is a boost of productivity for the monopolist which leads to a price reduction: i.e. a sale. This sale induces an increase in output within the competitive sector. This spillover effect is consistent with the findings of Beaulieu-Miron [1990] discussed below.

There are a number of extensions of this structure worth considering. First, here we have stressed final demand linkages between the sectors. In Cooper-Haltiwanger [1990a], we considered factor demand linkages as the basis for the co-movement across sectors. This model could be amended so that the monopolist requires an input from competitive upstream producers. If this input is not storable, then the downstream MRP will create fluctuations in demand for upstream products. To the extent that upstream inputs are not perfectly storable and/or discounting is important, production across sectors will be positively correlated.

This also highlights a second important simplification of our model -- the nonstorability of goods. Suppose that the competitive good was storable. Still, periods of machine replacement would coincide with greater demand for the monopolists goods and, as long as there were some costs to the holding of inventories, greater output by the competitive producers.

The model generates a number of empirical implications that can be related to observations. First, the model produces positive correlation in the level of output and employment across sectors. These features of the seasonal and business cycles are stressed by Long-Plosser [1983], Cooper-Haltiwanger [1990b], and Beaulieu-Miron [1990]. Second, productivity in the "manufacturing sector" is procyclical -- upturns are associated with the installation of new productive machines and as the economy falls into a downturn, productivity is declining as well.[22]

As a historical note, there was an effort in 1934 to shift the new model year of the automobile manufacturers. The spillover effects from this are described by Charles Roos ([1937], p.468), who was the Director of Research at the Cowles Commission and formerly the Director of Research for the National Recovery Administration, as:

> "Late in 1934 automobile manufacturers reached an agreement to introduce the 1935 new models in October instead of December so as to separate the new-model and spring demand and make possible steadier operation. Simple as the plan is, its effects should be tremendous -- regularization of employment in the automobile industry and to a lesser extent in steel, lumber and allied industries, and, as may readily be verified by existing statistics, intensification of seasonal demand for transportation. Moreover, without any additional capital outlay, productive capacities of the automobile and steel industries will be increased, demand for housing in Detroit, Flint and other

> automobile-manufacturing towns will be regularized and bank deposits throughout the country be changed seasonally. Also, farm workers, who have been accustomed to finding winter employment in the automobile industry, will have to look elsewhere. But despite all these economic changes, the net effect on the national economy should be beneficial."

This is a quite intriguing quote in two respects. First, it highlights the large spillover effects associated with retooling of automobile plants during this period. Second, it points once again to the extent of synchronization of retooling across manufactures since the movement to change the time of retooling was evidently industry-wide.

V. Machine Replacement with Multiple Producers

The previous section considered the spillover effects of machine replacement by focusing on the interactions of a single, non-convex firm on the remainder of the economy. This section focuses on the other dimension of multiple firms: the case of machine replacement with multiple producers each solving the machine replacement problem. This is important in that one might presume that the non-convexities at the firm level may be much less important as one aggregates. In fact, the smoothing by aggregation arguments implicit in general equilibrium models with indivisibilities and/or non-convexities in preferences and technology rests on the observation that an economy with multiple agents sufficiently dispersed across indivisible choices behaves very much like a convex economy.[23] The key in those results is the assumed dispersion or, applying that argument to our model, the assumed staggering of discrete decisions over time. Here we consider two classes of arguments bearing on the issue of timing of machine replacement with multiple producers. First, are arguments concerning the common nature of shocks and second are arguments which rest on the nature of the strategic interactions across agents.

Shocks and the timing of discrete decisions

One perspective on timing of discrete decisions, stressed in the work by Bertola-Cabellero [1990], is that the degree of synchronization is influenced by the correlation in the shocks influencing the tastes and preferences of the agents.[24] In an economy in which there is no interaction between agents, so that each individual solves an optimization problem in which payoffs are independent of the actions of others, synchronization can still occur if shocks are highly correlated. Consider the economy described in Section III, if there were multiple producers solving the machine replacement problem and their values of α_t were perfectly correlated, then clearly the entire economy would follow the solution of the representative agent. So, if July was a valued time for leisure by all agents, the model predicts that machine replacement will take place in that month.

This approach to timing consideration rests on the strong assumption of common shocks, just as aggregate models often rely on common shocks to explain observed co-movements in activity across sectors of an economy. An alternative approach is to rely on linkages across agents to produce observed co-movements, such as the synchronization of machine replacement and other discrete activities. We first discuss this alternative and then later turn to a discussion of evidence on the relative importance of these approaches.

Strategic interactions and the timing of discrete decisions

According to Cooper-Haltiwanger [1990a] the timing of discrete decisions will depend on the nature of the strategic interaction between the agents. That paper investigates a single good, endowment economy in which two agents have the rights to endowment streams that deterministically fluctuate between two states: high and low. Agents' payoffs depend jointly on their own endowment and that of the other agent and players choose whether to have their period of high endowment in even or odd periods. After the agents make these choices, nature determines whether the first period is odd or even. Cooper-Haltiwanger [1990a] proves that in this game if agents' payoffs display strategic complementarities (i.e. the payoff functions have positive cross partial derivatives), then the Nash equilibrium of the game will entail synchronization. Alternatively, the case of strategic substitutes will imply staggering. Thus smoothing by aggregation effects do not arise if the interactions between agents are characterized by strategic complementarities.[25]

Those results can be extended to the machine replacement problem in that following manner. Let $\pi(\theta,\theta')$ be the payoff to a single producer using a machine with productivity θ if all other firms in the economy produce using a machine of productivity θ'. Assume that this economy is symmetric in that all firms have identical payoff functions. As we are searching for conditions under which synchronization occurs, we assume that all other firms are behaving in an identical manner. The productivity of the machine is governed by (1.b) for each of the firms in this economy. To focus on the issue of timing of machine replacement, assume that replacement occurs every T periods. The issue is then whether or not a single producer will synchronize replacement with other firms.

Proposition 4: If $\pi_{12}>0$, then replacement will be synchronized in this economy.

The proof of the Proposition 4 repeatedly makes use of the following lemma.[26]

Lemma 4: If $\theta>\theta'$, $\sigma>\sigma'$ and $\pi_{12}>0$, then $[\pi(\theta,\sigma)-\pi(\theta',\sigma)]>[\pi(\theta,\sigma')-\pi(\theta',\sigma')]$.

Proof: This difference can be rewritten as the difference between two integrals, i.e.

$$\int_{\theta'}^{\theta} \pi_1(x,\sigma)dx - \int_{\theta'}^{\theta} \pi_1(x,\sigma')dx > 0.$$

Since $\sigma>\sigma'$ and $\pi_{12}>0$, the first term exceeds the second. QED.

Let $\theta'(t)$ be the sequence of machine productivities for all of the other firms in the economy and let $\theta(t)$ be the sequence for the firm under consideration. To prove Proposition 4, we need to show that profits for the remaining firm are highest if $\theta(t)=\theta'(t)$ for $t=1,2,...T$ assuming that replacement occurs every T periods.

Proof of Proposition 4:

Suppose that $\theta(t)\neq\theta'(t)$ for $t=1,2,...T$. Index time so that at $t=1$, $\theta'(1)=\hat{\theta}$. Hence $\theta(1)<\theta'(1)$. Since replacement for the first firm must occur before the other firms replace, we know that $\theta'(T)<\theta(T)$. Let t^* be the first time period such that $\theta(t)>\theta'(t)$, this is where the two productivity functions cross.

If $t^*\leq T/2$ create $\theta^1(\tau)$ for $\tau=1,2,...T$ by

$$\theta^1(\tau) = \theta(t^*+\tau-1) \quad \text{for } \tau = 1,2\ldots t^*$$
$$\theta^1(\tau+t^*) = \theta(\tau) \quad \text{for } \tau = 1,2,\ldots. t^*.$$
$$\theta^1(\tau) = \theta(\tau) \quad \text{for } T \geq \tau > 2t^*.$$

This new productivity series corresponds to $\theta'(t)$ for the first t^* periods and is constructed by swapping the first t^* periods of $\theta(t)$ with the t^* periods of $\theta(t)$ after period t^*. Using Lemma 4, we know that undiscounted profits for the T periods are increased since we have interchanged productivity variables by increasing productivity when other firms have high productivity, as in the lemma. This is also true for $\beta<1$ since the reduction in productivity is pushed into the future and the high productivity brought forward.

If $t^*>T/2$, then the interchange outlined above is not possible. In this case, create $\theta^1(t)$ by

$$\theta^1(\tau) = \theta(\tau) \quad \text{for } \tau = 1,2\ldots(2t^*-T-1)$$
$$\theta^1(2t^*-T+\tau-1) = \theta(t^*+\tau-1) \quad \text{for } \tau = 1,2\ldots.(T-t^*+1)$$
$$\theta^1(t^*+\tau) = \theta(2t^*-T+\tau-1) \quad \text{for } \tau=1,2..(T-t^*+1)$$

This process involves bringing the low values of $\theta(t)$ to end of the period to coincide with the low values of $\theta'(t)$. Again, from Lemma 4 we know that profits over the T periods is higher with $\theta^1(t)$ than with $\theta(t)$.

Starting with $\theta^1(t)$, this process of interchange can be repeated and thus higher values of $\theta^1(t)$ can be moved forward in time and lower values of $\theta^1(t)$ moved back in time to create $\theta^j(t)$ for j = 2,3.. The process will converge to $\theta'(t)$. From Lemma 4, each step of this process increases profits over the T periods. Thus,

$$\sum_{t=1}^{T} \beta^t \pi(\theta'(t),\theta'(t)) \geq \sum_{t=1}^{T} \beta^t \pi(\theta(t),\theta'(t))$$

for all $\theta(t)$ paths such that replacement occurs every T periods and $\theta(t+1)=\rho\theta(t)$.

Since profits are higher by synchronizing over the T periods, given that replacement occurs every T periods, the present discounted value of profits is higher under synchronization. QED.

The condition that $\pi_{12}>0$ implies a complementarity across the producers in terms of the productivity of their machines. When this condition holds, the value to a single producer of increasing the productivity of his machine is higher when others have more productive machines. In fact, the proof of the proposition follows this intuition by taking a potential equilibrium without synchronization by the representative firm and showing, through a sequence of modifications of the productivity path, that profits can be increased by having productive machines when other firms do.

This proposition rests on the condition that $\pi_{12}>0$, the condition of strategic complementarities emphasized by Cooper-John [1988]. The natural question is whether there are interesting economies for which this cross-partial derivative condition holds. Here we present several examples of economies for which synchronization of machine replacement will occur.

Example 1

First consider a partial equilibrium model of the interaction between two symmetric producers. Suppose that the profits of producer i=1,2 are given by $\pi^i = (p_i - (1/\theta_i))q_i$ where q_i is the demand for this product and is given by $q_i = \alpha - \beta p_i + \eta p_{-i}$ where p_{-i} is the price in the other sector. Here $1/\theta_i$ is labor productivity at firm i. Firms play a simultaneous move game in which price is the strategy variable. Computing the Nash equilibrium determines $\pi^j(\theta_1,\theta_2)$ for j=1,2. Using these reduced form profit functions, the sign of $\pi^j_{12}(\theta_1,\theta_2)$ is determined by η. If η is negative, so that the goods are complements, then the cross partial of the payoff function is positive. Hence we see that in an economy with complementary products, the requirement of Proposition 4 will be met and, adding the appropriate dynamics, replacement would be synchronized.

Note too that if products are substitutes, so that η is positive, then staggering will occur. In particular, if two firms, producing identical products, are playing this Bertrand game, then synchronization would imply zero profits each period so that the fixed cost of machine replacement could not be covered.

Example 2

This example makes use of the monopolistic competition structure found in Ball-Romer [1989], Blanchard-Kiyotaki [1987] and described in detail in Blanchard-Fischer [1989]. We follow

Blanchard-Fischer and consider a simple economy with N goods produced from labor time by N agents who have market power as sellers. Individuals produce a single good and consume all the goods produced in the other sectors of the economy and also demand real money balances. Blanchard-Fischer generate this demand by including real money balances in utility function. This is not critical for our discussion: real money balances could also be a non-produced good that is endowed to the agents. Using a specific form for utility, Blanchard-Fischer derive explicit demands for each of the N producers. Demand for product i depends on the aggregate level of real balances and the ratio of the price of product i to an aggregate price index.

Denote by $\sigma(\theta_i,P)$ the payoff to firm i if labor productivity at that firm is θ_i and the aggregate price level is P. This is the optimized value of profits for the firm given its productivity and the price level. Payoffs depend on the aggregate price level through the effects of P on the demand for product i and because the producer of product i is a consumer of all products produced in the economy. Using the conditions which determine the optimal price for firm i (essentially from equation (6) in Blanchard-Fischer ([1989], p. 379) taking account of productivity differences), one can show that the cross partial of $\sigma(\cdot)$ is negative. That is, the gains to improved productivity for a single firm are higher when the aggregate price level is lower. When P is low, sales of product i are relatively high so that productivity increases are more profitable than for higher values of P. Since the aggregate price level is a decreasing function of the aggregate state of productivity , an increase in productivity at all other firms implies that the marginal gain to increasing productivity at the remaining firm will increase; i.e. the condition on cross partial needed for Proposition 4 is met.[27]

Example 3

Example 2 highlights the effects of productivity gains throughout the economy on the behavior of the residual firm. In that economy, firms did not produce identical products. In contrast, in the partial equilibrium model given as Example 1, if $\eta>0$, firms produce substitute commodities and will stagger machine replacement due to strategic substitutability. Here we consider the implications of firms producing identical products in a general equilibrium setting and find that synchronization may still arise.

Cooper [1989] considers an economy with three goods, two produced and one endowed. The producers of the two produced goods consume the output produced by the other sector and the non-produced good. Further, the producers in sector one choose between two techniques: high fixed cost and high marginal product or low fixed cost and low marginal product. Cooper finds that if the share of expenditure on the other produced good is sufficiently high, multiple equilibria may arise in this economy. One equilibrium has sector 1 firms using the high fixed cost, high marginal product technology and the other has them all using the low fixed cost technology. The key to this result is that by producing with the more productive technology in sector 1, prices of sector 2 goods are reduced which increases the returns to sector 1 firms from adopting the high technology.

This structure can be modified to provide conditions on the profit functions used in Proposition 4. If all firms but one in a sector are producing using a high productivity machine, then the returns to increasing productivity by the residual firm will be higher as long as that firm's owners consume goods produced in other sectors. The point is that if all other firms have a high productivity technology, firms in the other sector will be induced to increase output so that the gains to having a high productivity technology for the remaining firm in sector 1 increase. As emphasized in Cooper [1989], this requires a spillover of increased productivity in one sector on the prices of goods produced in other sectors and a large expenditure share across sectors of the economy.

Proposition 4 is a statement about synchronization given that replacement occurs every T periods. We are also interested in the issue of frequency in this decentralized setting. Cooper-Haltiwanger [1990a] and Murphy, Shleifer and Vishny [1989] find that in economies with non-convexities such as that studied here, multiple equilibria might emerge indexed by the time between periods of discrete activities (production runs, buying a durable good). Coordination failures can then emerge if an economy is stuck with a time interval that is too long relative to the Pareto-optimum.

A full evaluation of the decentralized problem to include the determination of T is outside the scope of this paper. However, we would conjecture that, using the economy outlined in Example 2, some of the results reported in Cooper-Haltiwanger [1990a] could be extended to this model. First, there will exist a T^c defined as the optimal time between machine replacements obtained by allowing producers to jointly decide on T prior to playing a non-cooperative game in prices. We would conjecture that replacement every T^c periods would be a Nash equilibrium of a non-cooperative game in which individual firms decide on T at the start of the game and on the price of their product each period. It is quite possible, as in Cooper-Haltiwanger [1990a] and Murphy, Shleifer and Vishny [1989] that other equilibrium values of T will exist.

A second coordination problem may arise in this environment. Consider again the attempts described earlier to move the retooling period in the 1930s from December to the late Summer/early Fall. What was the motivation for the involvement of the Roosevelt Administration and the Automobile Manufacturers Association in this shift? It could be that the automobile producers, while replacing machines at a privately optimal frequency, were retooling at a socially sub-optimal time of the year. By retooling earlier in the year, relative to the burst of sales in the spring, the automobile manufacturers would have been able to smooth their own production and, through factor and final demand linkages, smooth production and employment elsewhere in the economy. These gains to production smoothing may not have been fully internalized by the automobile producers.[28]

The importance of strategic interactions relative to the correlation of shocks is relevant for understanding the synchronization of replacement by automobile manufacturers in July (see Section VI). If final demand linkages are sufficiently strong across producers (as in Examples 2 and 3 above) and/or there are significant non-convexities upstream from the automobile manufacturers, synchronization can emerge. Alternatively, following Bertola-Cabellero, one might argue that there are taste shocks for September cars, rationalizing replacement of machines in July with appropriate lags. Or, one might argue that July is a time of valuable leisure so that replacement in that period is appropriate. These taste shock explanations of the timing of replacement are consistent with the model presented in Section III of this paper. Note though that relying solely on the high value of leisure in July requires an explanation of the fact that the shutdown for retooling occured in early winter during the 1920s and early 1930s.

VI. Evidence

The main theoretical predictions of our model can be summarized as:

(1) The process of machine replacement creates fluctuations in output, employment and productivity and the variations in these variables are positively correlated.

(2) Machine replacement is most likely to occur during downturns where the resource cost of replacement is lower and just prior to upturns where the benefits of replacement are higher.

(3) To the extent that other activities in the economy are linked to those industries undertaking machine replacement (either through factor demand, final demand linkages or thick market effects), replacement in one sector will spillover to others.

(4) To the extent that parameter variations are common across agents or the reduced form payoffs of these agents exhibit strategic complementarities, periods of machine replacment by independent producers will be synchronized.

The purpose of this section is to relate these theoretical findings to existing empirical evidence on machine replacement and sectoral fluctuations. Our aim is modest: to argue that the theoretical processes explored in this paper are observed in the manufacturing sector of the U.S. economy. We do not maintain that our simple model of machine replacement is capable of reproducing all features of manufacturing fluctuations. In fact, there are some interesting aspects of monthly fluctuations that are not part of our model and this section ends with some discussion of the ways in which our model fails to match the data. Further, our emphasis here is on the implications of our model for seasonal fluctuations since this seems to be the most appropriate frequency for the machine replacement cycle.

Plant level observations

To obtain some perspective on the importance of machine replacement for observed fluctuations in output and productivity at the micro-level, we examine the behavior of seven automobile plants in the United States for the period 1978-85.[29] The data provide information on monthly production, sales, the number of days the plant operated during the month, the number of shifts operating at the plant during the month, and the number of days that the plant was shutdown for retooling.[30] For the automobile industry, the shutdown of plants for retooling enables the producer to introduce new machines for the production of a new design and, at the same time, to install more productive capital.[31]

Our analysis of this data is based upon the following accounting identity: monthly production is equal to the product of: (i) the number of cars produced per shift; (ii) the number of shifts per day; and (iii) the number of days the plant operates during the month.[32] The number of days that the plant operates can be further decomposed into the product of: (iv) the sum of the number of days the plant operates and the number of days the plant is shutdown for retooling; and (v) the ratio of the number of days of operation to the sum of days of operation and the days shutdown for retooling. We interpret variations in production driven by (v) as those associated with machine replacement. Given our general interest in discreteness, we also examine variations in production driven by (ii) and (iii), i.e., shifts and days (where the latter incorporates changes in production due to machine replacement).

Figure 3 plots actual monthly production (PROD) and the implied monthly production when only days for machine replacement ((v) in the above decomposition)) is allowed to vary ($PROD^M$) for each of the seven plants.[33] For each plant we observe relatively volatile production often characterized by large, discrete changes. Of particular interest is the role of machine replacement. Observe that for virtually every plant in every year there is a reduction in production in particular months due to machine replacement.[34] The magnitude and duration of the reduction in employment due to machine replacement varies considerably across plants and time. There is a tendency for the machine replacement to be scheduled during Summer months and a tendency for the magnitude of the production loss due to machine replacement in a particular month to be larger when production in adjacent months is low. In particular, the magnitude and duration of the downturn in production is especially pronounced during the business cycle slump in 1982. This evidence supports the arguments in Section III discussing the potential interaction between the fluctuations induced by machine replacement and the stage of the business cycle.

Additional quantitative evidence on these findings is provided in Table 1. First, observe that for each of these plants the monthly variance of production is greater than or equal to the variance of sales.[35] Second, excluding months involving machine replacement reduces the ratio of the variance of production to the variance of sales for six of the seven plants and actually results in the ratio being less than 1 for two of the plants. Thus we see that machine replacement is an important factor for the observation that the variance of production exceeds the variance of sales. Third, the mean fraction of the change in production accounted for by machine replacement varies between 4% and 50% across plants. The standard deviation of this fraction reveals considerable variation (as is illustrated in Figure 3) in the importance of machine replacement. The mean fraction of the change in production accounted for by changes in the number of days and the number of shifts is large and varies between 36% and 103%.[36] Simply put, it is large monthly variation in days and shifts that accounts for the observed large variance in production.

The correlation between monthly production and monthly sales is quite high for all seven plants. Variation in production due to machine replacement and sales is also positively correlated for six out of the seven plants. This suggests that machine replacement is typically scheduled during periods of lower than average sales as suggested by our theoretical model. Similarly, we find a large positive correlation between sales and monthly production variation due to adjustments in days (both for machine replacement and other purposes) and shifts.

The evidence presented in Figure 3 indicates that the important role of machine replacement in the observed high volatility of production is closely tied to seasonal factors. Table 2 reports monthly seasonal coefficients (estimated via seasonal dummies) for monthly production, sales, and production allowing only machine replacement to vary. Several patterns emerge from Table 2. First, the reported R^2s indicate that seasonal variation accounts for a significant fraction of the overall variation in production, sales, and machine replacement. For production, we tend to observe high production in the spring and the Fall with notable slowdowns in the Summer (particularly July and August) and the winter (particularly December). Interestingly, although sales exhibit a similar seasonal pattern it is much less pronounced. The exception appears to be plant 1 though here we know that the variance of production only slightly exceeds the variance of sales. The Summer

Figure 3

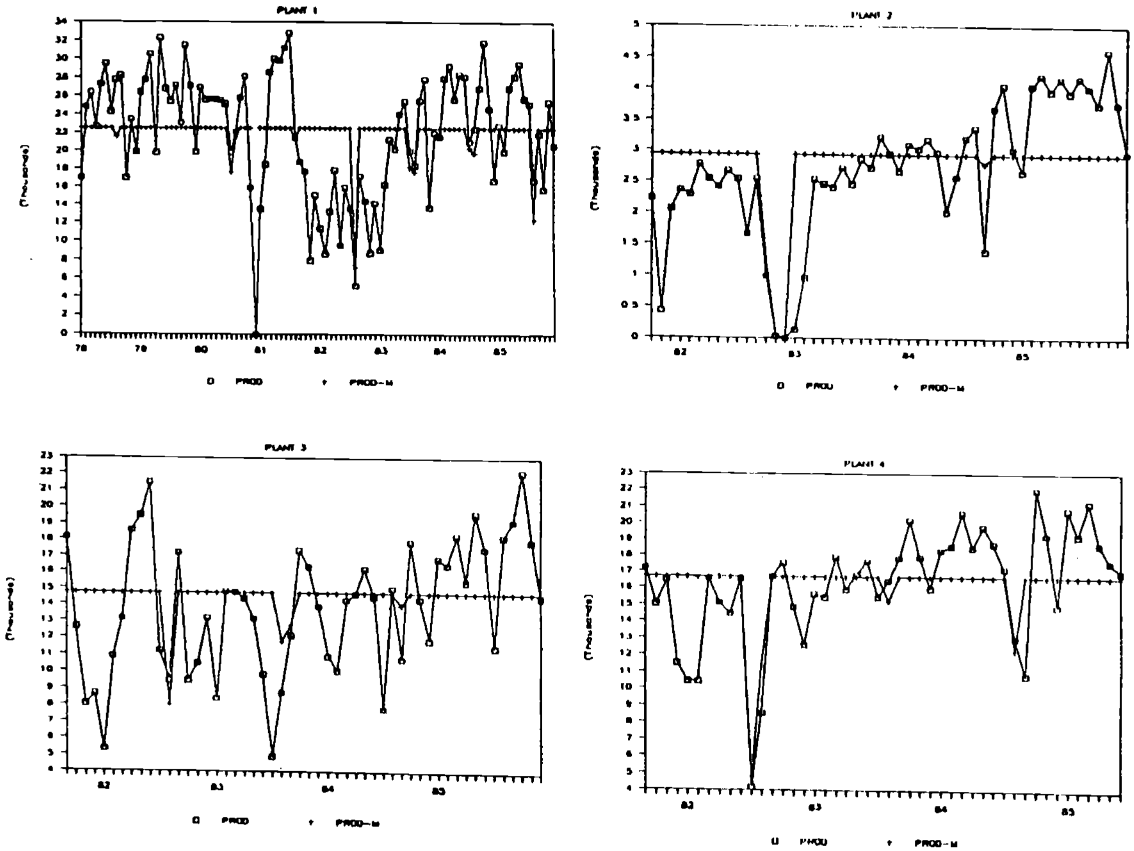


Figure 3 (cont.)

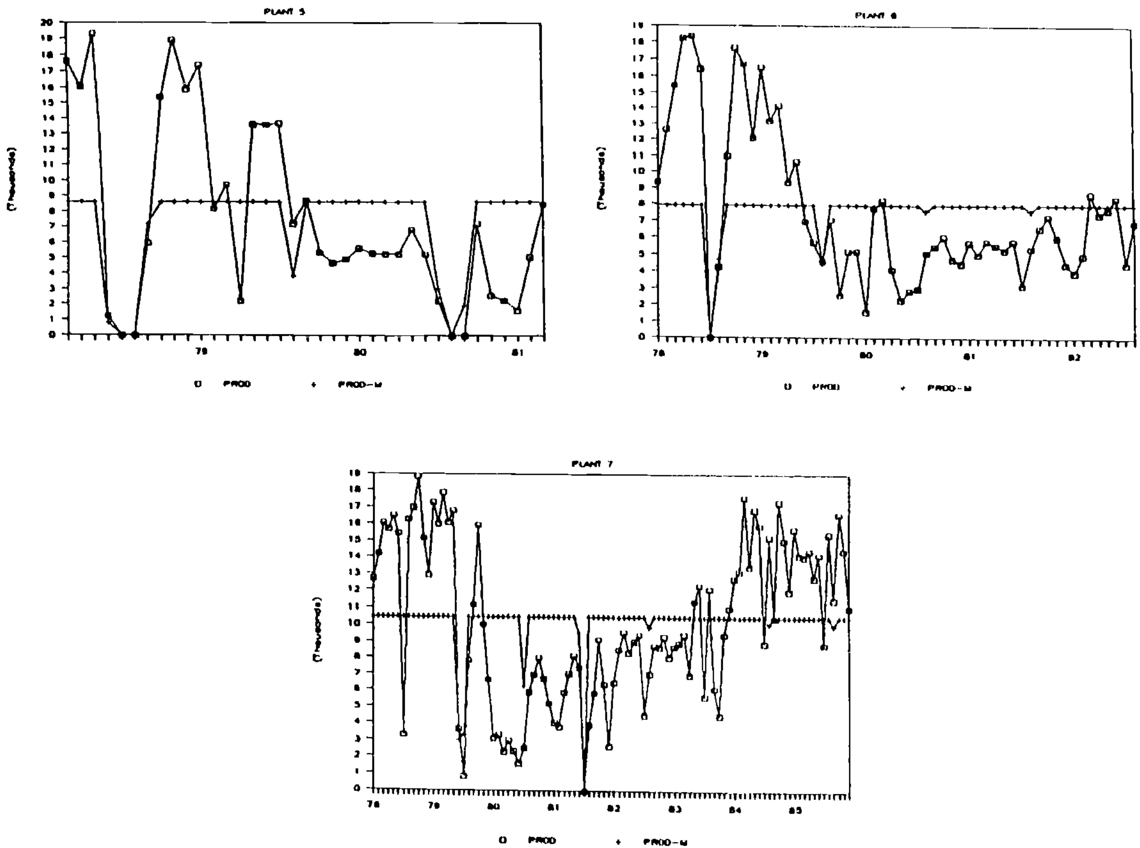


TABLE 1

Nature and Sources of Production Variability

	Plant 1	2	3	4	5	6	7
Var(PROD)/Var(Sales)	1.04	1.83	2.62	1.32	1.76	1.48	1.48
$Var(PROD^{NM})/Var(Sales^{NM})$ [1]	.85	1.33	2.97	.94	1.53	1.42	1.47
Mean of: [2]							
$\Delta PROD^{M} / \Delta PROD$	0.17	0.50	0.10	0.04	0.31	0.14	0.04
	(0.50)	(3.20)	(0.57)	(0.52)	(0.69)	(0.50)	(0.59)
$\Delta PROD^{D,S}$ [3] $/ \Delta PROD$	0.94	0.99	0.36	0.95	1.03	1.01	1.03
	(0.48)	(0.18)	(4.82)	(0.24)	(0.17)	(0.12)	(0.24)
Correlation of:							
PROD, SALES	0.55	0.72	0.57	0.70	0.57	0.75	0.81
$PROD^{M}$, SALES	0.12	0.27	0.33	0.40	0.12	-0.14	0.23
$PROD^{D,S}$, SALES	0.53	0.75	0.59	0.64	0.56	0.71	0.80

[1] $PROD^{NM}$ ($SALES^{NM}$) is production (sales) for all months excluding those in which retooling occured.

[2] The standard deviations are given in brackets.

[3] $PROD^{DS}$ is production allowing only days and shifts to vary.

TABLE 2

Seasonal Coefficients
(Thousands of cars)

PRODUCTION

Plant	R^2	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	.21	18.8	21.2	25.2	23.8	25.8	26.0	22.8	20.2	23.5	23.0	18.4	16.1
2	.12	2.1	2.6	3.2	3.0	2.8	3.0	3.1	3.0	2.6	3.1	2.2	2.2
3	.33	10.4	13.1	15.1	15.8	17.1	15.8	8.9	12.9	14.8	15.9	13.4	12.4
4	.34	16.3	15.9	19.1	17.1	17.2	17.5	12.3	12.7	15.1	18.7	17.2	13.8
5	.20	8.2	6.1	7.8	7.8	13.2	6.7	5.3	2.4	4.9	9.3	8.7	7.7
6	.17	6.9	8.7	10.4	8.9	8.9	8.1	3.2	5.2	7.5	8.4	8.2	6.5
7	.17	9.7	10.3	11.2	10.7	11.7	10.0	4.3	10.5	9.7	12.4	10.8	8.7

SALES

Plant	R^2	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	.31	19.3	25.1	31.1	23.8	24.1	21.7	20.7	19.9	19.0	22.1	17.1	18.4
2	.23	2.1	2.1	2.7	3.3	3.2	2.5	2.4	2.3	2.2	2.8	2.6	2.4
3	.49	11.8	13.8	17.0	15.2	16.1	16.1	13.2	12.5	12.8	15.9	13.9	11.9
4	.44	13.8	14.0	16.6	16.2	18.9	15.2	13.7	13.4	15.4	19.8	17.7	15.2
5	.25	6.2	6.3	10.6	11.5	12.4	11.3	8.3	8.3	8.5	6.6	7.0	6.7
6	.09	6.3	7.5	9.8	8.4	9.6	8.5	8.0	6.6	6.6	6.5	8.1	7.0
7	.08	8.6	9.5	11.7	9.8	10.6	9.5	8.8	8.7	10.6	12.2	10.9	10.1

$PROD^M$

Plant	R^2	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	.22	22.6	22.6	22.6	22.6	22.6	22.6	21.1	18.0	22.6	22.6	22.6	19.7
2	.16	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.5	2.4	2.4
3	.40	14.7	14.7	14.7	14.7	14.7	14.7	14.7	12.3	14.1	14.7	14.7	14.7
4	.34	16.7	16.7	16.7	16.7	16.7	16.7	12.7	13.0	16.7	16.7	16.7	16.7
5	.63	8.6	8.6	8.6	8.6	8.6	6.0	3.9	1.3	5.9	8.6	8.6	8.6
6	.24	7.9	7.9	7.9	7.9	7.9	6.3	6.4	7.9	7.9	7.9	7.9	7.9
7	.26	10.5	10.5	10.5	10.5	10.5	9.4	7.8	10.4	10.4	10.5	10.5	10.5

Average Pairwise Cross Plant Correlations of Coefficients from Seasonal Regressions

PROD	$PROD^M$	SALES
.47	.39	.57 ^Z

slowdown in production and sales is closely linked to the bunching of machine replacement in the Summer months. Most plants experience variation due to machine replacement in only a few of the months, often in July and August. The seasonal coefficients reveal considerable positive co-movement in the seasonal fluctuations across plants. Confirmation of this positive co-movement is provided by the fact that the average pairwise correlation of the seasonal coefficients across plants, reported at the bottom of Table 2, are all positive and large.

While we do not have direct evidence on productivity in our plants (we do not have employment data), Aizcorbe [1990] provides data on automobile plant level productivity for the 1978-1985 period. The data is monthly and is similar to that used in our tables. The employment data is the number of production-worker employees for the pay period which includes the twelfth of each month. Aizcorbe finds a positive correlation between line speed (the number of cars produced each hour) and the ratio of line speed to employment for each of the plants she considers (controlling for the number of shifts). This indicates that productivity tends to be positively related to output at the plant-level in the auto industry.

Evidence from the 1920s and 1930s

Related work by Kashyap-Wilcox [1990] provides further perspective on machine replacement during the interwar years. In a study of General Motors during the 1920s and 1930s, Kashyap-Wilcox [1990] discuss GM's attempt at production smoothing given large seasonals in demand.[37] Two observations from that paper are important to our work. First, Kashyap-Wilcox do find some support for the production smoothing model during this time period <u>if</u> the shutdown months are excluded from the data. Once these months are included in the data, the variance of production exceeds the variance of sales.

Second, shutdowns for the purposes of retooling occurred annually throughout the industry during this time period. I.e., retooling was synchronized during the interwar years. As mentioned earlier, Kashyap-Wilcox report that there was an effort in the early 1930s to move the shutdown period, which was approximately the same across GM plants and other producers, from the end of the year to the late Summer.[38] There is also evidence that the extent of the shutdown period was related to the stage of the business cycle: in each of the shutdowns of 1932 and 1933 about 100 cars were produced by GM while shutdowns in other years were not nearly as severe. These points are made in more detail in Kashyap-Wilcox [1990].

Our own evidence on this period is shown in Table 3. Using the industrial production series for automobiles, this table presents the seasonal dummies of the ratio of monthly production to average monthly production for the calendar year for the 1923-33 and the 1934-39 periods.[39] The split in the sample reflects the change in the retooling period occured in 1934. The monthly coefficients show the extent to which the entire industry underwent a coordinated change in the retooling period. In the early part of the sample, the period of retooling is December while in the second part of the sample the period of low output and retooling has moved back in the calendar year to August and September. Note that throughout this time period, there was a Spring burst of production just as in the 1978-85 period for the plant-level observations. Overall, there is a pronounced shift of production from the first 7 months to the last 5 months of the year.

From the perspective of understanding synchronization of machine replacement, the data clearly indicate that shutdowns were not staggered. Further, they were not driven by high values of leisure alone unless one argues that the value of a late Summer vacation was "discovered" in 1933. Understanding the basis for this change in the retooling time and its implications for sectoral production patterns during the Depression years are clearly important for a further evaluation of our model.

Table 3 also provides some evidence on the spillover effects of changing the retooling period for automobiles on other sectors. Seasonal dummies for iron and steel as well as for total manufacturing are reported using the same methodology used to generate the seasonal auto dummies. The results indicate that the seasonal production pattern of iron and steel was altered after 1933 -- production is lower in each of the first 7 months and higher in each of the last 5 months. Further, the same pattern emerges for overall manufacturing. This table clearly indicates the importance of automobile retooling on the timing of economic activity throughout manufacturing.

Machine Replacement and post World War II Aggregate Behavior

Table 3
Seasonal Production Patterns for Auto, Iron and Steel and Total Manufacturing, 1923-39

Month

	January	February	March	April	May	June	July	August	September	October	November	December
Automobiles												
1923-33	0.86	1.06	1.14	1.30	1.33	1.19	1.05	0.99	0.99	0.80	0.65	0.63
1934-39	0.96	1.03	1.20	1.11	1.18	1.13	0.98	0.68	0.51	0.81	1.13	1.28
Iron and Steel												
1923-33	1.03	1.16	1.13	1.09	1.05	0.99	0.97	0.97	0.95	0.96	0.90	0.86
1934-39	0.90	0.95	0.98	1.00	1.01	0.98	0.91	1.00	1.03	1.08	1.10	1.07
Total Manufacturing												
1923-33	0.96	1.01	1.02	1.03	1.03	1.02	0.99	1.00	1.03	1.02	0.97	0.92
1934-39	0.92	0.94	0.97	0.99	0.99	0.99	0.98	1.01	1.04	1.07	1.06	1.04

A final bit of evidence on the importance of machine replacement is discussed in a study of seasonality of manufacturing by Beaulieu-Miron [1990]. They find a strong decrease in activity throughout manufacturing during July. This is, of course, frequently a period of machine replacement for automobile plants. Beaulieu-Miron note that one explanation of the finding is the presence of synergies (strategic complementarities) that provide incentives for firms to synchronize reductions in activity. In Section V of this paper we find that the spillover effects of machine replacement can lead other sectors to decrease output and employment during periods of replacement. Beaulieu-Miron also note that labor productivity is positively correlated with output. This is also a property of the model we described in this paper and is true for the automobile industry too.

Beaulieu-Miron also report a expansionary phase in the Spring and a reduction of output during December. This was also true for our plant-level data and is not predicted by our model given that we concentrate solely on the replacement cycle.

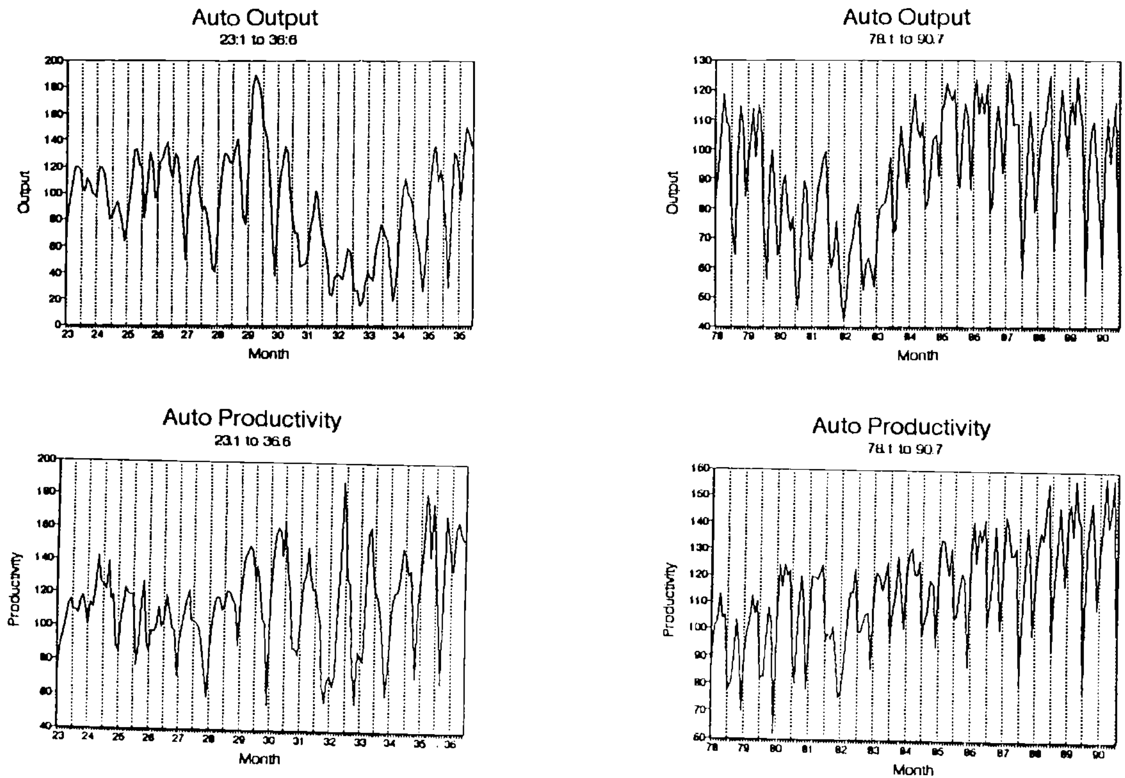
Evidence on Productivity

One of the motivations of this study was to understand the importance of machine replacement for observed productivity variations. While we rely on Aizcorbe [1990] for evidence of the procyclical nature of productivity in the plants we study, it is also important to consider other evidence on this issue. For that purpose, we evaluated the seasonal pattern of average labor productivity for the automobile sector in both the interwar and post WW II years. Our findings for output and average labor productivity are summarized in Figure 4.[40]

For both periods, we observe large seasonals for both output and productivity during a given year. Three variations in productivity seem important. First, in both sample periods, productivity is quite low during retooling periods. This is also a time of low output and employment. Second, for the 1978-1990 period, there is another dip in productivity around December. Finally, during periods of output expansions (such as those arising in the Spring for both sample periods) there appears to be a surge in productivity as well.

The puzzling aspect of the data is that no single explanation of the productivity series appears to be consistent with all of the observations. Instead the data might be best described as being a

Figure 4



Auto Output
23:1 to 36:6
Output
Month
Auto Output
78.1 to 90.7
Output
Month
Auto Productivity
23.1 to 36.6
Productivity
Month
Auto Productivity
78.1 to 90.7
Productivity
Month

consequence of an August retooling effect, labor hoarding (in December) and some increasing returns (in the Spring).[41] Productivity decreases during retooling periods are consistent with our model. However, our model does not predict the productivity slowdown during December as this is not generally a time of replacement.[42] Instead, this productivity decline might be the consequence of mismeasuring employment. Our employment series is for paid workers and thus includes workers on vacation so that "labor hoarding" might best describe this dip in productivity. Finally, the productivity jump in the Spring, coinciding with a production burst, is not a consequence of installing new machines but may reflect increasing returns either internal or external to these firms.[43]

VII. Conclusions

The point of this paper has been to study the discrete choice involving the replacement of obsolescent machines. When a single agent solves an intertemporal optimization problem which involves machine replacement, the solution displayed endogenous (seasonal) cycles with procyclical labor productivity. In a stochastic environment, machine replacement will occur near the end of economic downturns since the opportunity cost of displaced production workers is less than during good times. Through the spillover effects of machine replacement on other sectors, activity in other sectors will be positively correlated with the productivity of machines in the sector undertaking replacement. Finally, in the presence of strategic complementarities, multiple producers will synchronize machine replacement so that smoothing by aggregation will not occur. We also presented evidence: (i) of significant monthly output fluctuations due to machine replacement in the automobile industry, (ii) that these fluctuations matched some of the important seasonal fluctuations observed in manufacturing and (iii) that labor productivity is positively correlated with monthly output in the automobile industry.

A number of important issues remain. First, in our discussion of the decentralized economy, we focus on the two dimensions of timing separately: strategic interactions and the nature of the correlations in shocks to the agents' payoffs. It would be quite useful to consider a model in which both of these effects are present and then to attempt to identify the relative importance of these two influences on the timing of discrete decisions. This could be accomplished by merging the arguments in Cooper-Haltiwanger [1990a] and those contained in Section V of this paper with Bertola-Cabellero [1990].

A second, and much harder issue, concerns the relative importance of the effects considered here for business cycles. We have offered evidence that the effects modeled here are important for seasonal fluctuations. Further, both our model and the evidence suggest potentially important links between the seasonal fluctuations generated by machine replacement and the stage of the business cycle. One interesting way to evaluate the relative importance would be to produce a model of machine replacement that was capable of generating time series along the lines of Kydland-Prescott [1982] and then to compare the quantitative predictions of this model with those using exogenous shocks to generate fluctuations in a convex environment.[44]

Finally, a complete investigation of the basis for changing the retooling period during the 1930s and its implications for the seasonal pattern of production would be quite interesting. In particular, why was the retooling period changed and why did it require collective action? Further, how did the change in the retooling period impact on the seasonal production pattern of other industries?

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Endnotes

1. As argued in Barsky-Miron [1989] and Beaulieu-Miron [1990], one can learn about business cycles through the study of seasonal fluctuations. Further, as argued below, our seasonal cycles are dependent on the stage of the business cycle. Finally, to the extent that some costs of adjustment are non-convex, our model can be applied to a study of more general investment decisions.

2. Here we use the term non-convexity to include economies in which choice sets are not convex (as studied here) and economies in which technologies are not convex, as in Kiyotaki [1988] and Cooper-Haltiwanger [1990a].

3. This point is discussed in Cooper-Haltiwanger [1990a] and Murphy, Shleifer and Vishny [1989]. In some sense, the fluctuations induced by non-convexities are similar to those produced in models of non-linearities in that for both types of models optimal choices can be very sensitive to variations in the underlying environment. Shleifer [1986] analyzes a model of cycles driven by the synchronized introduction of new innovations.

4. The issue of replacement investment has received some attention, see, for eg., Feldstein-Rothschild [1974] and Nickell [1975]. The Feldstein-Rothschild analysis was mainly to understand the determinants of replacement investment and, in particular, to point out that conditions under which a constant replacement rate is optimal are quite restrictive. Using their terminology, our replacement entails scrapping capital due to deterioration. In cases, considered below, where replacement allows the firm to introduce a more productive vintage, then our model is also about depreciation due to technological obsolescence. Nickell focuses on issues of maintenance and the optimal time to scrap a machine. Relative to these papers and others in the literature, we focus on lumpy replacement processes and on the implications of replacement for activities in other sectors of the economy.

5. In an economy with technological progress, the productivity of a machine put into place in period t would exceed that of a machine installed in an earlier period.

6. Thus the paper differs from those in the large literature on convex costs of adjustment by assuming that replacement is a lumpy activity. See the interesting arguments for non-convex costs of adjustment in Rothschild [1971].

7. That is, these shocks are not needed to obtain the qualitative prediction of positive correlation between productivity and output. Whether an economy such as the one considered here can match the quantitative properties of U.S. time series is an open issue that we discuss in the conclusion. We use the term procyclical to indicate the productivity is positively correlated with employment keeping in mind that employment is cyclical in this model.

8. Note that here the cost of replacement is modeled as a resource cost rather than a simple utility cost. The latter approach, while simpler, misses congestion effects, operating through the disutility of work, associated with machine replacement.

9. This perspective on the model was suggested to us by S. Rao Aiyagari.

10. Equivalently, this could be introduced directly into the technology instead of the disutility of work by reducing the average and marginal products of labor during replacement.

11. So that the productivity of the machine in the previous period was θ/ρ.

12. Since $u(\bullet)$ and $g(\bullet)$ are assumed to be continuous, $n^N(\theta)$ is continuous and so will be $W^N(\theta)$ by the maximum theorem.

13. That is, both $n^I(\theta)$ and $n^N(\theta)$ are increasing in θ.

14. We are grateful to Marc Dudey for helpful discussions on this proposition.

15. See Murphy, Shleifer and Vishny [1989] for another example of this.

16. Because of the lag structure in the replacement, iid shocks are enough to generate interesting dynamics.

17. The implied intertemporal substitution of machine replacement during slumps is similar to the reallocation timing arguments in Davis-Haltiwanger [1990] and the shake-out mechanisms discussed in Blanchard-Diamond [1990]. Essentially, these theories together suggest that business cycle slumps are times in which the economy takes a "pit stop" in order to retool, reallocate, and restructure.

18. Here we assume that there is only a single agent solving the machine replacement problem. The next section considers the case of multiple agents solving that problem.

19. This assumption is not crucial but simplifies matters so that we need not solve a static labor supply and an intertemporal optimization problem jointly. The main point of this section, that upon replacement the relative price of the monopolist's good will fall and stimulate production in other sectors, should generalize to a setting with competitive agents living more than a single period.

20. This means that $v(\bullet)$ is not too concave and ensures that labor supply is an increasing function of the real wage.

21. This is an important aspect of this model given the observed co-movements in output and employment described by Beaulieu-Miron [1989] and Cooper-Haltiwanger [1990b]. See also the discussion in Fine [1963] about upstream linkages of the automobile industry and the resulting fluctuations caused by retooling.

22. An interesting empirical question concerns the behavior of productivity in the service sector at the monthly frequency.

23. See, for example, the arguments in Broom [1972] and Mas-Colell [1977].

24. See also, Ball-Romer [1989], on the influence of stochastic structure on timing in price change games.

25. See Cooper-John [1988] for a more complete discussion of strategic complementarities and their role in many macroeconomic models.

26. The relationship between the cross partial of the π function and the monitonicity of these differences is discussed by Milgrom-Roberts [1990] as a condition of supermodularity. This lemma is similar to their Theorem 2.

27. To find the relationship between the aggregate price level and productivity, simply solve for the symmetric Nash equilibrium in the Blanchard-Fischer model and note that P varies inversely with the labor productivity parameter.

28. In fact, Fine [1962, page 351] reports on failed unilateral attempts to change the timing of retooling in the auto industry in the 1920s.

29. These data have been collected and tabulated from Ward's Automotive Reports and Ward's Automotive Yearbook. The data are available upon request. Note that the sample period varies across plants. This is because one of the criteria for the plants selected for analysis is that they are sole producers of a particular model. This facilitates linking final sales to production.

30. Note that by sales here we mean final sales (not shipments to dealers).

31. Fine [1963, p5] provides a discussion of this process, "When the line was stopped at the end of the model run, the bulk of the production force would be laid off, new machinery would be installed, new dies moved into place, and the assembly line rearranged for the production of the new model."

32. The number of cars produced per shift depends on the line speed and the number of hours that a line operates during a shift.

33. That is, we fix at its mean (i), (ii) and (iv) in the above decomposition.

34. The summer slowdown in production observed in these plots is consistent with the seasonality in manufacturing production reported by Beaulieu and Miron [1989]. Our findings here suggest that at least part of the pervasive summer slowdown is due to machine replacement/retooling effects.

35. Since this data is at the plant level, it does not suffer from some of the data problems on testing whether the variance of production exceeds the variance of sales described in Fair [1989].

36. These numbers can exceed 100% if, say, a reduction in output due to a change in the number of shifts occurs at the same time that line speed is increased.

37. The change in GM's production policy as well as the retooling process is described in some detail by Sloan [1964].

38. One can see this by looking at figures on monthly production of passenger cars during the 1920s and 1930s reported in various issues of Automobile Facts and Figures.

39. The data is from the Federal Reserve Bulletin, August 1940 and is monthly without any seasonal adjustment.

40. For the January 1923 to June 1936 period, the output series was the monthly industrial production index for automobiles in the 1940 Federal Reserve Bulletin, not seasonally adjusted. The employment series comes from Beny [1936]. These data sources are the same as those used in Bernanke [1986] in his study of labor markets during the Depression. The output data for the period from January 1978 to July 1990 is from the industrial production index for passenger cars and trucks (industry #3711) and the total hours data covers the same industry group. The sample period for the post WWII period is chosen given its overlap with the plant level auto data presented above. The seasonal patterns for the entire post WWII period are very similar to that reported in Figure 4.

41. In an interesting study of productivity variations in the interwar years, Bernanke-Parkinson [1990] attribute part of the observed procyclical productivity to labor hoarding and the rest to increasing returns. In contrast to their effort, the data we consider is monthly. Further, we focus on the seasonal behavior of productivity.

42. Although some of the auto plants we examine (see Table 2) do experience machine replacement in December.

43. In fact, our preliminary investigation of the data indicates that not all of the spring productivity burst can be accounted for by internal increasing returns since there are observations in which Spring and Fall employment levels are approximately the same, yet Spring output and productivity is much higher.

44. We have made some progress on this already by simulating a single agent model as described in Section II of this paper. We need to augment that model by: allowing for some consumption smoothing through inventory holdings of finished goods, explicitly distinguishing investment from retooling and introducing stochastic elements into preferences and technology.