Geophys. J. R. astr. Soc. (1967) **13**, 61–70.

The Amplitude–Distance Curve for Short Period Teleseismic *P*-Waves

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Summary

An amplitude–distance curve has been computed for short period P waves in the range $\Delta = 30°–102°$ (and $h = 0$) using the amplitudes of explosion signals only. Effects of source asymmetry can thus be discounted and the problem of deciding what amplitude to measure is reduced because the maximum amplitude of explosion signals always occurs in the first few cycles. To increase the consistency of the measurements all the amplitudes have been measured by the same person.

Assuming the effect of distance is constant over a series of discrete intervals of distance, the amplitude–distance curve with its confidence limits has been estimated by least squares using data from 65 stations which recorded some or all of the explosions fired at six sites. The most striking features of the curve are: (1) a sharp maximum at about $\Delta = 35°$, (2) a minimum followed by a sharp increase at $\Delta = 75°$, and (3) a sharp minimum at $\Delta = 93°$. Over the remainder of its length the curve is similar to the Gutenberg and Richter curve. Station magnitude corrections have also been estimated.

1. Introduction

An amplitude–distance curve derived from explosion data should be more reliable than curves derived from earthquake data: explosions have radially symmetrical radiation patterns and their seismic signals are almost always the same shape with the maximum amplitude occurring in the first few cycles. Differences in the amplitude of signals recorded from a single explosion at two recording stations are unlikely to be due to source asymmetry (a significant factor if earthquakes are shears), and the difficulties of defining what amplitude to measure for magnitude determinations are reduced.

We have collected the available World Wide Standard Seismograph Network (WWSSN) and Canadian records of underground explosions and the amplitudes of the explosion signals have been read by one seismologist (Mr P. D. Marshall). The accumulated data has been used to estimate both an amplitude–distance curve and station magnitude corrections for short period teleseismic P waves.

2. The measurement of magnitude

The size of a seismic event is measured by its 'magnitude'. Seismic magnitude scales give the magnitude relative to an arbitrary baseline; the most used scale is the 'unified magnitude' of Gutenberg and Richter. On this scale the magnitude of an event is given by:

$$\log_{10}(A/T) + Q(\Delta, h)$$

where A is the half maximum peak–peak ground motion in microns, T the apparent period in seconds and $Q(\Delta, h)$ a factor to correct for the distance (Δ) of the event from the recording station and for the depth of the event focus (h).

The amplitude A required is half the maximum amplitude of the P phase; usually defined as the maximum amplitude in the first few cycles (L.R.S.M. 1963). On earthquake records P may not be clearly separated from later arrivals (Fig. 1); the seismologist has then to decide subjectively what amplitude to measure. For teleseismic explosion records, however, the P arrival is always clearly marked (Fig. 2), the maximum amplitude always occurs in the first few cycles and for a given site the maximum usually occurs at the same position on the record. Amplitude A is therefore easily defined and measured.

Defining and hence measuring T can also be difficult but the necessity of measuring T can be by-passed for short period observations ($T \approx 1$ s) provided the recording instruments have a gain inversely proportional to T over the frequency range of interest; this is true for the records used in the present study. The amplitude in millimeteres measured from the record is converted to ground motion in microns by dividing by the gain, G, at apparent period T. If $G = K/T$ where K is the gain at 1 c/s, dividing by G and then by T to get A/T is equivalent to dividing the measured amplitude in millimetres by the gain at 1 c/s, i.e. K. The period T then need not be measured.

The measurement of amplitudes will never be as objective as the measurement of arrival times, but by working only with explosions, avoiding the measurement of T and having one person read all amplitudes directly from original records (or good copies) a reliable body of $\log_{10}(A/T)$ data has been collected from which an amplitude–distance curve, essentially $Q(\Delta, h)$ for $h = 0$, can be derived.

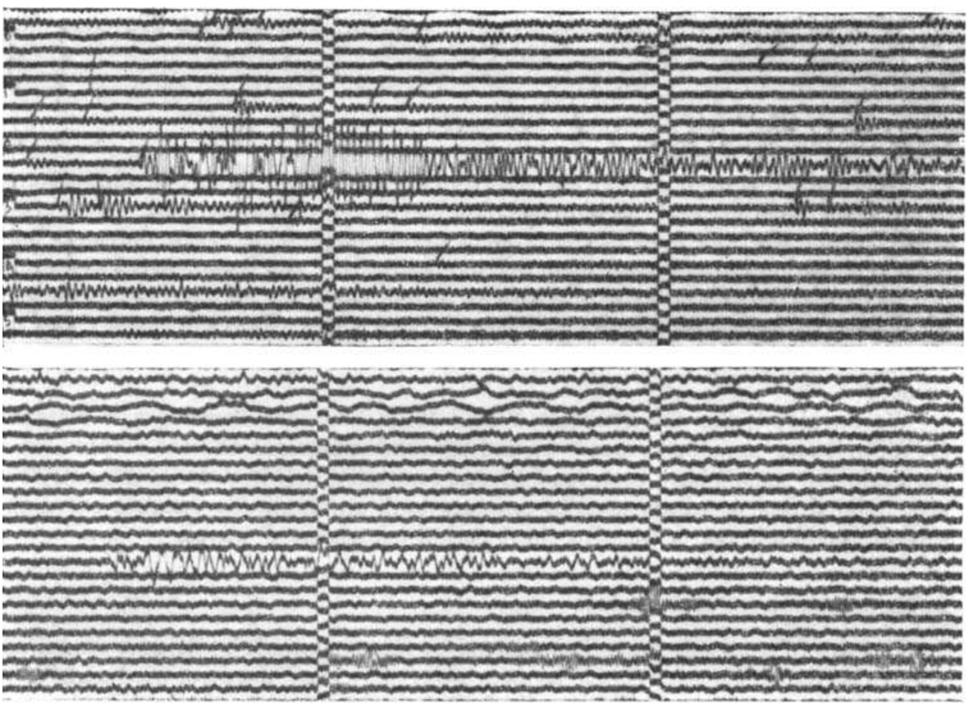

FIG. 1. Example of earthquake records from WWSSN stations. Two recordings of an earthquake on 1965 February 4 are shown, the upper record from Rapid City, South Dakota, the lower from Georgetown, Washington D.C. In each case the gain is 25 000.

3. The determination of the amplitude–distance relationship

The analysis has been carried out assuming that records from one station for different events fired at the same site and from different stations for each event, are linearly related. For this to be so: (1) any two events must generate Fourier spectra $g_i(\omega)$ and $g_j(\omega)$ such that in the frequency range of interest $g_i(\omega)=kg_j(\omega)$; (2) the effect of the recording station must be independent of the distance and azimuth of the firing site; (3) the propagation path effects must consist of two independent effects, one a function of distance only, $f(\Delta)$, the other a function of frequency only, $h(\omega)$. If $g(\omega)$ is the source spectrum then the signal spectrum at any distance is $f(\Delta)\,h(\omega)\,g(\omega)$;

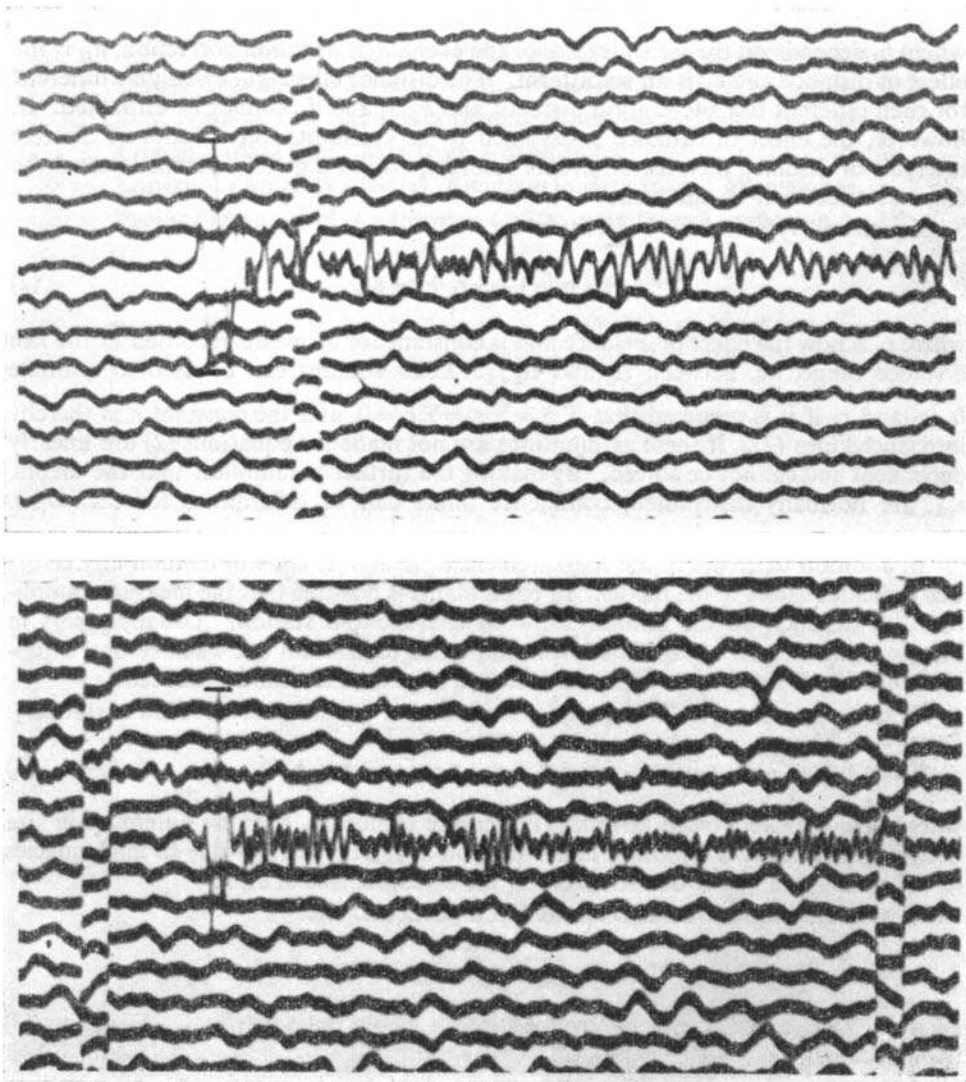

FIG. 2. Examples of explosion records from WWSSN stations. The upper recording is of a French event, 1965 February 27, recorded at Bulawayo. The lower recording is of a Russian event, 1964 November 16, recorded at College, Alaska. In each case the peak-to-peak amplitude represents a ground motion of about 180 mμ. The time scales are slightly different.

$h(\omega)$ takes care of non-elastic behaviour and implies the 'constant Q/T' model (Carpenter 1966). (Note that it is not assumed that amplitude is linearly proportional to yield.)

If the above assumptions are correct all explosion records from the same site should have the same shape. In general this is true (Thirlaway 1966); differences in shape can occur because of differences in the depth at which the explosions were fired but such differences are usually small. The assumptions made would seem to be fairly justified.

Now consider n explosions all fired at different sites and recorded at some or all of q stations. Then if m_{ij} is $\log_{10}(A/T)$ for the ith event recorded at station j:

$$m_{ij} = b_i + s_j + d_{ij} + \varepsilon_{ij}, \tag{1}$$

where b_i depends on the (seismic) size of the event, s_j is a station correction, d_{ij} is the effect of distance and ε_{ij} is an error term. The distance effect will usually be different for each different test site–station distance and b_i, s_j and d_{ij} cannot be estimated. If, however, the effect of distance is assumed to be constant over a series of discrete intervals of distance then for all events recorded within a given distance interval the distance effect will be the same.

Equation (1) can now be written:

$$m_{ijk} = b_i + s_j + r_k + c + \varepsilon_{ijk}, \tag{2}$$

where r_k is now the effect of distance and is constant for all events recorded in the kth distance range, i.e. between Δ_k and Δ_{k+1}. Least squares can be used to estimate b_i, s_j and r_k if it is assumed that $\sum^{i} b_i = \sum^{j} s_j = \sum^{k} r_k = 0$, and the constant c is thereby introduced into (2). If these assumptions are not made the equations (2) are linearly dependent and cannot be solved. By making the further assumption that the errors, ε_{ijk}, are normally distributed, confidence limits can be determined for each b_i, s_j and r_k.

In addition to errors in the measurement of A and T, ε_{ijk} will contain any errors due to inadequacies in the model. In particular it is possible that the measured amplitude, A, depends on the azimuth between test site and station. Usually errors in measurement cannot be separated from those due to azimuth but when several explosions have been fired at the same site errors in measurement can be reduced.

Thus if a_{pjh} is $\log_{10}(A/T)$ for explosion h fired at test site p as recorded at station j:

$$a_{pjh} = e_h + b_p + s_j + r_k + \zeta_{pjk} + \eta_{pjh}, \tag{3}$$

where b_p is the effect of an (arbitrary) reference explosion, e_h is the difference in size between the reference explosion and explosion h, ζ_{pjk} is the error due to inadequacies in the model and η_{pjh} is due to errors in the measurement of A/T.

If only one explosion has been fired at a particular site:

$$\zeta_{pjk} + \eta_{pj1} = \varepsilon_{pjk}.$$

Now b_p, s_j, r_k and ζ_{pjk} can be grouped into a single unknown m_{pjk}. Equation (3) then becomes

$$a_{pjh} = e_h + m_{pjk} + v + \eta_{pjh}$$

and m_{pjk}, e_h and v estimated in the presence of η_{pjh} by least squares. As with equation (2), it is necessary to introduce a constant, v, into equation (3), and assume that $\sum^{h} e_h = \sum^{j} m_{pjk} = 0$, otherwise the unknowns are linearly dependent. Combining the results for all explosions at a single test site in this way, gives a more accurate estimate of m_{pjk} from which to derive the amplitude–distance curve.

An alternative method of analyzing the data is to assume the variation of amplitude with distance can be represented by a polynomial. This means, for any reasonable order of polynomial, that the amplitude is assumed to vary relatively slowly with distance. This may not be so, and the method that assumes the curve is constant over a series of discrete intervals is therefore preferred.

Fig. 3 shows the amplitude–distance curve using data from 6 sites and 65 stations and assuming that the amplitude–distance curve is constant over intervals of 3° (between 3° and 102°). Where more than one explosion has been fired at a particular site the results for all the explosions at that site were combined before the amplitude–distance curve was determined.

The test sites and numbers of explosions used are:

USSR Kazakh Site: data from 8 explosions;

USSR Kazakh Site 2: data from 1 explosion;

U.S. Nevada Test Site: data from 5 explosions;

French Sahara Site: data from 4 explosions;

Longshot (Amchitka Island, Aleutians): data from 1 explosion;

Pacific Ocean off N. Californian Coast: data from 1 explosion.

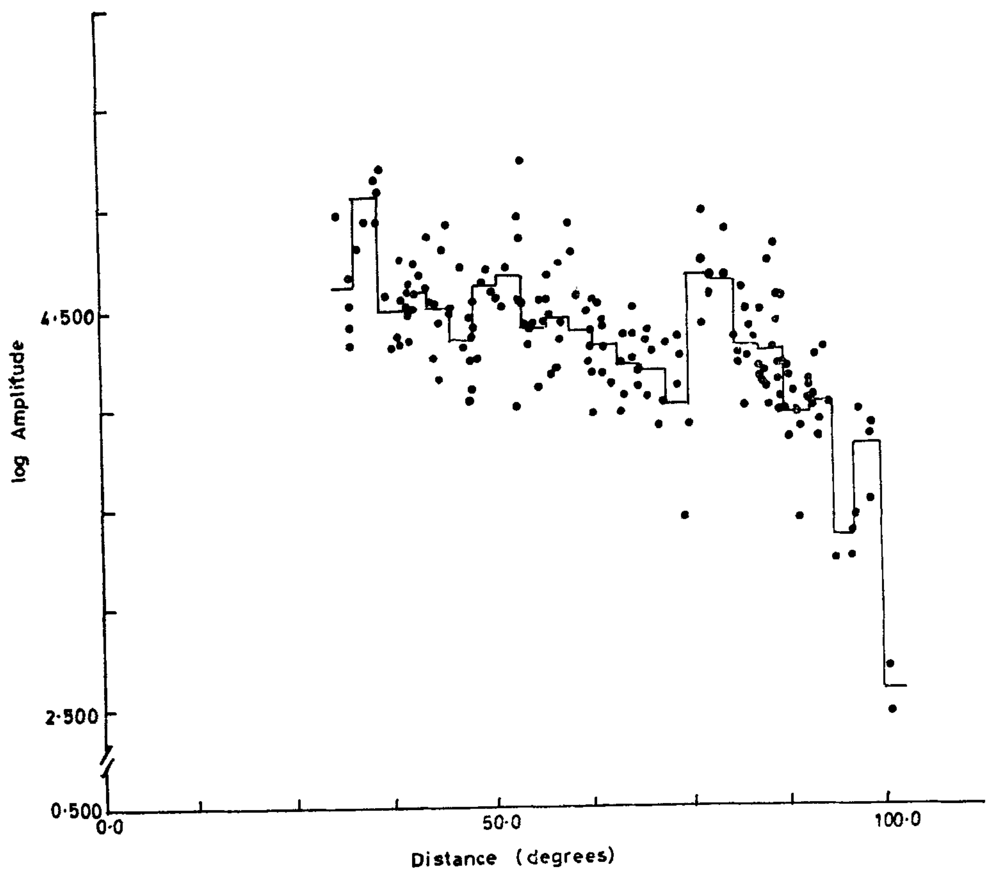

FIG. 3. Amplitude–distance curve and data points.

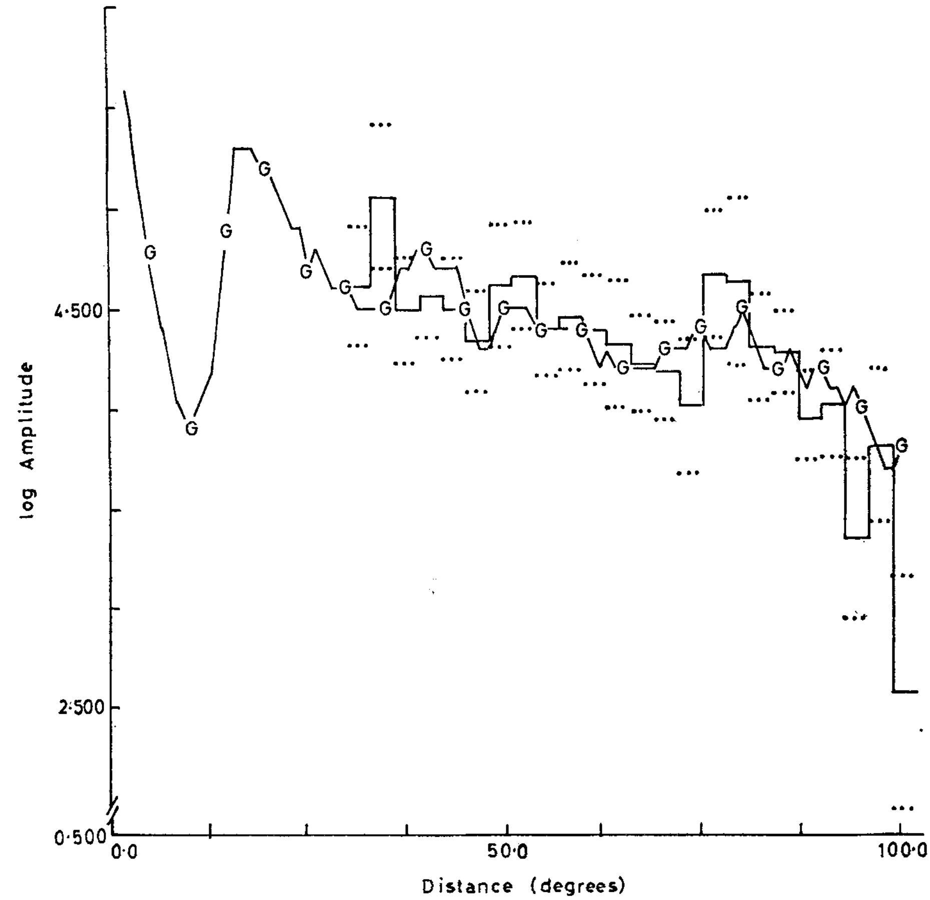

FIG. 4. Amplitude–distance curve with confidence limits and the Gutenberg–Richter curve. The Gutenberg–Richter curve is represented by —G—G—G— and the 95% confidence limits are shown by dotted lines.

From USCGS information most of the Kazakh explosions appear to have occurred at roughly the same site, but one explosion appears to have occurred at a significantly different site. The event has been treated separately and referred to as Site 2. Only stations that recorded events from at least two sites have been used to derive the amplitude–distance curves. As the absolute level of the derived curve is arbitrary the curve has been adjusted to coincide as nearly as possible with the Gutenberg and Richter curve (Fig. 4) (see Richter 1958).

A table of amplitude against distance is given in Table 1. Station corrections are given in Table 2 and residuals in Table 3.

4. Discussion of results

The amplitude–distance curve presented here has been derived using a simple model and straightforward least squares. But the results are encouraging: the curve shows considerable similarity to the Gutenberg and Richter curve except that the curve presented here shows (1) a sharp peak between 33° and 36°, (2) a slight minimum followed by a marked increase at about 75° and (3) a sharp minimum between 93° and 96°. No attempt has yet been made to interpret these sharp changes in the amplitude–distance curve in terms of velocity structure in the Earth but this will be attempted if further data shows these maxima and minima are real.

Table 1

Amplitude–distance table

Distance (degrees)	Log amplitude (with 95% confidence limits)
30·0	
	4·61±0·30
33·0	
	5·06±0·36
36·0	
	4·49±0·26
39·0	
	4·58±0·22
42·0	
	4·50±0·25
45·0	
	4·34±0·25
48·0	
	4·62±0·30
51·0	
	4·66±0·26
54·0	
	4·39±0·23
57·0	
	4·46±0·27
60·0	
	4·39±0·27
63·0	
	4·32±0·32
66·0	
	4·22±0·24
69·0	
	4·19±0·25
72·0	
	4·01±0·33
75·0	
	4·67±0·32
78·0	
	4·64±0·41
81·0	
	4·30±0·27
84·0	
	4·28±0·21
87·0	
	3·97±0·22
90·0	
	4·03±0·27
93·0	
	3·34±0·40
96·0	
	3·82±0·38
99·0	
	2·57±0·58
102·0	

Table 2

Station corrections
(and 95% confidence limits)

Station	Correction	Station	Correction
AAM	0·00±0·36	HNR	0·37±0·43
ADE	0·82±0·57	IST	−0·63±0·41
AKU	0·00±0·36	KEV	−0·16±0·45
ALE	−0·32±0·28	KIP	−0·14±0·44
ALQ	0·13±0·37	KON	0·19±0·46
AQU	−0·37±0·43	KTG	−0·04±0·36
ARE	0·36±0·36	LND	0·07±0·35
ATL	0·35±0·43	LON	−0·17±0·44
ATU	0·01±0·45	MAL	0·01±0·44
BAG	0·07±0·37	MBC	0·06±0·29
BEC	−0·19±0·46	MNT	0·13±0·39
BHP	0·14±0·35	MUN	0·12±0·51
BLA	−0·11±0·38	NAI	−0·13±0·38
BOZ	0·20±0·34	NDI	−0·22±0·46
BUL	−0·10±0·39	NOR	−0·39±0·35
CAR	0·09±0·32	NUR	−0·31±0·32
CHG	−0·47±0·45	PMG	−0·05±0·43
CMC	0·51±0·48	POO	−0·01±0·36
COL	−0·13±0·28	PRE	−0·08±0·36
COP	0·14±0·43	PTO	−0·14±0·36
COR	0·45±0·39	RCD	0·07±0·32
CTA	−0·30±0·44	RES	−0·22±0·30
DAL	0·08±0·35	SCH	0·29±0·32
EDM	0·11±0·35	SCP	0·15±0·30
EKA	0·28±0·33	SHI	0·07±0·42
ESK	0·32±0·45	SJG	0·19±0·44
FBC	0·12±0·34	STU	0·12±0·43
FLO	−0·04±0·40	TOL	0·28±0·43
GDH	−0·16±0·43	TUC	−0·40±0·46
GEO	0·08±0·28	VAL	−0·59∓0·37
GOL	−0·14±0·43	WIN	−0·15±0·37
HAL	−0·25±0·39	YKA	0·16±0·36
HKC	−0·11±0·43		

Table 3

Residuals

Station	USSR site 1	USSR site 2	French test site	Nevada test site	Off North California	Longshot
AAM	−0·08	−0·07				0·15
ADE		0·11				−0·11
AKU	0·01		0·41			−0·42
ALE	−0·02		0·21	0·08	−0·29	0·02
ALQ	0·10	−0·10	−0·02			0·02
AQU	−0·01			0·01		
ARE			−0·27	0·14	0·13	
ATL	0·09	0·03				−0·11
ATU	−0·09	0·09				
BAG	0·06	0·19				−0·26
BEC				−0·03		0·03
BHP			0·27	−0·35		0·08
BLA	−0·09	−0·18				0·28
BOZ	−0·11		0·15			−0·04
BUL	−0·06	0·10	−0·04			
CAR			−0·14	−0·16	0·14	0·16
CHG	−0·13		0·13			
CMC	−0·09	0·09				
COL	0·38	0·45	−0·52	−0·13	−0·18	

Table 3 (Continued)

Station	USSR site 1	USSR site 2	French test site	Nevada test site	Off North California	Longshot
COP	0·09	−0·09				
COR	−0·16	−0·10				0·26
CTA	0·26					−0·26
DAL	0·04	0·10	−0·22			0·09
EDM	0·07		−0·07			0·01
EKA	0·11	−0·25	−0·20			0·34
ESK	−0·09					0·09
FBC	0·09	−0·02		0·06		−0·12
FLO	−0·16	−0·08	0·24			
GDH	0·15					−0·15
GEO	−0·08	−0·06	0·17		0·02	−0·05
GOL	−0·04					0·04
HAL		−0·28	0·26		0·02	
HKC		−0·00				0·00
HNR				0·22		−0·22
IST	0·03	0·14				−0·17
KEV	−0·10			0·10		
KIP				−0·08		0·08
KON	−0·25			0·25		
KTG	0·02			0·27		0·29
LND	0·01	0·26	−0·28			
LON	0·17					−0·17
MAL	−0·04			0·04		
MBC	0·18	0·57	−0·57	−0·10	−0·08	
MNT				0·19	0·03	−0·21
MUN	0·11					−0·11
NAI	−0·15	0·00	0·15			
NDI			0·11			−0·11
NOR	0·28			0·03		−0·31
NUR	0·05			−0·01	−0·19	0·15
PMG	0·16					−0·16
POO	−0·28		−0·17			0·45
PRE	−0·11	0·19	−0·07			
PTO		−0·13		−0·10		0·23
RCD	−0·30	−0·15	0·52			−0·07
RES	−0·29		−0·12	0·05	0·37	
SCH	−0·16	0·13		−0·13		0·16
SCP	0·02		0·18		0·04	−0·25
SHI			−0·26			0·26
SJG			−0·01			0·01
STU	0·30			−0·30		
TOL	0·04			−0·04		
TUC		−0·29				0·29
VAL		−0·65	0·34			0·31
WIN	0·06	0·02	−0·08			
YKA	0·02		−0·09			0·08

The residuals (Table 3 and Fig. 3) show possible evidence of other sharp peaks and troughs in the curve but the data available is insufficient to warrant a more detailed analysis using narrower range intervals within which the curve can be assumed constant. However, as further data accumulates narrower range intervals will be used in an attempt to discover any fine structure of the curve. Large residuals that cannot be attributed to distance effects or station corrections will indicate azimuthal effects (or errors).

The station corrections range from −0·6 for Istanbul, Turkey, to +0·8 for Adelaide, Australia. The origin of these corrections has to be investigated; they might show a correlation with station travel time corrections derived by other workers (see, for example, Cleary & Hales 1966). It may also be possible to show a correlation between the station corrections and the acoustic impedance of the surface layers at the recording station (Carpenter 1966) and if this effect were removed the residual station corrections might be related to lateral variations of Q in the upper mantle.

Finally, it should be noted that the results described above are preliminary; they will be upgraded as more data becomes available. The preliminary results are presented here because of the interest shown in the work and also in the hope that seismologists will send more data to include in the analysis.

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