The analysis of geometrical and thermal errors of non-Cartesian structures

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Abstract

There is currently much interest in non-Cartesian machines in the machine tool industry. The analysis of errors arising from component variations in geometry and temperature is far more complex than in the case of the Cartesian machine. This is because the errors are no longer additive along each axis. This paper describes techniques leading to the development of general software which is capable of analysing the geometric and thermal errors in any structure consisting of measurable struts. This limitation is not so restrictive as it might appear, since solid components can usually be modelled as interconnected struts if large temperature gradients do not produce high stresses within their volume. The software also provides an animated display showing the moving structure. The techniques described constitute three efficient matrix methods which can be programmed in a general way. The approach is to break down a machine into sub-assemblies and to analyse each of these individually. The sub-assemblies are brought together into an overall model and the predictions of errors, such as the tilt of a spindle, are evaluated. Each sub-assembly can be assigned an average temperature or each strut can be treated separately. Hence, the errors arising from a source of heat, such as a spindle motor, can be evaluated, as can the effect of random errors in strut lengths. Three dimensional plots of errors in any plane are available.

Introduction

Non-Cartesian machines such as the hexapod have the advantage of stiffness and lightness compared with conventional machines, where bending moments of the structure provide the machining forces. A useful introduction

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to non-Cartesian machines is available on Internet. [1] In the hexapod the machining forces are along the struts and each member is uniformly stressed. The analysis of errors arising from component variations in geometry and temperature is far more complex than in the case of the Cartesian machine, Waldron. [2] This is because the errors are no longer additive along each axis. This paper describes techniques leading to the development of general software which is capable of analysing the geometric and thermal errors in any structure consisting of measurable struts. This limitation is not so restrictive as it might appear, since solid components can usually be modelled as interconnected struts if large temperature gradients do not produce high stresses within their volume.

The general problem is to be able to calculate easily the Cartesian co-ordinates of all nodes in a structure from a knowledge of the exact inter-nodal distances at any one point in time. The effect of changes in these distances due to temperature changes or other influences can then be easily accommodated. It is an assumption in the machines under consideration that they are fully triangulated. That is, all points in the machine can be found from three reference points and the necessary inter-nodal distances between the other points to fully define the geometry. However, care must be taken not to define too many inter-nodal distances since any extras will be redundant and this could easily lead to confusion in calculations.

It is common practice to estimate errors of this type by a technique involving estimation of the effect of each possible source of error by a laborious reasoning based on its effect on all points in the structure. This is error prone in itself and requires much patience and insight into the geometry of the mechanism. An alternative and more rigorous method is sought. It may appear to be a relatively simple task to calculate all the points in a triangulated structure from the reference points and the inter-nodal distances, but this is far from the case. There are several problems.

1. The geometrical calculations are difficult if carried out by conventional means.

2. The general triangulation problem of finding a point from its distances to three other known points leads to an ambiguous solution. The correct choice depends on which side of the plane containing the reference points the unknown point lies. This usually requires some engineering intuition.

3. It may not be convenient to treat every point in the structure in the same way. Some points may be permanently connected together and would be better treated as a substructure. Other points may be better fixed by other more empirical means.

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MATHEMATICAL ANALYSIS

The following describes several methods which have been developed to provide a flexible solution to these problems. First, there is a method of performing triangulation which gives two possible solutions, then there is a technique which removes the ambiguity by using a fourth redundant distance.

TRIANGULATION TO THREE NON-CO-LINEAR POINTS.

This technique requires the evaluation of the equation of the plane containing the three reference points shown in Fig. 1, which is similar to the diagram in our previous paper, [3] but the reference plane is no longer horizontal. A normal to the reference plane contains the desired point and intersects the plane at a point Q which it is possible to find. The unknown point X can be found using the direction cosines for the normal and the distance from point Q to the point X. This method gives two possible solutions depending on which side of the reference plane we put the unknown point. We must first find the equation of this reference plane, which can be determined from the co-ordinates of the three reference points which define the plane.

Let the plane be described by the equation: $ax + by + cz = 1$. Then we substitute the co-ordinates of the three points A, B and C to obtain in matrix notation

$$\begin{bmatrix} x_A & y_A & z_A \\ x_B & y_B & z_B \\ x_C & y_C & z_C \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{I}$$

where **I** is the unit vector. Then a, b and c can be determined by matrix inversion, thus giving the equation of the plane.

The next step is to find the point Q in the reference plane where the normal intersects the plane. This can be done by solving three simultaneous equations, including the equation above for the plane and two more which describe the normal. The latter are the equations of two of the three planes containing the circular loci. These can be found in the same way as those in the previous derivation for the case of the

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horizontal reference plane, [3] but all the terms in z must be retained since they are no longer zero.

TRIANGULATION TO ANY THREE POINTS

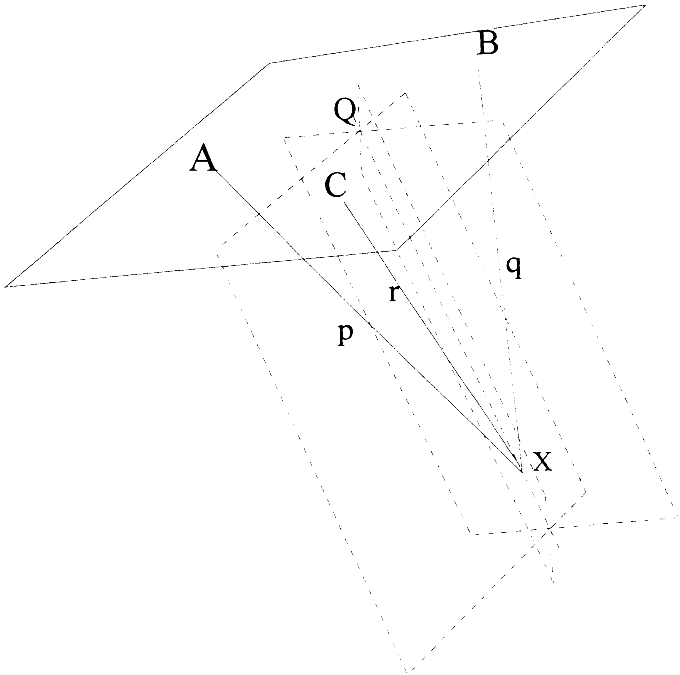

FIG 1

We therefore have

$$\begin{bmatrix} (x_b - x_a) & (y_b - y_a) & (z_b - z_a) \\ (x_c - x_a) & (y_c - y_a) & (z_c - z_a) \\ a & b & c \end{bmatrix} \begin{bmatrix} x_q \\ y_q \\ z_q \end{bmatrix} = \tfrac{1}{2} \left(\begin{bmatrix} p^2 - q^2 \\ p^2 - r^2 \\ 2 \end{bmatrix} - \begin{bmatrix} x_a^2 + y_a^2 + z_a^2 - x_b^2 - y_b^2 - z_b^2 \\ x_a^2 + y_a^2 + z_a^2 - x_c^2 - y_c^2 - z_c^2 \\ 0 \end{bmatrix} \right)$$

which can be solved for x_q, y_q and z_q by matrix inversion.

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Having found the point Q and knowing A and p it is now possible to find the distance XQ by Pythagoras,

$$XQ = \sqrt{p^2 - \left[(x_a - x_q)^2 + (y_a - y_q)^2 + (z_a - z_q)^2\right]}$$

It remains to find the point X by vector addition, for which the direction cosines l, m and n of the normal to the reference plane are required. It is convenient that these are the same as the normalised coefficients a, b and c of the plane.

Thus, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_q \pm ld_q \\ y_q \pm md_q \\ z_q \pm nd_q \end{bmatrix}$$

where $l = a / L$, $m = b / L$, $n = c / L$ and $L = \sqrt{a^2 + b^2 + c^2}$

The three reference points must not be co-linear since the two planes will then be identical and will not intersect in a line. The alternative solutions corresponding to X being on either side of the reference plane are evident in the +/- choice to be made in the last equation. The actual choice must be made having a knowledge of the problem under consideration using engineering reasoning and intuition. There is no way to remove the ambiguity using only three reference points. An obvious approach is to use four reference points as in the next section.

'QUADRILATION' TO 4 NON-COPLANAR POINTS

'Quadrilation' is a coined name for the use of four rather than three non-planar points as previously used for triangulation. There is great advantage in using four reference points along with the distances of these points from the unknown point. The dual solution that exists for triangulation no longer complicates the solution, there being only one point satisfying the criteria. Indeed, there is some redundancy in the data. Because of this care must be taken to ensure that the four distances and the co-ordinates of the four points are self consistent.

The four reference points A, B, C and D in Fig. 2 are taken in three pairs. Each pair is used in turn to find a plane at right angles to the line joining the two reference points and passing through the unknown point X. The intersection of the three resulting planes at X can then be found by treating the equations of the three planes as simultaneous linear equations and solving them by matrix inversion. This gives the co-ordinates of the point of intersection of the three planes at the unknown point.

QUADRILATION TO
FOUR NON - COPLANAR POINTS

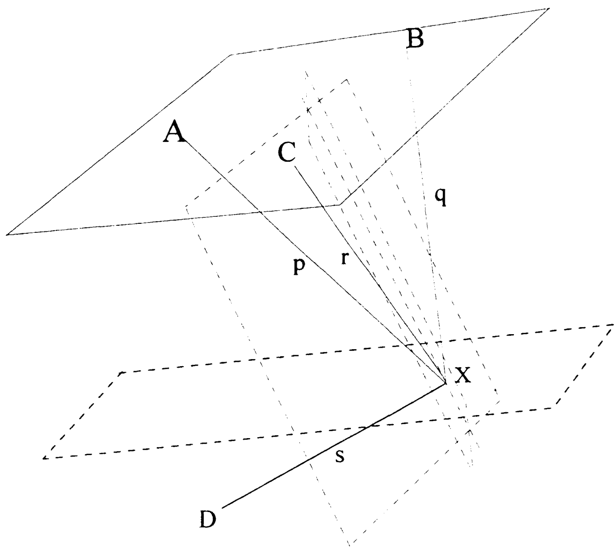

Fig 2.

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The three planes are defined in matrix form in the same way as were the two for triangulation, but incorporating a fourth point D distance s from X to provide the third equation.

The equations are:

$$\begin{bmatrix} (x_b - x_a) & (y_b - y_a) & (z_b - z_a) \\ (x_c - x_a) & (y_c - y_a) & (z_c - z_a) \\ (x_d - x_a) & (y_d - y_a) & (z_d - z_a) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \tfrac{1}{2} \left(\begin{bmatrix} p^2 - q^2 \\ p^2 - r^2 \\ p^2 - s^2 \end{bmatrix} - \begin{bmatrix} x_a^2 + y_a^2 + z_a^2 - x_b^2 - y_b^2 - z_b^2 \\ x_a^2 + y_a^2 + z_a^2 - x_c^2 - y_c^2 - z_c^2 \\ x_a^2 + y_a^2 + z_a^2 - x_d^2 - y_d^2 - z_d^2 \end{bmatrix} \right)$$

which can be solved for x, y and z, the co-ordinates of the unknown point X.

This method works well provided data is available for four points which have no inconsistencies. The matrix is never singular provided the four reference points are not coplanar. If they are coplanar then no solution exists using this method since the three planes then intersect in a common line rather than at a unique point.

ANALYSIS VIA SUBSTRUCTURES

A machine commonly consists of identifiable substructures which move relative to one another and may have movements within themselves. Such machines are best treated by analysing the geometry of each unit in its own local reference frame. This helps to avoid a problem that sometimes arises during triangulation of having less than three known distances from an unknown point to known points. A well-defined unit can be carefully analysed in a reference frame which is convenient for that unit. Later, inter-nodal distances found in the reference frame can be transferred to the overall model, thus filling in the missing information. This method works well in a non-Cartesian scanning machine where the mechanism to keep the sensor vertical is treated as a separate unit in this way. Its vertical swivel can be varied in the local frame, its inter-nodal distances found and these transferred into the main model.

APPLICATION OF TRIANGULATION AND 'QUADRILATION'

The approach to using the techniques above is to minimise the amount of programming for any one application by utilising data files containing almost all the information for that structure.

The stages in developing an analysis of a new machine design are :

1. Define one or more substructures of the machine.

2. Define at least three reference points in each substructure in local co-ordinates for the substructure.

3. Devise a method to calculate all other points in local co-ordinates in each substructure using the above methods. This will involve using say triangulation to find the fourth point, then possibly 'quadrilation' or triangulation to find each successive point in turn. The point found at each step can be used in later steps as one of the three or four reference points. Thus each substructure can be fully triangulated.. It may be necessary in some situations to calculate one or more points by normal geometrical methods. This is because a substructure may swivel, so its position must be fixed using prior knowledge of its angle.

4. Analyse the main machine by the same methods but using where necessary inter-nodal distances calculated from the already determined geometry of the substructures. It may again be necessary to calculate one or more points by normal geometrical methods.

5. When all points of the machine have been found, calculate any necessary variables such as the tilt of the head or spindle using the calculated co-ordinates by geometrical methods.

THERMAL AND GEOMETRICAL ERRORS

The Cartesian co-ordinates of the nodes of a non-Cartesian structure vary in a complex way if the individual struts change length. It is simple in the above procedure to introduce a change to the strut lengths due to thermal expansion or manufacturing errors and to re-evaluate the new nodal co-ordinates. Each sub-assembly can be assigned an average temperature or each strut can be treated separately. Hence, errors in the tilt of the spindle, arising from a source of heat such as a spindle motor, can be estimated , as can the effect of random errors in strut lengths. It is assumed that estimates of the temperature of the components of the machine are available. They can be estimated using thermal imaging methods, as described by Allen. [4]

The software allows output of the calculated errors in two dimensional arrays for any cross section of the volume of the machine. Thus three dimensional plots of errors in any plane are available.

APPLICATION TO A NON-CARTESIAN SCANNING MACHINE

Using the techniques described above software has been written in Pascal to analyse the errors in the vertical alignment of the sensor of a non-Cartesian scanning machine. The errors arise from slight inaccuracies in the lengths of the struts making up the mechanism which keeps the sensor vertical as the main struts move the sensor head through space. The calculated errors can in principle be used to correct the readings from the sensor in real-time. However, complexity and speed may restrict such an approach. If computational speed is a problem, then it is possible to build up an error map of the machine and save this in tables. The software must then perform three dimensional interpolation on the error map in real time. This is expensive on memory but is not computationally intensive.

A particularly useful aspect of the Pascal software is its ability to animate the structure as it moves. This is accomplished in a very simple way. Each strut in the structure which is to be drawn in the animation is defined in the data by its start and end node. After each calculation for the whole structure the software draws all the struts from the calculated nodal positions determined by triangulation methods. Using very simple co-ordinate transformations three projections are drawn, a plan and two elevations. The animation is very effective in helping to locate errors in the data, since the likely source of any problem is immediately obvious in the pictorial representations.

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