The Analysis of Opinion Evolution and Control Based on the Prisoner’s Dilemma Game in Social Networks

Xianyong Li,1 Jian Zhu,2 Yajun Du,1 and Qian Zhang1

1School of Computer and Software Engineering, Xihua University, Chengdu 610039, China
2Department of Mathematics and Physics, Xinjiang Institute of Engineering, Urumqi 830023, China

Correspondence should be addressed to Xianyong Li; lixy@mail.xhu.edu.cn

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1. Introduction

In social networks, the process of public opinion propagation is essentially the process of each netizen propagating
their opinions in their community. Based on infectious disease models, Daley and Kendall [1] proposed DK model,
thought that there is nothing critical in the process of rumors propagation, and analyzed the influence of randomness and
certainty on rumor propagation. Maki and Thomson [2] presented MK model and believed that the first dissemi
nators are controlled to suppress the propagation of rumors. Ising [3] connected the positive and negative of the particle
with the positive and negative of the view and put forward Ising model to describe opinion dynamics. Sznajd-Weron
and Sznajd [4] further proposed Sznajd model. In their model, they thought that one person will imitate their behavior around people and attract more people to imitate. On the basis of Sznajd model, Deffuant et al. [5, 6] established continuous opinion evolution model, i.e., Deffuant model and Hegselmann–Krause (HK) model. Moreno et al. [7, 8] made the simulation experiment on MK model and found that the clustering coefficient of network greatly influences the rumor propagation in scale-free networks. They further proposed a rumor dynamic model and made simulation experiments on homogeneous and heterogeneous networks. Martins [9] considered that all agents have inherent continuous opinions and external discrete behaviors and presented the continuous opinions and discrete actions (CODA) model. DeGroot [10] considered that the individual opinion is updated by the weighted average of all its neighbors’ opinions and proposed DeGroot model. Friedkin and Johnsen [11] introduced stubborn individuals, extended the DeGroot model, and proposed the Friedkin–Johnsen (FJ) model. In the FJ model, individual opinion is updated by the weighted average of the convex combination of all its neighbor nodes’ opinions and its opinion of innate belief. Gong et al. [12] improved the FJ model, proposed a structural-hole-based approach to control public opinion, and analyzed the influence of ordinary users and structural hole users on opinion evolution. Evidently, these researches
mainly focused on the opinion evolution rules and the opinion dynamics environments to depict the public opinion evolution laws in social networks.

To understand the ubiquitous existence of cooperative phenomena in the process of opinion evolution, some scholars introduced the game theory into the process of the opinion evolution and propagation in social networks. Liu et al. [13] thought that opinion propagation process is just the process of different strategy choices and applied the game theory to opinion evolution models. Li et al. [14] studied the rumor propagation based on the evolutionary game and investigated the influences of the penalty coefficient and negative message risk factor on the opinion propagation. Hilbe et al. [15] studied the evolution of extortion in iterated prisoner’s dilemma games and found that the extortion is not a stable outcome of evolution but can catalyze the emergence of cooperation. Xu et al. [16] studied the evolution of cooperation structured populations in the context of repeated games by unconditional cooperation, unconditional defection, and extortion strategies and found a nontrivial role of the population structure and the microscopic strategy dynamics in the evolution of cooperation. Nowak and May [17] first adopted the prisoner’s dilemma model to study the cooperation evolution of group organizations in rule network and found that the persons with the same strategies are gradually concentrated in a denser group by the self-organization evolution. Tang et al. [18] studied the effects of average degree on cooperation-based prisoner’s dilemma game in random networks, small-world networks, and scale-free networks.

To the best of our knowledge, few scholars studied that the opinion evolution and control based on the prisoner’s dilemma game. This paper aims to establish the opinion evolution control levels, and opinion control threshold values, providing a wide variety of control strategies for network regulators in social networks.

The remainder of this paper is organized as follows. Section 2 establishes the opinion evolution and control model and the corresponding algorithm based on prisoner’s dilemma game in detail. Section 3 carefully makes some experiments under the different initial positive opinion proportions, different opinion control levels, and same or different opinion control threshold values in scale-free networks. Finally, Section 4 summarizes this paper and gives some future research directions.

2. Opinion Evolution and Control Based on the Prisoner’s Dilemma Game

In a social network, users mainly propagate two different opinions: positive opinion and negative opinion. Considering that a user is greatly influenced by its neighbors’ opinions and inspired by the prisoner’s dilemma game [15], we take two arbitrary adjacent users as two players, and consider two strategies: positive opinion propagation and negative opinion propagation for them. When two users choose the positive opinion propagation (resp., negative opinion propagation), each player obtains the payoff \( R \) (resp., \( P \)). When two players choose different strategies, the player who chooses the cooperation strategy gains the payoff \( S \), and the other player obtains the payoff \( T \). In general, the relationship among the four payoffs is \( T > R > P > S \) and \( 2R > T + S \). In particular, the donation game is a game where the player who chooses cooperation strategy pays a cost \( c \) to provide a benefit \( b \) for the other player with \( 0 < c < b \), resulting in the parameters \( T = b, R = bc, P = 0, \) and \( S = c \). For simplicity, we set \( b - c = 1 \) in the following.

For two adjacent users \( u \) and \( v \), let \( PF_{uv} \) denote by the payoff of the user \( u \) from the user \( v \), and let the values of negative, neutral, and positive opinions of \( u \) be denoted by \( a_u = -1, 0, 1 \), respectively. Because the users holding neutral opinions are not susceptible to or influenced by others, we only consider the users holding positive and negative opinions and their interaction. Then, we can obtain the payoff of user \( u \) from user \( v \):

\[
P_{uv} = \frac{1}{4} (1 + a_u)(1 + a_v)(b - c) + \frac{1}{4} (1 + a_u)(1 - a_v)(-c)
\]

\[
+ \frac{1}{4} (1 - a_u)(1 + a_v)b + \frac{1}{4} (1 - a_u)(1 - a_v) 	imes 0
\]

\[
= \frac{1}{2} ((1 + a_u)b - (1 + a_v)c),
\]

(1)

where \( a_u, a_v \in \{-1, 1\} \).

Denote the set of all neighbors of user \( u \) by \( N(u) \). Then the total payoff of user \( u \), that is, from its all neighbors, denoted by \( PF_u \), is the following:

\[
PF_u = \sum_{v \in N(u)} PF_{uv} = \frac{1}{2} \sum_{v \in N(u)} [(1 + a_v)b - (1 + a_u)c]
\]

\[
= \frac{1}{2} \left[ \sum_{v \in N(u)} (1 + a_v)b - (1 + a_u)|N(u)|c \right].
\]

(2)

Let \( St_u \) and \( deg_u \) stand for the strategy, i.e., positive opinion propagation or negative opinion propagation, and the number of the neighbors of user \( u \), respectively. During the game, user \( u \) continues to update its strategy based on its neighbors’ payoffs and its payoff. Then we assume that \( u \) randomly chooses a strategy from one of their neighbors, say \( v \) with the strategy \( St_u \), as their updating strategy, and the updating probability can be defined as

\[
Pr_{uv} = \frac{PF_v - PF_u}{\max\{deg_u, deg_v\}(b + c)}
\]

(3)

In a social network excluding the nodes of neutral opinions, let \( pp \) (resp., \( pn \)) denote the proportion of the users holding positive opinions (resp., negative opinions), simplified as positive (negative) opinion proportion in the following, at some time. As users continue to change their opinion propagation strategies, the values \( pp \) and \( pn \) vary.
continuously. If the values \( pp > 0.5 \), \( pp = 0.5 \), and \( pp < 0.5 \) in the network, we call the network as positive, neutral, and negative opinion networks, respectively. For the neutral opinion network and negative opinion network, we need to employ some control strategies to change the networks into positive opinion networks.

When the proportion of the users holding positive opinions \( pp \) (resp., the proportion of the users holding negative opinions \( 1 - pp \)) reaches a control threshold value, denoted by \( L \in [0, 1] \), the network regulators should adopt the control strategy, such as persuading users, providing some positive information to change some users’ opinions, to reduce the proportion of negative opinions in the social network. For user \( u \) holding a negative opinion, we assume that the penalty variable \( C(k) \) that represents opinion control level \( k \) for the payoff of \( u \) is

\[
C(k) = y \times \text{deg} \times k,
\]

where \( y \) is a parameter, \( \text{deg} \) stands for the average degree of the network, and \( k = 0, 1, 2, 3, 4, 5 \). The higher the opinion control level, the greater the costs that the network regulators need. Obviously, when \( k = 0 \), \( C(k) = 0 \) implies that the user is not controlled by network regulators in the network. Then, we further get a new payoff with opinion control level \( k \) for user \( u \), denoted by \( PF_u(k) \) as follows:

\[
PF_u(k) = PF_u - y \times \text{deg} \times k,
\]

where \( k = 0, 1, 2, 3, 4, 5 \). Obviously, when \( k = 0 \), we have \( PF_u(0) = PF_u \). Similar to the former user’s strategy updating principle, a user \( u \) randomly chooses the strategy of one of their neighbors, say \( v \), with the strategy \( St_v \) as their updating strategy, and the updating probability is

\[
Pr_{u \leftarrow v, v \in N(u)}(k) = \frac{PF_u(k) - PF_v(k)}{\max\{\text{deg}_u, \text{deg}_v\} \times (b + c)}
\]

Now, we propose an opinion evolution and control algorithm based on prisoner’s dilemma game (PDG-OEC algorithm) as follows (Algorithm 1).

For the network regulators, they can control opinion evolution trends in social networks by flexibly changing opinion control levels and opinion control threshold values based on the PDG-OEC algorithm.

3. Experiment

As the connections of nodes (nodes’ degrees) in social networks obey power law distribution, we will adopt the scale-free network [19] to make simulation experiments on the proposed opinion evolution and control model and the PDG-OEC algorithm. To illustrate the effect of network topology on our model, we set the parameters of scale-free networks with 1000 nodes as follows. The number of the initial nodes is \( m_0 = 20 \); the number of newly added nodes in each time is \( m = 1, 3, 5, 7, 9 \), respectively. On the five scale-free networks, when the proportion of the initial positive opinion (initial proportion) is \( pp = 0.5 \), we obtain five opinion evolution trends as shown in Figure 1. By comparison, it is found that the positive opinion proportions are slowly descending to a stable state (even tend to zero) in all networks. The reason is that, under the opinion evolution model based on prisoner’s dilemma game, if two adjacent users choose different strategies, then the user who chooses negative opinion propagation gains more payoff compared to another user, resulting in users tending to choose negative opinion propagation in the networks. In fact, compared to true information, rumors (false information) are more easier to propagate from one user to others and to be accepted by users to some extent. When \( m = 5 \), the positive opinion proportion in the corresponding network declines the slowest compared with three other networks. For our experiment purpose, the scale-free network with the parameters \( m_0 = 20 \) and \( m = 5 \) will be chosen. For the newly added nodes, their initial strategies are randomly chosen from positive opinion propagation and negative opinion propagation.

In the following experiments, we directly set \( y \times \text{deg} = 2 \), and then the opinion control level \( k \) is \( C(k) = 2 \times k \) \((k = 0, 1, 2, 3, 4, 5) \), abbreviated as \( C = 0, 2, 4, 6, 8, 10 \).

Next, we analyze the opinion evolution trends in the scale-free network under the different initial proportions, opinion control levels, and opinion control threshold value \( L = 0.5 \) (see Figure 2). In Figure 2(a), when the initial proportion \( pp = 0.1 \), if \( C = 2 \), it has no effect on the trend of network opinions; if \( C = 4, 6, 8 \), the positive opinion proportions are increasing to a small extent and are controlled within the range of 0.1 to 0.2, respectively; if \( C = 10 \), the control effect is significant, and the positive opinion proportion continues to rise more than 0.5. It can be seen that, under the initial proportion \( pp = 0.1 \), the network regulators should adopt the opinion control level 5 to achieve positive opinion network. In Figure 2(b), when the initial proportion \( pp = 0.2 \), if \( C = 2 \), the positive opinion proportion is controlled about 0.15, a stable state; if \( C = 4 \), the positive opinion proportion increases continually in excess of 0.5; if \( C = 6, 8, 10 \), the positive opinion proportions go up to 1, keeping a stable state, respectively, and the larger \( C \) is, the faster it goes up. It can be seen that, under the initial proportion \( pp = 0.2 \), the network regulators should adopt the opinion control level 2 to achieve positive opinion network. In Figure 2(c), when the initial proportion \( pp = 0.3 \), if \( C = 2 \), the positive opinion proportion just exceeds 0.1, a stable state; if \( C = 4 \), the positive opinion proportion just exceeds 0.3, a stable state; if \( C = 6, 8, 10 \), the positive opinion proportions keep going up, respectively. After the 16th time step, the larger \( C \) is, the faster it goes up. It can be seen that, under the initial proportion \( pp = 0.3 \), the network regulators should adopt the opinion control level 2 to achieve positive opinion network. In Figure 2(d), when the initial proportion \( pp = 0.4 \), if \( C = 2 \), the positive opinion proportion quickly reaches about 0.52, a stable state, at the 7th time step; if \( C = 4, 6, 8, 10 \), the positive opinion proportions rapidly increase and then slowly go up to 1, a stable state, respectively. It can be seen that, under the initial proportion \( pp = 0.4 \), the network regulators should adopt the opinion control level 1 to achieve positive opinion network. In Figure 2(e), when the initial proportion \( pp = 0.5 \), if \( C = 2 \), the positive opinion proportion
slowly decreases to 0.2, a stable state; if \( C = 4, 6, 8, 10 \), the larger \( C \) is, the faster the positive opinion proportion increases; finally, the positive opinion proportions are stable at 0.6 for \( C = 4 \) and at the range of 0.9 and 1 for \( C = 6, 8, 10 \). It can be seen that, under the initial proportion \( pp = 0.5 \), the network regulators should adopt the opinion control level 2 to achieve positive opinion network. By comparison of Figures 2(a)–2(e), under the control threshold value \( L = 0.5 \), if the positive opinion proportion \( pp = 0.1 \), the network regulators need to choose the opinion control level 5; if the initial proportion \( pp \geq 0.2 \), they can choose opinion control level 2 (\( C = 4 \)) to change the negative opinion network and neutral opinion network into positive opinion networks.

Now we discuss opinion evolution trends under different positive opinion proportions, opinion control threshold values, and opinion control levels in the scale-free network. For simplicity, in the following experiments, we only consider opinion control levels 1, 3, and 5 (i.e., \( C = 2, 6, 10 \)), called low, medium, and high control levels, respectively. For the initial proportion \( pp = 0.1 \), from Figure 3(a), it is shown that if the network regulators adopt the low control level (\( C = 2 \)), while \( L = 0.4, 0.3, 0.2, 0.1 \), the positive opinion proportions add in small increments and then slowly go down below 0.1; meanwhile, when \( L = 0.5 \), the positive opinion proportion increases to about 0.15, a stable state. This implies that, under the initial proportion \( pp = 0.1 \), the network regulators using the low control level cannot change

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**Algorithm 1**: PDG-OEC algorithm.

**Input**: Social network \( G = (V, E) \) excluding the nodes of neutral opinions, \( o_u \in V(G), \gamma, L \)

**Output**: The final \( pp \)

1. Calculate the average degree of the graph \( \overline{\text{deg}} \);
2. Compute the total payoff \( PF_u = (1/2)|\sum_{v \in N(u)}(1 + o_v)b - (1 + o_u)|N(u)c| \) for each \( u \in V(G) \);
3. Update the strategy of the user \( u \) by randomly choosing a strategy of some \( v \in N(u) \) with the probability \( Pr_{u \leftarrow v \in N(u)} = ((PF_u - PF_v)/(\max\{\text{deg}_u, \text{deg}_v\})(b + c)) \);
4. Count the proportion of the users holding positive opinions: \( pp \)
5. while \( pp < L \) do
6. Output the value \( pp \)
7. while \( pp \geq L \) do
8. for \( k = 1, 2, 3, 4, 5 \) do
9. Compute the payoff with opinion control level \( k \) for each \( u \in V(G) \):
   \( PF_u(k) = PF_u - \gamma \times \text{deg} \times k \);
10. Update the strategy of user \( u \) by randomly choosing a strategy of some \( v \in N(u) \) with the probability \( Pr_{u \leftarrow v \in N(u)} = ((PF_u(k) - PF_v(k))/(\max\{\text{deg}_u, \text{deg}_v\})(b + c)) \);
11. end for
12. Output the final \( pp \) under different opinion control level \( k \), \( k = 1, 2, 3, 4, 5 \).

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**Figure 1**: The opinion evolution trends for the initial proportion \( pp = 0.5 \) in scale-free networks with \( m_0 = 20 \) and \( m = 1, 3, 5, 7, \) and 9.
Figure 2: Continued.
Figure 2: Continued.
Figure 2: The evolution trends for the initial proportions $p_p = 0.1, 0.2, 0.3, 0.4, \text{and} 0.5$, opinion control levels $k = 0, 1, 2, 3, 4, \text{and} 5$, and $L = 0.5$ in scale-free networks with $m_0 = 20$ and $m = 5$. (a) The initial proportion $p_p = 0.1$, $C = 2, 4, 6, 8, \text{and} 10$, and $L = 0.5$. (b) The initial proportion $p_p = 0.2$, $C = 2, 4, 6, 8, \text{and} 10$, and $L = 0.5$. (c) The initial proportion $p_p = 0.3$, $C = 2, 4, 6, 8, \text{and} 10$, and $L = 0.5$. (d) The initial proportion $p_p = 0.4$, $C = 2, 4, 6, 8, \text{and} 10$, and $L = 0.5$. (e) The initial proportion $p_p = 0.5$, $C = 2, 4, 6, 8, \text{and} 10$, and $L = 0.5$. 

Figure 3: Continued.
the network opinion trends whenever $L = 0.5,0.4,0.3,0.2,0.1$. From Figure 3(b), for the medium control level ($C = 6$) and $L = 0.5,0.4,0.3,0.2,0.1$, the positive opinion proportions all continuously increase over 0.5; in particular, for $L = 0.1$, the positive opinion proportion rises the fastest. It shows that, under the initial proportion $p_p = 0.1$ and $C = 6$, the network regulators should select $L = 0.5$ to begin to control the network state, a low cost. From Figure 3(c), for the high control level ($C = 10$) and $L = 0.5,0.4,0.3,0.2,0.1$, the positive opinion proportions all rapidly go up over 0.5 and reach the final stable state value about 0.94; in particular, for $L = 0.4$, the positive opinion proportion rises to the stable state at the first speed. This presents that the network regulators should opt $L = 0.5$ to start to control the network state, a low cost, for $C = 10$. Combining the above analysis, it is concluded that if the initial proportion $p_p = 0.1$, the network regulators should select medium control level and $L = 0.5$, which needs a low cost to change the negative opinion network into the positive opinion network.

For the initial proportion $p_p = 0.2$, from Figure 4(a), it is shown that if the network regulators adopt the low control level ($C = 2$), while $L = 0.5$ and 0.2, the positive opinion proportions decline about 0.15, a stable state; meanwhile,
Figure 4: Continued.
when $L = 0.4$, the positive opinion proportion presents small fluctuation and is finally stable at 0.2; when $L = 0.3$, the positive opinion proportion slowly rises to 0.24, a stable state; when $L = 0.1$, the positive opinion proportion slides to 0.04, a stable state. This implies that, under the initial proportion $pp = 0.2$, the network regulators using the low control level cannot change the network opinion trends into a positive opinion network whenever $L = 0.5, 0.4, 0.3, 0.2, 0.1$. From Figure 4(b), if the network regulators choose the medium control level ($C = 6$), while $L = 0.5, 0.4, 0.3, 0.2, 0.1$, all positive opinion proportions constantly increase between 0.9 and 1 and, respectively, reach different stable states; when $L = 0.1$, the positive opinion proportion first rapidly declines and then rises almost linearly over 0.6, an unsteady state. In this case, the network regulators should select $L = 0.5$ to begin to control the network state, a low cost. From Figure 4(c), for the high control level ($C = 10$), while $L = 0.5, 0.4, 0.3, 0.2, 0.1$, all positive opinion proportions increase at almost the same speed and run up to about 1, a stable state; when $L = 0.1$, the positive opinion proportion increases almost linearly at 0.75, a stable state. In this case, the network regulators should choose $L = 0.5$ to start the network control, changing the network into positive opinion network, a low cost. To sum up the analysis, it is concluded that if the initial proportion $pp = 0.2$, the network regulators should select medium control level and $L = 0.5$ to change the negative opinion network into the positive opinion network, a low cost.

For the initial proportion $pp = 0.3$, from Figure 5(a), if the network regulators choose the low control level ($C = 2$), while $L = 0.1$, the positive opinion proportion rapidly decreases to 0.09, a stable state; when $L = 0.2$, the positive opinion proportion rapidly descends to 0.2 and then slowly falls to 0.15, a stable state; when $L = 0.3$ and 0.4, the positive opinion proportions slowly go up to 0.35, a stable state; when $L = 0.5$, the positive opinion proportion slowly ascends to 0.41, a stable state. This implies that, under the initial proportion $pp = 0.3$, the network regulators using the low control level cannot change the network opinion trends into a positive opinion network whenever $L = 0.5, 0.4, 0.3, 0.2, 0.1$. From Figure 5(b), if the network regulators choose the medium control level ($C = 6$), while $L = 0.5, 0.4, 0.3, 0.2, 0.1$, the positive opinion proportions continuously increase over 0.5 and then reach the stable value 1 at almost the same time step; compared with the former three cases, while $L = 0.2$, the positive opinion proportion slowly goes up to 0.9, a stable state; when $L = 0.1$, the positive opinion proportion increases to 0.51 at the lowest speed, a stable state. It is concluded that, in this case, the network regulators should choose $L = 0.5$ to control the network opinion trends into a positive opinion network, a low cost. From Figure 5(c), if the network regulators choose the high control level ($C = 10$), while $L = 0.5$ and 0.4, the positive opinion proportions continuously increase over 0.5 and then reach the stable value 1 at almost the same time step; compared with the former two situations, while $L = 0.3, 0.2$, the positive opinion proportions go up to 0.94 at almost the same growth rate, a stable state; when $L = 0.1$, the positive opinion proportion slowly increases over 0.5 and does not reach a stable state. It concludes that, in this case, the network regulators should choose $L = 0.5$ to control the network opinion trends into a positive opinion network, a low cost. Combining the above analysis, it is concluded that if the initial proportion $pp = 0.3$, the network regulators should select medium control level and $L = 0.5$, which...
Figure 5: Continued.
Figure 5: The evolution trends for the initial proportion $pp = 0.3$, opinion control levels $k = 1, 3, \text{ and } 5$, and $L = 0.5, 0.4, 0.3, 0.2, \text{ and } 0.1$ in scale-free networks with $m_0 = 20$ and $m = 5$. (a) The initial proportion $pp = 0.3$, $C = 2$, and $L = 0.5, 0.4, 0.3, 0.2, \text{ and } 0.1$. (b) The initial proportion $pp = 0.3$, $C = 6$, and $L = 0.5, 0.4, 0.3, 0.2, \text{ and } 0.1$. (c) The initial proportion $pp = 0.3$, $C = 10$, and $L = 0.5, 0.4, 0.3, 0.2, \text{ and } 0.1$.
needs a low cost to change the negative opinion network into the positive opinion network.

For the initial proportion $p_0 = 0.4$, from Figure 6(a), if the network regulators choose the low control level ($C = 2$), while $L = 0.5, 0.4, 0.3, 0.2$, and $0.1$, the positive opinion proportions rapidly decrease and then slowly goes up to a stable value $0.37$; when $L = 0.3$, the positive opinion proportion first rapidly descends and then slowly decreases the stable value $0.37$. This presents that, under this case, the network regulators cannot change the network opinion trends into a positive opinion network whenever $L = 0.5, 0.4, 0.3, 0.2$, and $0.1$. From Figure 6(b), if the network regulators choose the medium control level ($C = 6$), while $L = 0.5, 0.4, 0.3, 0.2$, and $0.1$, the positive opinion proportions first rapidly descend, then continuously go up more than $0.5$ and $0.37$, and finally do not reach the stable state. This shows that, under this case, the network regulators should choose $L = 0.5$ to change the network opinion trends into a positive opinion network, a low cost.

Figure 6: The evolution trends for the initial proportion $p_0 = 0.4$, opinion control levels $k = 1, 3$, and $5$, and $L = 0.5, 0.4, 0.3, 0.2$, and $0.1$ in scale-free networks with $m_0 = 20$ and $m = 5$. (a) The initial proportion $p_0 = 0.4$, $C = 2$, and $L = 0.5, 0.4, 0.3, 0.2$, and $0.1$. (b) The initial proportion $p_0 = 0.4$, $C = 6$, and $L = 0.5, 0.4, 0.3, 0.2$, and $0.1$. (c) The initial proportion $p_0 = 0.4$, $C = 10$, and $L = 0.5, 0.4, 0.3, 0.2$, and $0.1$. 
Figure 7: The evolution trends for the initial proportion $pp = 0.5$, opinion control levels $k = 1, 3$, and 5, and $L = 0.5, 0.4, 0.3, 0.2$, and 0.1 in scale-free networks with $m_0 = 20$ and $m = 5$. (a) The initial proportion $pp = 0.5$, $C = 2$, and $L = 0.5, 0.4, 0.3, 0.2$, and 0.1. (b) The initial proportion $pp = 0.5$, $C = 6$, and $L = 0.5, 0.4, 0.3, 0.2$, and 0.1. (c) The initial proportion $pp = 0.5$, $C = 10$, and $L = 0.5, 0.4, 0.3, 0.2$, and 0.1.
From Figure 6(c), if the network regulators choose the high control level \((C = 10)\), while \(L = 0.5\) and 0.4, the positive opinion proportions quickly go up to the stable value 1 at almost the same speed; when \(L = 0.3, 0.2,\) and 0.1, the positive opinion proportions present a brief reduction and then continuously increase to different stable values 0.98, 0.92, and 0.62, respectively. This shows that, under this case, the network regulators should choose \(L = 0.5\) to change the network opinion trends into a positive opinion network, a low cost. To sum up the analysis, it is concluded that if the initial proportion \(pp = 0.4\), the network regulators should select medium control level and \(L = 0.5\), a low cost, to change the negative opinion network into the positive opinion network.

For the initial proportion \(pp = 0.5\), from Figure 7(a), if the network regulators choose the low control level \((C = 2)\), while \(L = 0.5\), the positive opinion proportion slowly increases between 0.5 and 0.6 and then slowly decreases to 0.5, no stable state; when \(L = 0.4, 0.3, 0.2,\) and 0.1, the positive opinion proportions first rapidly descend and then slowly decrease to the stable values 0.35, 0.19, 0.19, and 0.08, respectively. This presents that, under this case, the network regulators cannot change the network opinion trend into a positive opinion network whenever \(L = 0.5, 0.4, 0.3, 0.2,\) and 0.1. From Figure 7(b), if the network regulators choose the medium control level \((C = 6)\), while \(L = 0.5, 0.4,\) and 0.3, the positive opinion proportions quickly decrease and then continuously go up to the stable values 1, 0.98, and 0.8, respectively; when \(L = 0.2, 0.1\), the positive opinion proportions present a brief rise and then continuously increase to more than 0.5. This shows that, under this case, the network regulators should choose \(L = 0.5\) to change the network opinion trend into a positive opinion network, a low cost. From Figure 7(c), if the network regulators choose the high control level \((C = 10)\), while \(L = 0.5, 0.4, 0.3, 0.2,\) and 0.1, the positive opinion proportions all first decrease and then continuously go up. Practically speaking, while \(L = 0.5\) and 0.4, the positive opinion proportions rise to the stable value 1 at almost the same speed; when \(L = 0.3\), the positive opinion proportion increases to the stable value 0.98; when \(L = 0.2\), the positive opinion proportion goes up to more than 0.9 but does not reach a stable state; when \(L = 0.1\), the positive opinion proportion presents a linear increase and eventually exceeds 0.5. This shows that, under this case, the network regulators should choose \(L = 0.5\) to change the network opinion trends into a positive opinion network, a low cost. Combining with the analysis, if the initial proportion \(pp = 0.4\), to reduce network control cost, the network regulators should choose medium control level and \(L = 0.5\) to change the current opinion network into the positive opinion network.

According to the above experiments, it is concluded that, for the network, the lower the initial positive proportion is, the lower (higher) the opinion control level is, while the control opinion threshold value is chosen smaller (larger); the higher the initial positive proportion is, the lower the opinion control level is, while the control opinion threshold value is chosen larger.

### 4. Conclusion

As a user’s neighbors impact the opinion of the user strongly and the process of users’ opinion evolution can be considered the process of the prisoner’s dilemma game, this paper proposes an opinion evolution and control model based on the prisoner’s dilemma game and gives the corresponding opinion evolution and control algorithm, i.e., the PDG-OEC algorithm. For our purpose, based on the PDG-OEC algorithm, we first analyze the parameter selection of our experimental scale-free network. Then we make two types of simulation experiments in the scale-free networks. Under the different initial proportions, opinion control levels, and the same control threshold value, and under the different positive opinion proportions, opinion control levels, and opinion control threshold values in the scale-free network, the experiments show that if the initial positive proportion is lower, then the opinion control level needs to be lower (higher), while the control opinion threshold value is adopted smaller (larger); if the initial positive proportion is higher, then the opinion control level could be chosen lower, while the control opinion threshold value is chosen larger. The future work is finding more compatible game models to depict the opinion evolution and control in social networks and making some opinion evolution and control game models in the real social networks.

### Data Availability

Data sharing is not applicable to this article as no datasets were generated.

### Conflicts of Interest

All authors declare no conflicts of interest.

### Authors’ Contributions

The authors claim that the research was realized in collaboration with the same responsibility. All authors read and approved the last version of the manuscript.

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References


