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Three body forces in the Duflo-Zuker approach to nuclear masses

Andres Zuker*
IPHC Strasbourg
E-mail: apzuker@gmail.com

A critical reading of the successful but hitherto poorly understood Duflo Zuker mass models is proposed. The analysis concentrates on their simplest ten parameter version. It is found that the main interpretive problem rests on the need to introduce genuine three body interactions, detected by the model and crucial to its success but treated inconsistently. The necessary steps for an improved description are outlined.

More precisely, the model rests on a basic Liquid Drop Bethe-Weiszäcker form (LD), complemented by four "pillars": A) The possibility to extract fron the realistic potentials a "master term" that yields asymptotically the bulk energy of nuclear matter and produces shell effects *i.e.*, Harmonic Oscillator (HO) closures; B) a mechanism that transforms them into the observed Extruder-Intruder (EI) ones; C) Quartic forms that summarize Shell Model correlation effects; D) Separate treatment of deformed regions based on similar quartic forms in the number operators. In its present version, the model lumps together A and B in a single term and is forced by the data to introduce cubic forms to supplement C. Their origin can only be explained by invoking genuine three body interactions and their empirical necessity is demonstrated by comparing with results produced by a monopole Hamiltonian whose derivation is independent of masses. The general conclusion is that A and B must be reformulated by introducing genuine three body forces.

The presentation avoids technicalities with two exceptions: i) the very compact derivation of the quartic correlation term in Eq. (3.1) and ii) the cryptic remark on compressibility in Section 4.

Two subjects that would demand special attention are hardly touched: 1) Deformed regions, because DZ does quite well in them; 2) The surface energy, whose origin remains an open problem.

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*Speaker.

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1. Preliminaries

The 2003 Atomic Mass Evaluation [1] was closely followed by a review article in which different models were compared [2], leading to the conclusion that

"One mass formula stands above all others..."

The authors were referring to the work of Duflo and Zuker [3] [DZ, RMSD≈ 350 keV], noting that

" this does not mean that with Duflo-Zuker we have reached the end of history.."

In the mean time little has changed, but now there is a second paper [4] (MHZ) dealing with DZ10 [5] [RMSD≈ 550 keV], an invaluable summary of the DZ approach. It does not point to the end of history, but to a **Three Body** follow up of the story. Here we summarize and supplement the MHZ findings through some **critical remarks**. Bullets (•) point to aspects that need special attention and stars (★) to crucial ones.

1.1 Data I. BE: Shell effects + LD

The figure shows what has to be explained.

$$\mathrm{LD} = 15.5A - 17.8A^{2/3} - 28.6\frac{4T(T+1)}{A} + 40.2\frac{4T(T+1)}{A^{4/3}} - \frac{.7Z(Z-1)}{A^{1/3}}\ (\mathrm{MeV}). \qquad (1.1)$$

The information for both plots is the same. Magicity shows at $Z, N = (14), 28, 50, 82, 126$ and

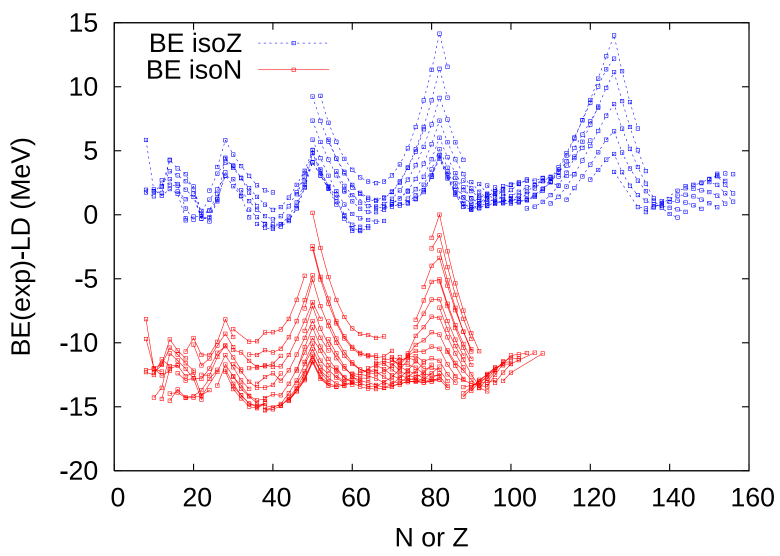

Figure 1: Shell effects (BE(exp)-E(LD)) along isotope and isotone lines (latter displaced by -14 MeV). Only even-even nuclei are shown.

with—very rare exceptions—nowhere else. The roughly parabolic shapes are interrupted by flat sections corresponding to well-deformed nuclei. The amplitude of the shell effects goes as $A^{1/3}$.

1.2 Structure and evolution of DZ

DZ10 contains 10 terms, the first six—referred to as macroscopic—correspond to an LD form.

• 1 Leading term. Contains basic shell effects and yields asymptotically the bulk energy of nuclear matter $\approx 15.5A$ MeV.

2. The surface term in $A^{2/3}$.

3,4. Asymmetry, $T(T+1)/A$ and Surface asymmetry, $T(T+1)/A^{4/3}$. That little or no shell effects are associated to these terms is an empirical fact emerging in Fig.2 below.

5,6. Pairing, $[mod(N,2)+mod(Z,2)]A^{-1/3}$ and Coulomb, $Z(Z-1)/A^{1/3}$

• 7,8,9. Spherical "correlation terms"

• 10. Deformation term

Fits yield

• For the first six "macroscopic" terms, RMSD=2.88 MeV poor compared with RMSD=2.35 MeV for LD. A puzzling result, **BUT**

★ For the first nine terms, RMSD=717 keV.

★ For the ten terms, RMSD=567 keV.

Conclusion: understanding DZ is equivalent to understanding the action of the spherical and deformation terms.

1.3 Data II: Alpha and Beta lines

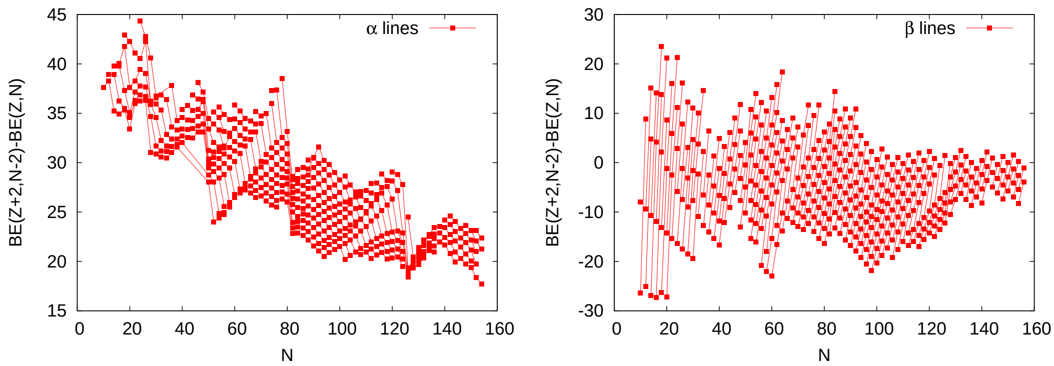

Figure 2: Four nucleon separation energies along lines at constant T, (α) and constant A, (β). Even-even nuclei.

Critical remark 1. The DZ work is conducted in a neutron-proton representation. Fig.2 indicates that an isospin representation is more adequate as shell effects are concentrated in the α lines which involve only number operators. The β lines depend only on the total isospin.

2. The master term

DZ identifies the collective term responsible for the bulk energy of nuclear matter and the basic shell effects, suggested in [6].

Calling m_p the number of particles in the major HO shell of principal quantum number p of degeneracy $D_p = (p+1)(p+2)$, the master term and a possible variant are

$$M_A = \frac{9}{A^{1/3}} \left(\sum_p \frac{m_p}{\sqrt{D_p}} \right)^2 \asymp \frac{9}{A^{1/3}} (p_f+2)^4 \approx 9(3/2)^{4/3} A = 15.45A \text{ (MeV)} \tag{2.1}$$

$$\text{Possible variant } v \qquad \frac{m_p}{\sqrt{D_p}} \longrightarrow \frac{m_p}{\sqrt{D_p}}\left(1 - \frac{.133}{\sqrt{D_p}}\right) \equiv \frac{m_p}{\sqrt{D_p}} u_p \tag{2.2}$$

leading to very different numerical asymptotic fits $M_A \asymp 15.35A - 18.73A^{1/3}$ and $M_A^v \asymp 15.54A - 9.51A^{2/3} - 4.62A^{1/3}$, but to similar shell effects at $N = 8$, 20, 40, 70, 112 and 168 in Fig. 2, whose

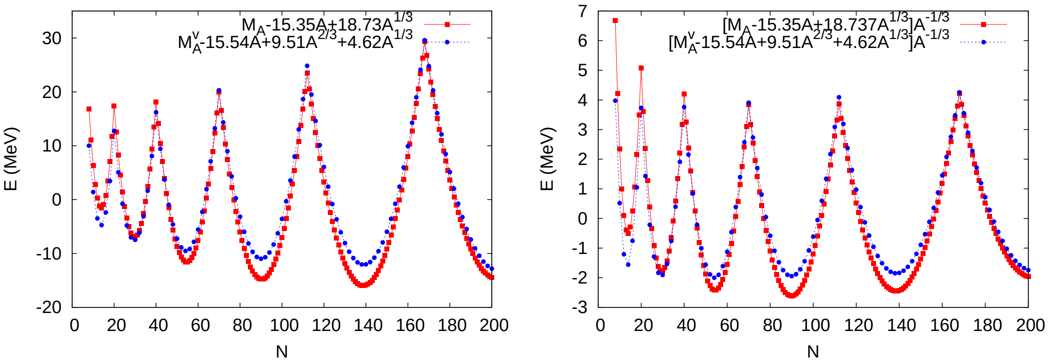

Figure 3: Master shell effects produced by M_A and M_A^v for $t = N - Z = 0$. See text.

second panel shows that their amplitudes scale asymptotically as $A^{1/3}$ as expected (see legends).

Critical remark 2. Both M_A and M_A^v are consistent with realistic potentials but in a complete formulation the master term should emerge from the variational interplay of kinetic and two plus three body potentials.

2.1 The HO-EI transition. The leading term

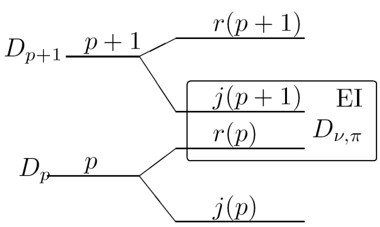

Figure 4: Harmonic oscillator and extruder-intruder (EI) shells.

The master term contains only m_p operators. To transform HO closures into extruder-intruder (EI) ones at $N, Z = 28, 50, 82$ and 126 we have to introduce some subshell structure which we restrict to j_p (the orbit of largest angular momentum in a major HO shell) and r_p (the rest of the orbits treated as a single one) as defined in Fig. 4 following the hints from Figs 1 and 2 . The operators that may trigger the HO-EI transition must involve linear, quadratic and cubic combinations of

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$$m_p = m_{j(p)} + m_{r(p)} \quad \text{and} \quad \Gamma_p = (pm_{j_p} - 2m_{r_p})/(2(p+2))$$

where Γ_p vanishes at closed HO shells, thus ensuring no asymptotic contributions.

Nowadays we know that cubics *i.e.*, three body (3b) interactions are essential [7]. At the time DZ was formulated only two body forces (2b) were considered. DZ28 used some twelve terms. In DZ10 Jean Duflo reduced them to a single leading one incorporated in the master term

$$M + S = M + \Sigma_p[u_p^1\Gamma_p + u_p^2 m_p \Gamma_p/\sqrt{D_p}]$$

where $u_p^{1,2}$ are scaling factors analogous to u_p in Eq. (2.2). Fig.5 gives an idea of the HO-EI transition.

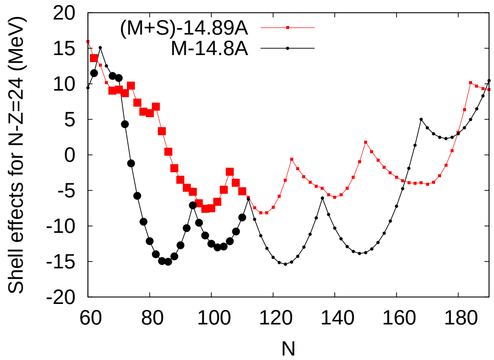

Figure 5: The evolution from HO (dots) to EI (squares) shell effects for $N - Z = 24$ even-even nuclei. The asymptotics are roughly represented by a simple A term. Heavier marks refer to species whose masses have been measured.

Critical remark 3. We insist: the HO-EI mechanism(s) in DZ cannot be correct: they must involve three body forces.

3. DZ10 correlations: Shell Model in EI spaces.

Once the macroscopic terms have defined model spaces bounded by the EI closures we estimate the average form of the correlation energies produced in a shell model calculation. The exact ground states $|\overline{0}\rangle$ are obtained by acting with k body operators A_k on the unperturbed ones $|0\rangle$. A_1 is omitted as its effects can be incorporated in the monopole Hamiltonian H_m.

$$|\overline{0}\rangle = (1 + \sum_{k>2} \hat{A}_k)|0\rangle \Longrightarrow E = \langle 0|H_m|0\rangle + \langle 0|H_M \hat{A}_2|0\rangle \tag{3.1}$$

We proceed in two steps: first we take $|0\rangle$ to be generic states in the model space that are "dressed" through the correlation term to produce an effective two body interaction. We separate its monopole contribution which goes into H_m. Then Eqs. (3.1) are reinterpreted, as the result of a diagonalization. The correlation term now amounts to a four body operator which must vanish at closed shells and single particle and single hole states, which defines uniquely the quartic term S_4 below. Pairing correlations are expected to produce the quadratic S_2 while the cubic S_3 is demanded by the data but we have no explanation for it, unless it is accepted as a genuine three body contibution.

$$S_2 = \frac{m_v \bar{m_v}}{D_v \rho}; \quad S_3 = \frac{m_v \bar{m_v}(m_v - \bar{m_v})}{D_v^{2-a}\rho}; \quad S_4 = \frac{m_v^{(2)} \bar{m_v}^{(2)}}{D_v^{3-a}\rho} \tag{3.2}$$

Where we have used $\bar{m} = D - m$ (D is the degeneracy of the space), $m^{(2)} = m(m-1)$, $\rho = A^{1/3}$. Setting $a = 0$ ensures proper $A^{1/3}$ scaling. Comments:

• No S_2 is requested by the data

★ S_4 and S_3 are both crucial (remember RMSD evolution at the end of Section 1.2) but the latter changes sign at around $A = 100$ and both have the anomalous A scaling *i.e.*, $a = 1$.

★ Deformation is associated with 4neutron-4proton excitations and a single quartic of type S_4 with proper scaling. No problem here.

4. The DZ scaling problem. GEMO

To understand the S_3 and S_4 scaling anomaly we call upon a totally different approach, based on GEMO, an independently determined monopole Hamiltonian that describes strictly shell effects. It is obtained by fitting the excitation energies of single particle and single hole states on doubly magic nuclei [8] The only free parameter entering the GEMO estimates in Fig. 6 is an overall con-

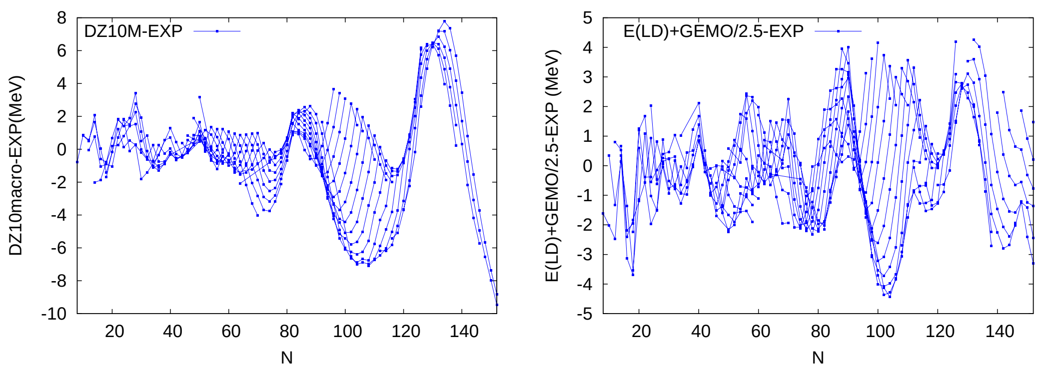

Figure 6: Differences between experimental and calculated binding energies: (left) Macroscopic DZ10 (RMSD=2.86 MeV), (right) GEMO (RMSD=1.69 MeV). Even-even nuclei. Lines join points at constant $t = N - Z$. In comparing beware of different y-axis scales.

traction by a 2.5 factor that simulates correlations that in DZ have been incorporated (hidden) in the master term. GEMO leads to larger monopole shell effects because it provides correct compressibility by placing $1\hbar\omega$ excitations at $1\hbar\omega \approx 40A^{-1/3}$ MeV: A non trivial result that explains why the monopole isoscalar resonance is indeed at $2\hbar\omega$.

From Fig. 6 it is obvious that the DZ10 pattern cannot possibly have correct $A^{1/3}$ scaling, due to an inconsistent treatment of the strong asymptotic $A^{1/3}$ contribution to M_A [see under Eq.(2.2)], spuriously compensated by the surface term in $A^{2/3}$. The problem does not arise for GEMO, which explains its much smaller RMSD (see the figure caption), in spite its more irregular behavior.

The truly significant point about Fig. 6 is the similarity between DZ10 and GEMO. In particular the pronounced cubic patterns that emerge for the heavier nuclei.

Concluding critical remark The success of the DZ formulation is mostly due to to its spontaneous capacity to identify the need of three body forces. Progress will depend on a consistent implementation of critical remarks 1-3.

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