

# The Anelasticity of the Mantle\*

Don L. Anderson

## *Summary*

The attenuation of seismic waves provides the most direct data regarding the non-elastic properties of the Earth. Recent experimental results from body waves, surface waves and free oscillations provide estimates of the anelasticity in various regions of the Earth. Results to date show that the upper mantle is more attenuating than the lower mantle, the maximum attenuation is in the vicinity of the low-velocity zone, a rapid increase in attenuation occurs in the vicinity of the C-region of the mantle and compressional waves are less attenuated than shear waves. A frequency dependence of  $Q$  has not yet been discovered.

Most laboratory measurements of attenuation have been performed at ultrasonic frequencies on pure specimens of metals, glasses, plastics and ceramics. A general feature of laboratory measurements is an exponential increase of attenuation with temperature on which are superimposed peaks which can be attributed to dislocation or other defect phenomena. Measurements on natural rocks at atmospheric pressure can be attributed to the presence of cracks. The intrinsic attenuation of rocks as a function of temperature and pressure is not known. However, on other materials grain boundary phenomena dominate at high temperature. This can be attributed to increased grain boundary mobility at high temperatures. High pressure would be expected to decrease this mobility. If attenuation in the mantle is due to an activated process it is probably controlled by the diffusion rate of defects at grain boundaries. Estimates of attenuation in the lower mantle then yield an estimate of the activation volume of the defects contributing to the loss. If the lower mantle is assumed homogeneous the estimated activation volume is a small fraction of the presumed molal volume of materials making up the lower mantle. Stress induced migration of small point defects is a possible loss mechanism consistent with the observations.

## 1. Introduction

Seismology provides the most powerful tools for the study of the Earth's interior. The classical method of determining the velocity structure of the Earth from arrival times of body waves has recently been supplemented by study of the periods of free oscillation and the dispersion of surface waves. These techniques are complementary and have provided a fairly detailed and reasonably complete description of the elastic properties within the Earth and their radial and lateral variations. Since the Earth is not perfectly elastic, seismic waves and free oscillations decay in space and time, the

\* Contribution No. 1410, Division of Geological Sciences, California Institute of Technology, Pasadena.

rate of decay being a measure of anelasticity. This is a seismic parameter that has not been fully utilized in discussions of the Earth because of the considerable difficulties involved in amplitude studies. These difficulties are being resolved or circumvented and seismic amplitude data is now yielding the anelasticity as a function of depth in the Earth. This branch of seismology is relatively new and results to date are sparse. Even so, information regarding the variation of anelasticity with depth is ahead of our ability to interpret it in terms of defect structure, intergranular phenomena or temperature.

The elastic properties of the Earth and their variation with depth are controlled primarily by the interatomic forces and, therefore, yield information about the crystalline structure and composition of the Earth. Interatomic forces are affected also by temperature and pressure. To a first approximation these forces depend only on the atomic spacing, and, therefore, on the volume or density, and mean atomic weight of the material.

The effect of pressure on the compressional and shear wave velocity is now known for a wide variety of rocks. The effect of temperature is less well determined, but can be estimated from thermodynamic considerations. The influence of porosity and macroscopic defects such as cracks, which tend to mask intrinsic properties, can be suppressed by studying single crystals, sintered ceramic synthetics or by performing experiments at pressures greater than about 3 kilobars. Ultrasonic and static compression determinations of elastic properties of rocks are now being supplemented by shock wave experiments. The general tendency of elastic properties is to increase with pressure and decrease with temperature, and to be relatively independent of frequency. Although much high pressure, high temperature, shock wave and theoretical work remains to be done, we are now in a position to utilize the seismically determined elastic properties of the Earth in fairly detailed discussions of composition and phase of the various regions of the interior.

In addition to interatomic forces the anelastic properties of a material are controlled by grain boundary and defect phenomena which depend critically on temperature. Regardless of the loss mechanism or mechanisms, which have not yet been isolated for the Earth, the damping of seismic waves should supply one of the more direct estimates of temperature in the Earth. It is premature with present data to attempt this task, particularly in light of the complete absence of attenuation data on rocks at high pressure and temperature, but it can be shown (Anderson & Archambeau 1964) that present estimates of temperature are not inconsistent with the observed damping if this damping is attributed to activated processes with low activation energies and volumes. Depending on the mechanism, damping in solids is also sensitive to grain size and orientation, impurity, defect or vacancy concentration, dislocation sizes, orientations and densities, previous mechanical and radiation history, and the thermal and elastic contrast between grains. The relative importance of these parameters depends on the dominant loss mechanism which can only be determined if the frequency, amplitude, temperature and pressure dependence is known, if then. The relative attenuation of shear and compressional waves will also help to isolate the loss mechanism. Several mechanisms possibly contribute to the observed damping and the dominant mechanism may vary with depth.

There are very few experimental data that are pertinent to the problem of anelastic attenuation in the Earth's interior. The pores and open cracks that are present in all rocks at low pressure overwhelm the intrinsic crystalline or intercrystalline phenomena which contribute to the damping of seismic waves at depth in the mantle. This is particularly true at the kilocycle and megacycle frequencies, which are used in ultrasonic experiments. The difficulties inherent in high pressure measurements of ultrasonic damping are obvious. Because of differential thermal expansion between grains the effect of temperature is also difficult to measure on rocks at low pressures. Even though there are hundreds of laboratory determinations of ultrasonic attenuation on

rocks, only a few are pertinent to the problem of seismic attenuation in the mantle. A direct determination of the effect of temperature and pressure on the intrinsic attenuation in a rock has not been made. Powdered rocks, reconstituted by sintering under pressure are a potential source of low-porosity, fine-grained material for ultrasonic anelasticity measurements when the corresponding rock is not naturally available. Experiments will probably still have to be performed under modest pressure, even when temperature is the variable of interest, because of differential thermal expansion of the grains. Indeed, macroscopic defect phenomena may always dominate intrinsic effects at pressures obtainable in the laboratory. Synthetic, doped crystals may be the best raw material for studying the intrinsic attenuation of Earth materials, particularly since seismic wave lengths are typically greater than  $10^4$  times laboratory wave lengths. This places severe constraints on particle size in sintered specimens in studies of mechanisms where the grain size to wavelength ratio or the jump time to period ratio is important or where an attempt is made to eliminate the scattering mechanism. Quasistatic creep or hysteresis measurements, such as low-frequency, low-amplitude cycling of a rock in torsion under high confining pressure, is a possible way to obtain data at seismic frequencies.

The phenomenology and physics of internal friction, the mathematics of wave propagation in nonelastic solids, the statistical mechanics and thermodynamics of activated processes and the experimental determination of internal friction in the laboratory under extreme conditions of temperature and pressure offer many challenges to the theoretician and experimenter and indicate the complexity and richness of the field.

## 2. Phenomenology of anelasticity

The theory of elasticity, appropriate for infinitesimal stresses which give rise to instantaneous and reversible strains, is well developed and unambiguous. Hooke's Law, combined with various conservation and continuity conditions, is the only statement regarding the relation between stress and strain that is required. The situation deteriorates rapidly when time dependent, finite amplitude, or irreversible deformations of a solid are to be considered. Plastic, viscous, viscoelastic, viscoplastic, Maxwell, Kelvin, Voigt, Bingham, standard linear, creep and Boltzmann are just a few of the names that have been attached to phenomenological equations which purport to describe the nonelastic response of solids. A vast mathematical literature has grown out of these equations even though they represent *ad hoc* modifications of Hooke's Law, rather than empirically verified relationships. The catchall terms *internal friction* and *anelasticity* are often used when referring to processes which result in the attenuation of acoustic, ultrasonic or seismic energy.

The lack of uniform terminology and units for the measured loss, further complicates the situation. In a recent review article, the first five figures gave the loss as attenuation ( $s^2/cm$ ),  $Q$ , attenuation factor (dB/ft), attenuation factor ( $cm^{-1}$ ) and decrement; later figures used attenuation, (N/cm) and  $Q^{-1}$ . Other common terms used in studies of attenuation are loss angle,  $\delta$ ,  $\tan \delta$ , attenuation coefficient, percent energy loss per cycle, specific damping capacity, specific loss and dissipation.

The most common expressions for internal friction, for low damping, are inter-related as follows:

$$\sin \delta \cong \tan \delta \cong \delta = \Delta E / 2\pi E = Q^{-1} = \Delta / \pi = \Delta \omega / \omega = M^* / M = 2k^* / k,$$

where  $\delta$  is the lag of strain behind stress in a cycling experiment,  $\Delta E / E$  is the fractional energy loss per cycle, *specific loss* or, *specific damping capacity*,  $\Delta$  is the *logarithmic decrement*,  $\Delta \omega / \omega$  is related to the breadth of a resonance peak,  $M^*$  and  $M$  are the imaginary and real parts of a storage modulus,  $k^*$  and  $k$  are the imaginary (attenuation coefficient) and real parts of a wave number and  $Q$  is the *quality factor*.

These expressions (except for  $M^*/M$ ) and their interrelationship, do not depend on details of the attenuation mechanism or its frequency dependence, but require that the damping be small and be independent of amplitude.

Many attempts have been made to generalize the equations of classical elasticity theory to include anelastic effects. The works of Maxwell, Voigt and Kelvin, in this direction are well known. Their phenomenological models have met with limited success in describing the macroscopic properties of solids. These models form the building blocks of more complicated relationships between stress and strain and their first time derivatives which are usually visualized as networks of perfectly elastic springs and perfectly viscous dashpots. The most general linear homogeneous equations of this type, proposed by Zener (1948), is called the *standard linear solid* or a *linear visco-elastic solid* and is discussed in many textbooks and review articles on the subject. The mechanical elements of the standard linear model are two springs and one dashpot and is, therefore, described by three constants: these are usually taken as the fully relaxed elastic modulus  $M_r$ , the relaxation time under constant strain  $\tau_1$  and the relaxation time under constant stress  $\tau_2$ . For very rapid vibrations the pertinent modulus,  $M_u$ , the unrelaxed elastic modulus is

$$M_u = (\tau_2/\tau_1) M_r. \quad (1)$$

For periodic variations in stress, such as occur in a standing or propagating wave, the stress and strain amplitude are connected by a complex modulus,  $\mathbf{M}$ , where

$$\mathbf{M} = \frac{1 + i\omega\tau_2}{1 + i\omega\tau_1} M_r. \quad (2)$$

Putting  $\tau = \sqrt{(\tau_1 \tau_2)}$ , the tangent of the phaselag angle  $\delta$  is

$$\tan \delta = \frac{M_u - M_r}{\sqrt{(M_r M_u)}} \frac{\omega\tau}{1 + \omega^2 \tau^2} = \Delta_0 \frac{\omega\tau}{1 + \omega^2 \tau^2}, \quad (3)$$

where  $\Delta_0$  is called the *modulus defect*, or *relaxation strength*.

The phaselag, or the loss, is maximum for  $\omega = 1/\tau$  and

$$(\tan \delta)_{\max} = \frac{1}{2} \Delta_0.$$

It follows that

$$|\mathbf{M}| = M_r \quad \omega \ll 1/\tau \text{ (low frequency)} \quad (4)$$

$$= (\tau_2/\tau_1) M_r = M_u \quad \omega \gg 1/\tau \text{ (high frequency)}. \quad (5)$$

For small  $\delta$ ,  $\tan \delta \approx \delta$  and

$$Q^{-1} = \Delta_0 \frac{\omega\tau}{1 + (\omega\tau)^2} \quad (6)$$

$$= \Delta_0/(\omega\tau) \quad \omega \gg 1/\tau \quad (7)$$

$$= \Delta_0 \omega\tau \quad \omega \ll 1/\tau. \quad (8)$$

A peak-shaped attenuation curve can be explained physically as well as described phenomenologically. It is observed whenever the frequency range of measurement spans a natural frequency of the lattice-defect system; for instance, the mean jump frequency of an interstitial or the resonant frequency of a dislocation line. Consider interstitials with a jump frequency  $\tau^{-1}$  where  $\tau$  is the mean time of stay in a given lattice site in a solid which is subjected to an alternating stress of frequency  $\omega$ . For  $\omega\tau \gg 1$  the frequency is so high that the interstitials do not have time to change

sites in reaction to the applied stress; stress and strain are completely in phase, resulting in no phase lag, no hysteresis and no loss. For  $\omega\tau \sim 1$  many of the interstitials will be able to change sites, but the stress varies too rapidly for equilibrium conditions to prevail at any particular value of the stress leading to a lag between stress and strain and a loss. For  $\omega\tau \ll 1$  the applied frequency is so much less than  $1/\tau$  that the defect population will always be in equilibrium with the applied stress and no loss occurs. The strain for a given stress in this case is the elastic plus the anelastic strain which yields a smaller effective modulus than for the high frequency case which has only the elastic strain. The high frequency modulus and low frequency moduli are called unrelaxed and relaxed respectively. If the stress is relaxed by a heat flow process rather than by impurity relaxation, the moduli are called, respectively, *adiabatic* and *isothermal*.

In many cases the relaxation involves atomic movement by diffusion and is strongly temperature dependent, viz.

$$\tau = \tau_0 e^{H/RT}, \quad (9)$$

where  $H$  is an *energy of activation*. In such cases the frequency of maximum damping also depends on temperature and the whole relaxation curve may be observed by varying the temperature over a few tens of degrees instead of varying the frequency over many octaves. For thermally activated processes that have very long relaxation times

$$Q^{-1} \approx (\Delta_0/\omega\tau_0) e^{-H/RT}, \quad (10)$$

where  $\Delta_0$  is the relaxation strength which is independent of temperature. This equation describes the features of high temperature damping very well.

In addition to the *relaxation process* there also exists, in solids containing defects, the possibility of losses due to *resonance*. An example would be the losses associated with a viscously damped oscillating dislocation. In this case the internal friction is

$$Q^{-1} = \frac{\omega/\tau_r}{\sqrt{[(\omega_0^2 - \omega^2)^2 + \omega^2/\tau_r^2]}}, \quad (11)$$

where  $\tau$  is now a viscous time constant and  $\omega_0$  is the resonant frequency. Here again the damping has a maximum at  $\omega = \omega_0$ .

The width of the damping peak is proportional to  $\tau^{-1}$  but the frequency of damping depends little on  $\tau$ , contrary to the relaxation process. Intensive variables, such as temperature and pressure, have a large effect on  $\omega_{\max}$  in relaxation damping (since they occur in exponentials) and little effect on  $\omega_0$  in resonance damping (since their main effect is on the restoring force, such as the line tension of a dislocation). Both of these mechanisms are described by linear equations and, therefore, do not depend on amplitude. This mechanism is also phenomenological since the source of the viscous damping has not been specified.

Dislocations can move through a crystal with or without thermal agitation. At high temperatures a stress can cause a dislocation to move over an energy barrier between minimum energy positions with a consequent dissipation of energy.

The *standard linear* model predicts an attenuation that is strongly frequency dependent. Attenuation peaks are often observed in very pure or well ordered substances, particularly at low temperatures. The peaks can usually be attributed to some microscopic phenomena with a well-defined relaxation time.

Some relaxation processes, such as the interstitial solute atom relaxation, can be closely approximated by the standard linear elastic solid with a single relaxation type. Other processes, such as grain boundary relaxation, have  $1/Q$  curves that are much broader than a single relaxation curve. A phenomenological model with a distribution of relaxation times can be invoked to approximate these broader peaks.

The assumption that all the relaxations are associated with shear deformations and none with compressional motions appears to be a good approximation for such mechanisms as interstitial atom relaxation and grain boundary relaxations but is not valid for deformations of coiled chain solids such as polymers. For the former processes it can be assumed that the shear modulus is complex and the bulk modulus is real. This leads to an attenuation for longitudinal waves that is less than for shear waves.

Many materials exhibit a frequency independent dissipation function. Knopoff & MacDonald (1958, 1960) proposed a non-linear phenomenological model based on a solid friction loss mechanism that had this property. Futterman (1962) showed that a  $Q$  roughly independent of frequency range was consistent with linear theories of attenuation.

A distribution of relaxation times can also be invoked to broader relaxation peaks to give a slowly varying attenuation over a wide frequency band.

### 3. Attenuation mechanisms

Anelasticity as an important property of materials has received the attention of metallurgists, solid state physicists, communication engineers, polymer scientists and ceramicists. As a measure of the defect solid state it has been utilized by physicists primarily on ultra-pure specimens at very low temperatures and very high frequencies. The science received its major impetus, particularly in the field of metallurgy, with the appearance of Zener's definitive work in 1948. Zener listed and analysed several loss mechanisms; for example thermoelastic effect, ordering of solute atoms under stress deformation at crystal boundaries and eddy currents generated by mechanical vibration. Although emphasis in this monograph and many later papers is on metals, most of the loss mechanisms discovered in metals are also operative in non-metals. The metallurgical discussions of dislocation and stress induced ordering mechanisms are quite general and those kinds of losses are very common in other materials. The effects of irradiation, cold work, annealing and alloying have been especially well studied on metals and illustrate the complexity and multiplicity of phenomena that require attention in discussions of anelasticity. Much experimental work on ultra pure, carefully prepared samples is necessary to isolate the loss mechanism even in the laboratory. Temperature, *per se*, is an important variable in studies with engineering applications in mind but is usually used as a convenient way to extend the frequency range in studies of damping peaks. Low-frequency, high-temperature measurements,  $T > T_m/2$ , on polycrystalline materials are of particular interest to geophysicists.

There are many logical ways to categorize damping mechanisms: intrinsic vs non-intrinsic; thermally activated vs non-thermally activated; single crystal vs polycrystal; dislocation, point defect and thermoelastic; and relaxation, resonance, hysteresis and viscous are some of the possibilities. Metals, non-metallic crystals, glasses and polymers are usually discussed separately.

Van Bueren (1960, pp. 399–401) summarizes some of the internal friction mechanisms in metals. Among mechanisms which are important at low frequencies and high temperature and high-temperature dislocation damping, visco-elastic after effect and microcreep, temperature activated hysteresis, grain boundary, Snoek damping due to solute atoms, stress induced ordering in substitutional solid solutions, Köster damping, phase transition damping, and thermoelastic damping. Additional mechanisms which become important at high frequencies are dislocation relaxation, dislocation resonance, dislocation hysteresis, magnetoelastic relaxation, electron attenuation and magneto acoustic resonance. Mechanisms which become dominant at low temperatures are dislocation relaxation, transient cold-work internal friction, deformation hysteresis, point defect internal friction, magnetoelastic relaxation, ferromagnetic relaxation, electron attenuation and magnetoacoustic resonance. Van Bueren also summarizes

the frequency, amplitude, temperature, cold work and impurity dependence of the various mechanisms, where known. In general, the influence of all of these variables must be studied in order to isolate the loss mechanism. The relative losses in pure shear and pure compression also help to isolate the mechanism. The influence of pressure is particularly important for geophysical applications but, this is a relatively new parameter for the physicist and has received little study.

In addition to the monographs by Zener and Van Bueren, the subject of anelasticity has been reviewed by Mason (1958) and Entwistle (1962). Summaries also appear in the Physical Acoustics series edited by W. P. Mason (1965), *The Progress in Non-Destructive Testing* series edited by Stanford & Fearon (1966) and *Acta Metallurgica* (1962 April). Current research is reported in the *Journal of Applied Physics*, *Journal of the American Ceramic Society*, *Acta Metallurgica*, *Journal of Nuclear Materials*, and *Physical Review*. The present review draws heavily on all these sources.

#### 4. Thermal conductivity losses

One of the best known mechanisms of relaxation is a consequence of thermal conductivity and is known as *thermoelasticity*, the reverse of *thermal expansion*. Thermal gradients are induced in the solid due to a stress wave. The relaxation time for the establishment of thermal equilibrium is given by

$$\tau \sim \frac{\rho d^2 C_v}{K},$$

where  $K$  is the thermal conductivity,  $C_v$  is the specific heat at constant volume,  $\rho$  is the density and  $d$  is an appropriate dimension over which the heat must flow. If the solid is homogeneous then  $d$  is one-half the wave length, i.e. the distance from the heated compressed regions to the cooled dilatational regions. Damping is a maximum when  $\omega\tau \sim 1$ , i.e.

$$\frac{\pi^2 V^2 C_v}{K} \sim \omega.$$

Values appropriate for seismology give  $\omega \sim 10^{16}$  c/s. The relaxation frequency for thermoelastic damping is clearly too high to make this an important damping mechanism in the solid parts of the Earth. It is even difficult to measure this component of damping on metals at low temperature and laboratory frequencies. At other frequencies this loss mechanism satisfies the familiar relation

$$Q^{-1} = \Delta_0 \frac{\omega\tau}{1 + (\omega\tau)^2},$$

where

$$\Delta_0 \approx \frac{M_s - M_T}{M_T} = \frac{C_p - C_v}{C_v}$$

is the *modulus defect* and is the relative difference between the adiabatic and isothermal moduli. This mechanism does not contribute to shear losses.

In polycrystal materials the flow of heat may take place between adjacent grains because of differential compression. This can occur even in solids where the grains are of identical composition if they are anisotropic and unoriented. This latter situation gives

$$Q^{-1} = \left[ \frac{C_p - C_v}{C_v} \right] R \frac{\omega\omega_0}{\omega^2 + \omega_0^2},$$

where the first term on the right is the relative difference between the specific heats,  $R$  is a factor that increases with the anisotropy of the grains and  $\omega_0$ , the intergranular relaxation frequency, is

$$\omega_0 \sim \frac{K}{\rho C_p d^2},$$

where  $d$  is now a mean grain diameter.

Again the damping is a maximum when  $\omega_0 \tau \sim 1$  or approximately

$$\omega^{-1} = \tau \sim 10^3 d^2.$$

The factor  $\Delta_0 R$  is of the order of  $10^{-3}$  for very anisotropic crystals giving, as an order of magnitude estimate,

$$Q \sim 10^3$$

at maximum damping. The grain size in the upper mantle cannot be estimated but for  $d = 1$  mm to  $10^2$  cm the critical wave period for maximum attenuation is  $10^{-1}$  to  $10^7$  s. The range of grain sizes and thermal properties in a common rock would tend to broaden the relaxation peak. Zener has shown that in the nearly adiabatic high frequency case the thermal currents are confined to the immediate vicinity of the grain boundaries and the internal friction varies as  $\omega^{-\frac{1}{2}}$ ; this for wavelengths short compared to the grain dimensions. An approximate calculation gives

$$Q^{-1} = \Delta_0 \Omega (K/C_v)^{\frac{1}{2}} (2\omega)^{-\frac{1}{2}},$$

where  $\Omega$  is the total surface area to volume ratio.

In the other extreme, isothermal vibrations (small grain sizes and low frequency) the internal friction is proportional to the first power of frequency.

Savage (1965, 1966) has computed the thermoelastic attenuation due to local stress concentrations in granular material and around cracks. By appropriate choice of parameters, primarily the distribution of grain sizes, he can obtain a  $Q$  that is frequency independent over a broad spectrum. Savage notes that the internal friction,  $Q^{-1}$ , increases slightly faster than linearly with temperature for thermoelastic mechanisms. Except for the contribution of open cracks, which presumably dominates the damping at low pressure, the effect of pressure can be expected to be slight. Intergranular thermoelastic stresses are suppressed by pressure primarily to the extent that anisotropy and mechanical heterogeneity are reduced.

Thermoelastic losses are intimately related to changes in volume and, therefore, are usually more effective for compressional waves than for pure shear waves. However, when inhomogeneities are imbedded in a material, shear waves as well as compressional waves generate local dilatational stress inhomogeneities and, therefore, can have thermoelastic components in their damping. In fact, it is possible to contrive geometries which lead to higher shear wave attenuation than longitudinal wave attenuation. Several examples are discussed by Savage (1966). An example is a spherical cavity in a plane strain field.

Thermoelastic losses undoubtedly exist in materials as inhomogeneous as those thought to make up the mantle. Anisotropic grains with random orientation, the close association of grains with different elastic and thermal properties and the possible existence of molten regions all would tend to introduce local fluctuations in stress with an attendant thermal diffusion. The wide spectrum of grain sizes, anisotropies, orientations and physical properties of rock forming minerals would tend to smear out the thermoelastic peak and broaden the  $Q$  spectrum over a wide frequency band. Detailed calculations appropriate for, say, dunite with randomly oriented olivine crystals (one of the most anisotropic of major rock forming minerals), or for a multiminerale ultra-basic rock, or for a granular model with intergranular melt would be enlightening



regarding the possible importance of this mechanism in the damping of seismic waves in the mantle. Savage (1966) using mechanical and thermal properties appropriate for granite with open cracks estimated  $Q_\alpha \sim 200$  and  $Q_\beta \sim 350$ , these estimates being appropriate for near surface conditions. This model represents a limiting case of stress concentration due to embedded heterogeneities; losses due to embedded fluid or solid heterogeneities would be much less since their effectiveness as stress concentrators would be less. In the other limiting case, where the heterogeneities have the same mechanical properties as the matrix there is, of course, no attenuation, i.e.  $Q^{-1}$  (thermoelastic) = 0. The relatively small effect of temperature and pressure on this mechanism may make it possible to use the Earth itself as the critical test as to the importance of thermoelastic damping.

Anisotropy and heterogeneity of the grains in a rock also cause scattering of elastic waves when the wavelength is of the order of a grain size, the quality factor,  $Q$ , is proportional to  $(df)^{-3}$  where  $d$  is the average grain diameter and  $f$  is the frequency. This effect is probably not important at seismic frequencies except for scatterers of the dimensions of geological provinces or continents. The scattering mechanism, if present at seismic frequencies, should be easily identified by its strong dependence on frequency.

### 5. Dislocation mechanisms

Dislocations play a significant role in both the plastic behaviour and damping properties of single crystals. They are also involved in the properties of polycrystalline substances. There are basically three kinds of mechanical losses which are associated with dislocations: *hysteresis*, *resonance* and *relaxation* losses. Some dislocation mechanisms give rise to damping that is independent of amplitude and varies with frequency; other mechanisms depend on the amplitude of the stress wave and vary only slowly with frequency. The decrement and modulus changes due to dislocation motions depend on the orientation of the crystal since only the shear component of an applied stress in the slip direction contribute to dislocation motion.

The motion of dislocations which are pinned down by impurities or other dislocations is a common loss mechanism in metal single crystals. The damping depends sensitively on the amount of previous cold work and on the purity of the metal. Increasing the number of impurity atoms or increasing the number of dislocations by cold work provides more anchoring points and the internal friction decreases. Decreases in internal friction are accompanied by increases in the elastic moduli. The pinning of dislocations by point defects is especially effective at low temperatures and high frequencies since the ability of an impurity atom to follow an alternating stress is controlled by the slow process of diffusion. Amplitude dependence is attributed to the breakaway of the dislocation line from the impurity atoms at large strain amplitudes.

The name *hysteresis* is given to losses associated with the occurrence of *irreversible changes* of state. Typical of *hysteresis* are those in which dislocations are irreversibly torn away from pinning points such as impurity atoms or other point defects. This mechanism is frequency-independent and depends on the amplitude of applied stress. It is suppressed by annealing, by increasing the impurity concentration or by irradiation with neutrons or fast electrons, all of which serve to more effectively pin the dislocation and prevent breakaway. The binding energy of an impurity atom to a dislocation is rather small, about 0.1 eV in face-centred cubic metals and, therefore, the dependence of damping on temperature is rather small.

*Resonance* motion of dislocations within their equilibrium potential valleys can contribute to the damping of waves with frequencies comparable to the resonant frequency of the pinned dislocation segment. Because of the presumed shortness of dislocation loops, this will be important only at high frequencies, say, megacycles per second.

The *relaxation* dislocation mechanism is related to the thermally activated formation of kinks in the dislocation line which allow the dislocation to change from one potential valley to another. A shear stress can move a dislocation from one valley to another by sideways motion of the kink. The activation energy for the process is proportional to the height of the barrier between the equilibrium positions (the *Peierls barrier*) and the length of the dislocation line. An alternating stress produces fluctuations in the distribution of kinks which are accompanied by energy dissipation. If the frequency,  $\omega$ , of the applied stress is much less than the frequency of thermal formation of kinks,  $\omega_0$ , the kinks are always in thermal equilibrium and no losses occur; this will also be the case for frequencies much higher than  $\omega_0$ . Hence, the mechanism follows a typical relaxation process with maximum loss at  $\omega \sim \omega_0$ . This is called *Bordoni* or *low-temperature* internal friction. These peaks are absent in well annealed specimens but develop after cold working or plastic strain or even after the passage of a high amplitude stress wave. Annealing reduces the height of the peak, recrystallization removes it. An increase of frequency of vibration moves the peak to higher temperatures indicating a thermally activated process. The peak temperature and, therefore, the relaxation time is relatively insensitive to dislocation density, dislocation line length and the impurity concentration. The peak height, however, increases with dislocation density and is decreased by impurities. The production of point defects by irradiation reduces the size of the peak. This damping is approximately independent of strain amplitude.

An ultrasonic attenuation and a lowering of the elastic modulus can occur because of the displacement of pinned dislocations by an applied stress. The internal friction due to this mechanism for frequencies lower than the frequency ( $\sim 10^9$  c/s) of the dislocation loop is  $Q^{-1} = A\omega$  where  $A$  is a constant that contains information about the crystal lattice and the size and density of the dislocations. This linear dependence of  $Q^{-1}$  on frequency is not observed on metals except at megacycle frequencies. The internal friction is often more accurately represented by

$$Q^{-1} = B + C\omega \quad (12)$$

(Mason 1958). Mason showed that the frequency independent  $Q^{-1}$  implied by this relationship for low frequencies follows if the hysteresis lag for the displacement of the dislocation is proportional to the displacement. Mason attributed this kind of loss to an irreversible change in the bonding mechanism not involving breakaway from pinning atoms. A stress induced irreversible reordering or reorientation of bonds is not unreasonable in the vicinity of a grain boundary.

The *Köster* effect is a special kind of dislocation damping that appears when large amounts of cold work are applied to metals. The effect diminishes with time or can be annealed out. It is attributed to pinned dislocations which become shorter with time. The effect is independent of strain, temperature, and frequency at low strains and temperatures.

## 6. High temperature thermally activated internal friction

At high temperatures attenuation increases rapidly with temperature. Mason (1958) shows many examples where the internal friction follows

$$Q^{-1} = Q_0^{-1} e^{-H/RT} \quad (13)$$

at high temperatures. This effect was first observed by Ké (1950) using low-frequency torsion pendulum techniques on polycrystalline Mg, Al, Cu, Fe and  $\alpha$ -brass. It is particularly obvious on single crystals when the superimposed grain boundary peak is absent. The effect seems to be nearly independent of frequency.

A background loss increasing with temperature has been ascribed to the increased production of vacancies at high temperatures. The increase in the number of vacancies,  $\delta n$ , is given by

$$\delta n = n_0 e^{-H/RT}, \quad (14)$$

where  $n_0$  is the number of lattice sites and  $H$  is the energy to form a vacancy. The damping effect of dragging a dislocation through a concentration of vacancies increases as the number of vacancies and the attenuation increases exponentially with temperature. In the absence of vacancies the background attenuation has been attributed to the thermal breakaway of dislocations from their pinning points. The rate at which dislocation loops become unpinning is also proportional to  $e^{-H/RT}$ . The applied stress can modulate the number of thermal breakaways producing a non-elastic strain inversely proportional to the frequency of vibration. Mason (1958) also discusses a frequency independent high temperature internal friction caused by momentary breakaway of a dislocation loop from a pinning point. This mechanism gives

$$Q^{-1} = 2b^2 \sqrt{\frac{3}{2}} V_s N_0 e^{-H/RT} / \pi^2, \quad (15)$$

where  $b$  is the Burger's vector,  $V_s$  is the shear velocity and  $N_0$  is the dislocation loop concentration. This temperature-activated breakaway loss is proportional to the number  $N_0$  of dislocation loops and, therefore, should increase with impurity content.

Entwistle (1962) reviews a high temperature mechanism caused by stress-induced migration of polygonized boundaries which gives

$$Q^{-1} \sim (l^2 T f)^{-1} \exp(-H/RT), \quad (16)$$

where  $l$  is the sub-grain size,  $H$  is the activation energy for self-diffusion and  $f$  is the frequency of vibration. Typical activation energies for metals range from 4 to 20 kcal/mole.

Regardless of its origin the high temperature 'background' damping seems to dominate other mechanisms when measurements are performed at sufficiently high temperature. Therefore, it is particularly important in geophysical discussions.

## 7. Point defect internal friction

The passage of a stress wave through a crystalline solid can cause a stress induced diffusive motion and reordering of such point defects as vacancies, interstitial or solute impurity atoms, and substitutional atoms. This reordering can take place in a single crystal if the point defect produces distortions which have a lower symmetry than that of the lattice (Berry 1962). In polycrystals defects undergo a mass transport between regions of different dilatational strain. A characteristic feature of atomic diffusion is the relatively long time of relaxation and its strong dependence on temperature. The relaxation time for the process is of the order of the inverse jumping frequency of the defect. This frequency,  $\omega_R$ , depends on the temperature as

$$\omega_R = \omega_0 e^{-H/RT},$$

where  $H$  is a heat of activation and  $R$  is the gas constant. The heats of activation for elements in metals are of the order of tens of kilocalories per mole. For very rapid varying stresses, or for very low temperatures, the defects do not have a chance to move and stress and strain are in phase and no losses occur. For very slowly varying stresses or for very high temperatures the diffusion frequency can be greater than the applied frequency, again giving no delay and no loss. For intermediate frequencies and temperatures the motion of defects is out of phase with the applied stress and a typical relaxation peak occurs. Typical relaxation times are 1 s for C or N diffusion

in  $\alpha$ -iron at room temperature, 0.5 s for C diffusion in steel or nickel near 250°C,  $10^{-5}$  s for oxygen diffusion in silicon at 1030°C,  $0.16 \times 10^{-2}$  s for zinc diffusion in brass at 420°C, about 0.4 s at 60°C for Ti interstitials in  $\text{TiO}_2$ , and 0.1 to 5000 s for nitrogen in niobium for temperatures between 300°C and 150°C. The minimum value for  $Q$  in the vicinity of the  $Q^{-1}$  peak for Ti interstitials in rutile single crystals is typically about 500. For temperatures in the Earth and for seismic frequencies the jump time of point defects is probably faster than the periods of the stress cycle if the above estimates of relaxation times are appropriate.

Interstitial solutes give rise to a typical relaxation damping peaks which are called Snoek peaks. These have been found in many alloys and solid solutions. The study of this kind of internal friction gives one of the most accurate methods of determining the diffusivity of interstitial atoms in metals at room temperature or below where ordinary diffusion techniques would require millions of years. This effect has been studied on many alloys including carbon, nitrogen, hydrogen, and deuterium in iron; carbon, oxygen and nitrogen in tantalum, vanadium and niobium; interstitial nickel in cold worked pure nickel; and Mn, Cr, Mo and V in iron–nitrogen alloys. The peaks are often broader than the theoretical width for a single value of relaxation time, particularly for high solute concentrations. This has been attributed to interaction between solute atoms or between interstitial and substitutional solute atoms. The relaxation times for these interacting pairs, or larger groups, are close to but not exactly the same, as for single atoms. Sometimes separate peaks can be resolved for single diffusing atoms and pairs of atoms. The Snoek peak can be used for solubility measurements since its height depends on the solute concentration.

The stress-induced ordering and rotation of solute atoms or groups of atoms is also a loss mechanism in substitutional solid solution alloys. These are called Zener peaks and have been found in such alloys as Cu–Zn, Au–Ni, Al–Cu, Fe–Al and Li–Mg, among many others. They can be seen in homogeneous single-phase solid solutions under conditions of equilibrium; the peak height increases roughly as the square of the concentration and the peak temperature shifts with frequency according to an Arrhenius type equation. The strength of the relaxation depends on orientation and the size of the solute, the state of order present in the alloy and the presence of precipitates. Substitutional elements diffuse more slowly than interstitial elements and the relaxation peak occurs at higher temperatures.

A related peak, the Köster peak, is probably due to solute atoms along dislocations and occurs only in plastically deformed material, being proportional to the square root of the plastic strain. Annealing or recrystallization reduces the height of this peak. This peak may also be due to the interaction between vacancies and solute atoms.

## 8. Grain boundary losses

In addition to thermoelastic and scattering losses associated with grain boundaries in polycrystals, there are relaxation effects related to the dislocation mechanism at grain boundaries. Boundaries between grains of different orientations or composition are regions of vacancies, impurities, high strain, general disorder and, at high temperatures, partial melting. Losses associated with grain boundaries are high compared to other processes. The simplest kind of grain boundary is simply a plane separating slightly misoriented crystals. Such a boundary can be considered to be a series of edge dislocations which can be made to glide or to migrate on the application of a shearing stress. Other simple cases are tilt boundaries and twist boundaries (Mason 1958, p. 282).

More complicated boundaries can migrate by diffusion of atoms if the temperature is high enough to overcome the activation energy of diffusion. As in other relaxation mechanisms the temperature of the maximum attenuation shifts as the driving frequency is changed. For high frequency vibrations, when the relaxation of stress by

diffusion cannot keep up with the applied stress, the internal friction increases with temperature according to a heat of activation type law and increases with the grain boundary area per unit volume, i.e. inversely with grain size.

The energy dissipated across a slip boundary is proportional to the product of relative displacement and shear stress. At low temperatures relative displacement between grains is small and at high temperatures the shear stress across boundaries is essentially relaxed at all times. Appreciable energy dissipation by grain boundary slip occurs only at intermediate temperatures where there is both displacement and shear stress between grains. This temperature range is between about 200 or 400°C for 1 c/s on aluminium polycrystals. The maximum value of the damping in this intermediate temperature range is nearly independent of the grain size, temperature and frequency although the peak temperature depends on grain size.

There is no fully developed theory of grain-boundary damping, the often used term *viscous grain boundary losses* is a catchall designation, not an explanation.

Grain boundary damping was initially attributed to sliding at grain boundaries which relaxed the shear stress across the boundary. The boundary was interpreted as a region of disorder which would behave as a film of viscous fluid. The activation energy measured for grain boundary damping, however, shows good correlation with values measured for grain boundary diffusion and creep. This suggests that the grain boundary 'sliding' is actually due to ordering and reordering of atoms under a stress bias in the boundary by diffusion. Representative values for the activation energies associated with given boundary damping are 46 kcal/mole for iron and 22 kcal/mole for silver. These are close to measured values for grain boundary diffusion. At low stress levels a stress induced migration of arrays of dislocations by dislocation climb is probably a more appropriate description of grain boundary losses than is grain-boundary sliding. This climb would be controlled by self-diffusion rates in the boundary region. It would indeed be fortunate if this mechanism were responsible for both creep and seismic attenuation in the mantle since it would then be possible to estimate geophysical creep rates from seismic data.

Although grain boundary damping is clearly an activated process, the peaks are too broad to be fit by a unique value for the relaxation time, even in pure substances. A range of activation energies is indicated. An activation energy distribution from 27 to 35 kcal is required to account for the width of the peak in polycrystalline aluminium. The differences in orientation and/or composition between grains lead to boundaries with different structure and mechanical relaxation times. A spectrum of activation energies can be expected for polycrystalline substances leading to broad internal-friction peaks. Data is not available on rocks but a broad spread of activation energies is probable because of the many possibilities. This could lead to a  $Q$  roughly independent of frequency over a wide frequency band.

Changes in the high temperature 'background damping' are often correlated with changes in the grain boundary peak for example, both decrease with an increase in grain size. In such cases the *background* is probably also a grain boundary effect. The so-called *background damping* increases with temperature.

The effect on the grain boundary peak caused by substitutional and impurity defects suggests that it is due to grain boundary migration rather than sliding. Substitutional impurities increase the activation energy for the grain boundary peak and reduce its height. Migration of atoms from one grain to another driven by the applied stress is identical to grain boundary diffusion.

Grain boundary damping is not really a separate phenomenon from those previously discussed but represents the increased effectiveness of these mechanisms in the boundary region. The most plausible explanation of grain boundary damping is essentially the same as stress induced migration of atoms or reordering of the atoms and atomic bonds in and across the boundary.

## 9. Other mechanisms

There are other loss mechanisms which appear when the stoichiometry and perfection of crystals is near ideal, such as magnetoelastic interactions in ferromagnetic insulators. Except for carefully grown synthetic crystals stress-induced ordering processes and various mechanical imperfections usually dominate over the latter mechanism. Flux-grown single crystal garnets are receiving much attention because of their extremely low damping. The acoustic  $Q$  of yttrium iron garnet, for instance, increases with the magnetic field and is typically about  $4 \times 10^5$  (see for example LeCraw & Comstock 1965). The simultaneous presence of  $\text{Fe}^{2+}$  and  $\text{Fe}^{3+}$  results in elastic losses by a stress-induced relaxation mechanism due to 'electron hopping' which introduces a damping peak at about  $200^\circ\text{K}$  at  $1.34 \text{ Mc/s}$  in yttrium iron garnet with a minimum  $Q$  of about 300. A typical activation energy is less than  $0.3 \text{ eV}$  and a typical electron hopping time is less than  $10^{-10} \text{ s}$ . For this material  $Q^{-1}$  is nearly linear with frequency between 7 and  $28 \text{ Mc/s}$  at  $300^\circ\text{K}$ . LeCraw and Comstock point out that  $Q$  is essentially independent of frequency for  $\alpha \text{ Al}_2\text{O}_3$  from 500 to  $3000 \text{ Mc/s}$ .

Microwave ultrasonics using frequencies in the range  $10^9$ – $10^{11} \text{ c/s}$  is a recent development that is being used to study anharmonic, electron and spin forces between atoms and various lattice electron and spin interactions with the sound wave.

The anharmonicity of real crystal lattices leads to attenuation even in ideal single-crystal insulators. This is an intrinsic phonon mechanism and represents the interaction of an elastic wave with thermal lattice waves. An elastic wave varies the thermal phonon frequencies, thereby disturbing the original Boltzmann distribution of thermal phonons. At high temperatures these frequencies are all much higher than seismic, or even ultrasonic, frequencies. At extremely low temperatures anharmonic processes are negligible and the elastic scattering of phonons by imperfections becomes the dominant phonon mechanism. Anharmonic processes become increasingly important as the temperature is raised from absolute zero and the attenuation increases with temperature.

Phonon–phonon interactions are particularly conveniently studied on the semiconductors germanium and silicon because they can be manufactured with high purity and acoustic attenuation due to dislocations is negligible between  $0^\circ\text{K}$  and room temperature. The majority of dislocations in diamond type structure has a high activation energy and the energy loss associated with the motion of dislocations is negligible for temperatures below  $500^\circ\text{K}$  (Mason 1965).

## 10. Anelasticity of oxides

Measurements of the internal friction of low-porosity sintered ceramics such as  $\text{TiO}_2$ ,  $\text{MgO}$  and  $\text{Al}_2\text{O}_3$  are at present the most pertinent data available, particularly since data exist for high temperature. Here again, however, attention has been focussed on high purity, carefully doped specimens, in order to isolate the loss mechanism. These materials are, of course, more closely related to the mantle than are metals or polymers. Natural and artificial quartz has also received much attention because of its very high  $Q$ .

Chang (1961) measured the internal friction of  $\text{Al}_2\text{O}_3$  crystals at  $11 \text{ c/s}$  and  $41 \text{ c/s}$  up to about  $1700^\circ\text{C}$ . The internal friction,  $Q^{-1}$ , varied from about  $1 \times 10^{-3}$  at  $1200^\circ\text{C}$  to  $7 \times 10^{-3}$  at  $1500^\circ\text{C}$  at  $11 \text{ c/s}$  and  $5 \times 10^{-3}$  at about  $1730^\circ\text{C}$  at  $41 \text{ c/s}$ . The corresponding values for  $Q$  are 1000, 142 and 200. The general form of the variation is an exponential rise with temperature interrupted by slight knees, presumably unresolved peaks, at  $1400^\circ\text{C}$  ( $11 \text{ c/s}$ ) and  $1540^\circ\text{C}$  ( $41 \text{ c/s}$ ). Such a background internal friction is quite common on measurements made at high temperature and complicate the resolution of individual peaks. In the case of  $\text{Al}_2\text{O}_3$  the peaks were assumed to be Bordoni peaks with an activation energy of  $2.6 \text{ eV}$ . Chang (1961) also found Bordoni

peaks in pure and Fe doped MgO crystals. These peaks occurred between  $\pm 20^\circ\text{C}$ , increased with increasing amounts of cold work, decreased with annealing, were independent of strain amplitude, were composed of a number of individual peaks, were broadened by impurities and shifted to lower temperatures for lower frequency measurements. The activation energy for the peak was about 0.15 eV. Measured  $Q_s$  varied from 5000 to 1000.

Chung (1961) measured the temperature variation of Young's modulus and damping in high purity (99.97%)  $\text{Al}_2\text{O}_3$  and alumina with controlled amounts of impurity. The internal friction on the pure alumina was considerably lower than measured by Wachtman (1957) and Chang (1961). The 'background  $Q^{-1}$ ', averaged about  $5.2 \times 10^{-4}$  between  $25^\circ\text{C}$  and  $800^\circ\text{C}$ , increasing slowly with temperature. An internal friction peak was observed at about  $1080^\circ\text{C}$  where the  $Q^{-1}$  was about  $153 \times 10^{-4}$  (5000 c/s). The grain size for this specimen was about 4 microns. A small secondary peak appeared at about  $700^\circ\text{C}$  when small amounts of silica ( $\sim 1\%$ ) were introduced; the size of the peak increased with the amount of silica. The silica is believed to react with the  $\text{Al}_2\text{O}_3$  to form mullite,  $3\text{Al}_2\text{O}_3$ . This peak was assigned to grain boundary slip in the mullite phase. The  $1000^\circ\text{C}$  peak was assigned to grain boundary slip in the alumina phase, influenced perhaps by carbon impurities. The energy loss below  $850^\circ\text{C}$  was believed due to stress inhomogeneity between adjacent grains due to grain anisotropy. The 'background'  $Q^{-1}$  increased with temperature in all specimens; this could be attributed to a true exponential increase of damping with temperature or another unresolved peak beyond  $1200^\circ\text{C}$ . The former interpretation satisfies Chang's (1961) data to  $1700^\circ\text{C}$ .

Crystal quartz has a number of relaxation peaks due to impurities—many of these occur below room temperature. The work of Brommel, Mason & Warner (1955) on internal friction in an AT-cut crystal from 10 to  $120^\circ\text{K}$  reveals a peak at  $20^\circ\text{K}$  which appears to be due to a phonon-phonon interaction and a peak at  $50^\circ\text{K}$  which is probably due to sodium impurities (Mason 1965). The impurity peak can be eliminated by X-irradiation, by reducing the rate of growth of the synthetic crystal or by electrically sweeping the impurities out of the specimen. Natural quartz crystals have only a very small relaxation peak near  $50^\circ\text{K}$ . Lithium and potassium impurities produce smaller peaks, again at temperatures below room temperature for Mc/s frequencies. Impurities also cause relaxation peaks at higher temperatures in crystal quartz which are superposed on an exponentially rising internal friction background which is correlated with the doping impurity.

Because of their absence of ductility it might be expected that the importance of dislocations will be less for these materials than for metals. In fact, it is found that the number of dislocations in quartz and related non-metallic crystals is significantly less than for metals. Attenuation in single crystal quartz is very small, values for  $Q$  range from  $10^4$  to  $10^7$  for frequencies between 0.2 and 100 Mc/s. The internal friction increases in proportion to the frequency.

Much experimental work has been done on the internal friction of inorganic glasses. Here the general trend is an experimental increase of damping interrupted by diffuse relaxation peaks. These relaxation peaks have been attributed (Hopkins & Kurkjian 1965) to short-range deformation of the glass network, stress-induced ordering of the constituent ions by diffusion and large-scale relaxation and flow of the whole network in the glass transition range. These mechanisms are considered to be the same as those responsible for the viscous flow of glasses. The network deformation loss involves a bending of elongated Si-O-Si bonds (in silica glass) which permits the hopping of oxygen from one equilibrium position to another. Activation energies for this process are:  $\text{SiO}_2$ , 600 cal,  $\text{GeO}_2$ , 3000 cal and  $\text{B}_2\text{O}_3$ , 5100 cal. The peak near  $40^\circ\text{K}$  in fused silica becomes less pronounced and migrates to lower temperatures as the frequency decreases from megacycles to kilocycles.

The losses in shear and longitudinal vibrations can both be attributed to losses in shear, i.e. a complex shear modulus. The variation of  $Q$  with relaxation frequency and absolute temperature indicates a range of atomic diffusion processes with an average activation energy of 1300 cal/mole.

The high temperature measurements of Marx & Sivertsen (1953) at 37 kc/s on clear fused silica are of particular interest. Up to 300°C the decrement ( $\Delta = \pi/Q$ ) is fairly flat with temperature but then it rises to a diffuse peak at 646°C. This peak is followed by an exponential rise satisfying an equation

$$Q^{-1} = 0.37 e^{20\,000/RT}.$$

The data extends up to 1000°C. Similar trends are exhibited by pyrex glass and soft glass. The final exponential rise has not been definitely explained but it may be related to a sort of grain boundary relaxation between masses of crystallites in the amorphous body. Lower temperature peaks near 317°C in pyrex and soft glass have been attributed by Marx & Sivertsen (1953) to the migration of sodium atoms. The activation energy for this process is about 18 kcal/mole.

### 11. High temperature measurements

There are only a few crystalline materials for which measurements on the behaviour of  $Q$  at temperatures approaching the melting point have been performed; low melting point alloys and ice have been studied. While the materials are not pertinent to the mantle, the behaviour of internal friction near the melting point may be representative.

Mizutani & Kanamori (1964) measured the elastic and anelastic properties of a Pb-Bi-Sn-Cd alloy of melting point 72°C from 10 to 130°C at near megacycle frequencies. The elastic velocities decrease slowly with temperature, the decrease accelerates near the melting point and is most pronounced for the shear wave. The compressional wave velocity decreased about 20% on passing through the melting point; a shear wave was undetectable in the melt. The elastic parameter  $\lambda$  seems to pass smoothly through the transition. The quality factor for  $P$  waves,  $Q_p$ , increases almost linearly with frequency between 0.5 and 3.0 Mc/s, as appropriate for a Maxwell material. The factor  $Q_\alpha$  decreased slowly with temperature up to within about 20°C of the melting point and then dropped approximately exponentially with a further increase. The  $Q_\alpha$  in the liquid was about twenty times less than the  $Q_\alpha$  in the low temperature range ( $\sim 10^\circ\text{C}$ ).

The most detailed study of internal friction at temperatures near the melting point was carried out on various forms of ice (Kuriowa 1964). Kuriowa studied the damping in flexure of rods made of pure H<sub>2</sub>O, D<sub>2</sub>O, lake glacier and doped ice between 0 and -180°C over a range of frequencies between 170 and 965 c/s. Doping impurities included NaCl, HCl, NaOH, HF, NH<sub>3</sub> and NH<sub>4</sub>F as well as natural contaminants. In the pure H<sub>2</sub>O and D<sub>2</sub>O polycrystalline specimens a sharp peak centred near -20 to -40°C, depending on the frequency, was superimposed on an attenuation curve that increased rapidly with temperature. As the excitation frequency increased, the peak value moved to higher temperatures. The relaxation times associated with the peak were of the order of 10<sup>-3</sup>-10<sup>-4</sup> s and their variation with temperature agreed with dielectric relaxation experiments. Both relaxation processes were attributed to the same mechanism, diffusion of protons with an activation energy of about 13 kcal/mole. A typical value of  $Q$  at the damping peak is 120 near -30°C. Typical values of the 'background'  $Q$  are 1000 near -60°C, 160 near -50°C and 100 near -20°C. These values increase as the frequency decreases. All the measurements were made in what can be considered the high temperature range for ice, i.e.  $T > T_m/2$  where  $T_m$  is the melting temperature. The background attenuation was attributed to grain boundary effects.



The introduction of NaCl into the ice broadened the peaks, shifted them to lower temperatures and increased the peak damping. The high temperature grain boundary internal friction also increased. Another broad peak appeared at  $-145^{\circ}\text{C}$ ; the amplitude of this peak varied with frequency but its location did not. It decreased with annealing. The minimum  $Q$  near the melting point for NaCl ice was of the order of 60. Commercial polycrystalline ice gave a  $Q$  of 40 near the melting point. The background damping increases with increasing NaCl content. In these experiments NaCl can be considered a typical neutral impurity that concentrates at grain boundaries during the freezing process. Grain boundary melting occurs above  $-22.4^{\circ}\text{C}$ . The background damping was not present for experiments on single crystals which strengthened the grain boundary interpretation.

## 12. Effect of pressure

The effect of pressure on anelasticity is extremely difficult to measure and there is little direct data on the subject. There are, however, data on the closely related phenomena of diffusion and creep. At temperatures well below the melting point atomic motion is confined to grain boundaries and dislocations. At high temperatures and in disordered solids diffusion is due to vacancy and interstitial mechanisms. The interstitial mechanism dominates for high-temperature diffusion of small atoms in open lattices, i.e. for nitrogen and carbon in iron. The vacancy mechanism of diffusion appears to dominate at high temperatures in close-packed solids such as oxides.

The temperature dependence of the diffusion,  $D$ , in simple systems obey an Arrhenius type relation

$$D = D_0 \exp(-\Delta H_m/RT),$$

where  $D_0$  is a factor that contains information about the lattice frequency, crystal structure, lattice parameter, the relative concentration of atoms capable of diffusing and the entropy changes associated with the jump;  $\Delta H_m$  is the enthalpy of activation for the jump of an atom over a potential saddle point. The concentration  $C$  of imperfections in a pure material also depends on temperature

$$C = C_0 \exp(-\Delta H_f/RT).$$

This is constant for interstitial impurities.

The enthalpy  $\Delta H$  can be written  $E^* + P\Delta V^*$  where  $E^*$  is the activation energy and  $\Delta V^*$  is the activation volume. One, therefore, also obtains an exponential variation of diffusion with pressure.

$\Delta V_m^*$  and  $\Delta V_f^*$  can be interpreted as the changes in volume associated with the motion and creation, respectively, of the defect. For example, the  $\Delta V_f^*$  associated with the creation of a mole of vacancies would be equal to  $V_M$ , the molal volume, if there were no relaxation around the vacancy. The existence of relaxation causes  $\Delta V_f^*$  to be appreciably different from  $V_M$ ; in metals  $\Delta V^*$  is less than the molal volume. The ratio of activation volume to molar volume for self diffusion is usually between about 0.4–0.9. The activation volume apparently decreases slowly with pressure; the ratio of activation volume to molar volume, of course, decreases even more slowly.  $\Delta V_m^*$  is a measure of the local expansion required for motion and represents the difference between a mole of atoms at positions of relative stability in the lattice and a mole of atoms at potential saddle points, i.e. the dilatation attending the elementary diffusion jump. The work that must be done against the external pressure is  $P\Delta V_m^*$ .  $\Delta G = \Delta H - T\Delta S$  is the Gibbs free energy for the process and expresses the isothermal work required to raise a diffusing atom to the saddle point.

Plastic flow or creep is also a thermally activated process rate limited by the diffusive jumps of atoms.

Table 1 gives activation volumes measured for diffusion, creep and internal friction. Also tabulated for comparison are the molecular volumes for the diffusing atom. Pressure would tend to suppress the importance of the larger ions in favour of the smaller ones which can diffuse with less work being expended.

**Table 1**  
*Activation volumes*

Substance	Component	$\Delta V^*$ (cm <sup>3</sup> /mole)	$V_M$ (cm <sup>3</sup> /mole)
Li	$\Delta V_f + \Delta V_m$	3.4	13.1 (Li)
Na	$\Delta V_f + \Delta V_m$	12.4	24.0 (Na)
P	$\Delta V_f + \Delta V_m$	30.0	68.0 (P)
Sn	$\Delta V_f + \Delta V_m$	5.0	27.0 (Sn)
Ag	$\Delta V_f + \Delta V_m$	9.2	10.3 (Ag)
Au			
Pb	$\Delta V_f + \Delta V_m$	13.0	18.2 (Pb)
	$\Delta V_f + \Delta V_m$	15.5	18.2 (Pb)
AgZn	$\Delta V_f + \Delta V_m$	5.4	10.0 (Ag)
V-O	$\Delta V_m$	1.7	2.0 (O)
V-N	$\Delta V_m$	1.1	2.0 (N)
Fe-C			2.0 (C)
Fe-N			2.0 (N)
Au	$\Delta V_m$	1.5	10.2 (Au)
NaCl	$\Delta V_m$ (Na vac.)	7.7	5.4 (Na)
KCl	$\Delta V_m$ (K vac.)	7.0	10.8 (K)
AgBr	$\Delta V_f$	16.0	9.0 (Ag)
	$\Delta V_f$	4.4	29.0 (AgBr)
	$\Delta V_m$ (Ag int.)	2.7	9.0 (Ag)
	$\Delta V_m$ (Ag vac.)	7.4	9.0 (Ag)
Hg (liq.)		0.57	14.8
Ga (liq.)		0.55	11.5

The activation energy and activation volume for diffusion in liquids are much smaller than for solids. The activation volume is of the order of 5% of the atomic volume. Although no measurements have been made of the attenuation coefficient in the Earth's liquid outer core, it can be assumed that it will be roughly a constant.

Techniques and results of high pressure diffusion experiments have recently been reviewed by Lazarus & Nachtrieb, Keyes and Lawson (see Paul & Warschauer 1963). Most of the data in Table 1 is from these sources.

### 13. Summary

All of the effects just discussed contribute to the damping of seismic waves in the Earth's mantle. The question that cannot be answered yet with any finality is their relative importance. Attention must be focussed on those mechanisms which are important at high temperatures and low frequencies. Even the shortest seismic wavelengths which have been used to study the mantle have wavelengths of the order of a kilometre. Most of the attenuation measurements use wavelengths of the order of 100–3000 kilometres. Since the temperature throughout the mantle is relatively close to the melting point, we probably are dealing with a mechanism that increases exponentially with temperature, since this is the typical behaviour of materials that have been tested at high temperatures. Although no complete theory has been advanced for this 'high temperature background loss' it seems to be related to grain boundary phenomena and it dominates other processes which are better understood. The term *background* expresses our general ignorance of the object. The effectiveness of the

process is related to the mobility of the atoms in the boundary which depends exponentially on temperature and pressure. Since the average atomic spacing is larger, the interatomic attractive forces are smaller in the vicinity of the grain boundary than elsewhere. This, plus the tendency of impurities to concentrate at grain boundaries, lead one to suspect that stress induced reordering of atoms in the boundary region may dominate the other possible mechanisms. Because of the general disorder and lattice distortions in these regions, one might also anticipate that the reordering may be irreversible and be a *hysteresis*, rather than a *relaxation* process.

#### 14. Results from seismology

The amplitude spectrum of a seismic signal depends on the amplitude spectrum of the source, spreading losses which include reflection, refraction and scattering effects, and the loss of energy due to absorption along the path. Special techniques must be invoked in order to study only the latter effect. The details of these techniques are too specialized to discuss here. They are described in the original references and in Anderson (1966). The most general technique is the *spectral ratio method*, which has been applied to both surface waves and body waves. The ratio of the spectrums of two seismic signals is related to the attenuation over that part of the path which is not common to the two signals. For example, the spectral ratio of the signal that corresponds to two successive passages of a surface wave past a seismometer, say  $R_1$  and  $R_3$ , gives the average attenuation for Rayleigh waves over a great circle path. The spectral ratio between successive shear wave reflections off the core, say  $ScS_I$  and  $ScS_{II}$ , give the average shear wave attenuation for the whole mantle. Likewise, such events as  $S$  and  $SS$ ,  $P$  and  $pP$  can be used to study the attenuation in the upper mantle and the study of the spectrums of body waves as a function of distance can be used to determine the attenuation as a function of depth. These methods serve to eliminate or reduce the effect of the source. For body waves the method is complicated by the possibility that the radiation pattern of the source is frequency dependent and by the transfer function of the crust which is dependent on frequency and angle of incidence.

Estimates of damping in various regions of the Earth have been made by Gutenberg (1958), Wadati & Hirona (1956), Fedotov (1963), Asada & Takano (1963), Press (1956, 1964), Anderson & Kovach (1964), Kovach & Anderson (1964), Anderson & Archambeau (1964), and Anderson, Ben-Menahem & Archambeau (1965).

Asada & Takano (1963) studied the spectra of high frequency 1–7 c/s compressional waves as a function of earthquake depth and distance. Although they had to assume a source spectrum, their results were consistent with a low  $Q$  upper mantle and a high  $Q$  lower mantle.

Press (1956), Anderson & Kovach (1964), and Kovach & Anderson (1964) estimated the average  $Q$  for the mantle from multiply-reflected shear waves reflecting between the Earth's surface and the boundary of the core. Press obtained  $Q_\beta$  of 500 for 0.09 c/s waves; Anderson & Kovach obtained 508 for 0.04 c/s shear waves, and Kovach & Anderson obtained 600 for the frequency range 0.02–0.07 c/s. Considering the experimental uncertainties, these results are consistent with frequency independent  $Q$  for shear waves in the period range 10–50 s. These estimates of the average  $Q$  of the whole mantle are also consistent with the decay of long period torsional oscillations which have periods of the order of 2000 s. A frequency dependent internal friction cannot be completely ruled out on the basis of these data, but the implication is strong that  $Q$  varies less rapidly than frequency or the square of frequency, as predicted by the simple relaxation mechanisms. The constancy of  $Q$  is also consistent with the apparent absence of dispersion in body waves. Additional evidence is provided by high frequency laboratory measurements on crustal rocks which result in estimates of  $Q$  similar to those measured in the crust by much lower frequency seismic methods.

More direct measurements of the frequency variation of  $Q$  in the Earth are definitely required. This is essential to the understanding of the attenuation mechanism.

Multiply-reflected core phases have also been used to estimate the average  $Q$  above and below the focus of deep earthquakes. Anderson & Kovach (1964) determined an average  $Q$  of 151–185 for the upper 600 km of the Earth and 1430 for the rest of the mantle by assuming the energy radiated upward from the earthquake was the same as radiated downward. Kovach & Anderson (1964) determined 200 and 2200 for the same regions by assuming that  $Q$  was independent of frequency. The spread of these results is typical of the present uncertainty in attenuation measurements.

The attenuation of seismic waves can be handled most conveniently by allowing the propagation parameters, such as velocity, wave number or frequency to be complex. The energy of a propagating damped body wave can be written

$$E(\omega) = GE_0(\omega) e^{2i(\omega t - \mathbf{k}x)} = GE_0(\omega) e^{-2k^*x} e^{2i(\omega t - kx)},$$

where

$E_0(\omega)$  = the energy spectrum at the source

$E(\omega)$  = energy spectrum at distance  $x$

$G$  = frequency independent geometric spreading factor

$\mathbf{k}$  = complex wave number

$k, k^*$  = real and imaginary parts of wave number

$k^*$  = spatial attenuation coefficient

$\omega$  = radial frequency

$t$  = propagation time.

The damping,  $Q^{-1}$ , is defined as the ratio of the energy dissipated per wavelength,  $k^{-1}$ , to the total energy carried or (for non-dispersive waves)

$$\begin{aligned} Q^{-1} &= \frac{\Delta E}{2\pi E} = 1 - \exp(-2k/k^*) \\ &= 2k^*/k \quad k^* \leq k. \end{aligned}$$

This is valid as long as the waves damp out exponentially in space and does not depend on assumptions regarding the loss mechanism.

Lamb (1962) and Futterman (1962) showed that body wave dispersion is very small, even though some is required in linear theories of attenuation.

The ratio of the energy spectrums for two seismic signals is

$$\ln [E_1(\omega)/E_2(\omega)] = \ln [G_1 E_1(\omega)/G_2 E_2(\omega)] - \omega \left[ \frac{t_1 Q_2 - t_2 Q_1}{Q_1 Q_2} \right],$$

where  $t_1$  and  $t_2$  are the travel times for the two events. If  $Q$  is independent of frequency a plot of the log of the spectral ratio versus angular frequency then gives a measure of the relative differences in attenuation between the two paths.

The purest experimental technique is to measure the spatial rate of decay of surface waves or the time rate of decay of free oscillations. The method for inverting this kind of data in terms of attenuation versus depth has been given by Anderson & Archaibeau (1964).

The advantages of the surface wave or free oscillation method have been summarized by Anderson *et al.* (1965):

1. The long period waves suffer less degradation due to inhomogeneities, i.e. Rayleigh-type scattering.
2. A given bundle of energy can be sampled at several points along its own ray, i.e. the surface of the Earth.

3. The geometric spreading factor can be calculated more accurately or cancelled out completely.
4. Velocity reversals and horizontal discontinuities lead to no particular difficulty.
5. Successive passages of longer-period surface waves from large earthquakes past a given instrument can be analysed, removing completely the effects of source asymmetry, instrument, local geology and geometric spreading.
6. A single seismogram contains sufficient information for a fairly detailed interpretation.

The disadvantages of this method are the limited depth resolution, the restriction to large earthquakes, and the masking of any intrinsic frequency dependence by the variation of anelasticity with depth. Theoretical and numerical difficulties associated with attenuation of dispersed waves have been overcome.

There are three parameters involved in the dispersion of surface waves in an elastic waveguide; compressibility,  $K$ , rigidity,  $\mu$ , and density,  $\rho$ . The variation of these parameters with depth lead to a unique relationship between frequency,  $\omega$ , and wave number,  $k$ . The corresponding problem in the presence of attenuation can be formally treated by allowing any or all of the physical variables to become complex and solving for complex wave numbers which represent damped sinusoidal waves. In practice, the attenuation is attributed to complex moduli and the density is taken as a real variable. The damping of standing waves or free oscillations is treated by determining complex  $\omega$  for real order number,  $n$ . The travelling wave and standing wave experiments are completely equivalent and yield the quality factor:

$$Q^{-1} = 2\omega^*/\omega \quad \text{standing wave}$$

$$Q^{-1} = 2(U/c)k^*/k \quad \text{travelling wave}$$

where  $\omega^*$ ,  $k^*$  and  $\omega$ ,  $k$  represent the imaginary and the real parts of frequency and wave number and  $U$  and  $c$  are the group and phase velocities. The quantities  $\omega^*$

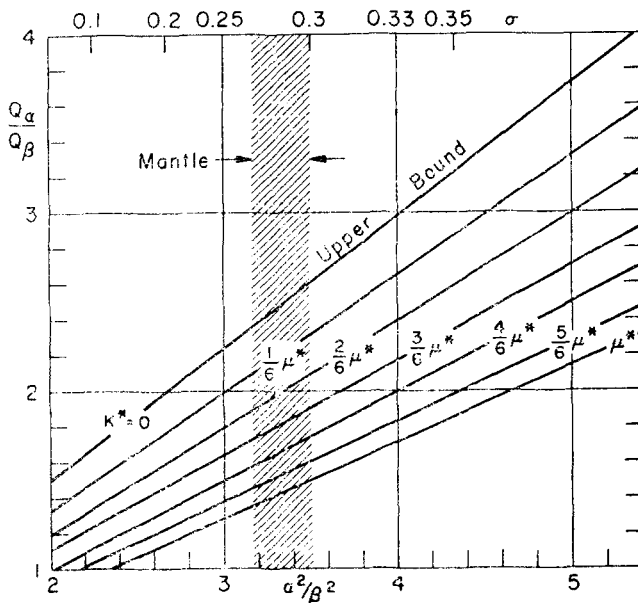


FIG. 1. Ratio of longitudinal to shear wave quality factors as a function of the square of the longitudinal to shear wave velocity ratio with complex compressibility as a parameter (after Anderson *et al.* 1965).

Downloaded from https://academic.oup.com/gji/article/14/1-4/135/739107 by U.S. Department of Justice user on 16 August 2022

Table 2

Layer thickness,  $H$ , and shear wave dissipation functions  $Q$   
(after Anderson & Archambeau 1964)

$H$ (km)	$Q(F)$	$Q(G)$	$Q(H)$	$H$ (km)	$Q(MM8)$
56.5	200	200	100	38	450
100	60	80	100	22	60
240	80	80	80	10	80
50	100	100	200	55	100
250	300	300	400	375	150
300	800	900	800	100	180
1900	1600	1700	2000	100	250
				100	450
				100	500
				100	600

and  $k^*$  are the temporal and spatial attenuation factors respectively. In a homogeneous material the intrinsic attenuation can be written in terms of the material constants

$$Q_K^{-1} = K^*/K, \quad Q_\mu^{-1} = \mu^*/\mu, \quad Q_\rho^{-1} = \rho^*/\rho.$$

For body waves

$$Q_\alpha^{-1} = \frac{K^* + 4\mu^*/3}{K + 4\mu/3}$$

$$Q_\beta^{-1} = \mu^*/\mu$$

if the loss is due to complex moduli which is an assumption regarding the mechanism. If there are no losses in pure compression, i.e.  $K^*=0$

$$Q_\alpha^{-1}/Q_\beta^{-1} = (\frac{4}{3})(\beta/\alpha)^2.$$

This ratio is shown in Fig. 1 for various assumptions regarding  $K^*/\mu^*$  and  $\alpha/\beta$ . Most of the losses discussed in previous sections, except thermoelastic, give  $Q_\alpha > Q_\beta$ .

In a layered or inhomogeneous waveguide the measured  $Q$  is a weighted combination of the intrinsic  $Q$ 's; the weighting factors are related to the appropriate energy distribution, compressional shear or kinetic in the mode under consideration.

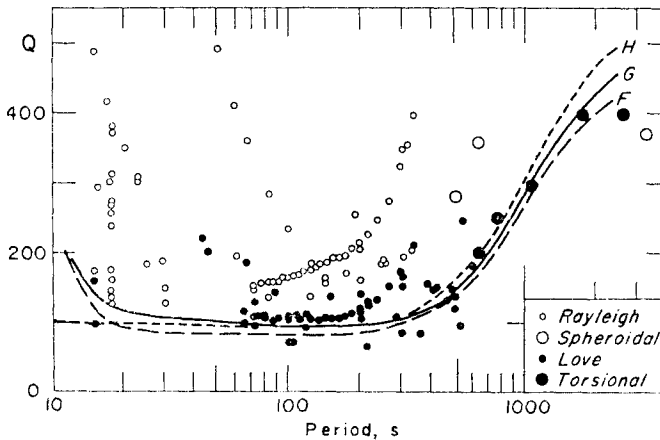


FIG. 2. Love wave, Rayleigh wave, spheroidal oscillation and torsional oscillation data for the Earth as a function of period (after Anderson & Archambeau 1964).

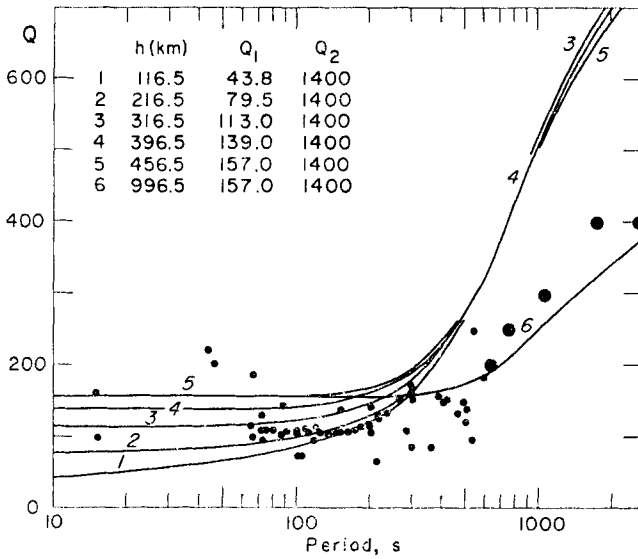


FIG. 3. Love wave attenuation data and theoretical two layer  $Q$  models of the mantle.

Fig. 2 shows measured  $Q$  values as a function of period for Love and Rayleigh waves and torsional and spheroidal oscillations. The curves are theoretical results for Love waves and torsional oscillations for the models listed in Table 2. A low  $Q_\beta$  zone in the upper mantle is the main feature of these results. The rapid increase of  $Q$  for Love waves for periods greater than some 300 s implies a rapid increase in  $Q$  with depth at some depth below 400 km. Fig. 3 shows the variation of Love wave  $Q$  as a function of period for six simple two-layer models of the upper mantle which have average upper and lower mantle attenuations similar to those determined by Anderson & Kovach (1964). These curves indicate that a simple two-layer model is not adequate even to explain this limited data, that an average  $Q_\beta$  as low as 80 in the upper 200 km

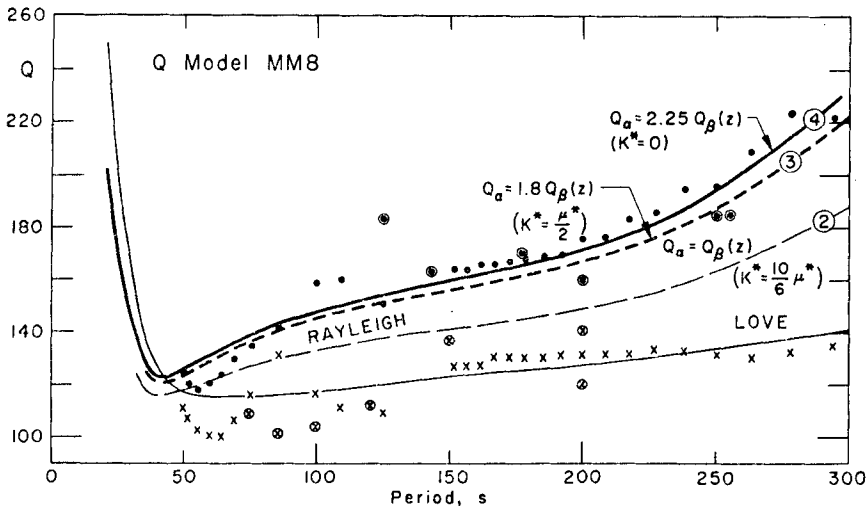


FIG. 4. Experimental and theoretical Love wave and Rayleigh wave attenuation results (after Anderson *et al.* 1965).

may be required to satisfy the shorter period ( $T < 200$  s) data, and that there must be a depth where  $Q$  increases rather rapidly in order to satisfy the free oscillation data.

A later study (Anderson *et al.* 1965) utilized both Rayleigh and Love wave data between about 50 and 300 s. The major  $Q$  discontinuity was placed at a depth of 700 km. The variation of  $Q$  with depth that satisfied this data given in Table 2 (Model MM8).

Fig. 4 gives more Love and Rayleigh wave attenuation data. The Rayleigh wave  $Q$  is systematically higher than the Love wave  $Q$ . Since Rayleigh wave motion involves both compression and shear and Love waves involve only shear motions, this immediately suggests that losses in shear are greater than losses in compression. This situation is complicated by the fact that Rayleigh and Love waves sample the Earth differently and to different effective depths. The broad plateau, for example, in the Love wave data between 150 and 300 s is due to trapping in the low velocity channel, which is more effective for Love waves than for Rayleigh waves.

Model MM8, Table 2, was designed to satisfy the Love wave data of Fig. 4. The crustal  $Q_\beta$  value of 450 was taken from Press (1964), and is based on the attenuation of short period ( $\sim 1$  s)  $Lg$  waves. With this value for the crust the  $Q_\beta$  in the upper 80 km of the mantle is between 60 and 100 in agreement with the earlier determinations of Anderson & Archambeau (1964). Between 125 and 700 km the  $Q$  in shear is between 100 and 250. Below 700 km the  $Q_\beta$  is greater than about 450. The variation of anelasticity in the upper mantle is about a factor of seven or eight. The variation in rigidity for the same depth range is less than a factor of four. Attenuation is, therefore, much more sensitive to composition or temperature than the elastic properties which is not unexpected, if attenuation is a thermally activated process.

Once the variation of  $Q_\beta$  is determined as a function of depth from Love wave data, an assumption about the ratio  $Q_\alpha/Q_\beta$  gives the variation of  $Q$  for Rayleigh waves as a function of period. The results of three such assumptions are shown in Fig. 4. The assumption  $K^* = 0$ , i.e. there are no losses in pure compression, gives satisfactory agreement with the Rayleigh wave attenuation data. Rayleigh waves are more sensitive to the shear properties of the Earth than they are to the compressional properties, so this is a very indirect and inaccurate method of estimating  $Q_\alpha$ . More direct evidence, however, is not yet available. The conclusion that  $Q_\alpha > Q_\beta$  is probably firm on the basis of this data. The well known experimental fact that shear waves are longer period than  $P$  waves is consistent with this relationship, although this observation may be a result of properties of the source.

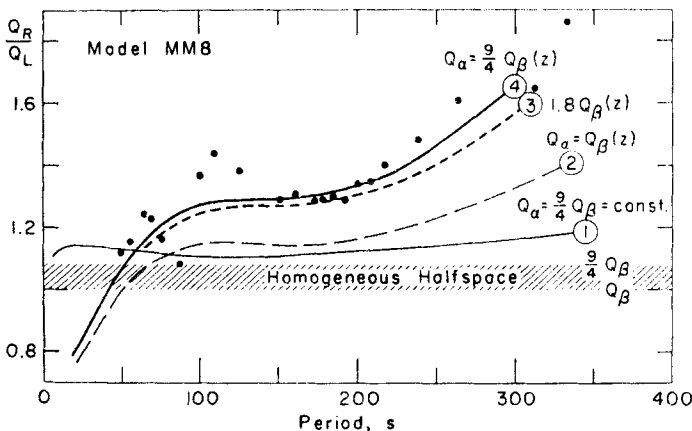


FIG. 5. The ratio  $Q_R/Q_L$  for various  $Q$  distributions with depth (after Anderson *et al.* 1965).



Although the assumption that  $Q_\alpha/Q_\beta$  is a constant with depth is clearly an oversimplification, a more complicated depth dependence is not justified by the present data. The ratio of  $Q$  (Rayleigh) to  $Q$  (Love) as a function of period, gives  $Q_\alpha/Q_\beta$  if the elastic properties and  $Q_\beta$  are known as a function of depth. This relationship is shown in Fig. 5 for a homogeneous half-space, for a realistic Earth model with a constant  $Q_\beta$  with depth and for a realistic Earth model with the  $Q_\beta$  structure of model MM8. Also shown are experimental  $Q_R/Q_L$  ratios. The curve  $Q_\alpha = (\frac{9}{4}) Q_\beta$  corresponds to  $K^* = 0$  for the case where  $\lambda = \mu$  and is a satisfactory fit to the data.

Model MM8 gives an average  $Q_\beta$  of 144 for the upper 600 km of the Earth. This can be compared with the estimates of 185, 151 and 200 given by Anderson & Kovach (1964), and Kovach & Anderson (1964). Fedotov (1963) evaluated the absorption coefficient of 0.2 to 0.5 c/s shear waves in the Earth's crust and upper mantle in the vicinity of the southern Kurile Islands. From his values we obtain  $Q_\beta$  of roughly  $400 \pm 200$  for the crust,  $80 \pm 30$  at 70 km and  $130 \pm 60$  at 120 km. Model MM8 has values of 450, 80 to 100 and 100 to 150 at the corresponding depths. The frequencies utilized in the construction of MM8 were from 0.02 to 0.003 c/s, i.e. one to two orders of magnitude less than used by Fedotov. Although the surface wave determination refers to average world-wide conditions which are not necessarily the same as existing beneath the Kuriles, the agreement lends further support to the frequency independent  $Q$  hypothesis. More direct measurements of the frequency dependence of  $Q$  along a given ray would be quite valuable.

## 15. Interpretation

The interpretation of the presently available experimental determination of anelasticity in terms of damping mechanism or temperature is premature. This is partly due to the limited amount of data and the questions still outstanding regarding the ratio  $Q_\alpha/Q_\beta$ , the frequency dependence and a direct determination of lower mantle anelasticity. Sparse as this data is it is still more complete for the Earth itself than it is for appropriate rocks measured under appropriate conditions of high temperature and pressure.

Except possibly for the low-velocity zone the general trend of the variation of elastic properties as a function of depth in the crust and upper mantle, is controlled by variations in composition and phase—the effects of temperature and pressure are superimposed on the composition and phase control. In the lower mantle the general trend of elastic properties is consistent with the effects of increasing pressure. We do not know the effect of composition and phase on attenuation. If we ignore the known inhomogeneities we can investigate the effect of temperature and pressure on anelasticity by assuming an activation energy and an activation volume, taken as constant throughout the mantle. This implies that attenuation is due to an activated process and, therefore, depends exponentially on temperature and pressure and further, that a single loss mechanism dominates throughout the mantle. Changes in composition, phase and loss mechanism will result in different activation and pre-exponential parameters in the various regions of the Earth. It is not yet even clear if a thermally activated mechanism is indeed appropriate. Anderson & Archambeau (1964), assuming a high temperature activation mechanism, showed that a low  $Q$  upper mantle and a  $Q$  increasing rapidly with depth in the lower mantle, is a natural result of any process of this type with reasonable estimates of the parameters. In fact, there is little difficulty in fitting almost desired depth distribution with something as versatile as an exponential function. Fig. 6 shows the average  $Q$  in various regions of the Earth for various activation energies and volumes. The general trend is due to the dominant influence of temperature in the upper mantle and the dominance of pressure in the lower mantle. We can suppress the effect of these parameters simply by decreasing the activation energy or the activation volume.

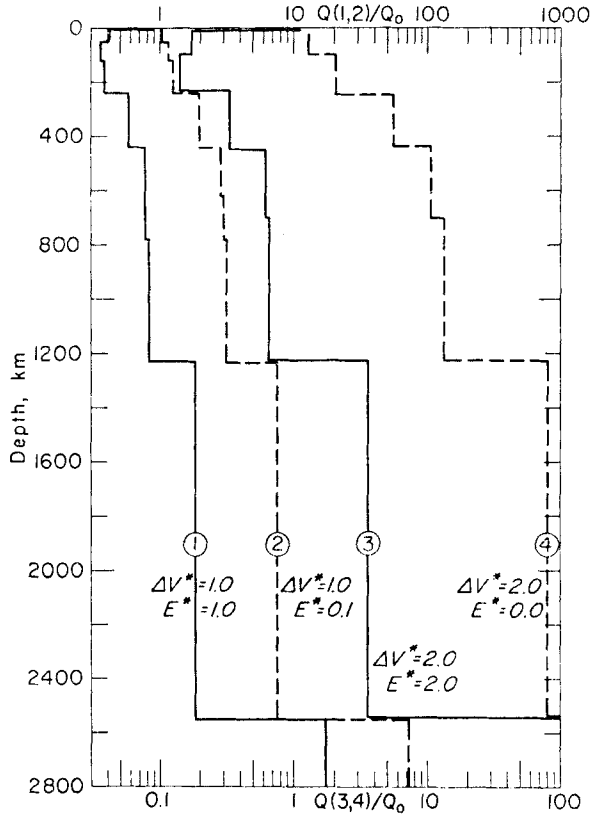


FIG. 6. The relative  $Q_\beta$  in various regions of the Earth assuming an activated process with various values for the activation energy,  $E^*$  (kcal/mole) and activation volume,  $\Delta V^*$  ( $\text{cm}^3/\text{mole}$ ) (after Anderson & Archambeau 1964).

Let us assume that the internal friction in the mantle is an activated process of the type found in metals, glasses and oxides at high temperature, i.e.

$$Q^{-1}(T) = Q_0^{-1} \exp -E^*/RT.$$

As discussed in previous sections, the effect of hydrostatic pressure is to increase the energy required in an elementary diffusion jump by  $P\Delta V^*$  where  $\Delta V^*$  is the volume increase associated with the motion, or activation volume. Then

$$Q^{-1}(T, P) = Q_0^{-1} \exp -E^*/RT \exp -\Delta V^* P/RT.$$

Table 3

Average  $Q_\beta$  in lower mantle versus  $Q$  (600 km) and activation volume,  $\Delta V^*$  in  $\text{cm}^3/\text{mole}$

$\Delta V^*$	$Q(600)$				
	100	200	300	400	500
1	443	885	1330	1770	2210
2	870	1740	2600	3480	4350
3	1310	2620	4220	5240	6550

In homogeneous regions of the Earth we can assume that  $Q_0^{-1}$ ,  $E^*$  and  $\Delta V^*$  are roughly constant. The magnitudes of these quantities depend on the details of the attenuation mechanism as discussed previously. The ratio of the attenuations at two levels in a homogeneous region can be written

$$\begin{aligned} \frac{Q_1^{-1}(T_1, P_1)}{Q_2^{-1}(T_2, P_2)} &= \exp \frac{E^*}{R} \left[ \frac{1}{T_1} - \frac{1}{T_2} \right] \exp \frac{\Delta V^*}{R} \left[ \frac{P_1}{T_1} - \frac{P_2}{T_2} \right] \\ &= \exp \frac{E^* + \Delta V^* P_1}{RT_1} \exp \frac{-E^* - \Delta V^* P_2}{RT_2} \end{aligned}$$

These expressions can be used to determine the average  $Q$  in homogeneous regions of the Earth. For example, from the ScS data we know the average  $Q_\beta$  in the lower mantle below 600 km. This is also probably the most homogeneous part of the mantle. We also know from surface wave measurements that the  $Q_\beta$  at 600 km probably lies between 200 and 400. This information is sufficient to estimate the activation volume of the lower mantle if the attenuation is due to an activated process, even if we do not know the details of the mechanism.

Table 3 gives the average  $Q_\beta$  in the lower mantle as a function of the  $Q_\beta$  at 600 km and the activation volume  $\Delta V^*$  in  $\text{cm}^3/\text{mole}$ . Since the average  $Q_\beta$  of the lower mantle is probably between 1400 and 2200, we see that an activation volume greater than about  $2 \text{ cm}^3/\text{mole}$  seems to be ruled out. If the  $Q$  at 600 km is 200 the average  $Q$  of the lower mantle is 1740 and the  $Q$  at the base of the mantle is  $1.25 \times 10^6$  for an activation volume of  $2 \text{ cm}^3/\text{mole}$ . Even higher values are obtained at the base of the mantle

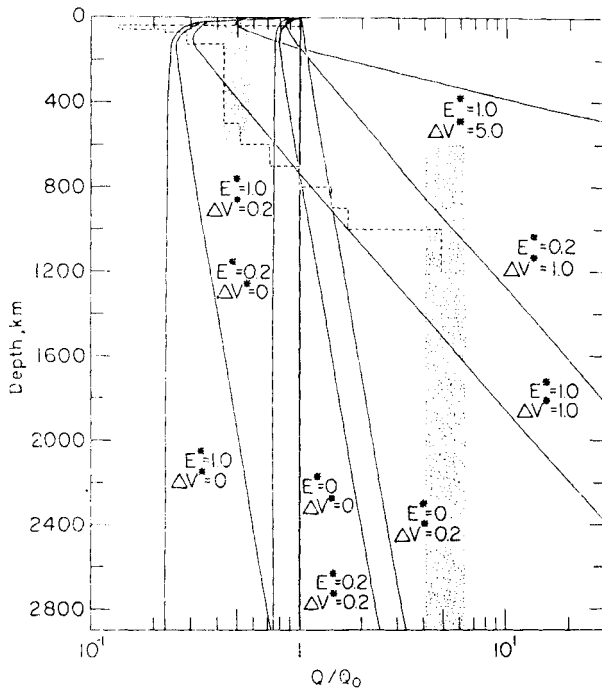


FIG. 7. Experimental and theoretical variation of  $Q_\beta$  in the mantle normalized to a surface  $Q_\beta = Q_0$  of 450. Stippled regions represent average values in the upper and lower mantle (Anderson & Kovach 1964, Kovach & Anderson 1964). Dashed curve is surface wave model MM8 (Anderson *et al.* 1965). Temperatures after Gutenberg (1959).

for larger activation volumes. Although data is sparse there have been no suggestions that the lower mantle  $Q$  is this high. Therefore, the  $Q$  at 600 km is probably greater than 200 and the activation volume is probably less than  $2\text{ cm}^3/\text{mole}$ .

Fig. 7 shows the ratio  $Q/Q_0$  as a function of depth for several values of the activation energy and activation volumes. Gutenberg's estimates of temperature have been used. The surface wave and ScS data are also shown, normalized to a surface value  $Q_\beta = 450$ .

If the activation energy and the intrinsic  $Q$  does not change across the crust-mantle interface, an activation energy of about 1–2 kcal/mole gives reasonable results for the low value of  $Q$  in the upper mantle. An activation volume of about  $2\text{ cm}^3/\text{mole}$  brings the  $Q$  up to the required values in the lower mantle. The molecular volume of the materials in the mantle is probably of the order of 7–11  $\text{cm}^3$  per oxygen atom; 5–7  $\text{cm}^3/\text{mole}$  is a representative range for the mean volume per mole at atmospheric pressure. The volume of the metallic ions is typically of the order of one-tenth the total volume of an oxide, roughly the order of the inferred activation volume. With these assumptions, the stress induced reordering of metallic interstitials or substitutional ions possibly at grain boundaries is consistent with the inferred variation of  $Q$  with depth in the lower mantle.

### Acknowledgments

This research was supported by the Advanced Research Projects Agency and was monitored by the Air Force Office of Scientific Research under contract AF-49(638)-1337.

The author wishes to also thank the Sloan Foundation for its generous support during the time that this manuscript was being prepared.

*California Institute of Technology,  
Seismological Laboratory,  
Pasadena,  
California.*

1966 June.

### References

- Anderson, Don L., 1967. *The Earth's Mantle*, Chapter III. Academic Press, New York and London.
- Anderson, Don L. & Archambeau, C. B., 1964. *J. geophys. Res.*, **69**, 2071.
- Anderson, Don L., Ben-Menahem, A. & Archambeau, C. B., 1965. *J. geophys. Res.*, **70**, 1441.
- Anderson, Don L. & Kovach, R. L., 1964. *Proc. natn. Acad. Sci.*, **51**, 168.
- Asada, T. & Takano, K., 1963. *J. Phys. Earth*, **11**, 25.
- Berry, B. S., 1962. *Acta metall.*, **10**, 271.
- Brommel, H. E., Mason, W. P. & Warner, A. W., 1955. *Phys. Rev.*, **99**, 1894.
- Chang, R., 1961. *J. appl. Phys.*, **32**, 1127.
- Chung, D. H., 1961. *State University of New York, College of Ceramics Report No. 297*. Alfred, N.Y.
- Entwhistle, K. M., 1962. *Metall. Rev.*, **7**, 175.
- Fedotov, S. A., 1963. *Izv. Akad. Nauk SSSR, Ser. Geofiz.*, 509.
- Futterman, W. I., 1962. *J. geophys. Res.*, **67**, 5279.
- Gutenberg, B., 1958. *Bull. seismol. Soc. Am.*, **48**, 269.
- Hopkins, I. L. & Kurkjian, C. R., 1965. *Physical Acoustics* (ed. W. P. Mason), II B Academic Press, New York and London.

- Ké, T. S., 1950. *J. appl. Phys.*, **21**, 414.
- Knopoff, L. & MacDonald, G. J. F., 1958. *Rev. mod. Phys.*, **30**, 1178.
- Knopoff, L. & MacDonald, G. J. F., 1960. *J. geophys. Res.*, **65**, 2191.
- Kovach, R. L. & Anderson, Don L., 1964. *Bull. seismol. Soc. Am.*, **54**, 1855.
- Kuroiwa, D., 1964. *Contributions from the Institute of Low Temperature Science*, **A**, (18), 1.
- Lamb, G. L., Jr, 1962. *J. geophys. Res.*, **67**, 5273.
- LeCraw, R. C. & Comstock, R. L., 1965. *Physical Acoustics* (ed. W. P. Mason), III B. Academic Press, New York and London.
- Marx, J. W. & Sivertsen, J. M., 1953. *J. appl. Phys.*, **24**, 81.
- Mason, W. P., Jr, 1955. *J. acoust. Soc. Am.*, **27**, 643.
- Mason, W. P., 1958. *Physical Acoustics and The Properties of Solids*. Van Nostrand, Princeton, N.J.
- Mason, W. P., 1965. *Physical Acoustics* (ed. W. P. Mason), III B. Academic Press, New York and London.
- Mizutani, H. & Kanamori, H., 1964. *J. Phys. Earth*, **12**, 43.
- Paul, W. & Warschauer, D. M., 1963. *Solids Under Pressure*. McGraw-Hill, New York.
- Press, F., 1956. *Science*, **124**, 1204.
- Press, F., 1964. *J. geophys. Res.*, **69**, 4417.
- Savage, J., 1965. *J. geophys. Res.*, **70**, 3935.
- Savage, J., 1966. *J. geophys. Res.*, in press.
- Stanford, E. G. & Fearon, J. H., 1966. *Progress in Non-Destruction Testing*, Series. MacMillan, New York.
- Van Bueren, H. G., 1960. *Imperfections in Crystals*. North Holland, Amsterdam.
- Wachtman, J. G., 1957. U.S. Air Force, Wright Air Development Center, Report WADS-TR-57-526.
- Wadati, K. & Hirona, T., 1956. *Geophys. Mag.*, **27**, 1.
- Zener, C., 1948. *Elasticity and Anelasticity of Metals*. University of Chicago Press.

### Discussion

In the analysis of seismic attenuation data the assumption that the intrinsic damping of the material traversed is frequency independent has been made. Laboratory data on rocks taken at room temperature and pressure do show a large, frequency independent damping. However, this damping originates in the sliding of unbonded interfaces in the rock and is suppressed by a pressure of a few kilobars (Birch & Bancroft 1938). It is an important source of seismic attenuation only in the upper 50 km or so of the Earth. The relevant laboratory data for understanding the damping of the mantle are experiments at high temperature on sintered oxides and observations on the damping in granular solids containing interstitial fluids. In the solid, polycrystalline oxide material the principal source of internal friction is viscous grain boundary damping (Nelson & Gordon 1966) which, to a close approximation, shows the anelastic behaviour of a standard linear solid and is therefore strongly frequency dependent. In any region of the mantle which may be partially melted the stress induced flow of melt through intergranular channels is expected to be an important source of damping. This source of internal friction is also characteristically frequency dependent (Biot 1956). In the determination of a damping profile for the mantle from seismic observations, then, it is very important to assess the dependence of the results on the assumption of frequency independence of the intrinsic internal friction.

If physically meaningful activation volumes are to be deduced from seismic attenuation data it is essential that the measured activation volume be defined in

such a way that it can be compared with the actual activation volumes of thermally activated processes occurring in the material traversed by the seismic wave. In general,  $V^* = (\partial G^* / \partial P)_T$  where  $G^*$  is the activation free energy.  $V^*$  may be measured directly for thermally activated atom movements in crystals by means of diffusion or internal friction experiments carried out at high pressure. To relate  $V^*$  determined by laboratory measurements, or by theory, to seismic attenuation requires establishment of a specific relation between  $Q^{-1}$  and the thermally activated process in question. In the case of grain boundary damping, for example,  $V^*$  can be related to  $Q^{-1}$  through the equations for the internal friction of a standard linear solid. To assume an arbitrary relation between  $Q$  and  $V^*$  not related to any specific damping mechanism,

$$Q^{-1}(T, P) = Q_0^{-1} \exp [E^*(T_0 - T) / kTT_0] \exp (PV^* / kT)$$

for example, will not yield physically meaningful values of  $V^*$  in the sense that they can be compared to the activation volumes of any atomic processes occurring in the material of the mantle.

### References

- Biot, M. A., 1956. *J. acoust. Soc. Am.*, **28**, 168.  
 Birch, F. & Bancroft, D., 1938. *J. Geol.*, **46**, 59.  
 Nelson, C. W. & Gordon, R. B., 1966. *Anelastic Properties of the Earth*, ONR Technical Report Contract 609 (54). Yale University, New Haven, Connecticut.

R. B. GORDON