

## THE APPARENT ATTENUATION OF A SCATTERING MEDIUM

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### ABSTRACT

We report the results of some numerical experiments that bring out the low-pass characteristics of a purely elastic medium with a heterogeneous velocity structure. Although the typical fluctuation is spatially confined within less than a wavelength, waves propagating over a sufficiently long path suffer major cumulative effects. We summarize the removal of high frequencies during transmission by a frequency-independent apparent  $Q$ , and show that attenuation by intrinsic friction and scattering are approximately additive. We propose some diagnostics that might help to distinguish the presence of velocity fluctuations and resultant scattering from the presence of anelasticity and true dissipation. When scattering dominates over intrinsic friction: (1) the coda of a transmitted wave contains relatively higher frequencies than the initial pulse; (2) the attenuation deduced from the power spectrum of the transmitted wave is greater than that deduced from the phase spectrum; (3) compressional and shear wave apparent  $Q$ 's are approximately equal; and (4) estimates of apparent  $Q$  made from reflected coda vary with frequency, while estimates made from the transmitted waves do not. We also outline several topics in the theory of wave propagation that will be relevant in a satisfactory interpretation of short-period observations, if the amplitude of such signals is affected by scattering.

### INTRODUCTION

It is known that the  $P$ -wave velocity in the Earth's crust has fluctuations on all spatial scales. An example is shown as the left-hand curve of Figure 1, and on the right of this figure is shown an idealization that approximates the measured velocity profile. The approximation has over 40 layers, and hence is far more complicated than most of the crustal models customarily used to generate synthetics. But obviously, the approximation does not begin to approach the detailed complexity of the real crust, even if we assume plane layering.

This paper addresses directly the question of what happens to a body wave when it is transmitted vertically through layering as complex as that suggested by well logs. Note that if we take just the product of transmission coefficients, we would be multiplying hundreds of numbers together, all of them less than one (presuming we work with energy flux), so that the true direct wave (the first arrival) has negligible amplitude. Instead, one must recognize that the problem concerns forward scattering, and it is the combination of multiples that forms the apparent transmitted wave.

This point was well made, and examined in some detail, by O'Doherty and Anstey (1971). From talks given at national meetings of the Society of Exploration Geophysicists in recent years, and from many relevant papers in the Society's journal, *Geophysics*, it is apparent that an understanding of scattering and attenuation losses has high priority in connection with the processing and interpretation of multi-channel reflection data. The literature is now extensive.

As O'Doherty and Anstey (1971) pointed out, the type of velocity fluctuation most effective in removing high frequencies from the earliest portion of a transmitted wave train is the fluctuation in which high- and low-velocity layers alternate. In the

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following section, we describe several numerical experiments, some of which emphasize a statistical distribution of velocities with layers of fixed vertical travel time, and some fixed values of velocity but a statistical distribution of layer thicknesses. In general, we look at the changing shape of a pulse that initially was just a single spike, as it propagates over increasing path length. We point out some possibilities for discriminating between scattering and dissipation, and conclude the paper with a review of some of the implications not so much for exploration geophysics (where the apparent attenuation caused by scattering is under intensive study) as for other aspects of seismology.

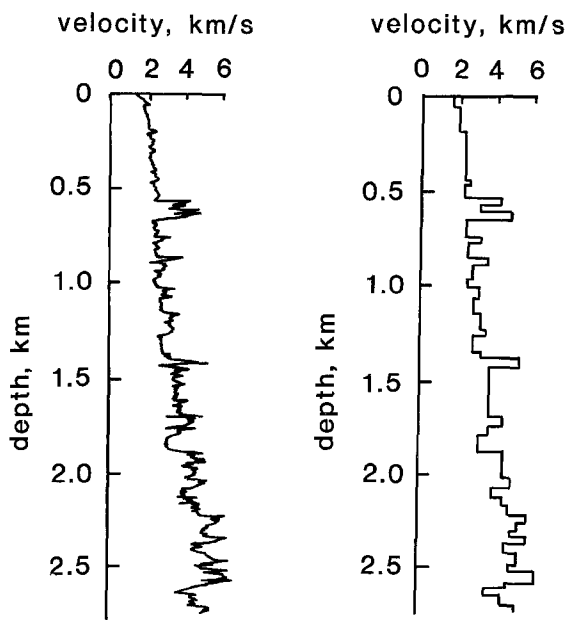


FIG. 1. (Left) Typical acoustic velocity log from borehole. (Right) Idealization of log using about 40 plane layers.

### NUMERICAL EXPERIMENTS

*Propagation through different thicknesses of gaussian random media.* We work here with a medium having layers of equal vertical travel time, the velocity in each layer being chosen from a gaussian distribution. An example of such a velocity profile, with mean 3.0 km/sec and standard deviation  $\frac{1}{4}$ , is shown in Figure 2. Starting out as a vertically propagated single spike, a transmitted wave train is shown in Figure 3 after crossing 100, 200, 300, 400, and finally 500 layers. Prominent features are the significant coda and the pulse-broadening that increases with propagation distance. At greater distances, there is also a diminishing amplitude of motion associated with the very first point of each wave train. It is this amplitude that is derived simply by forming the product of transmission coefficients for all interfaces crossed. After 500 have been crossed, the main feature apparent to the eye has a maximum about two (travel time) units after the first arrival. It is, therefore, composed of constructive multiples. The delay itself—two units, with respect to the first arrival—after a total travel time of 500 units, is also of interest.

The filtering effects are brought out in Figure 4 (for passage across 100 layers) and Figure 5 (after 400 layers). Note the overall fall-off with higher frequency, for

amplitude spectra of the early portion of the wave trains. This fall-off is greater for the longer propagation path (compare the left-hand spectra of Figures 4 and 5). Because this is an elastic medium, the higher frequencies are not lost: some are back scattered, and some forward scattered to show up later in the coda. Figures 4

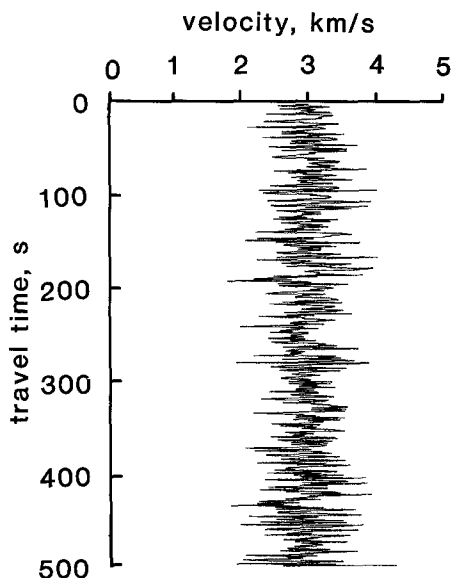


FIG. 2. Synthetic acoustic velocity log containing 500 layers of equal travel time. Velocities are gaussian distributed with mean 3.0 km/sec and standard deviation 0.25. (In all our experiments, density was always given the same value, so velocity is proportional to impedance.)

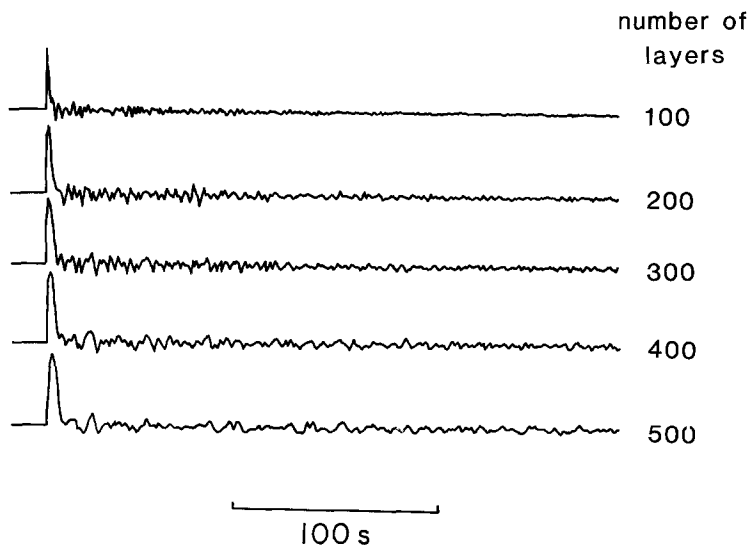


FIG. 3. Transmission response of a medium similar to that shown in Figure 2. (In fact, the mean velocity here is 3.0 km/sec, but the standard deviation is 0.5.) The incident wave is a single spike. Note the significant coda and pulse-broadening. Individual traces have been normalized to unit amplitude.

and 5 do show relative enrichment of high frequencies for the coda as compared to the main arrival. This is the first indicator, suggested by our numerical results, to discriminate between dissipation and scattering. The phenomenon is apparent in Figure 6, a short-period seismogram recorded digitally in the Caribbean.

For another realization of this same gaussian random medium, we show in Figure 7 the transmitted wave train after it has crossed 200, 400, and finally 800 layers. Also shown is the result of convolving the wave train, after passing 200 layers, with itself. The outcome, labeled 200\*200, is very similar to the wave train after 400

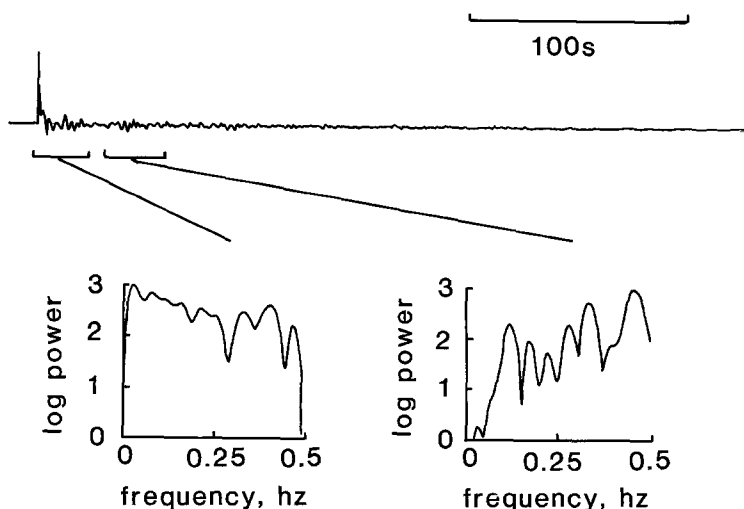


FIG. 4. (Top) Transmission response of upper 100 layers of log used in Figure 3. (Bottom) Power spectra of initial portion of pulse and later coda. Note loss of higher frequencies from initial pulse.

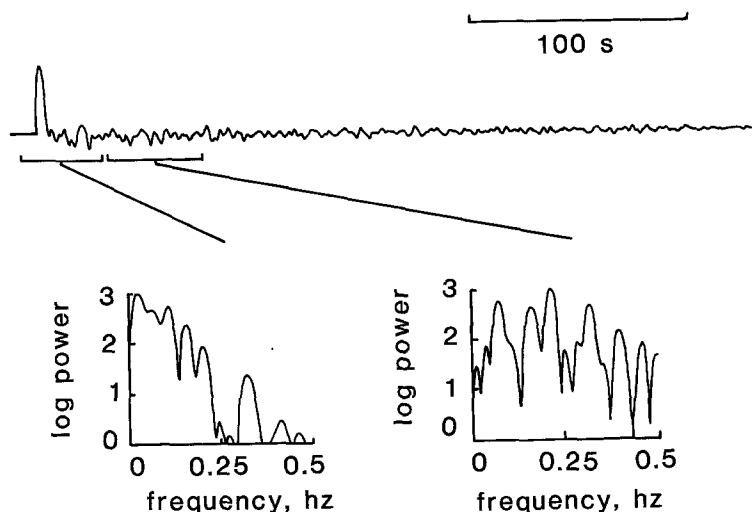


FIG. 5. (Top) Transmission response of upper 400 layers of log used in Figure 3. (Bottom) Power spectra of initial portion of pulse and later coda.

layers. There are some minor differences late in the coda, due probably to the neglect, in 200\*200, of multiples formed between the first and last groups of 200 layers. Convoluting 200\*200 again with itself, the outcome is seen to be similar to the wave train after 800 layers.

Stating these approximate results in the frequency domain, so that convolutions become products, the spectrum at distance  $nx$  is roughly equal to the  $n$ th power of the spectrum at distance  $x$ . If this result is exactly true, for all  $n$ , the spectrum

must be given by  $\exp(\lambda x)$  for some  $\lambda$ . Measuring  $x$  in units of wavelength, and recognizing that the quantity  $\lambda$  must be negative (amplitude decays with distance), we are naturally led to compare the spectrum with approximations of the form  $\exp[-\omega x/2\bar{v}Q]$ . Here,  $\bar{v}$  is the mean velocity of the medium, and " $Q$ " is the

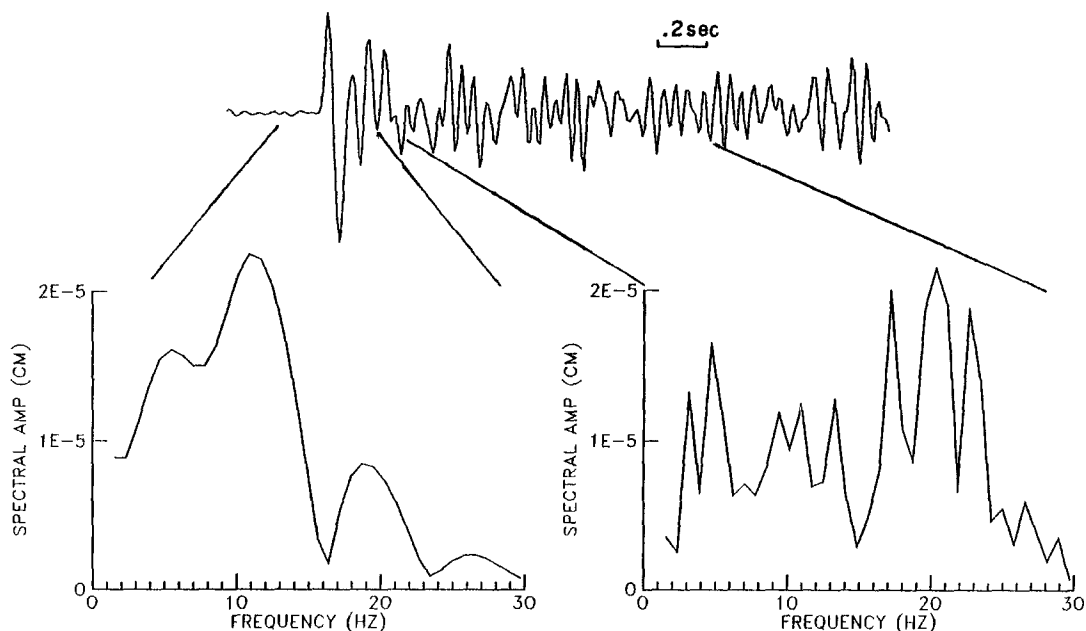


FIG. 6. (Top) Seismogram of magnitude 1.9 microearthquake in the Anagada Passage, observed with a 2-Hz geophone in St. John, Virgin Islands. The event has focal depth of about 20 km and was recorded at a range of about 30 km. (Bottom) Spectra of the velocity of ground motion; for the initial portion of the seismogram and separately for a portion of the coda. Note the relative enrichment of the coda in higher frequencies (figure taken from Frankel, 1982).

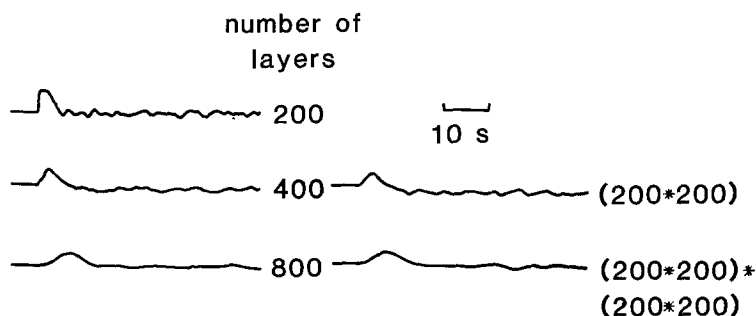


FIG. 7. (Left) Transmission response of stacks of 200, 400, and 800 gaussian random layers with mean velocity 3.0 km/sec and standard deviation 0.75. (Right) Approximations to response of 400 and 800 layers obtained by convolving the 200-layer response with itself.

apparent  $Q$  in the sense that a homogeneous anelastic medium with phase velocity  $\bar{v}$  would lead to the spectral decay  $\exp[-\omega x/2\bar{v}Q]$ .

To assess the apparent attenuation ( $1/Q$ ) for our elastic gaussian random medium, we computed the transmitted wave train across up to 900 layers, in a medium with mean velocity 3 km/sec and standard deviation 0.25. The resulting pulse can be seen, in Figure 8, losing amplitude and broadening as it crosses more

and more layers. Looking at the power spectra of such pulses (Figure 9), a straight line corresponds to an apparent  $Q$  that is constant for different distances  $x$  and frequencies  $\omega$ . We computed many pulses and spectra such as those of Figures 8 and 9, fitting the spectra with a straight line and attempting to see how the apparent  $Q$  changed with differing degrees of inhomogeneity in the elastic medium. The results can be summarized in terms of the mean square of the reflection coefficients at the interfaces,  $\sigma_c^2$  (say). In experiments where  $\sigma_c$  varied from 0.02 to 0.20, we found

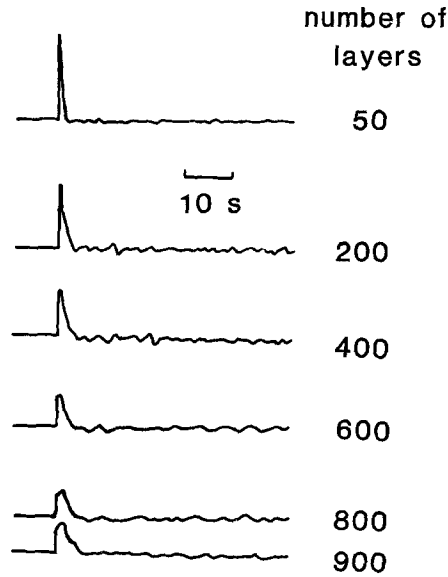


FIG. 8. Transmission response of stacks of gaussian random layers with mean velocity 3.0 km/sec and standard deviation 0.25.

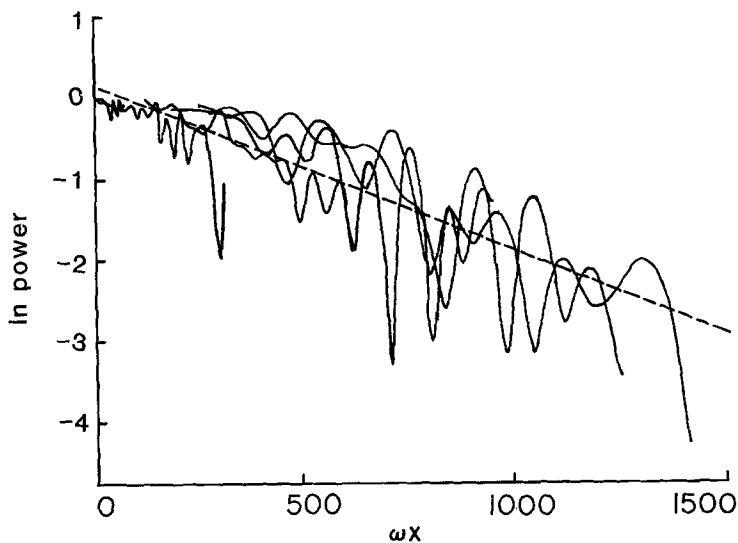


FIG. 9. Power spectra (solid lines) of traces in Figure 8 plotted against the product of frequency and propagation distance. Spectra fall about straight line (dashed) predicted by power  $\propto \exp[-\omega x/\bar{v}''Q]$ .

roughly that the apparent attenuation was

$$\frac{1}{\text{"}Q\text{"}} \approx 0.0005 \bar{v} \sigma_c^2$$

(where  $\bar{v}$  is expressed in kilometers/second).

If a pulse is propagating merely in a homogeneous anelastic medium with constant  $Q$ , it is possible to estimate the value of  $Q$  either from the amplitude or from the phase spectrum. We tried to estimate the apparent  $Q$  from the amplitude and phase spectra for the pulses shown in Figure 10 (the heterogeneity of the medium underlying this figure is greater than that for Figure 8), again using an elastic random medium with layers of equal time, and velocities (impedances) drawn from

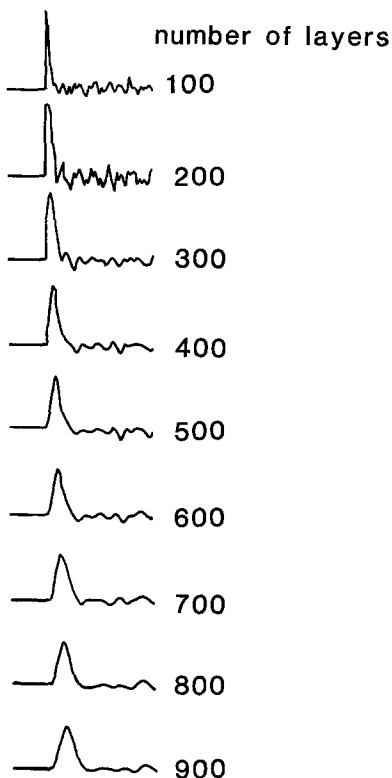


FIG. 10. Transmission response of stacks of gaussian random layers with mean velocity 3.0 km/sec and standard deviation 0.5.

a gaussian distribution. These pulses are obviously losing high frequencies, and fitting  $\exp[-\omega x/\bar{v}\text{"}Q\text{"}]$  to the power spectrum of Figure 11, the estimated apparent  $Q$  is roughly 40. To make an estimate from the phase spectrum of pulses shown in Figure 10, we assume that the spectrum is given by  $\exp(i\omega x/\bar{v})$  in which the phase velocity,  $\bar{v}$ , is weakly dependent on frequency as

$$\frac{\bar{v}}{\bar{v}_0} = 1 + \frac{1}{\pi \text{"}Q\text{"}} \ln \left( \frac{\omega}{\omega_0} \right).$$

This is the dispersion law known to be valid for causal propagation within an anelastic constant  $Q$  medium (see, e.g., Aki and Richards, 1980, chapter 5:  $\bar{v}_0$  is the

phase velocity at reference frequency  $\omega_0$ ). Fitting this logarithmic law to the pulses of Figure 10, we conclude that the apparent  $Q$  is roughly 700 (see Figure 12). This relatively high value is probably due to the pulse retaining a fairly symmetric shape as it broadens (see Figure 10).

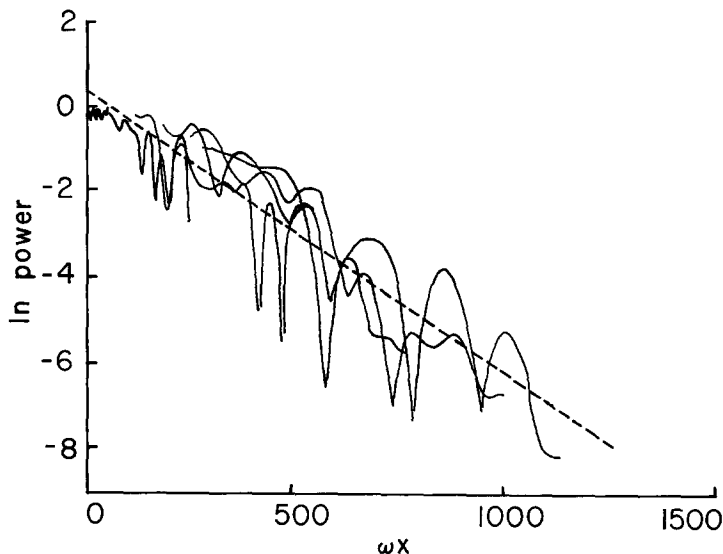


FIG. 11. Power spectra (solid lines) of traces in Figure 10 plotted against the product of frequency and propagation distance. Note the faster fall-off of power with  $\omega x$  of pulses in this strongly scattering medium, as compared to Figure 9.

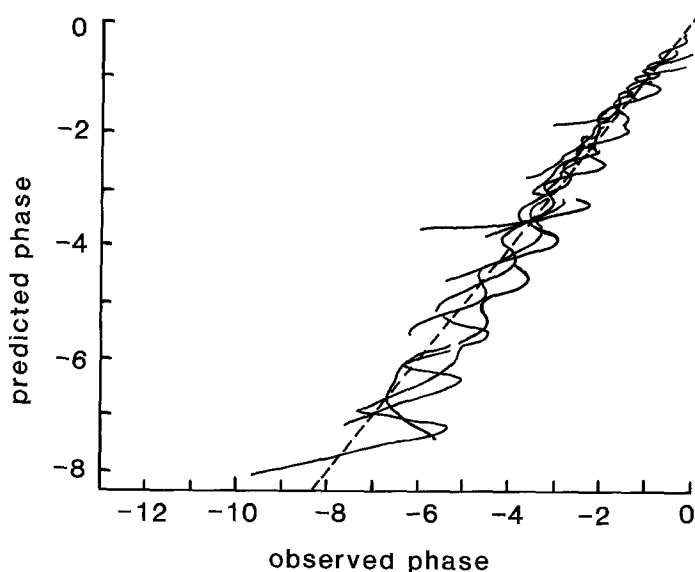


FIG. 12. " $Q$ " is estimated from the phase spectra by first computing the phase of pulses in Figure 10 and then subtracting the major ramp component,  $\omega x/v_0$ . This leaves a residual phase,  $\Delta\phi$ , for which we find the constants  $a$  and  $b$  in a least-squares fit of the form  $\Delta\phi$  (predicted) =  $a\omega x - b\omega \times \ln(\omega x)$ . The constant  $a$  gives a slight correction to the ramp. The constant  $b$ , associated with logarithmic dispersion, is  $(\bar{v}_0\pi"Q")^{-1}$  if we insist on interpreting the phase spectrum in terms of causal wave propagation in an anelastic medium. From our best fit, we infer " $Q$ " in this experiment to be about 700. This relatively high value (compared to that derived from the amplitude spectrum) may be due to its derivation from pulse shapes that are broadened in a very symmetric way. The figure shows our best fit for the predicted residual phase versus the residual phase itself. That the fit is quite good may be seen from the clustering of the data (solid lines) around a line of unit slope through the origin (dashed line).

Thus, we have the preliminary result that, for a random medium, the apparent  $Q$  derived from loss of high frequencies in the initial pulse shape is much less than the apparent  $Q$  derived from pulse-broadening and the phase spectrum. This may be a second indicator to distinguish between losses due to dissipation and losses due to scattering.

A third indicator might be derived from a comparison between  $P$  and  $S$  waves. For vertically traveling waves in the media we have considered so far, the apparent  $Q$  would be about the same for  $P$  and  $S$ . (In fact, it is easy to show the apparent  $Q$ 's are exactly the same if the ratio of  $P$ - and  $S$ -wave velocities is everywhere the same, i.e., if Poisson's ratio is constant.) But of course, the  $Q$  due to intrinsic friction is typically much higher for compressional waves than for shear waves.

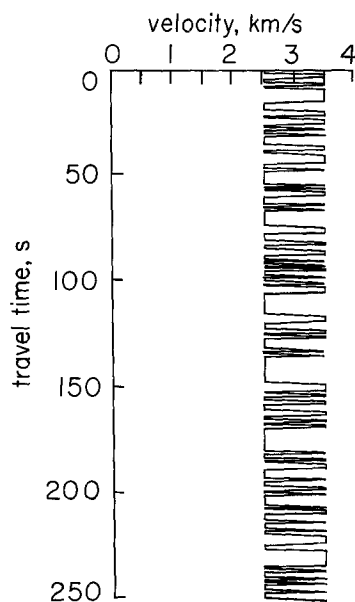


FIG. 13. Acoustic velocity log built from alternating layers of two different media (e.g., sandstone and shale) with thickness distributed as a Poisson process. Each layer has been approximated by an integral number of sublayers of equal travel time.

*Propagation through media of alternating velocities with varying layer thicknesses.* Rather than a gaussian distribution of velocities, in layers of equal travel time, one can choose to emphasize varying layer thicknesses. In practice, we usually used alternating layers with thickness determined first from a Poisson distribution and then rounded up to an integral multiple of the basic unit of vertical travel time. An example of such a velocity profile is shown in Figure 13, and the transmitted wave at various distances for this profile is given in Figure 14. The phenomena of high-frequency removal and pulse-broadening are again apparent.

It is of interest to allow for intrinsic friction in each type of layer and then to study the scattering from many layers. It is particularly simple to do the relevant computations for the case we are considering here (i.e., with a limited number of layer types and integral thicknesses) because the propagator matrix is built up from a limited number of basic layer matrices, and these can be computed just once and then stored. The outcome of one such numerical experiment is shown in Figure 15, based on a medium with 100 lamellae, and it indicates the weakness of one of the

diagnostics proposed above to distinguish between scattering and dissipation. Thus, the upper trace was computed with no intrinsic attenuation, leading to high frequencies later in the coda. But when true dissipation (with  $Q = 100$ ) was added in each layer, the coda high frequencies were effectively removed (Figure 15, *lower trace*) although pulse broadening, for the main arrival, was about the same in each case. Further experiments to investigate the joint effect of scattering and intrinsic friction are summarized in Figures 16 and 17. A practical conclusion is that the total attenuative effect is usually given adequately by simple addition of the two contributing attenuations.

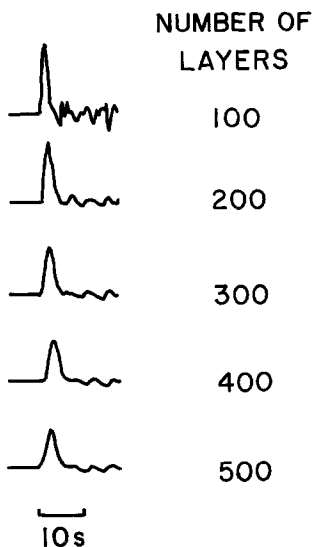


FIG. 14. Transmission response of medium with Poisson distributions of thickness ( $\alpha = 2.5$  and  $3.5$  km/sec; mean thickness =  $1.0$  km).

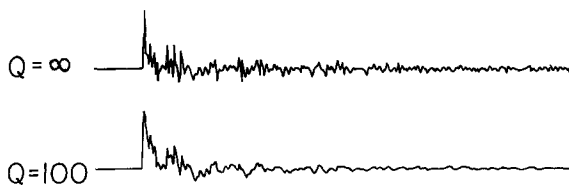


FIG. 15. Transmission response of 500 layers with a Poisson distribution of thicknesses when the layers are completely elastic ( $Q = \infty$ ) and when intrinsic  $Q = 100$ . Note the loss of high frequencies in the coda of the wave that has propagated through the anelastic medium.

For a given number of alternating layers, pulse-broadening and coda formation is more developed when the layer types are present in roughly equal amounts, as compared to the limited effects when one of the layer types is present only in very thin layers. This claim is quantified in Figure 18, showing the transmitted wave trains for media built up in each case from a pair of layers (see Figure 18, *inset*). Each trace is computed for a medium in which the doublet layer, repeated 60 times, has a particular percentage of slow layer thickness. We see that if the slow layer is very thin, say only 5 per cent of the total, the transmitted wave is dominated by a single pulse. The same holds when the slow layer is very thick, say 95 per cent. But in between, with layers of comparable thickness or travel time, the pulse indeed is effectively broadened, and the coda is significant and complex.

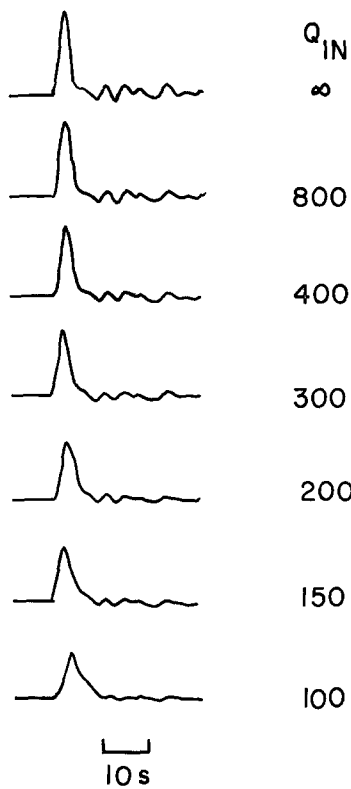


FIG. 16. Transmitted wave trains at a fixed distance, in media of varying intrinsic friction. The *first trace* is the result of an elastic medium with Poisson-distributed layer thicknesses, after crossing 500 layers. The apparent  $Q$  (derived from an amplitude spectrum) is about 74. Subsequent traces show the wave train as the medium is made more anelastic in steps. Note that the familiar asymmetry of pulse shape, associated with propagation in a causal anelastic medium, can be seen when intrinsic  $Q(Q_{IN})$  is low.

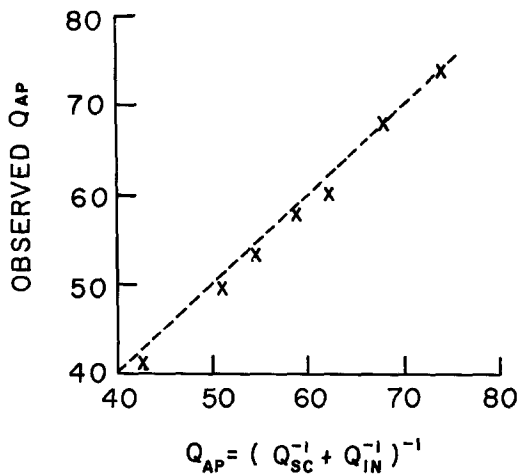


FIG. 17. Shown here is a test of the hypothesis that the combined effect of scattering and intrinsic friction is summarized by

$$\frac{1}{Q_{TOTAL}} = \frac{1}{Q_{SCATTERING}} + \frac{1}{Q_{INTRINSIC}}.$$

From amplitude spectra of the main pulses shown in Figure 16, we found an apparent (i.e., total)  $Q$ . The intrinsic  $Q$  is known as an input for the various anelastic media underlying Figure 16, and the attenuation due to scattering was presumed to be given by its value in the elastic case. This figure shows that the summation formula works fairly well, except for some bias when attenuation is very high.

*Reflected waves.* The waves that are backscattered out of a stack of layers have often been thought of in terms of two distinct components: the primary reflections (waves that have been transmitted downward to a particular reflector and then retransmitted back up), and the multiply scattered waves. But in the randomly fluctuating models we have discussed above, primary reflections are typically negligible because they are proportional to a long product of transmission coefficients. One would like to generalize the idea of a primary reflection to include all the energy that has traveled in the leading pulse of a wave during transmission and has undergone one additional and identifiable reflection. But it is difficult to quantify this idea rigorously, since the multiple scattering inherent in the transmission process interacts in too complex a way for a single "reflection" to be singled out. Nevertheless, one can expect that the frequency content of the reflections should evolve toward lower frequencies at later times, since the transmitted waves exhibit pulse-broadening with increasing distance of propagation. Numerical calculations indicate that this frequency shift does in fact occur.

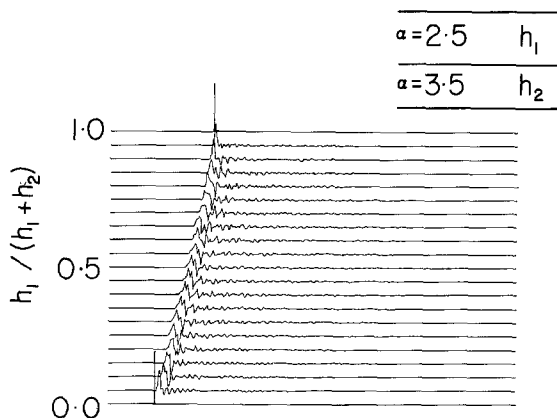


FIG. 18. Transmission response of 120 layers with alternating velocities and thicknesses (see *inset*), as a function of the relative thickness of the slower layer. The layers have equal travel time when the ratio equals 0.41. Note that pulse-broadening occurs to some degree for a wide range of relative thicknesses.

This effect has implications for interpreting estimates of  $Q$  made by the coda wave technique (Aki and Chouet, 1975; Aki, 1981). This technique, originally developed for very weakly scattering media, ascribes the coda of locally recorded earthquakes to backscattered waves that have undergone loss by anelastic attenuation during transmission and one reflection. The reflected waves (or coda) initiated from a point source then have a power spectrum  $P$  that evolves with time as

$$P(\omega | t) = S(\omega)t^{-2}\exp\{-\omega t/Q\}.$$

The function  $S(\omega)$  depends upon the spectrum of the earthquake and any frequency dependence of the single scattering processes. During transmission between source and scatterer and scatterer and receiver, the waves undergo geometrical spreading (the term  $t^{-2}$ ) and anelastic attenuation (the exponential). Intrinsic  $Q$  is estimated by fitting this formula to observed earthquake coda.

As long as the medium is anelastic and is in fact so weakly scattering that only the primary reflections are important, this method gives reliable estimates of  $Q$ . However, if the heterogeneity is sufficiently strong that multiple scattering is important and causes attenuation, these estimates of anelastic  $Q$  may be biased. For

example, in Figure 19 we give the exact reflection response of a stack of random elastic layers with different degrees of heterogeneity. We then compare them with the response predicted by the single backscattering model (Figure 20), obtained by summing the primary reflections from each interface. Note that the coda of the

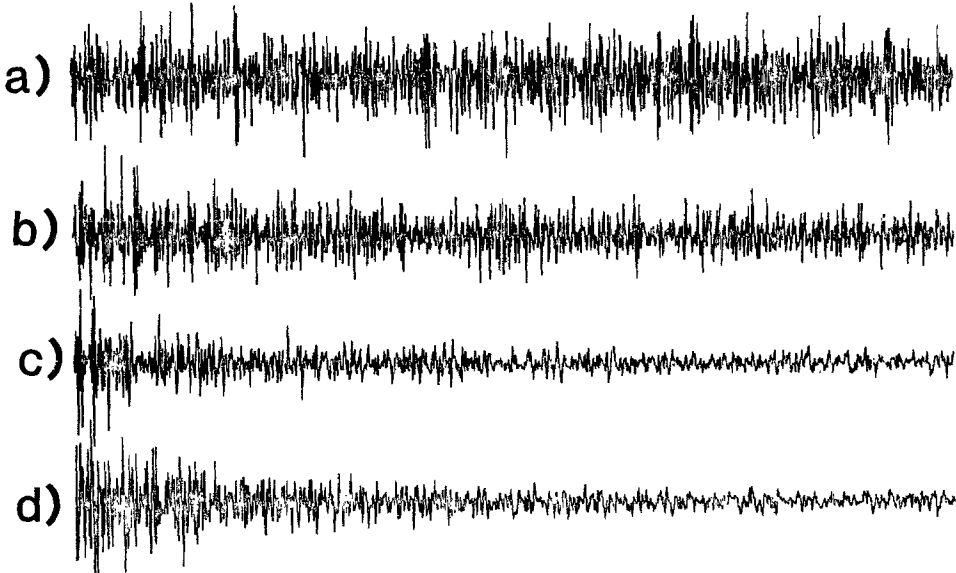


FIG. 19. Reflection response of a stack of gaussian random layers with mean velocity 3.0 and standard deviation (a) 0.01, (b) 0.05, (c) 0.1, and (d) 0.15.

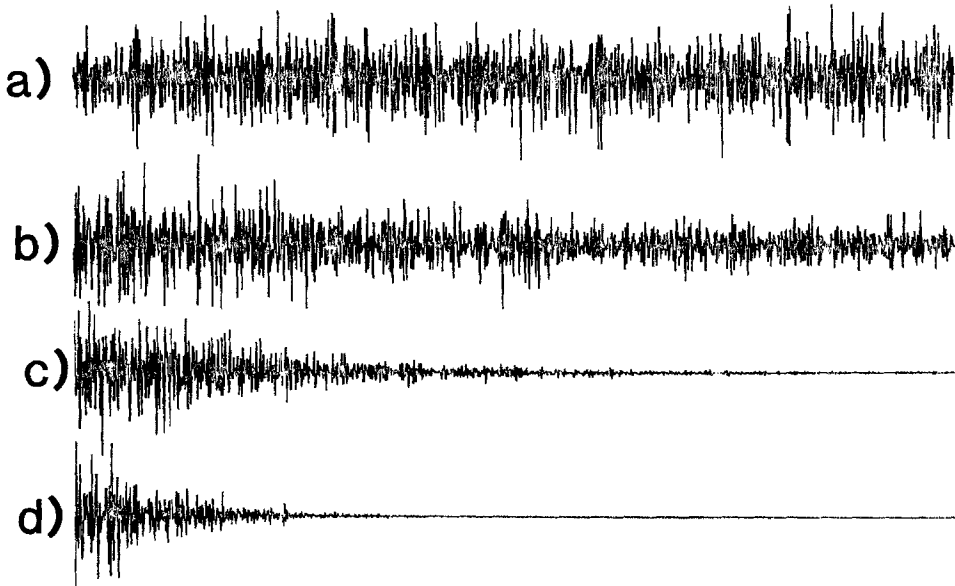


FIG. 20. Approximate reflection response based on the single scattering approximation. Compare with Figure 19. Note the faster fall-off of amplitude in the coda.

exact response is more energetic and of lower frequency than that of the approximate response. There is an evolution of frequency content with time even though intrinsic  $Q = \infty$ .

While some regions are evidently sufficiently weakly heterogeneous for the single

scattering approximation to give valid results (Aki and Chouet, 1975), some regions have been shown to violate this condition. For instance, Gir Subhash (1979) has estimated that 2-Hz waves propagating through the Jura, West Germany, region lose energy at a rate of 35 per cent/sec, which violates the weak scattering approximation by a considerable margin.

Some authors (Gir Subhash, 1979; Dainty, 1981) have sought to interpret estimates of  $Q$  made with the coda wave technique as an apparent or total  $Q$  representing attenuation by both anelasticity and scattering. For such an interpretation to be meaningful, this estimate of  $Q$  must be equal to estimates made from transmitted waves. (The above coda wave formulas do, after all, ascribe the attenuation to processes that occur during transmission.) However, these estimates will be equal only to the extent that the notion of a "generalized primary wave" correctly describes the bulk of backscattered energy. We feel, however, that this notion is fundamentally inconsistent, since it applies multiple scattering in different ways to transmission and reflection processes. (The transmission process we are describing in this paper is necessarily one which includes all forward multiples. But the "generalized primary" is only a subset of the backward multiples.) We, therefore, feel that the estimates of apparent  $Q$  made by the coda wave technique may not give values consistent with measurements based on the frequency content of the transmitted pulse.

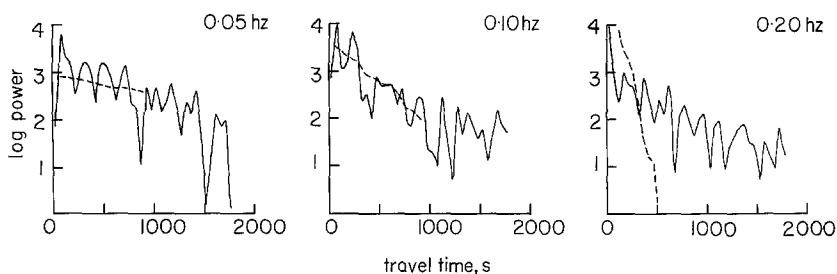


FIG. 21. Fall-off of power with travel time for the reflected waves (solid) and initial pulse of the transmitted waves (dashed) for a model with 900 random layers ( $\nu = 3$ , standard deviation = 0.5). The slope of the fall-off is proportional to  $1/Q$ . The power in the reflected wave is calculated by bandpassing the exact response and then integrating the power in a window centered about a given travel time. The power in the transmitted wave is calculated from the spectra of pulses transmitted through media of various thicknesses. The generalized primary reflection model predicts that at all frequencies, the slopes of the reflected and transmitted waves will be parallel. In fact, they are parallel only at lower frequencies and shorter travel times.

In support of this conclusion, we have computed fall-off of power with travel time for both the reflected and transmitted waves for a stack of 900 layers (Figure 21). The predictions of the generalized primary wave model seem to work for low frequencies, but fail at high frequencies and at later times in the coda. Furthermore, while the " $Q$ " estimated from the transmitted waves is constant, the " $Q$ " estimated from the reflected coda seems to increase with frequency. This important difference may provide a fourth diagnostic that can be used to distinguish scattering from intrinsic attenuation. However, it should be noted that coda is enhanced, in our plane wave analysis, as compared to the coda set up with point sources. In the latter case, for a basic study on coda wave data, see Rautian and Khalturin (1978).

#### DISCUSSION AND CONCLUSIONS

We have demonstrated with numerical examples some of the ways in which a pulse shape is affected by scattering. Of particular interest is the case of an elastic

medium in which the scattering is accomplished by many weak inhomogeneities so that the cumulative effect, due to forward-scattered multiples, is not apparent unless the path length is long.

The resulting removal of high frequencies, from the early portion of the transmitted wave train, has long been of interest to exploration geophysicists working with seismic waves propagated through the Earth's upper crustal layers. These shallow layers can often be studied best with instruments arrayed vertically (Wuenschel, 1976; Hauge, 1981; Kan *et al.*, 1981; Spencer *et al.*, 1982). But this option cannot be followed for studies of structure below about 0.05 per cent of depth to the Earth's center, because of the expense.

One point of some interest is that the two different types of velocity fluctuations we have considered (gaussian in velocity, with equal layer travel time; and Poisson in layer thickness, with only two different velocities) yield the same overall effect, an apparent  $Q$  that is insensitive to frequency. Note, however, that we worked with wavelengths that are long relative to the fine-scale details of the velocity variations. If the spectrum of incident wavelengths overlapped significantly with the spectrum of spatial variations in the velocity, there would be interactions and an effect upon the transmitted and reflected spectra that would be far more complicated than frequency-independent apparent  $Q$ . It will be interesting to investigate the apparent  $Q$  from fine-scale velocity variations, in media where the mean velocity itself varies systematically (e.g., increases with depth). A preliminary study of such cases shows little that is new, for vertically traveling waves. But one may expect new phenomena for oblique incidence, particularly when total internal reflections are present at some depth.

The key question for seismologists working with body waves transmitted over many hundreds of kilometers, is whether the mantle too has fine-scale spatial inhomogeneities that lead to significant losses due to scattering rather than intrinsic friction alone. There has, in recent years, been great interest among petrologists and isotope geochemists in chemical inhomogeneities in the mantle, but the access we have to mantle materials has not yet led to direct measurements of fine-scale physical inhomogeneities. [It would be of interest to attempt physical measurements in the (admittedly altered) blocks of mantle such as the Ronda ultramafics of southern Spain. Dickey *et al.* (1979) state that "the mafic segregations, which occur as layers 1 cm to 3 m thick, were formed when the massif lay at great depth (70 km)."]

It is fascinating to speculate on what mechanisms within the Earth are responsible for seismic wave attenuation by intrinsic friction [see Heinz *et al.* (1982) for references and an analysis of bulk attenuation in polycrystalline materials]. But caution is required in applying a theory of aggregates to obtain an overall anelasticity that then is compared in a quantitative way with published estimates of seismic  $Q$ . Both the model material (some form of aggregate) and the Earth itself scatter waves, as well as display anelasticity. The total attenuation observed seismically is, therefore, not amenable to explanation solely in terms of anelastic model processes.

Once the statistical nature of the propagation medium is acknowledged, various problems of sampling will arise. Some are reviewed by Morlet *et al.* (1982a, b). A series of papers by Schoenberger and Levin (1974, 1978, 1979) reached some disconcerting conclusions that illustrate particular problems of sampling. These authors showed an apparent lack of convergence in the overall process of sampling well logs at finer and finer intervals, and computing for each sampling the resulting pulse shape transmitted through a piecewise constant profile passing through the

sampled points. Specifically, they found changes both in pulse shape and in pulse delay, between samplings of a well each so fine that one might certainly have expected the essential features of the well log to have been captured.

The overall picture of the Earth's interior that we must pursue is a combination of deterministic and statistical elements. Broadly, the  $P$  and  $S$  velocities increase with depth. But upon this (deterministic) background is imposed a fine-scale fluctuation—the statistics of which are themselves of considerable interest to geophysicists. The field of wave propagation that perhaps has seen the greatest progress along these lines is the study of sound transmission in a fluctuating ocean. A book on this subject by Flatté (1979) reports progress in three major ways: in accounting for velocity inhomogeneities superposed on a background mean (the deterministic element) that is a function of water depth; in allowing for anisotropic correlation functions, so that, for example, the basic inhomogeneities might tend to have greater lateral than vertical extent; and in handling scattering that is strong, as well as that which is weak or intermediate. However, these considerable successes rely heavily on applying the parabolic approximation to the underlying scalar wave equation, in ways that do not appear possible when both  $P$  and  $S$  waves are present.

Given the difficulties of computing the effects of statistically distributed inhomogeneities, scientists from many different backgrounds have turned to manufacturing analog models in the laboratory (e.g., French, 1974; Dainty and Toksöz, 1975; Gelchinsky and Karaev, 1980a, b). The importance of ultrasonic waves in medical diagnosis and in nondestructive testing of mechanical structures has led to progress that may well have direct benefit in understanding seismic scattering.

We take the view that many papers reporting values of  $Q$  within the Earth have failed to adhere adequately to a precise meaning of this quantity. Attenuation, as we and many before us have shown, can be caused by scattering as well as anelasticity. Theories of anelastic wave propagation that neglect scattering are liable in practice to fail for high-frequency waves. In this paper, we have been able to examine in some detail the consequences of scattering upon the amplitude and shape of a pulse transmitted vertically through a medium with fine horizontal layering. There is a limit to how far such modeling of the Earth should be pursued, for lateral heterogeneities frequently cannot be ignored. The demonstrated importance of forward-scattered multiples, together with geometrical spreading effects that arise in laterally varying media, suggest that later-arriving short-period waves might, in practice, be raised to observable levels due to focusing. For a given source-receiver pair, if the coda indeed is dominated by those arriving multiples that are favorably focused (near caustic), then useful interpretation of the statistical properties of coda may not be practical. But it may still be possible to infer from the main phase itself (i.e., from its amplitude and shape) something of the statistical fluctuations of the medium through which it has propagated.

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