

# THE APPLICATION OF THE HYBRID INTERVAL ROUGH WEIGHTED POWER HERONIAN OPERATOR IN MULTI-CRITERIA DECISION-MAKING

Dragan Pamučar <sup>1\*</sup>, Aleksandar Janković <sup>2</sup>

<sup>1</sup> Department of logistics, Military Academy, University of Defence in Belgrade, Belgrade, Serbia

<sup>2</sup> Ministry of Transport and Maritime Affairs, Directorate for Transport, Montenegro

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**Abstract:** *In this paper, a new multi-criteria model which enables the processing of uncertainty and inaccuracy data through the application of interval rough numbers (IRN) is presented. The multi-criteria model represents the integration of the Power Aggregator (PA) and the Weighted Heronian Mean (WHM) operators. The goal of the forming of a hybrid Weighted Power Heronian Mean (WPHM) is to integrate the advantages of both operators into a single multi-criteria model, which has the following advantages: 1) it eliminates the influence of unreasonable arguments; 2) it takes into account the degree of support between input arguments and 3) it takes into account the interconnectedness of input arguments. Based on the mathematical concept of the IRN, the hybrid WPHM operator was extended and the IRNWPHM multi-criteria model was created. The IRNWPHA multi-criteria model enables objective decision-making in the case of imprecise input parameters in the initial decision matrix. Also, the IRNWPHA model allows flexible decision-making and the verification of the robustness of results through a variation of the  $p$  and  $q$  parameters. The IRNWPHM model was tested on a real-world multi-criteria example. The results showed that the IRNWPHM operator enabled a successful transformation of the uncertainties and inaccuracies that exist in group decision-making.*

**Key words:** *interval rough numbers; Heronian mean; multi-criteria decision-making; power operator.*

## 1. Introduction

The information that appears in real-world problems is often very difficult to quantify, since many facts, such as the complexity of phenomena and the ambiguity of human reasoning, represent significant limitations. In the multi-criteria modeling of decisions, different decision-makers are likely to use the linguistic expressions of a

\* Corresponding author.

dpmucar@gmail.com (D. Pamucar), aleksandar.jankovic@gmail.com (A. Janković)

different precision to express their preferences (Herrera and Martínez, 2000). In such situations, uncertainty theories, such as fuzzy sets (Zadeh, 1965), rough sets (Pawlak, 1982) and the other generalizations of the mentioned theories, are a good tool for presenting uncertainty.

In order to reach the best solution in group multi-criteria models, operators for the aggregation of group preferences and the calculation of the criterion functions of alternatives have been developed. In general, aggregation operators are important tools for the fusion of information into multi-criteria problems, which can also be used to evaluate alternatives. To date, many information fusion operators that can be used in decision-making models have been developed (Beliakov et al., 2007; Xu et al., 2012; Liu et al., 2015), including: the Bonferroni mean (Bonferroni, 1950), the Hamy mean (Hara et al., 1998), the Dombi operators (Dombi, 1982), the Maclaurin mean (Maclaurin, 1729), the Heronian mean (Beliakov, 2007), the Muirhead mean (Muirhead, 1902), Power aggregation (Yager, 2001) and numerous hybrid forms of aggregation operators (Pamučar et al., 2020; Sinani et al., 2020).

A better understanding of correlations between attributes can be very important for making objective decisions, so it is necessary to take into account the fact that relationships between attributes can be a significant determinant of an aggregated outcome. Therefore, the operators that have this feature have attracted significant attention in multi-criteria decision-making. Based on the analysis presented by Liu et al. (2016), it can be concluded that some aggregation operators only take into account the significance of the information presented in a decision matrix, while the interrelationships between data are neglected. Although there are aggregation operators which respect interrelationships between data, there are still significant shortcomings of some aggregation operators that need to be highlighted. For example, when aggregating data, power aggregation (Yager, 2001) only takes into account the influence of a change in the vector of the weight coefficients of criteria on aggregated values. At the same time, Power Aggregation (PA) does not take into account the relationships between aggregated arguments. On the other hand, Bonferroni mean (BM) operators respect the correlation between the attributes  $C_i$  and  $C_j$  ( $i \neq j$ ,  $C = \{C_1, C_2, \dots, C_n\}$ ), but ignore the relationship between the attributes  $C_i$  and itself. Considering the correlations between the attributes using the BM may also lead to redundancy (Liu et al., 2016). BM operators consider the correlation between  $C_i$  and  $C_j$  ( $i \neq j$ ) and the simultaneous correlation between  $C_j$  and  $C_i$  ( $i \neq j$ ), which may result in potential redundancy (Dutta et al., 2015).

Some of the requirements for decision-making in real-world systems include flexible decision-making, respect for the mutual influence between decision attributes and the elimination of the influence of awkward data. To achieve this goal, the integration of the PA and the weighted Heronian mean (WHM) operators into a hybrid WPHA operator is presented in this paper. The HM operator is a very useful tool, which takes into consideration the relationships between the attributes being aggregated. The WPHA operator combines the advantages of the PA and HM operators, and is a powerful tool with the following features: 1) it eliminates the influence of unreasonable arguments; 2) it takes into account the degree of support between input arguments, and 3) it takes into account the interconnectedness of input arguments. In recent years, the advantages of aggregation operators have been implemented

through multi-criteria models in a number of uncertainty theories: fuzzy sets (Pamucar et al., 2020; Ecer and Pamucar, 2020), intuitionistic fuzzy sets (Xu and Yager, 2011; He and He, 2016), interval- valued intuitionistic fuzzy sets (Liu and Li, 2017), hesitant fuzzy sets (He et al., 2015), rough sets (Sremac et al., 2018; Pamucar et al., 2018; Yazdani et al., 2020) and so on. To the authors' knowledge, no study considering the fusion of the PA and WHM aggregators in an interval rough environment has been carried out to date. Therefore, the logical goal and motivation for this study imply the presentation of a hybrid IRNWPHM operator. In this paper, Interval rough numbers were used to exploit uncertainties and inaccuracies, as they have certain advantages over traditional fuzzy sets (Yazdani et al., 2020). These advantages are especially evident when IRNs are applied in group decision-making.

The rest of the paper is organized into the next six sections. After the Introduction, the preliminaries of IRNs are presented in the second section of the paper. In the third section, the mathematical integration of the WHM and PA operators in an IRN environment is presented. In the fourth section of the paper, the structure of the multi-criteria IRN WPHM model is presented. In the fifth section, the model was tested on a real-world example, and the results were validated through the variation of the  $p$  and  $q$  parameters. Finally, the concluding remarks are given in the sixth section of the paper.

## 2. Interval Rough Numbers

Assume that  $U$  is the universe containing all the objects registered in an information table. Assume that there is a set of the  $k$  classes representing the DM's preferences  $R = (J_1, J_2, \dots, J_k)$  provided that they belong to the row that satisfies the condition  $J_1 < J_2 < \dots < J_k$  and another set of the  $k$  classes that also represent the DM's preferences  $R^* = (I_1, I_2, \dots, I_k)$ . Assuming that all the objects are defined in the universe and related to the DM's preferences. In  $R^*$ , every class of objects is represented by the interval  $I_i = \{I_{li}, I_{ui}\}$ , provided that  $I_{li} \leq I_{ui}$  ( $1 \leq i \leq m$ ), and  $I_{li}, I_{ui} \in R$  are satisfied. Then,  $I_{li}$  denotes the lower interval limit, while  $I_{ui}$  denotes the upper interval limit of the  $i$  class. If both class limits (the lower and the upper limits) presented so as  $I_{l1}^* < I_{l2}^* < \dots < I_{lj}^*, I_{u1}^* < I_{u2}^* < \dots < I_{uk}^*$  ( $1 \leq j, k \leq m$ ) are satisfied, respectively, then the two new sets containing the lower class  $R_l^* = (I_{l1}^*, I_{l2}^*, \dots, I_{lj}^*)$  and the upper class  $R_u^* = (I_{u1}^*, I_{u2}^*, \dots, I_{uk}^*)$  can be defined, respectively. If that is the case, then for any class  $I_{li}^* \in R$  ( $1 \leq i \leq j$ ) and  $I_{ui}^* \in R$  ( $1 \leq i \leq k$ ), the lower approximation of  $I_{li}^*$  and  $I_{ui}^*$  can be defined as follows (Pamucar et al., 2018):

$$\underline{Apr}(I_{li}^*) = \bigcup \{Y \in U / R_l^*(Y) \leq I_{li}^*\} \tag{1}$$

$$\underline{Apr}(I_{ui}^*) = \bigcup \{Y \in U / R_u^*(Y) \leq I_{ui}^*\} \tag{2}$$

The above-mentioned approximations of  $I_{li}^*$  and  $I_{ui}^*$  are defined by applying the following equation:

$$\overline{Apr}(I_{li}^*) = \bigcup \{Y \in U / R_l^*(Y) \geq I_{li}^*\} \tag{3}$$

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

$$\overline{Apr}(I_{ui}^*) = \bigcup \{ Y \in U / R_u^*(Y) \geq I_{ui}^* \} \quad (4)$$

Both object classes (the upper and the lower classes ( $I_{li}^*$  and  $I_{ui}^*$ , respectively)) are defined by their lower limits  $\underline{Lim}(I_{li}^*)$  and  $\underline{Lim}(I_{ui}^*)$ , and by their upper limits  $\overline{Lim}(I_{li}^*)$  and  $\overline{Lim}(I_{ui}^*)$ , respectively:

$$\underline{Lim}(I_{li}^*) = \frac{1}{M_L} \sum R_l^*(Y) | Y \in \underline{Apr}(I_{li}^*) \quad (5)$$

$$\underline{Lim}(I_{ui}^*) = \frac{1}{M_L} \sum R_u^*(Y) | Y \in \underline{Apr}(I_{ui}^*) \quad (6)$$

where  $M_L$  and  $M_L^*$  denote the number of the objects contained in the lower approximations  $I_{li}^*$  and  $I_{ui}^*$ , respectively. The upper limits  $\overline{Lim}(I_{li}^*)$  and  $\overline{Lim}(I_{ui}^*)$  are defined by the equations (7) and (8), as follows:

$$\overline{Lim}(I_{li}^*) = \frac{1}{M_U} \sum R_l^*(Y) | Y \in \overline{Apr}(I_{li}^*) \quad (7)$$

$$\overline{Lim}(I_{ui}^*) = \frac{1}{M_U} \sum R_u^*(Y) | Y \in \overline{Apr}(I_{ui}^*) \quad (8)$$

where  $M_U$  and  $M_U^*$  denote the number of the objects contained in the upper approximations  $I_{li}^*$  and  $I_{ui}^*$ , respectively.

For the lower class of objects, the rough boundary interval from  $I_{li}^*$  is represented as  $RB(I_{li}^*)$  and denotes the interval between the lower and the upper limits:

$$RB(I_{li}^*) = \overline{Lim}(I_{li}^*) - \underline{Lim}(I_{li}^*), \quad (9)$$

while for the upper object class, the rough boundary interval  $I_{ui}^*$  is obtained based on the following equation:

$$RB(I_{ui}^*) = \overline{Lim}(I_{ui}^*) - \underline{Lim}(I_{ui}^*) \quad (10)$$

Then, the uncertain class of the objects  $I_{li}^*$  and  $I_{ui}^*$  can be expressed by using their lower and upper limits, as follows:

$$RN(I_{li}^*) = \left[ \overline{Lim}(I_{li}^*), \underline{Lim}(I_{li}^*) \right] \quad (11)$$

$$RN(I_{ui}^*) = \left[ \overline{Lim}(I_{ui}^*), \underline{Lim}(I_{ui}^*) \right] \quad (12)$$

It can be seen that every class of objects is defined by its lower and upper limits that create the interval rough number that can be defined as:

$$IRN(I_i^*) = \left[ RN(I_{li}^*), RN(I_{ui}^*) \right] \quad (13)$$

Interval rough numbers are characterized by specific arithmetic operations, which differ from those dealing with typical rough numbers. Arithmetic operations between

two interval rough numbers  $IRN(A) = ([a_1, a_2], [a_3, a_4])$  and  $IRN(B) = ([b_1, b_2], [b_3, b_4])$  are done by applying the following expressions (14), (15), (16), (17) and (18) (Pamučar et al., 2019):

(1) The addition of interval rough numbers “+”

$$IRN(A) + IRN(B) = ([a_1, a_2], [a_3, a_4]) + ([b_1, b_2], [b_3, b_4]) \\ = ([a_1 + b_1, a_2 + b_2], [a_3 + b_3, a_4 + b_4]) \tag{14}$$

(2) The subtraction of interval rough numbers “-”

$$IRN(A) - IRN(B) = ([a_1, a_2], [a_3, a_4]) - ([b_1, b_2], [b_3, b_4]) \\ = ([a_1 - b_4, a_2 - b_3], [a_3 - b_2, a_4 - b_1]) \tag{15}$$

(3) The multiplication of interval rough numbers “×”

$$IRN(A) \times IRN(B) = ([a_1, a_2], [a_3, a_4]) \times ([b_1, b_2], [b_3, b_4]) \\ = ([a_1 \times b_1, a_2 \times b_2], [a_3 \times b_3, a_4 \times b_4]) \tag{16}$$

(4) The division of interval rough numbers “/”

$$IRN(A) / IRN(B) = ([a_1, a_2], [a_3, a_4]) / ([b_1, b_2], [b_3, b_4]) \quad \text{and} \\ = ([a_1 / b_4, a_2 / b_3], [a_3 / b_2, a_4 / b_1]) \tag{17}$$

(5) The scalar multiplication of interval rough numbers, where  $k > 0$

$$k \times IRN(A) = k \times ([a_1, a_2], [a_3, a_4]) = ([k \times a_1, k \times a_2], [k \times a_3, k \times a_4]) \tag{18}$$

### 3. Interval Rough Weight Power Heronian Operator

The Power Aggregation (PA) operator proposed by Yager (2001) is a very significant aggregation operator, which eliminates the impact of unreasonable arguments. The traditional PA operator can be defined as follows:

*Definition 1* (Yager, 2001): Let  $(\xi_1, \xi_2, \dots, \xi_n)$  be a set of non-negative numbers and  $p, q \geq 0$ . If

$$PA(\xi_1, \xi_2, \dots, \xi_n) = \frac{\sum_{i=1}^n (1 + T(\xi_i)) \xi_i}{\sum_{i=1}^n (1 + T(\xi_i))} \tag{19}$$

where  $T(\xi_i) = \sum_{j=1, j \neq i}^n Sup(\xi_i, \xi_j)$  and  $Sup(\xi_i, \xi_j)$  denote the degree of the support that  $\xi_i$  received from  $\xi_j$ , where  $Sup(\xi_i, \xi_j)$  satisfies the following axioms:

- 1)  $Sup(\xi_i, \xi_j) = Sup(\xi_j, \xi_i)$ ;
- 2)  $Sup(\xi_i, \xi_j) = [0, 1]$ ;
- 3)  $Sup(\xi_i, \xi_j) > Sup(\xi_i, \xi_k)$ , if  $|\xi_i - \xi_j| < |\xi_i - \xi_k|$ .

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

The Heronian Mean (HM) operator was proposed by Beliakov (2007). The HM takes into account the interconnectedness between input arguments (Liu and Pei, 2012). The HM operator can be defined as follows:

*Definition 2* (Yu, 2013): Let  $p, q \geq 0$ ,  $(\xi_1, \xi_2, \dots, \xi_n)$  be a set of non-negative numbers. If

$$HM^{p,q}(\xi_1, \xi_2, \dots, \xi_n) = \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \xi_i^p \xi_j^q \right)^{\frac{1}{p+q}} \quad (20)$$

then  $HM^{p,q}$  is called the Heronian Mean (HM) operator.

*Definition 3* (Zhao, 2019): Let  $p, q \geq 0$  and  $(\xi_1, \xi_2, \dots, \xi_n)$  represent a set of non-negative numbers. Then, the Weight Heronian Mean (WHM) operator for averaging can be defined as follows:

$$WHM^{p,q}(\xi_1, \xi_2, \dots, \xi_n) = \left( \frac{2}{n(n+1)} \sum_{i=1, j=i}^n \left( (nw_i \xi_i)^p \cdot (nw_j \xi_j)^q \right) \right)^{\frac{1}{p+q}} \quad (21)$$

where  $WHM^{p,q}$  is called the Weighted Heronian Mean (WHM) operator.

Based on the defined settings of the PA and WHM operators, Eqs. (19) and (21), in the following part a hybrid Interval Rough Weighted Power Heronian Aggregation (IRWPHA) operator was developed.

*Definition 4*: Set  $\xi_i = ([\xi_i^L, \xi_i^U], [\xi_i^L, \xi_i^U])$  ( $i = 1, 2, \dots, n$ ) as a collection of IRNs in  $\Omega$ ; then the IRWPHA can be defined as follows:

$$IRNWPHA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) = \left( \frac{2}{n(n+1)} \sum_{i=1, j=i}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j \right)^q \right) \right)^{\frac{1}{p+q}} \quad (22)$$

$$\text{where } w_i = \frac{(1+T(\xi_i))}{\sum_{i=1}^n (1+T(\xi_i))}, \quad T(\xi_i) = \sum_{j=1, j \neq i}^n \text{Sup}(\xi_i, \xi_j) \quad \text{and} \quad \sum_{i=1}^n w_i = 1,$$

where  $\text{Sup}(\xi_i, \xi_j)$  denote the degree of the support that  $\xi_i$  received from  $\xi_j$ , where  $\text{Sup}(\xi_i, \xi_j)$  satisfies the following axioms (Đorđević et al., 2019):

- 1)  $\text{Sup}(f(\xi_i), f(\xi_j)) = \text{Sup}(f(\xi_j), f(\xi_i))$ ;
- 2)  $\text{Sup}(f(\xi_i), f(\xi_j)) = [0, 1]$ ;
- 3)  $\text{Sup}(f(\xi_i), f(\xi_j)) > \text{Sup}(f(\xi_i), f(\xi_k))$ , if  $d(\xi_i, \xi_j) < d(\xi_i, \xi_k)$ , where  $d(\xi_i, \xi_j)$  represents the distance between the numbers  $\xi_i$  and  $\xi_j$ .

Then  $IRNWPHA^{p,q}$  represents the IRN weight power Heronian aggregation operator.

*Theorem 1*: Set  $\xi_i = ([\xi_i^L, \xi_i^U], [\xi_i^L, \xi_i^U])$  as a collection of IRNs in  $\Omega$ ; then, according to Eq. (22), aggregation results are obtained for IRNs, and the following aggregation formula can be developed:

$$\begin{aligned}
 IRNWPFA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) &= \left( \frac{2}{n(n+1)} \sum_{i=1, j=i}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j \right)^q \right) \right)^{\frac{1}{p+q}} \\
 &= \left[ \begin{aligned}
 &\left( \frac{2}{n(n+1)} \sum_{i=1, j=i}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^L \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^L \right)^q \right) \right)^{\frac{1}{p+q}}, \\
 &\left( \frac{2}{n(n+1)} \sum_{i=1, j=i}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^U \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^U \right)^q \right) \right)^{\frac{1}{p+q}}, \\
 &\left( \frac{2}{n(n+1)} \sum_{i=1, j=i}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^L \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^L \right)^q \right) \right)^{\frac{1}{p+q}}, \\
 &\left( \frac{2}{n(n+1)} \sum_{i=1, j=i}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^U \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^U \right)^q \right) \right)^{\frac{1}{p+q}}
 \end{aligned} \right]
 \end{aligned}
 \tag{23}$$

*Proof:*

The proof for *Theorem 1* is presented in the following section. Based on the equations (19) and (22), the following is obtained:

$$\begin{aligned}
 \text{a) } nw_i \xi_i &= n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i \text{ and } nw_j \xi_j = n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j ; \\
 \text{b) } \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i \right)^p &= \left[ \begin{aligned}
 &\left[ n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^L, n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^U \right]^p ; \\
 &\left[ n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^L, n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^U \right]^p
 \end{aligned} \right] ; \\
 \text{c) } \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j \right)^q &= \left[ \begin{aligned}
 &\left[ n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^L, n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^U \right]^q ; \\
 &\left[ n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^L, n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^U \right]^q
 \end{aligned} \right] ;
 \end{aligned}$$

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

d)

$$\left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j \right)^q = \left[ \begin{array}{c} \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^L \right)^p \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^L \right)^q \\ \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^U \right)^p \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^U \right)^q \\ \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^L \right)^p \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^L \right)^q \\ \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^U \right)^p \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^U \right)^q \end{array} \right]$$

Finally, the equation for IRN is obtained by means of the weight power Heronian operator ( $IRNWPHA^{p,q}$ ) for aggregation, as follows:

$$IRNWPHA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) = \left( \frac{2}{n(n+1)} \sum_{i=1, j=1}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j \right)^q \right) \right)^{\frac{1}{p+q}}$$

$$= \left[ \begin{array}{c} \left( \frac{2}{n(n+1)} \sum_{i=1, j=1}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^L \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^L \right)^q \right) \right)^{\frac{1}{p+q}} \\ \left( \frac{2}{n(n+1)} \sum_{i=1, j=1}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i^U \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j^U \right)^q \right) \right)^{\frac{1}{p+q}} \\ \left( \frac{2}{n(n+1)} \sum_{i=1, j=1}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^L \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^L \right)^q \right) \right)^{\frac{1}{p+q}} \\ \left( \frac{2}{n(n+1)} \sum_{i=1, j=1}^n \left( \left( n \frac{nw_i w_i}{\sum_{t=1}^n w_t w_t} \xi_i'^U \right)^p \cdot \left( n \frac{nw_i w_j}{\sum_{t=1}^n w_t w_t} \xi_j'^U \right)^q \right) \right)^{\frac{1}{p+q}} \end{array} \right]$$

So, *Theorem 1* is true.

*Theorem 2* (Idempotency): Set  $\xi_i = \left( \left[ \xi_i^L, \xi_i^U \right], \left[ \xi_i'^L, \xi_i'^U \right] \right)$  as a collection of IRNs in

$\Omega$ ; if  $\xi_i = \xi$ , then  $IRNWPHA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) = IRNWPHA^{p,q}(\xi, \xi, \dots, \xi)$ .

*Proof:*

Since  $\xi_i = \xi$ , i.e.  $\xi_i^L = \xi^L$ ,  $\xi_i^U = \xi^U$ ,  $\xi_i'^L = \xi'^L$ ,  $\xi_i'^U = \xi'^U$ , then



$$IRNWPHA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) = IRNWPHA^{p,q}(\xi, \xi, \dots, \xi)$$

$$= \left[ \begin{array}{l} \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi_i^L))}{\sum_{t=1}^n (1+T(\xi_t^L))} \xi_i^L \right)^p \left( \frac{n(1+T(\xi_j^L))}{\sum_{t=1}^n (1+T(\xi_t^L))} \xi_j^L \right)^q \right)^{\frac{1}{p+q}}, \\ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi_i^U))}{\sum_{t=1}^n (1+T(\xi_t^U))} \xi_i^U \right)^p \left( \frac{n(1+T(\xi_j^U))}{\sum_{t=1}^n (1+T(\xi_t^U))} \xi_j^U \right)^q \right)^{\frac{1}{p+q}}, \\ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi_i'^L))}{\sum_{t=1}^n (1+T(\xi_t'^L))} \xi_i'^L \right)^p \left( \frac{n(1+T(\xi_j'^L))}{\sum_{t=1}^n (1+T(\xi_t'^L))} \xi_j'^L \right)^q \right)^{\frac{1}{p+q}}, \\ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi_i'^U))}{\sum_{t=1}^n (1+T(\xi_t'^U))} \xi_i'^U \right)^p \left( \frac{n(1+T(\xi_j'^U))}{\sum_{t=1}^n (1+T(\xi_t'^U))} \xi_j'^U \right)^q \right)^{\frac{1}{p+q}} \end{array} \right]$$

$$= \left[ \begin{array}{l} \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi^L))}{\sum_{t=1}^n (1+T(\xi_t^L))} \xi^L \right)^p \left( \frac{n(1+T(\xi^L))}{\sum_{t=1}^n (1+T(\xi_t^L))} \xi^L \right)^q \right)^{\frac{1}{p+q}}, \\ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi^U))}{\sum_{t=1}^n (1+T(\xi_t^U))} \xi^U \right)^p \left( \frac{n(1+T(\xi^U))}{\sum_{t=1}^n (1+T(\xi_t^U))} \xi^U \right)^q \right)^{\frac{1}{p+q}}, \\ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi'^L))}{\sum_{t=1}^n (1+T(\xi_t'^L))} \xi'^L \right)^p \left( \frac{n(1+T(\xi'^L))}{\sum_{t=1}^n (1+T(\xi_t'^L))} \xi'^L \right)^q \right)^{\frac{1}{p+q}}, \\ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( \frac{n(1+T(\xi'^U))}{\sum_{t=1}^n (1+T(\xi_t'^U))} \xi'^U \right)^p \left( \frac{n(1+T(\xi'^U))}{\sum_{t=1}^n (1+T(\xi_t'^U))} \xi'^U \right)^q \right)^{\frac{1}{p+q}} \end{array} \right]$$

$$= \left[ \begin{array}{l} \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( n \frac{1}{n} \xi^L \right)^p \left( n \frac{1}{n} \xi^L \right)^q \right)^{\frac{1}{p+q}}, \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( n \frac{1}{n} \xi^U \right)^p \left( n \frac{1}{n} \xi^U \right)^q \right)^{\frac{1}{p+q}}, \\ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( n \frac{1}{n} \xi'^L \right)^p \left( n \frac{1}{n} \xi'^L \right)^q \right)^{\frac{1}{p+q}}, \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n \left( n \frac{1}{n} \xi'^U \right)^p \left( n \frac{1}{n} \xi'^U \right)^q \right)^{\frac{1}{p+q}} \end{array} \right]$$

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

$$= \left[ \left[ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n (\xi_i^L)^{p+q} \right)^{\frac{1}{p+q}}, \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n (\xi_i^U)^{p+q} \right)^{\frac{1}{p+q}} \right], \left[ \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n (\xi_i'^L)^{p+q} \right)^{\frac{1}{p+q}}, \left( \frac{2}{n(n+1)} \sum_{i=1}^n \sum_{j=i}^n (\xi_i'^U)^{p+q} \right)^{\frac{1}{p+q}} \right] \right] = \xi$$

The *Theorem 2* proof is completed.

*Theorem 3* (Boundedness): Set  $\xi_i = ([\xi_i^L, \xi_i^U], [\xi_i'^L, \xi_i'^U])$  as a collection of RNs in  $\Omega$ ; let  $\xi^- = ([\min(\xi_i^L), \min(\xi_i^U)], [\min(\xi_i'^L), \min(\xi_i'^U)])$  and  $\xi^+ = ([\max(\xi_i^L), \max(\xi_i^U)], [\max(\xi_i'^L), \max(\xi_i'^U)])$ , then  $\xi^- \leq IRNWPFA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) \leq \xi^+$ .

*Proof:*

Let  $\xi^- = \min(\xi_1, \xi_2, \dots, \xi_n) = ([\min(\xi_i^L), \min(\xi_i^U)], [\min(\xi_i'^L), \min(\xi_i'^U)])$  and  $\xi^+ = \max(\xi_1, \xi_2, \dots, \xi_n) = ([\max(\xi_i^L), \max(\xi_i^U)], [\max(\xi_i'^L), \max(\xi_i'^U)])$ ; then, it can be said that  $\xi_i^{L-} = \min(\xi_i^L)$ ,  $\xi_i^{U-} = \min(\xi_i^U)$ ,  $\xi_i^{L+} = \max(\xi_i^L)$ ,  $\xi_i^{U+} = \max(\xi_i^U)$ ,  $\xi_i'^{L-} = \min(\xi_i'^L)$ ,  $\xi_i'^{U-} = \min(\xi_i'^U)$ ,  $\xi_i'^{L+} = \max(\xi_i'^L)$ ,  $\xi_i'^{U+} = \max(\xi_i'^U)$ . Based on that, the following inequalities can be formulated:

$$\begin{aligned} \xi^- &\leq \xi_i \leq \xi^+; \\ \min(\xi_j^L) &\leq \xi_j^L \leq \max(\xi_j^L); \\ \min(\xi_j^U) &\leq \xi_j^U \leq \max(\xi_j^U); \\ \min(\xi_j'^L) &\leq \xi_j'^L \leq \max(\xi_j'^L); \\ \min(\xi_j'^U) &\leq \xi_j'^U \leq \max(\xi_j'^U). \end{aligned}$$

According to the above-shown inequalities, it can be concluded that  $\xi^- \leq IRNWPFA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) \leq \xi^+$  holds.

*Theorem 4* (Commutativity): Let the interval rough set  $(\xi_1', \xi_2', \dots, \xi_n')$  be any permutation of  $(\xi_1, \xi_2, \dots, \xi_n)$ . Then,  $IRNWPFA^{p,q}(\xi_1, \xi_2, \dots, \xi_n) = IRNWPFA^{p,q}(\xi_1', \xi_2', \dots, \xi_n')$ . *Proof:* This property is obvious.

#### 4. The IRNWPFA Model for Multi-Criteria Decision-Making

Based on the IRNWPFA operator, a model for the group multi-criteria evaluation of alternatives that includes the following steps can be defined:

*Step 1. The formation of the initial decision matrix.* Defining a set of the experts  $E_e$  ( $1 \leq e \leq t$ ,  $t$  represents the number of the experts) who evaluate alternatives and form expert correspondent matrices  $X^e = [x_{ij}^e; x_{ij}^e]_{m \times n}$  ( $1 \leq e \leq t$ ). The aggregation of the expert matrices  $X = [x_{ij}]_{m \times n}$  into the initial decision matrix was performed by using the IRN power Heronian aggregator (Đorđević et al., 2019).

*Step 2. The initial decision matrix normalization.* The normalization of the initial decision matrix is performed by applying the equation (24). Thus, the normalized matrix  $N = [IRN(n_{ij})]_{m \times n}$  is formed.

$$n_{ij} = \begin{cases} x_{ij} / \sum_{i=1}^n x_{ij} & \text{if } j \in B \\ 1 - x_{ij} / \sum_{i=1}^n x_{ij} & \text{if } j \in C \end{cases} \quad (24)$$

*Step 3. The determination of the criterion function alternatives.* By using IRNWPHA (23), the score function values  $H(n_i) = IRNWPHA^{p,q,r}(IRN(n_1), IRN(n_2), \dots, IRN(n_m))$  are obtained, representing the final values of the preferences by the alternatives.

*Step 4. Ranking alternatives.* The ranking of the alternatives  $\{A_1, A_1, \dots, A_m\}$  is done based upon the value of the criterion function  $H(n_i)$ , where the alternative that has a higher value  $H(n_i)$  is preferable.

### 5. Case Study

In the following section, the application of the IRNWPHA multi-criteria model for solving real-world problems is discussed. The IRNWPHA model was applied to evaluate the work of the advisors in dangerous goods transport. The criteria accounted for in Table 1 were taken from a study by Pamucar et al. (2019), in which the application of a linguistic neutrosophic methodology in order to evaluate advisors' work was considered.

Table 1. The criteria for the evaluation of advisors' work (Pamucar et al. 2019)

Number	Criteria	Type
1.	The knowledge of regulations and professional development	Benefit
2.	The analytical processing of the established requirements	Benefit
3.	The quality of the proposed measures	Benefit
4.	The level of implementation of the proposed measures	Benefit
5.	The quality of the professional training of the employees	Benefit
6.	A response to situations of emergency	Benefit
7.	The preparation of documents	Benefit
8.	The method for solving professional questions	Benefit
9.	Activity in professional bodies	Benefit

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

A total of eight experts participated in the research ( $e_i, i = 1, 2, \dots, 8$ ). The experts used the following nine-point scale to evaluate the work of the ten advisors ( $A_i, i = 1, 2, \dots, 10$ ): 1 – Very low (VL); 2 – Medium low (ML); 3 – Low (L); ... ; 8 – High (H); 9 – Very high (VH). The weighting coefficients of the criteria were taken from the Pamucar et al. (2019) study:

$$w_j = (0.1178, 0.0875, 0.1020, 0.1087, 0.1302, 0.0904, 0.0838, 0.1163, 0.1632)^T.$$

In the following section, the application of the IRNWPFA is presented through the steps defined in the previous section of the paper:

*Step 1 - The formation of the initial decision matrix:*

Eight experts evaluated the advisors using a nine-point scale. Expert correspondent matrices with evaluation of advisors are shown in Table 2.

Table 2. The expert correspondent matrices

Expert 1								
Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A1	(3;3)	(5;6)	(7;8)	(1;2)	(5;6)	(3;4)	(3;4)	(5;6)
A2	(8;9)	(7;8)	(5;6)	(9;9)	(5;6)	(5;6)	(7;8)	(5;6)
A3	(6;5)	(3;4)	(1;2)	(3;4)	(3;3)	(5;6)	(9;9)	(5;6)
A4	(4;5)	(3;3)	(3;4)	(7;8)	(5;5)	(5;6)	(9;9)	(5;6)
A5	(7;7)	(7;8)	(9;9)	(5;6)	(7;7)	(5;6)	(5;6)	(5;6)
A6	(5;5)	(3;4)	(5;6)	(1;2)	(3;3)	(3;4)	(3;4)	(5;6)
A7	(5;5)	(5;5)	(3;4)	(1;1)	(7;7)	(5;6)	(1;1)	(3;4)
A8	(6;7)	(9;9)	(5;6)	(1;1)	(7;8)	(5;6)	(5;6)	(5;5)
A9	(5;5)	(3;4)	(3;4)	(1;2)	(3;4)	(5;6)	(3;4)	(5;6)
A10	(4;5)	(5;5)	(5;6)	(3;3)	(5;5)	(3;4)	(5;6)	(7;7)
...								
Expert 8								
Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A1	(5;6)	(9;9)	(7;8)	(7;8)	(1;2)	(3;3)	(8;9)	(7;8)
A2	(9;9)	(9;9)	(9;9)	(8;9)	(9;9)	(9;9)	(9;9)	(7;7)
A3	(7;8)	(3;4)	(5;6)	(7;8)	(8;9)	(8;9)	(7;8)	(7;8)
A4	(9;9)	(8;9)	(8;9)	(9;9)	(8;9)	(8;9)	(9;9)	(5;6)
A5	(7;8)	(7;8)	(5;6)	(7;8)	(8;9)	(5;6)	(7;8)	(5;6)
A6	(7;8)	(3;4)	(5;5)	(7;8)	(8;9)	(5;5)	(8;9)	(8;9)
A7	(7;8)	(5;6)	(5;5)	(1;1)	(8;9)	(8;9)	(8;9)	(8;9)
A8	(7;8)	(7;8)	(5;6)	(1;2)	(9;9)	(9;9)	(7;8)	(9;9)
A9	(5;6)	(1;2)	(1;1)	(7;8)	(3;4)	(5;6)	(7;7)	(8;9)
A10	(5;5)	(5;6)	(5;6)	(7;8)	(1;2)	(7;8)	(7;8)	(5;6)

The dilemmas and uncertainties that exist during the expert evaluation of the advisors are shown by using the values given in Table 2. Thus, for example, for the expert E8 in the position A1-C1, the value is (5;6). This means that, during the evaluation of the advisor A1 (for the criterion C1), the E8 expert had a dilemma between the two values from the nine-point scale, i.e. there was a dilemma between the values 5 and 6 from the scale. Also, with the expert E8 in the position A1-C2, it is

possible to notice that there was no dilemma about the choice of the values from the composition, so the value (9;9) was assigned.

In the next part, the transformation of uncertainty into an IRN was performed by using the equations (1) – (13). After the aggregation of the IRN expert correspondence matrices, an aggregated IRN initial decision matrix was obtained, as in Table 3. The aggregation was performed by using the equation (21).

Table 3. The aggregate initial decision matrix

Alt.	C1	C2	C3	...	C9
A1	[[4.00,6.00],[4.63,6.70]]	[[5.63,7.41],[6.58,8.06]]	[[3.87,6.60],[4.86,7.33]]		[[5.85,7.50],[6.58,8.28]]
A2	[[8.35,8.95],[8.35,8.95]]	[[7.58,8.63],[8.44,8.89]]	[[5.09,7.75],[5.85,8.28]]		[[5.16,7.64],[5.91,8.59]]
A3	[[6.20,7.77],[7.19,8.33]]	[[4.36,7.38],[5.31,7.91]]	[[3.70,4.90],[4.70,5.90]]		[[6.26,7.90],[7.29,8.63]]
A4	[[6.49,8.37],[6.55,8.61]]	[[5.81,8.38],[6.54,8.77]]	[[5.77,8.22],[6.65,8.71]]		[[5.85,7.50],[6.58,8.25]]
A5	[[6.62,7.82],[7.35,8.38]]	[[5.86,7.67],[6.89,8.39]]	[[5.21,6.63],[5.75,7.42]]		[[6.62,7.82],[7.39,8.55]]
A6	[[4.49,6.37],[4.84,7.28]]	[[3.86,6.33],[4.79,7.26]]	[[4.68,7.40],[5.29,7.98]]		[[5.15,7.65],[6.20,8.38]]
A7	[[6.64,7.80],[6.70,8.16]]	[[5.90,7.67],[6.08,8.10]]	[[3.97,6.56],[4.63,6.84]]		[[4.24,6.30],[5.01,7.04]]
A8	[[5.85,7.50],[6.11,7.88]]	[[5.78,8.10],[6.22,8.41]]	[[4.71,6.40],[5.47,7.43]]		[[4.33,6.10],[5.00,7.06]]
A9	[[4.20,5.36],[4.84,6.30]]	[[2.38,5.04],[3.34,5.92]]	[[3.33,5.97],[3.85,6.96]]		[[5.06,6.88],[6.06,7.88]]
A10	[[6.17,8.26],[5.81,8.29]]	[[5.87,7.26],[5.97,7.59]]	[[4.58,6.40],[5.47,7.43]]		[[6.18,7.60],[6.84,8.26]]

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

*Step 2 – The initial decision matrix normalization:*

The normalization of the elements of the initial decision matrix is a logical step in the multi-criteria models in which criteria are represented by the different units of measurement and/or in which there are two types of criteria (Benefit and Cost). In this paper, the normalization of the value of the initial decision matrix is omitted because: 1) a nine-point scale was used to evaluate all the alternatives, i.e., all the criteria are presented by the same units, and 2) all the criteria belong to the group of the benefit criteria, i.e. there are no two types of the criteria.

*Steps 3 and 4 - The determination of the criterion function  $H(n_i)$  of alternatives and the ranking of alternatives*

By using the IRNWPHA, Equation (23), the alternatives of the criterion functions are obtained as in Table 4.

Table 4. The ranking of the alternatives

Alt.	IRN $H(n_i)$	Crisp $H(n_i)$	Rank
A1	[[4.12,6.18],[4.96,7.05]]	5.58	9
A2	[[6.52,8.04],[7.13,8.54]]	7.57	1
A3	[[5.46,7.34],[6.38,8.03]]	6.83	4
A4	[[6.38,8.12],[7.04,8.59]]	7.55	2
A5	[[6.16,7.52],[6.93,8.22]]	7.22	3
A6	[[4.43,6.7],[5.19,7.54]]	5.96	8
A7	[[4.97,6.93],[5.55,7.6]]	6.26	7
A8	[[5.15,7.2],[5.8,7.84]]	6.50	6
A9	[[3.91,6.1],[4.67,6.93]]	5.40	10
A10	[[5.21,7.39],[5.73,7.97]]	6.57	5

The calculation of the IRN value from Table 4 is shown in the next section. Table 5 shows the values of the alternative A1 according to the criteria C1-C9.

Table 5. The values of the alternative A1

Criterion	IRN value
C1	[[4.00,6.00],[4.63,6.70]]
C2	[[5.63,7.41],[6.58,8.06]]
C3	[[3.87,6.60],[4.86,7.33]]
C4	[[2.04,4.96],[3.04,5.96]]
C5	[[3.07,5.13],[4.07,6.13]]
C6	[[2.87,4.93],[3.50,5.90]]
C7	[[5.32,7.03],[6.32,8.03]]
C8	[[3.63,5.51],[4.36,6.57]]
C9	[[5.85,7.50],[6.58,8.28]]

Since  $IRN H(n_i) = ([n_i^L, n_i^U], [n_i^L, n_i^U])$  consists of the four segments,  $IRN H(n_i)$  aggregation will be performed separately for each of the segments, i.e.  $IRNWPHA^{1,1}(n_i^L(C1); n_i^L(C2), \dots, n_i^L(C9))$ ,  $IRNWPHA^{1,1}(n_i^U(C1); n_i^U(C2), \dots, n_i^U(C9))$ ,  $IRNWPHA^{1,1}(n_i^L(C1); n_i^L(C2), \dots, n_i^L(C9))$  and

$IRNWPHA^{1,1}(n_1^U(C1); n_1^U(C2), \dots, n_1^U(C9))$ . The segment calculation  $IRNWPHA^{1,1}(n_1^L(C1); n_1^L(C2), \dots, n_1^L(C9))$  is shown in detail in the next section:

Step 1: Normalized number functions are calculated:

$$f(n_1^L(C1)) = \frac{4}{4+5.63+\dots+5.85} = 0.11; \quad f(n_1^L(C2)) = \frac{5.63}{4+5.63+\dots+5.85} = 0.16, \dots,$$

$$f(n_1^L(C9)) = \frac{5.85}{4+5.63+\dots+5.85} = 0.16.$$

Step 2: The calculation of the degree of support for numbers:

$$Sup(f(n_1^L(C1)), f(n_1^L(C1))) = 0.045, \quad Sup(f(n_1^L(C1)), f(n_1^L(C3))) = 0.003,$$

$$Sup(f(n_1^L(C1)), f(n_1^L(C4))) = 0.054, \dots, \quad Sup(f(n_1^L(C8)), f(n_1^L(C9))) = 0.061$$

Step 3: By applying Equation (23),  $IRNWPHA^{p=1, q=1}(n_1^L(C1); n_1^L(C2), \dots, n_1^L(C9))$  is calculated as follows:

$$IRNWPHA^{p=1, q=1}(4.00; 5.63; 3.87; 2.04; 3.07; 2.87; 5.32; 3.63; 5.85) =$$

$$= \frac{2}{9(9+1)} \left( \left( 9 \cdot \frac{0.118 \cdot (1+0.257)}{0.118(1+0.257) + 0.088 \cdot (1+0.409) + 0.102 \cdot (1+0.254) + 0.109 \cdot (1+0.495) + \dots + 0.163 \cdot (1+0.450)} \cdot 4.00 \right)^1 \cdot \right. \\ + \left( 9 \cdot \frac{0.118 \cdot (1+0.257)}{0.118(1+0.257) + 0.088 \cdot (1+0.409) + 0.102 \cdot (1+0.254) + 0.109 \cdot (1+0.495) + \dots + 0.163 \cdot (1+0.450)} \cdot 4.00 \right)^1 + \\ \left( 9 \cdot \frac{0.088 \cdot (1+0.409)}{0.118(1+0.257) + 0.088 \cdot (1+0.409) + 0.102 \cdot (1+0.254) + 0.109 \cdot (1+0.495) + \dots + 0.163 \cdot (1+0.450)} \cdot 5.63 \right)^1 \cdot \\ \left( 9 \cdot \frac{0.088 \cdot (1+0.409)}{0.118(1+0.257) + 0.088 \cdot (1+0.409) + 0.102 \cdot (1+0.254) + 0.109 \cdot (1+0.495) + \dots + 0.163 \cdot (1+0.450)} \cdot 5.63 \right)^1 + \\ + \dots + \\ \left( 9 \cdot \frac{0.163 \cdot (1+0.450)}{0.118(1+0.257) + 0.088 \cdot (1+0.409) + 0.102 \cdot (1+0.254) + 0.109 \cdot (1+0.495) + \dots + 0.163 \cdot (1+0.450)} \cdot 5.85 \right)^1 \cdot \\ \left( 9 \cdot \frac{0.163 \cdot (1+0.450)}{0.118(1+0.257) + 0.088 \cdot (1+0.409) + 0.102 \cdot (1+0.254) + 0.109 \cdot (1+0.495) + \dots + 0.163 \cdot (1+0.450)} \cdot 5.85 \right)^1 \Bigg)^{\frac{1}{1+1}}$$

$$= 4.124$$

The remaining segments are calculated in the same manner:  $IRNWPHA^{1,1}(n_1^U(C1); n_1^U(C2), \dots, n_1^U(C9))$ ,  $IRNWPHA^{1,1}(n_1^L(C1); n_1^L(C2), \dots, n_1^L(C9))$

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

and  $IRNWPFA^{1,1}(n_1^U(C1); n_1^U(C2), \dots, n_1^U(C9))$ , so  $IRN H(n_1) = ([4.12, 6.18], [4.96, 7.05])$  is obtained. Thus, the remaining values  $IRN H(n_i)$  from Table 4 are obtained. For an easier ranking of the alternatives, the IRN values  $IRN H(n_i)$  were transformed into crisp  $H(n_i)$  values, and the following rank of the advisors was defined:  $A2 > A4 > A5 > A3 > A10 > A8 > A7 > A6 > A1 > A9$ .

The previous research (Pamucar et al., 2018) showed that changes in the  $p$  and  $q$  parameters had led to changes in the structure of the Heronian function, which further led to changes in the values of the decision model. Since it is inevitable that there is an influence of the parameters  $p$  and  $q$  on the results of the functions, it is necessary to check their influence on the results of the model. The initial rank shown in Table 4 was obtained based upon the values of the parameters  $p=q=1$ . In the next part, two scenarios were formed. In the first scenario, the influence of changing the parameter  $p$  in the interval  $p \in [1, 100]$  was considered, while the value of the parameter  $q$  did not change ( $q=1$ ), see Figure 1.

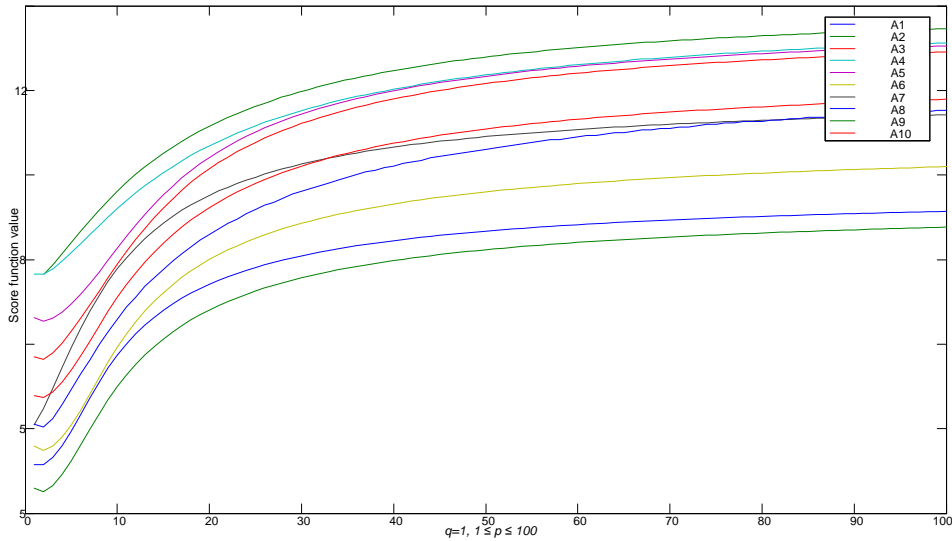


Figure 1. The influence of the parameter  $p$  on the ranking results

In the second scenario, the influence of changing the parameter  $q$  in the interval  $q \in [1, 100]$  was considered, while the value of the parameter  $p$  did not change ( $p=1$ ), see Figure 2.



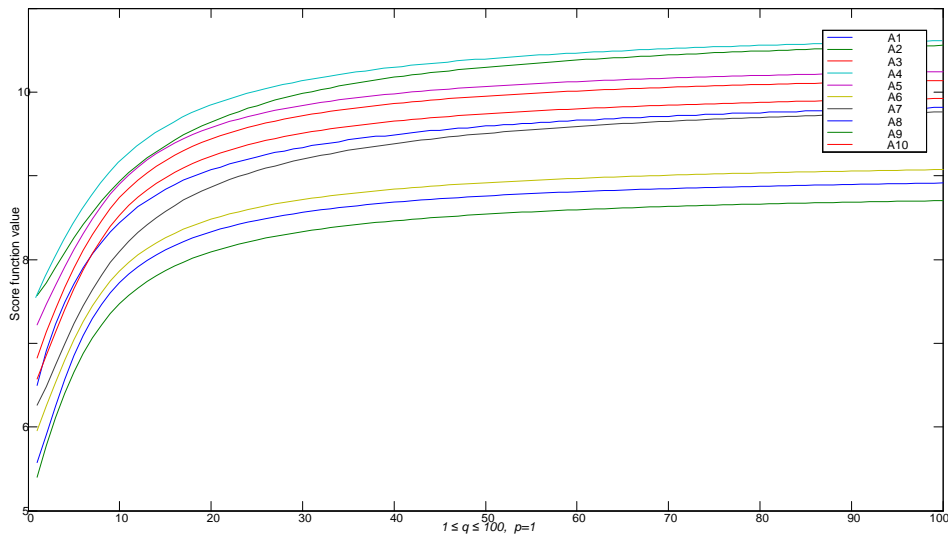


Figure 2. The influence of the parameter  $q$  on the ranking results

Both scenarios confirmed the expectations that a change in the values of the parameters  $p$  and  $q$  leads to a change in the values of criterion functions. Also, with a change in parameter values, the calculation of criterion functions becomes more complicated, since a larger number of mutual connections between criteria are simultaneously considered. Both scenarios showed that, when the values of the parameters  $p$  and  $q$  changed, there were minor changes in the ranks of the considered alternatives. According to Figures 1 and 2, it is also clear that there are no changes in the ranks of the first four ranked alternatives (A2, A4, A5 and A3). From this, it can be concluded that there is a satisfactory advantage between the considered alternatives, and that the alternatives A2 and A4 stand out as dominant from the considered set. Based on all the above-said, it is possible to conclude that the obtained rank  $A2 > A4 > A5 > A3 > A10 > A8 > A7 > A6 > A1 > A9$  is both confirmed and credible.

## 6. Conclusions

The application of the original IRNWPHA multi-criteria model for the evaluation of advisors in dangerous goods transport is presented. The model modified the weighted Heronian aggregator by using a power aggregator in an interval rough environment. The IRNWPHA multi-criteria model enables objective decision-making in the case of uncertain and imprecise input parameters in the initial decision matrix. Also, the IRNWPHA model allows flexible decision-making and the verification of the robustness of the results through the variation of  $p$  and  $q$  parameters. The IRNWPHA combines the advantages of the PA and WHM operators, and is a powerful decision-making tool characterized by the following features: 1) it eliminates the impact of unreasonable arguments; 2) it takes into account the degree of support between input arguments. and 3) it takes into account the interconnectedness of input arguments.

Since this is a new multi-criteria model, whose application has successfully been demonstrated in real research, it can be concluded that there is justification for the development of the presented methodology. Future research may be based upon

combining the IRNWPFA methodology with other MCDM techniques so as to improve the characteristics of the traditional MCMD methods. Future research may also focus on integrating IRNs with D numbers, which would allow for the creation of reasoning algorithms in uncertainty conditions. At the same time, an approach based upon IRN and D numbers would be a concept for the intelligent management of decision-making processes. In addition, improving the model by using neutrosophic fuzzy values might be an option for further research work.

## References

- Beliakov, G. (2007). Aggregation Functions: A Guide for Practitioners. *Studies in Fuzziness & Soft Computing*, 139-141.
- Beliakov, G., Pradera, A., & Calvo, T. (2007). *Aggregation Functions: A Guide for Practitioners*, Springer-Verlag, Berlin.
- Bonferroni, C. (1950). Sulle medie multiple di potenze. *Bollettino Matematica Italiana*, 5, 267-270.
- Dombi, J.A. (1982). general class of fuzzy operators, the demorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. *Fuzzy Sets and Systems*, 8, 149–163, doi:10.1016/0165-0114(82)90005-7.
- Đorđević, D., Stojić, G., Stević, Ž., Pamučar, D., Vulević, A., & Mišić, V. (2019). A New Model for Defining the Criteria of Service Quality in Rail Transport: The Full Consistency Method Based on a Rough Power Heronian Aggregator. *Symmetry*, 11(8), 992.
- Dutta, B., Guha D., & Mesiar, R. (2015). A model based on linguistic 2-tuples for dealing with heterogeneous relationship among attributes in multi-expert decision making, *IEEE Transactions on Fuzzy Systems*, 23, 1817–1831.
- Ecer, F., & Pamucar, D. (2020). Sustainable supplier selection: A novel integrated fuzzy best worst method (F-BWM) and fuzzy CoCoSo with Bonferroni (CoCoSo'B) multi-criteria model. *Journal of Cleaner Production*, <https://doi.org/10.1016/j.jclepro.2020.121981>
- Hara, T., Uchiyama, M., & Takahasi, S.E. (1998). A refinement of various mean inequalities. *Journal of Inequalities and Applications*, 2(4), 387-395.
- He, Y.D., & He, Z. (2016). Extensions of Atanassov's intuitionistic fuzzy interaction Bonferroni means and their application to multiple attribute decision making. *IEEE Transactions on Fuzzy Systems*, 24(3), 558–573.
- He, Y.D., He, Z., & Wang, G. (2015). Hesitant fuzzy power Bonferroni means and their application to multiple attribute decision making. *IEEE Transactions on Fuzzy Systems*, 23(3), 1655–1668.
- Herrera, F., & Martínez, A. (2000). 2-tuple fuzzy linguistic representation model for computing with words, *IEEE Transactions on Fuzzy Systems*, 8, 746–752.
- Liu, H., & Pei, D. (2012). HOWA operator and its application to multi-attribute decision making. *Journal of Zhejiang Sci-Tech University*, 25, 138-142.

Liu, J. P., Chen, H. Y., Zhou L. G., & Tao, Z. F. (2015). Generalized linguistic ordered weighted hybrid logarithm averaging operators and applications to group decision making, *International Journal of Uncertain Fuzzy Knowledge-Based Systems*, 23, 421–442.

Liu, P.D., Liu J.L., & Chen, S.M. (2018). Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making. *Journal of Operational Research Society*, 69(1), 1–24.

Liu, X., Tao, T., Chen, H., & Zhou, L. (2016). A MAGDM Method Based on 2-Tuple Linguistic Heronian Mean and New Operational Laws. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 24(4), 593-627.

Maclaurin C. A. (1729). second letter to Martin Folkes, Esq.; concerning the roots of equations, with demonstration of other rules of algebra, *Philos Trans Roy Soc London Ser A*, 36, 59–96.

Muirhead, R.F. (1902). Some methods applicable to identities and inequalities of symmetric algebraic functions of n letters. *Proceedings of the Edinburgh Mathematical Society*, 21(3), 144-162.

Pamucar, D., Božanić, D., Lukovac, V., & Komazec, N. (2018). Normalized weighted geometric Bonferroni mean operator of interval rough numbers – application in interval rough DEMATEL-COPRAS. *Facta Universitatis, series: Mechanical Engineering*. 16(2), 171-191

Pamučar, D., Chatterjee, K., & Zavadskas, E.K. (2019). Assessment of third-party logistics provider using multi-criteria decision-making approach based on interval rough numbers. *Computers and industrial engineering*, 127, 383-407.

Pamucar, D., Deveci, M., Canitezd, F., & Bozanic, D. (2020). A Fuzzy Full Consistency Method-Dombi-Bonferroni Model for Prioritizing Transportation Demand Management Measures. *Applied Soft Computing*, 87, <https://doi.org/10.1016/j.asoc.2019.105952>.

Pamučar, D., Deveci, M., Canitezd, F., & Bozanic, D. (2020). A Fuzzy Full Consistency Method-Dombi-Bonferroni Model for Prioritizing Transportation Demand Management Measures. *Applied Soft Computing*, 87, <https://doi.org/10.1016/j.asoc.2019.105952>

Pamučar, D., Mihajlović, M., Obradović, R., & Atanasković, P. (2017). Novel approach to group multi-criteria decision making based on interval rough numbers: Hybrid DEMATEL-ANP-MAIRCA model, *Expert Systems with Applications*, 88, 58-80.

Pamucar, D., Sremac, S., Stevic, Z., Cirovic, G., & Tomic, D. (2019). New multi-criteria LNN WASPAS model for evaluating the work of advisors in the transport of hazardous goods. *Neural Computing and Applications*, 31(9), 5045-5068.

Pawlak, Z. (1982). Rough sets. *International Journal of Computer & Information Sciences*, 11(5), 341–356.

Sinani, F., Erceg, Živko, & Vasiljević, M. (2020). An evaluation of a third-party logistics provider: The application of the rough Dombi-Hamy mean operator. *Decision Making: Applications in Management and Engineering*, 3(1), 92-107.

Sremac, S., Stević, Ž., Pamučar D, Arsić, M., & Matić, B. (2018). Evaluation of a Third-Party Logistics (3PL) Provider Using a Rough SWARA–WASPAS Model Based on a New Rough Dombi Agregator. *Symmetry*, 10(8), 305.

The application of the hybrid interval rough weighted Power Heronian operator in multi-criteria decision-making

Xu Z.-S., & Yager, R.R. (2011). Intuitionistic fuzzy Bonferroni means, *IEEE Transactions on Systems, Man, and Cybernetics B: Cybernetics*, 41, 568-578.

Xu, Y. J., Merigó, J. M., & Wang, H.M. (2012). Linguistic power aggregation operators and their application to multiple attribute group decision making, *Applied Mathematical Modeling*, 36, 5427–5444.

Yager, R.R. (2001). The power average operator. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 31, 724-731.

Yazdani, M., Tavana, M., Pamucar, D., & Chatterjee, P. (2020). A Rough Based Multi-Criteria Evaluation Method for Healthcare Waste Disposal Location Decisions. *Computeras and Industrial Engineering*, 143, 106394. <https://doi.org/10.1016/j.cie.2020.106394>

Yu, D. (2013). Intuitionistic fuzzy geometric Heronian mean aggregation operators. *Applied Soft Computing*, 13, 1235-1246.

Zadeh, L.A. (1965). Fuzzy sets, *Information and Control*, 8, 338-353.

Zhao, S., Wang, D., Liang, C., Leng, Y., & Xu, J. (2019). Some Single-Valued Neutrosophic Power Heronian Aggregation Operators and Their Application to Multiple-Attribute Group Decision-Making. *Symmetry*, 11, 653.

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