The Arrow of Time in Cosmology

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Abstract

Scientific cosmology is an empirical discipline whose objects of study are the large-scale properties of the universe. In this context, it is usual to call the direction of the expansion of the universe the 'cosmological arrow of time'. However, there is no reason for privileging the 'radius' of the universe for defining the arrow of time over other geometrical properties of the space-time.

Traditional discussions about the arrow of time in general involve the concept of entropy. In the cosmological context, the direction past-to-future is usually related to the direction of the gradient of the entropy function of the universe. But entropy is a thermodynamic magnitude that is typically associated with subsystems of the universe: the entropy of the universe as a whole is a very controversial matter. Moreover, thermodynamics is a phenomenological theory. Geometrical properties of space-time provide a more fundamental and less controversial way of defining an arrow of time for the universe as a whole.

We will call the arrow defined only on the basis of the geometrical properties of space-time, independently of any entropic considerations, the 'cosmological arrow of time'. In this paper we will argue that: (i) it is possible to define a cosmological arrow of time for the universe as a whole, if certain conditions are satisfied, and (ii) the standard models of contemporary cosmology satisfy these conditions.

1.- Introduction

Scientific cosmology is an empirical discipline whose object is the study of the large scale properties of the universe. In this context, it is usual to call the direction of the expansion of the universe "cosmological arrow of time": this arrow is global in an expanding model of the universe, but in an expanding-contracting model it has a sectional nature, pointing in different directions at different sections of the evolution. This terminology privileges the "radius" of the universe over other geometrical properties of the space-time for defining the arrow of time. But, what is the reason for accepting this terminology?

On the other hand, the traditional discussions about the arrow of time in general involve the concept of entropy. In the cosmological context, the direction past-to-future is usually related to the direction of the gradient of the entropy function of the universe; hence, the problem is conceived in terms of "ordered" or "disordered" boundary conditions. But entropy is a thermodynamic magnitude: a single value of entropy is compatible with many different configurations of the system. The question is, then, whether there is a more fundamental way of defining an arrow of time for the universe as a whole.

We will call the arrow defined only on the basis of the geometrical properties of space-time, independently of any entropic considerations, "cosmological arrow of time". In this paper we will argue that: (i) if certain conditions are satisfied, then it is possible to define a cosmological arrow of time for the universe as a whole, and (ii) the standard models of contemporary cosmology satisfy these conditions.

2.- What is cosmology?

From the perspective cosmology, the universe is a physical object that must be described in scientific terms[1]. In order to formulate an adequate description, the cosmologist has to use the laws of physics and to construct a model.

a) Laws. Cosmology is one of the more fruitful applications of general relativity: it is commonly accepted that scientific cosmology was born in 1917, when Einstein and de Sitter formulated the first cosmological models for the universe. In general relativity, a *space-time* is a Riemannian manifold, that is, an ordered pair (M, g), where M is a four-dimensional differentiable manifold, and g is a Lorentzian metric for M such that $\nabla g=0$. For any point $x\in M$, M_x is the tangent space to M at x. The set C_x of all vectors $\boldsymbol{v}\in M_x$ such that $g(\boldsymbol{v}, \boldsymbol{v})=0$ is the *null cone* at x. A vector $\boldsymbol{v}\in M_x$ is *timelike* (*null*, *spacelike*) if $\boldsymbol{v}$ lies inside (on, outside) C_x. General relativity also provides the

[1] At present, cosmologists tend to agree about conceiving cosmology as the science of the universe as a whole. However, there are exceptions; for example, towards the end of his book about the origin of the universe, Barrow (1994) claims that the aim of cosmology is to explain the structure of the *observable* universe.

connection between the geometrical properties of space-time and the distribution of energy-matter in the universe, through the well-known *Einstein's field equation* (here with $c=1$):

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The left-hand side of this equation is a geometrical expression: $g=g_{\mu\nu}$ is the *metric tensor*, function of the point $(x^0, ..., x^3)$, $R_{\mu\nu}$ is the *Ricci tensor*[2], R is the *Ricci scalar*, and Λ is the *cosmological constant*[3]. On the right hand side of the equation, $T_{\mu\nu}$ is the *energy-momentum tensor*, which describes the energy and momentum content of matter and radiation, G is the Newtonian gravitational constant, with appropriate units to connect the quantities at both sides of the equation, and the constant 8π is added in order to recover Newton's law of gravity in the Newtonian limit.

b) Models. For a given distribution of matter-energy, the set of field equations can be solved in principle. However, in all except the simplest circumstances, the resulting non-linear equations are prohibitively difficult. For this reason, the cosmologist is forced to introduce some approximations and to work with models, that is, idealized toy-universes for which the field equations can be solved. Of course, empirical adequacy constraints the acceptability of cosmological models: when the laws are applied to the model of interest, their solutions must correspond to physical and astronomical observations. As more realistic situations are considered, the degree of idealization has to be reduced: the models become progressively more complex and, in many cases, it is necessary to appeal to other theories, such as particle physics and quantum gravity.

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[2] The *Riemann curvature tensor*, $R^{\alpha}{}_{\beta\chi\delta}$, has the geometrical interpretation of telling how much the direction of a vector changes when it is parallel-transported around a closed curve –then, it is zero for a flat space-time–. The Ricci tensor is obtained by contracting the first and the third indices of the Riemann tensor, and the Ricci scalar is obtained by contracting the two indices of the Ricci tensor (for details, *cf*. Bergström and Goobar, 1999).

[3] The cosmological constant Λ was introduced by Einstein in order to make a static universe possible; when the Hubble expansion was accepted, Einstein regretted the introduction of this constant, calling it his "great mistake". Since then and for many years, the cosmological constant disappeared from almost all cosmological models; however, it was recently reintroduced as a result of astronomical observations which indicate that its value is non-zero (*cf*. Ostriker and Steinhardt, 1995). At present, the cosmological constant term is not yet conceived as a geometrical term, but rather as a contribution of energy –vacuum energy– which corresponds to negative pressure (*cf*. Bergström and Goobar, 1999).

Cosmology also presupposes two principles, linked with a general idea of simplicity that proscribes the introduction of elements lacking empirical or explanatory consequences. Even though these principles have no empirical content –they are unconfirmable and irrefutable by any possible experience–, they play a relevant methodological role in the construction of cosmological models. These principles are:

- *Uniqueness: There is only one universe; then, space-time is unique*. Topologically, totally disconnected space-times are possible: in this case, an event in one of them is not connected to any event in the other one by a timelike, a lightlike or a spacelike curve. But such a state of affairs would be absolutely unverifiable.
- *Universality: The laws of physics are always and everywhere the same*. It is logically possible that laws might change in time and be different from place to place, but there is no observable evidence for that, and there is no way of discovering such changes in unobservable times and regions of the universe.

Since the birth of scientific cosmology, many models of the universe have been proposed and discarded on observational grounds. At present, there are two scientific hypotheses, based on empirical evidence, which no cosmological model can ignore:

- *The cosmological principle*. This principle asserts that the large scale features of the universe are the same in all spatial directions and at all points of space, that is, the universe is spatially homogeneous and isotropic. The acceptance of the cosmological principle relies on two observational facts. First, apart from local inhomogeneities like stars, galaxies, clusters, etc., on the average matter seems to be smoothly distributed everywhere. Secondly, careful measurements show that the cosmic microwave background radiation is almost isotropic: it deviates from isotropy by about 1 part in 1000 (about 1 part in 10^5 if we eliminate the dipolar effect). Of course, these observational facts do not prove the cosmological principle: it is logically possible that the observed homogeneity and isotropy were due to our special position in the universe. However, as a result of a kind of Copernican principle, our position should not have a particular significance: in this case, the spatial homogeneity and isotropy of the universe as a whole is the best explanation for the observed large scale distribution of matter and radiation.

- *The expansion of the universe*. The universe is not static: the space expands in such a way that galaxies and clusters of galaxies seem to move away from each other with velocities proportional to their relative distance according to $v=H_0d$, where H_0 is the present day value of the Hubble constant. The expansion of the universe is not a directly observable fact, but it is inferred from the observed redshift in the spectra of distant galaxies[4].

Both scientific hypotheses must be taken into account in the construction of cosmological models. Since the universe expands, the worldlines of the galaxies fan outwards. It was first suggested by Hermann Weyl that the distribution of matter in the universe could be described by a bundle of non-intersecting timelike worldlines, diverging from a common point in the past. It is then possible to find a one parameter family of spacelike hypersurfaces which are everywhere orthogonal to the worldlines. This fact permits to choose a reference frame, described as *comoving*, such that the matter is everywhere at rest relative to it. From the comoving frame, the universe –in particular, the microwave background– looks maximally isotropic: an observer at rest in this frame will see the isotropic expansion of the universe.

From these properties it can be shown that the metric of space-time may be represented in the simple *Robertson-Walker* form:

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = dt^2 - a(t)^2\, [dr^2\, (1\text{-}k\, r^2) + r^2\, d\theta^2 + r^2\, sen^2\theta\;\; d\phi^2]$$

where (r, θ, ϕ) are the spherical coordinates, t is the cosmic time, k is a constant factor, and $a(t)$ is a scale factor usually called the "radius" of the universe. The geometry of the three-space hypersurfaces of constant t is determined by the factor k, which is only permitted to take three values, *+1*, *-1* and *0*[5]. With the Robertson-Walker metric, the field equations of general relativity

[4] Historically, there was a great debate about the interpretation of the observed redshift; nevertheless, the theory of the expansion of the universe gained the consensus of the scientific community (for historical details, *cf*. Smith, 1982).

[5] If $k=+1$, the curvature of the three-space is positive –spherical space–; the space is then closed and has finite volume, analogously to the surface of a sphere of two dimensions. If $k=-1$, the curvature is negative and the volume is infinite –hyperbolic space–. $k=0$ corresponds to a flat –Euclidean– space.

can be solved. The corresponding solutions describe the isotropic and homogeneous *Friedmann-Lemâitre-Robertson-Walker models* (*FLRW*, for short), which are the standard models of present day cosmology and provide the basis for the *Big-Bang* theory, that has been very successful for explaining many important features of the observable universe[6].

In conclusion, methodological principles and scientific hypotheses constraint the choice of the acceptable cosmological models. In other words, scientific cosmology is not the science of all possible universes but the study of the universe where we live and which we can study by the methods of empirical sciences. This theoretical context supplies the background for discussing the problem of the arrow of time in cosmology.

3.- Disentangling problems

The problem of the arrow of time owes its origin to the intuitive asymmetry between past and future. We experience the time order of the world as "directed": if two events are not simultaneous, then one of them is earlier than the other one. Moreover, we view our access to past and future quite differently: we remember the past and predict the future. The problem of the arrow of time arises when we seek a physical correlate of this intuitive asymmetry: does physics pick out a preferred sense of time?

The main difficulty to be encountered in answering this question lies in our anthropocentric perspective: the difference between past and future is so deeply rooted in our language and our thoughts that it is very difficult to shake off these asymmetric assumptions. In fact, philosophical discussions around the question are usually subsumed under the label "the problem of the direction of time", as if we could find an exclusively physical criterion for singling out *the* direction of time, identified with what we call "the future". But there is nothing in physics that distinguishes, in a non-arbitrary way, between past and future as we conceive them. It might be objected that physics implicitly assumes this distinction with the use of asymmetric temporal expressions, like

[6] At present, only a small minority of cosmologists do not accept the theory of the Big-Bang (*cf*., for example, Burbidge, 1989).

"future light cone", "initial conditions", "increasing time", and so on. However this is not the case, and the reason can be understood in simple conceptual terms.

Two entities are *formally identical* when there is a symmetry transformation between them that does not change the properties of the system to which they belong or in whose description they are involved. In physics it is usual to work with formally identical entities: the two lobes of a light cone, the two spin senses, etc. When we call two formally identical entities by different names, we are establishing a *conventional* difference between them; this is the case, for instance, when we call the two lobes of a light cone "past lobe" and "future lobe", or the two spin senses "up" and "down". By contrast, the difference between two entities is *substantial* when they are not formally identical: we assign different names to them in virtue of such a difference (*cf*. Penrose, 1979; Sachs, 1987). When asymmetric temporal expressions appear in the discourse of fundamental physics, they are used in a completely conventional way: if we exchanged each of them for its symmetric correlate, the resulting discourse would be indistinguishable from the original one, at least to the extent that the "directionality" of time is not introduced from the outside, that is, from our natural language.

Once this point is accepted, the problem cannot yet be posed in terms of singling out the future direction of time: the problem of the arrow of time becomes the problem of how to find a temporal asymmetry only grounded on physical arguments. But if this is our central problem, we cannot project our independent intuitions about past and future for solving it without begging the question. In spite of the fact that these observations are simple, they are usually ignored in the philosophical discussions. This is particularly evident in the widespread reductionistic attitude regarding the arrow of time, whose ambition consists in identifying or reducing the relation of temporal priority to some lawlike or *the facto* feature of the physical world: it is supposed that there is a non-temporal asymmetric relation R between events such that $R(e_1, e_2)$ holds iff $E(e_1, e_2)$ holds, where E is the temporal relation "is earlier than". For some reductionists, the connection between R and E is a lawlike association. For others, the connection between R and E has a definitional nature. Nevertheless, both approaches rely on assuming our previous intuitions about what "earlier" means. The reductionistic approach has been largely criticized in the literature on this subject (*cf*. Sklar, 1974; Earman, 1974), and this in not surprising. These attempts of solving the problem of the arrow of time are doomed to failure because they are misguided regarding what the problem is: how to find a temporal asymmetry only grounded on physical arguments.

If we want to address the problem of the arrow of time from a perspective purged of our temporal intuitions, we must avoid the conclusions derived from subtly presupposing time-asymmetric notions. As Huw Price (1996) claims, it is necessary to stand at a point outside of time, and thence to regard reality in atemporal terms[7]. This atemporal standpoint prevents us from using the asymmetric temporal expressions of our natural language in a non-conventional way. But then, what does "the arrow of time" mean when we accept this constraint? Of course, the traditional expression coined by Eddington has only a metaphorical sense: its meaning must be understood by analogy. We recognize the difference between the head and the tail of an arrow on the basis of its geometrical properties; therefore, we can distinguish between both senses, head-to-tail and tail-to-head, independently of our particular perspective. Analogously, we will conceive the problem of the arrow of time in terms of *the possibility of distinguishing between the two senses of time on the basis of exclusively physical arguments*.

But since physics comprises many different theories, there are in principle a number of distinct ways of distinguishing between the two temporal senses; in other words, there may be many different arrows of time. Here we are concerned with the possibility of defining an arrow of time for the universe as a whole; therefore, it is particularly relevant what general relativity has to say in this respect, that is, whether and under what conditions it is possible to distinguish between the two senses of time. If general relativity has the resources for establishing such a distinction for the universe as a whole, the resultant arrow will be the cosmological arrow since it will be defined by the geometrical properties of space-time. Of course, even if this problem could be solved, it would still remain the task of finding the relationships between the cosmological arrow of time and the other arrows, in order to determine whether there is a single "master arrow" from which all the other arrows can be derived. But this is a new problem which exceeds the limits of the present paper.

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[7] Our agreement with Price's general proposal of adopting an atemporal viewpoint does not amount to a complete agreement: as the present paper will show, we do not accept the conclusions drawn by Price in the cosmological context.

Another related issue is to know whether the arrow of time refers to a property of time itself or to a property of the arrangement of things in time. From our perspective, this is not a question that must be answered *a priori*: to the extent that it is possible to find a criterion by which to distinguish between the two temporal senses, no matter where such criterion comes from. Nevertheless, in the context of general relativity, both conceptually different cases are closely related: the field equations express the lawlike connection between the geometrical properties of the space-time and the distribution of matter-energy throughout that space-time.

Finally, it is worth while to emphasize the difference between the problem of the arrow of time, as it is considered here, and other issues traditionally subsumed under the label "the problem of the direction of time". In these discussions, the most invoked concepts have been the concepts of irreversibility and of time-reversal invariance. *Time-reversal invariance* is a property of laws: loosely speaking, a law is time-reversal invariant when it is expressed by a differential equation which is invariant under the transformation $t \to -t$ (we will return to this point later). By contrast, *irreversibility* is a property of processes: a process is irreversible if it is always observed in the same temporal order, and never in the inverse one. The problem of irreversibility consists in finding out how irreversible processes can be explained by means of time-reversal invariant laws. This problem may be related, in certain cases, to the problem of the arrow of time, but this fact does not cancel the conceptual difference between them.

4.- Cosmology and thermodynamics

When, in the late nineteenth century, Boltzmann developed the probabilistic version of his theory in response to the objections raised by Loschmidt and Zermelo (for historical details, *cf*. Brush, 1976), he had to face a new challenge: how to explain the highly improbable current state of our world. In order to answer to this question, Boltzmann offered the first cosmological approach to the problem. Since this seminal work, the traditional discussions about the arrow of time in cosmology have usually related the time direction past-to-future to the gradient of the entropy function of the universe: it has been usually assumed that the only way of distinguishing between the two temporal senses is by means of the second law of thermodynamics. Even the authors who admit the existence of many arrows of time, tend to suppose that thermodynamics supply the fundamental criterion for temporal asymmetry: all the other arrows must be derived, in some sense, from the entropic arrow. In a well known article about this subject, John Earman (1974, p.15) points out **"what has been taken as an unquestionable dogma: considerations about entropy are absolutely crucial to every aspect of the problem"**.

One of the most philosophically influential works on the problem of the arrow of time was *The Direction of Time* of Hans Reichenbach. In his classical book, Reichenbach *defines* the future direction of time as the sense of the increase of the entropy of the majority of *branch systems*, that is, systems which become isolated from the main system during certain period. However, Reichenbach perfectly knew Loschmidt's irreversibility and Zermelo's recurrence objections: if a system in a state of less than maximum entropy is overwhelmingly likely to evolve to a state of higher entropy in the future, it is also overwhelmingly likely to have evolved from a state of higher entropy in the past; on the other hand, if in an isolated system any state will be revisited to arbitrarily closeness, entropy cannot be a monotonically decreasing function of time. For these reasons, Reichenbach realized that his definition did not imply the existence of a global time direction for the universe: **"we cannot speak of a direction for time as a whole […] whether there is only one time direction or whether time direction alternate, depends on the shape of the entropy curve plotted by the universe"** (Reichenbach, 1956, pp.127-128). Paul Davies also appeals to the notion of branch systems, but from his own interpretation of Reichenbach's theses. Instead of conceiving, like Reichenbach, branch systems as independent systems whose parallelism regarding entropy increase must be proved, Davies considers that branch systems emerge as the result of a chain or hierarchy of branchings which expand out into wider and wider regions of the universe[8]. Therefore, **"The origin of the arrow of time always refers back to the cosmological initial conditions. There exists an arrow of time only because the universe originated in a less-than-maximum entropy state"** (Davies, 1994, p.127).

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[8] On the basis of a detailed analysis of Reichenbach's argument for parallelism, Skar (1993) concludes that it merely imposes parallelism on the systems without explaining it. In his *Physics of Time Asymmetry*, Davies (1974) also appeals to the notion of branch system, but from a perspective closer to Reichenbach's original view.

Reichenbach and Davies are only two of the many authors, coming from philosophy and physics, who approach to the problem of the arrow of time in cosmology in terms of entropy (*cf*. also Feynman *et.al*, 1964; Layzer, 1975). This approach rests on two assumptions: that it is possible to define entropy for a complete instantaneous cross-section of the universe, and that there is an only time for the universe as a whole. However, both assumptions involve difficulties. In the first place, to confidently transfer the concept of entropy from the field of thermodynamics to cosmology is a very dangerous move. The definition of entropy in cosmology is still a very controversial issue, even more than in thermodynamics: there is not a consensus among physicists regarding how to define a global entropy for the universe. In fact, it is usual to work only with the entropy associated with matter and radiation because there is not yet a clear idea about how to define the entropy due to the gravitational field. But even leaving aside this problem, if the entropy of the universe is considered as an entropy out of equilibrium, there are different definitions for it (*cf*. Mackey, 1989). In the second place, when general relativity comes into play, time becomes a dimension of a four-dimensional structure: it is not yet acceptable to conceive time as a background parameter which, as in pre-relativistic physics, is used to mark the evolution of the system. Therefore, the problem of the arrow of time in cosmology cannot be posed, from the beginning, in terms of the entropy gradient between the two ends of a linear and open time.

Nevertheless, these difficulties are not the main reason for denying the central role played by entropy in the problem of the arrow of time in cosmology: there is a conceptual argument for abandoning the traditional approach. Entropy, as defined by thermodynamics, is a phenomenological property: a given value of entropy is compatible with many different configurations of a system. The question is whether there is a more fundamental property of the universe which allows us to distinguish between both temporal senses. From our perspective, it is possible to address the problem of the arrow of time in cosmology in terms of the geometrical properties of the space-time, independently of thermodynamic arguments. In this sense, we follow Earman's "Time Direction Heresy", according to which the arrow of time, if it exists, is an intrinsic feature of space-time **"which does not need and cannot be reduced to nontemporal features"** (Earman, 1974, p.20). In other words, the geometrical approach has conceptual priority over the entropic approach, since the geometrical properties of the universe are more basic than its thermodynamic properties: the definition of entropy and the calculation of the entropy curve for the whole universe are possible only if the space-time has certain definite geometrical features. Therefore, to insist on entropic considerations for distinguishing between both temporal senses on the cosmological level can only be the result of the reductionistic attitude and its attempt of reducing temporal relations to non-temporal relations between events.

5.- Conditions for a cosmological arrow of time

Let us consider an object like, for example, the pyramid of Keops. Let us call the vertical axis “z-axis”. If we divide the pyramid by a plane parallel to its base and which intersects the middle point of its height, we cannot deny that the object is spatially asymmetric regarding this plane. Of course, this asymmetry does not distinguish between the two spatial senses along the z-axis, because the pyramid could change its orientation in space and because there are many other objects in the universe, in particular, other pyramids not pointing to the same sense. But, what would happen if the whole universe were reduced to the pyramid of Keops? In this case, the own space would be confined to the pyramid-universe. It would be, then, possible to distinguish between both directions along the z-axis: the direction base-to-vertex and the direction vertex-to-base. We could even fix the z-coordinate of a point by measuring its distance to, for instance, the base. In other words, the pyramid-universe is an object, and the geometrical structure of this object as a whole is what establishes the difference between the two senses in one of the dimensions of space.

For the cosmologist, the universe is a four-dimensional spatio-temporal object. As this kind of object, it may be symmetric or asymmetric along the temporal dimension. When the problem of the cosmological arrow of time is considered, the question is to find out whether the universe is asymmetric along the temporal dimension: this temporal asymmetry is what would allow us to distinguish between the two temporal senses. What does it mean that the universe is a temporally asymmetric object? This means that the distribution of matter-energy in space-time is not symmetrically arranged along the temporal dimension. But we know that, according to the field equations, there is a close connection between the distribution of matter-energy and the geometrical properties of space-time. Therefore, the temporal asymmetry of the universe amounts to the asymmetry of the geometrical properties of the space-time along the temporal dimension.

Of course, things are not so simple. Many different space-times, of extraordinarily varied topologies, are consistent with the field equations. And some of them have features that do not permit to define the two senses of time in a global way, or even to speak of a unique time for the universe as a whole. But even if these difficulties were overcome, other questions, not directly derived from the general relativistic features of the universe, must be answered: is it possible to describe a temporally asymmetric universe with time-reversal invariant laws?; if the answer is affirmative, how to decide between the two resultant descriptions, each one of which is the temporal mirror image of the other? In the following sections we will address each one of these matters.

6.- Temporal orientability

In a Minkowski space-time, there are two relevant sets of events relative to each event: the set of the events included in the future lobe of the light cone at the specified event, and the set of those included in its past lobe –where the labels "future" and "past" are conventional–. In general relativity, the metric can always be reduced, in small regions of space-time, to the Minkowski form. However, on the large scale, we do not expect the manifold to be flat because gravity can no longer be neglected. Many different topologies are consistent with Einstein's field equations; in particular, the possibility arises of space-time being curved along the spatial dimension in such a way that the spacelike sections of the universe become the three-dimensional analogous of a Moebius band; in technical terms it is said that the space-time is temporally non-orientable. A space-time is *temporally orientable* if there exists a continuous non-vanishing vector field on it which is timelike with respect to its metric. This definition implies that, in a temporally non-orientable space-time, it is possible to transform a future pointing timelike vector into a past pointing timelike vector by means of a continuous transformation that always keeps timelike vectors timelike (*e.g.* going around a spacelike "Moebius band"); therefore, the distinction between future and past lobes is not drawable on a global level (a classical example of a non-orientable space-time is the elliptic de Sitter model). The temporal orientability of space-time is a precondition for defining a cosmological arrow of time, since if space-time is not temporally orientable, it is not possible to distinguish between two temporal senses for the universe as a whole.

Earman (1974) was one of the first authors who emphasized the relevance of temporal orientability to the problem of the arrow of time. For him, a temporal orientation can be defined in a globally consistent manner only if space-time is temporally orientable, and this is enough to justify a *Principle of Precedence* (PP) according to which: **"Assuming that space-time is temporally orientable, continuous timelike transport takes precedence over any method (based on entropy or the like) of fixing time direction; that is, if the time senses fixed by a given method in two regions of space-time (on whatever interpretation of "region" you like) disagree when compared by means of transport which is continuous and which keeps timelike vectors timelike, then if one sense is right, the other is wrong"** (Earman, 1974, p.22). Of course, PP is not a method for fixing a temporal orientation, but rather it comes into play only after we already have a given method: PP works as an adequacy criterion to which any method must fit[9]. On the basis of PP, Earman argues against Reichenbach's entropy method: it is physically possible and in many cases highly probable that disagreements arise when time senses fixed by the entropy method in two regions of space-time are compared by means of continuous timelike transport; then, either (a) there is not right or wrong about temporal orientation, or (b) the entropy method yields the wrong result somewhere in space-time[10].

However, not all accept that temporal orientability is relevant for the problem of the arrow of time. Geoffrey Matthews (1979) argues that the importance of the global structure of the universe for the solution of the problem has been overestimated. Matthews admits that we must rule out non-temporally orientable space-times if we want to define the arrow of time for the universe as a whole. But, for him, the problem can be posed in exclusively local terms; so,

[9] This remark may seems superfluous, but the point has been misunderstood: Matthews (1979, p.90) misinterprets PP by quoting Earman's principle as **"continuous timelike transport takes precedence over any *other* method"** where Earman actually says **"takes precedence over any method"**.

[10] We mention temporal orientation because Earman and Matthews use this concept, but here we are interested in temporal orientability. We do not formulate the problem of the arrow of time in terms of temporal orientation because this seems to suggest that there is a privileged sense of time. In fact, following Earman's definition of orientation (1974, p.18), the choice of an orientation is the choice of the set of the *future* pointing timelike vectors; but, what does "future" mean in a non-conventional sense?

considerations of general relativity are of little or no importance. Matthews' argument is based on the distinction between global and regional directions of time[11]:

Def: A space-time (M, g) has a *global direction of time* iff:

(i) (M, g) is temporally orientable

(ii) For some $x \in M$, (M, g) has a direction of time at x, that is, there is a non-arbitrary way of choosing the future lobe C_x^+ of the null cone C_x at x

(iii) For all $x, y \in M$ such that (M, g) has a direction of time at both x and y, if the timelike vector $\boldsymbol{u}$ lies inside C_x^+ and the timelike vector $\boldsymbol{v}$ lies inside C_y^+, then $\boldsymbol{u}$ and $\boldsymbol{v}$ have the same direction, that is, the vector resulting from parallel transport of $\boldsymbol{v}$ to x lies inside C_x^+.

Def: A space-time (M, g) has a *regional direction of time* iff there is an open subset U of M such that:

(i) U has compact closure, U^-

(ii) $(U, g/_U)$ has a global direction of time, where $g/_U$ is the metric g restricted to U.

(iii) The complement of U^-, $M - U^-$, is not empty.

On the basis of these definitions, Matthews argues against Earman's Principle of Precedence: PP assumes that if there is any direction of time at all in a space-time, the direction must be a global one; but this assumption rules out the plausible possibility that a space-time has a regional direction of time but not a global one. Such a situation might happen if we use a local criterion for singling out the time direction in different regions but not in the universe as a whole. According to Matthews, this is precisely the case when the method of defining the direction of time is based on local boundary conditions or on non-gravitational physical laws. Therefore, **"the direction of time may only be present in one or two finite regions of spacetime. If this is the case, it would be highly artificial to give the entire spacetime a direction by means of parallelly transported future-pointing vectors"** (Matthews, 1979, p.92). This argument leads Matthews to agree with Reichenbach's implicit rejection of PP: **"only certain sections of time have directions and these directions are not the same"** (Reichenbach, 1956, p.127).

[11] He also introduces definitions for local, strictly regional and strictly global directions of time, but they are not necessary for our discussion.

A space-time may have a regional but not a global arrow of time only if the arrow of time is defined by means of local considerations. In turn, if we exclusively appeal to local considerations, we are giving up the possibility of defining an arrow of time in terms of gravitational laws. This amounts to refuse the arrow of time defined as an intrinsic feature of space-time, in particular, as a *temporal* feature of space-time. Even though Matthews does not explicitly reject the global approach, his argument tends to assume that local criteria offer the appropriate solution to the problem. But this is precisely the assumption that we want to challenge here: local criteria have not priority over global ones; on the contrary, when we want to consider the arrow of time which results from the geometrical properties of space-time, global considerations are indispensable and temporal orientability becomes a necessary condition. The insistence on the local –usually entropic– approach as the right way of solving the problem can only be motivated by the reductionistic position according to which temporal relations between events must be identified with or reduced to non-temporal relations resulting from some feature of the material happenings of the universe.

But we have admitted that there may be arrows of time other than the cosmological arrow, in particular, the arrow defined by the local phenomenological law of entropy increase. Why not? The fact is that, even in this case, temporal orientability cannot be avoided: the possibility of a time arrow pointing to different senses in two regions of space-time is not acceptable for contemporary cosmology. Let us consider the arrow defined by a time-reversal non-invariant law L, which is local in the sense that it is valid in small regions of the space-time. Let us suppose that the senses of the arrows defined by L in two regions of the space-time disagree when compared by means of continuous timelike transport: the trajectory of the transport will pass through a frontier point between both regions. In a region around this frontier point the sense of the arrow will be not univocally defined, and this amounts to a breakdown of the validity of L in the frontier point. But this fact contradicts the methodological principle of universality, according to which the laws of physics are valid in all points of the space-time. The strategy to escape this conclusion would consist in refusing to assign any meaning to the timelike continuous transport. This strategy would only be acceptable if the two regions with different arrows were physically isolated; this amounts to the disconnectedness of the space-time. But this fact would contradict the methodological principle of uniqueness, according to which there is only one universe and completely disconnected space-times are not allowed. These arguments show that the possibility of time's arrows pointing to opposite senses in different regions of the space-time not only demands a local definition of these arrows, but also requires to ignore every global consideration coming from cosmology.

Astronomical observations provide empirical evidence that makes implausible the temporal non-orientability of our space-time. In particular, observational evidence in favor of the standard FLRW models plays the role of indirect evidence for temporal orientability, since the space-times with Robertson-Walker metric are temporally orientable. Of course, all this does not prove that we live in a temporally orientable universe: it is logically possible that the distribution of matter and radiation in the unobservable part of the universe and the correlated curvature of the space-time were such that the FLRW models would loose their applicability. But this is not an alternative seriously considered in contemporary cosmology.

7.- Cosmic time

As it is well known, general relativity replaces the older conception of space-through-time by the concept of space-time, where space and time are inextricably interwinned. In one sense, time becomes a dimension of a four dimensional manifold. But when the time measured by a physical clock is considered, each particle of the universe has its own *proper time*, that is, the time registered by a clock carried by the particle. Since the curved space-time of general relativity can be considered locally flat, it is possible to synchronize the clocks fixed to particles whose parallel trajectories are confined in a small region of space-time. But, in general, the synchronization of the clocks fixed to all the particles of the universe is not possible. Only in certain particular cases all the clocks can be temporally coordinated by means of a *cosmic time*, which also has the features necessary to play the role of the temporal parameter in the evolution of the universe.

This problem can also be posed in geometrical terms. A space-time may be such that it is not possible to partition the set of all events into equivalence classes –disjoint and exhaustive– such that: (i) each one of the disjoint classes is a spacelike hypersurface, and (ii) the hypersurfaces can be ordered in time. This is the case when there are closed or almost closed timelike curves in space-time or, even without closed or almost closed timelike curves, when it is impossible to find a smooth function which attaches to each event a real number, the time of the event, such that the number assigned to e_1 is less than that assigned to e_2 whenever there is a causal signal propagable from e_1 to e_2. In such cases, the space-time is not globally splittable into spaces-at-a-time, that is, into spacelike hypersurfaces each one of which contains all the events simultaneous with each other.

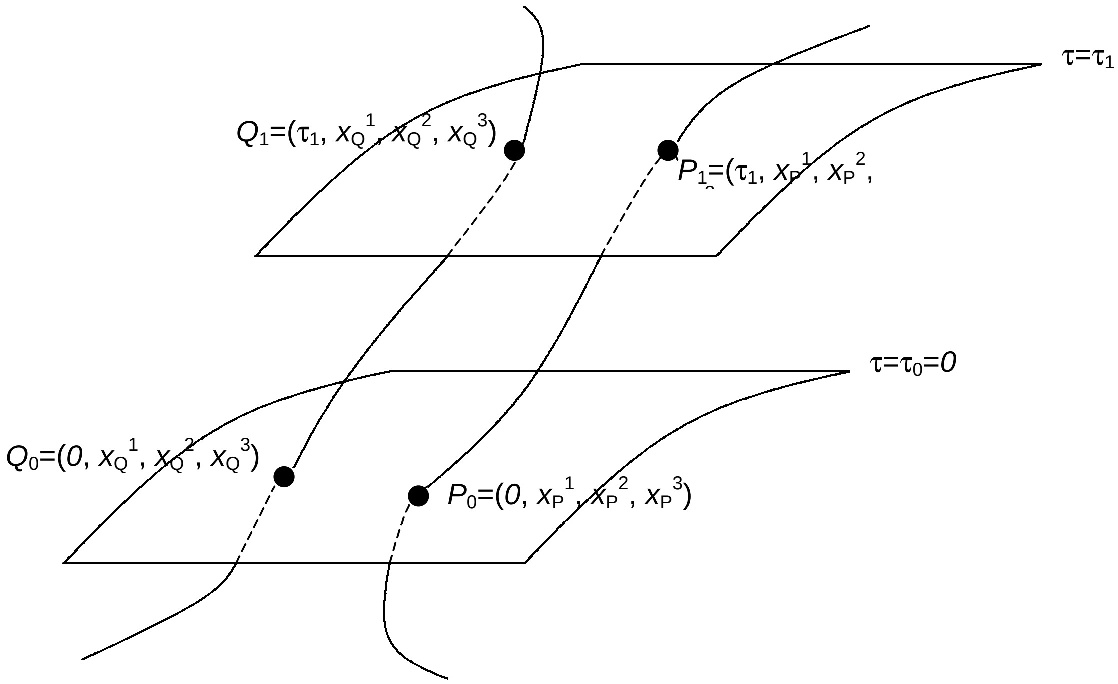
There is a hierarchy of conditions which, applied to a temporally orientable space-time, avoid the "anomalous" features just described (*cf*. Hawking and Ellis, 1973). Taking the space-time (M, g) to be temporally orientable, and using the words “future” and “past” in a conventional way, one can define:

- For $x \in M$, the *chronological future* (*past*) of x, $I^+(x)$ ($I^-(x)$), is the set of all $y \in M$ such that y can be reached from x by a future directed (past directed) timelike curve.
- For $x \in M$, the *causal future* (*past*) of x, $J^+(x)$ ($J^-(x)$), is the set of all $y \in M$ such that y can be reached from x by a future directed (past directed) non-spacelike curve (timelike or lightlike).

The first condition of the hierarchy is the *chronology condition*, which holds on (M, g) iff $\forall x \in M$, $I^+(x) \cap I^-(x) = \varnothing$; this first condition rules out closed timelike curves. The second condition, stronger than the previous one, is the *causality condition*, which holds on (M, g) iff $\forall x \in M$, $J^+(x) \cap J^-(x) = \varnothing$; this condition rules out closed non-spacelike curves. If we also want to exclude cases where there are non-spacelike curves which return arbitrarily close to their point of origin, we must strengthen the requirement imposed on the space-time: the *strong causality condition* holds on (M, g) iff $\forall x \in M$, every neighborhood of x contains a neighborhood of x which no non-spacelike curve intersects more than once. We may also want to exclude situations with spacelike curves which pass arbitrarily close to another spacelike curve which passes arbitrarily close to the origin of the first curve. As Hawking and Ellis (1973) point out, there is an infinite hierarchy of higher degree causality conditions depending on the number and order of the limiting processes involved. Even if the strong causality condition rules out closed or almost closed non-spacelike curves, it does not exclude the possibility that the slightest variation of the metric of the space-time leads again to such temporal “pathologies”. In order to avoid this possibility, the space-time must have some form of stability, defined as a property of “nearby” space-times. The *stable* *causality condition* holds on (M, g) iff strong causality condition holds on (M, g), and for every metric h which is sufficiently close to g, (M, h) has not closed timelike curves[12]. It can be proved that the stable causality condition holds on (M, g) iff (M, g) possesses a *global time function*, that is, iff there is a function t: $M \rightarrow \Re$ whose gradient is everywhere timelike. In other words, the value of the global time function increases along every future directed non-spacelike curve; the existence of such a function guarantees that the space-time is globally splittable into spaces-at-a-time, that is, into hypersurfaces of simultaneity (t=*const.*) which define a foliation of the space-time (*cf*. Schutz, 1980).

However, the fact that the space-time admits a global time function does not yet permit to define a notion of simultaneity in an univocal manner and with physical meaning. Let us consider the set of the worldline curves of all the particles of the universe. Let us also consider a particular foliation, defined by a function $f=\tau$, such that the worldline curves traverse all the hypersurfaces τ=*const*. Now, let us endow the hypersurface corresponding, say, to $\tau=0$ with a system of coordinates (x^i), with $i=1$ to 3. Each worldline curve is labelled by the coordinates (x^1, x^2, x^3) of the point where it intersects the hypersurface $\tau=0$. Now, we endow each hypersurface τ=*const*. with a system of coordinates (x^i) such that the point of intersection between the curve (x^1, x^2, x^3) and the hypersurface also has the coordinates (x^1, x^2, x^3) and this holds for all worldline curves. In this way, we can construct a system of coordinates for the whole manifold which assigns the coordinates (τ, x^1, x^2, x^3) to each point in such a way that, given the curve (x^1, x^2, x^3), τ increases along it and all its points have the same value of the coordinates (x^i).

[12] Of course, in order to give a precise meaning to "close", it is necessary to define a topology on the set of all space-times (*cf*. Hawking and Ellis, 1973).



In this coordinate system, the metric reads:

$$ds^2 = g_{00}\, d\tau^2 + 2\, g_{0i}\, d\tau\, dx^i + g_{ij}\, dx^i\, dx^j$$

where $g_{\mu\nu} = g_{\mu\nu}(\tau, x^1, x^2, x^3)$. It is quite clear that this procedure does not allow us to single out a preferred coordinate system for the whole manifold: we can construct as many coordinate systems as different foliations can be chosen; each foliation defines its corresponding notion of simultaneity. In order to avoid ambiguities in the notion of simultaneity, we must choose a particular foliation. The foliation τ according to which all the worldline curves are orthogonal to all the hypersurfaces τ=*const*. is the proper choice, because orthogonality recovers the notion of simultaneity of special relativity for small regions –tangent hyperplanes– of the hypersurfaces τ=*const*. (for the necessary formal conditions, *cf*. Misner *et al*., 1973). In this case, $g_{0i}=0$ and, then, the metric reads:

$$ds^2 = g_{00}\, d\tau^2 + g_{ij}\, dx^i\, dx^j$$

Nevertheless, this is not yet sufficient for defining a cosmic time which permits to coordinate the proper times corresponding to all the worldlines; this can be seen as follows. The proper time $\Delta\tau$

elapsed between τ_1 and τ_2 along the curve (x^1, x^2, x^3) can be computed as the distance Δs between the hypersurfaces $\tau=\tau_1$ and $\tau=\tau_2$ along such a curve:

$$\Delta\tau \;=\; \Delta s\,(\tau_1, \tau_2, x^1, x^2, x^3) \;=\; \int ds \;=\; \int [g_{00}(\tau, x^1, x^2, x^3)]^{1/2}\, d\tau$$

Since, in general, g_{00} is a function of (x^1, x^2, x^3), the proper time $\Delta\tau$ also depends on (x^1, x^2, x^3). This means that the proper time interval between two hypersurfaces of simultaneity depends on the particular worldline curve considered for computing it. If we want to avoid this situation, we have to impose the constraint $g_{00}=g_{00}(\tau)$: when g_{00} is just a function of τ, the proper time interval between two hypersurfaces $\tau=\tau_1$ and $\tau=\tau_2$ is the same for all worldline curves. Therefore, it is possible to define a *cosmic time t*:

$$dt^2 = g_{00}(\tau)\, d\tau^2$$

In this case, the metric results:

$$ds^2 \;=\; dt^2 + h_{ij}\, dx^i\, dx^j$$

where $h_{ij}=h_{ij}(t, x^1, x^2, x^3)$ is the three-dimensional metric of each hypersurface of simultaneity. Only the existence of a cosmic time guarantees that all the processes of the universe can be coordinated by a single time, recovering the temporal structure of pre-relativistic physics.

Of course, the existence of a cosmic time imposes a significant topological and metric limitation on the space-time. In a completely general case it is not possible to define a cosmic time in terms of which the history of the universe as a whole can be conceived as the temporal sequence of the instantaneous states of the universe. In other words, with no cosmic time there is not a single time which can be considered as the parameter of the evolution of the universe and, therefore, it is nonsensical to speak of the two senses of time for the universe as a whole. This means that the possibility of defining a cosmic time is a precondition for meaningfully speaking of a global arrow of time.

This fact supplies a strong argument against the entropic approach to the problem of the arrow of time in cosmology. By defining the time sense past-to-future in terms of the gradient of the entropy function of the universe, the entropic approach takes for granted the possibility of defining such a function. But this amounts to the assumption that: (i) the space-time can be partitioned in spacelike hypersurfaces on which the entropy of the universe can be defined, and

(ii) the space-time possesses a cosmic time or, at least, a global time on which the entropy gradient can be computed. When the possibility of space-times with no cosmic time is recognized, it is difficult to deny the conceptual priority of considerations about the geometrical structure of space-time over entropic considerations in the context of our problem.

The question about the existence of a cosmic time has not a single answer for all possible relativistic universes. But, what can we say about our universe? Cosmology offers a simple answer on the basis of the cosmological principle and the assumption of expansion. Since the universe is spatially homogeneous and isotropic on the large scale, each point of the space-time has a privileged velocity at which an observer will see isotropic expansion. This is the comoving frame: an observer at rest in this frame can, then, label time by the sequence of states that the universe passes through on the large scale as it expands. And this time can be made common to all points of the space-time because all comoving observers see the same sequence of states. In geometrical terms, it is possible to find a family of spacelike hypersurfaces which can be labelled by the proper time of the worldlines that thread through them. Since the hypersurfaces are orthogonal to all the worldlines, these labels define the cosmic time. In the Robertson-Walker metric, the cosmic time is represented by the variable t, and the scale factor a is a scalar only function of t; this is the time by means of which cosmologists estimate the age of the universe. In this sense, FLRW models recover a notion of time analogous to the conception of pre-relativistic physics, where time is an ordering parameter with respect to which the evolution of a system is described.

8.- Asymmetric universes

As Grünbaum correctly points out, temporal orientability is merely a necessary condition for defining an arrow of time for the universe as a whole, since **“it assumes only the globally consistent mere oppositeness of two senses of time”** (Grünbaum, 1973, p.790). But temporal orientability does not provide a physical, non-arbitrary criterion for distinguishing between both senses of time. It is usually accepted that we would have a straightforward method for defining an arrow of time if physics included some time-reversal non-invariant law. The puzzle seems to arise because the fundamental laws of physics are invariant under time reversal[13]. Nevertheless, this common perspective deserves deeper attention.

In Section 3 we have said that time-reversal invariance is a property that a law possesses when it is invariant under the transformation $t \rightarrow -t$. But this is not a precise characterization of the concept, because it suggests that time-reversal invariance is a syntactic property whose applicability only depends on the formal structure of the corresponding equation. The correct test for time-reversal invariance requires, not only the inversion of the sign of t, but also the inversion of the sign of all the dynamical variables whose definitions are not invariant under the inversion of the sign of t. This means that the concept of time-reversal invariance is not a merely syntactic concept, since the definitions of the involved dynamical variables depend on the theory under consideration. When precisely defined, time-reversal invariance can be characterized in terms of dynamically possible evolutions –solutions of the dynamical laws–. An evolution –sequence of states– $S_i \rightarrow S_j$ is *dynamically possible* relative to the law L if it is consistent with L, that is, if it is a solution of L. Let $\mathbf{T}(S)$ be the time-reversed state of the state S: as we have said, the time-reversal transformation $\mathbf{T}$ depends on the theory to which L belongs. The law L is *time-reversal invariant* when the following situation holds: the evolution $S_i \rightarrow S_j$ is dynamically possible relative to L iff its reversed evolution $\mathbf{T}(S_j) \rightarrow \mathbf{T}(S_i)$ is also dynamically possible relative to L (*cf.* Savitt, 1995)[14].

When characterized in this terms, it is easy to see that the time-reversal invariance of a law does not imply the symmetry of the dynamically possible evolutions described by it. A time-reversal invariant law may be such that all or most of the possible evolutions relative to it are

[13] The only exception seems to be the laws of certain elementary processes which involve the K meson; however, it is usually accepted that the effect of such processes is not relevant for macroscopic temporal asymmetry.

[14] Savitt also introduces a different concept of time-reversal invariance according to which a law L is time-reversal invariant iff should S_i evolve to S_j according to L, then $\mathbf{T}(S_j)$ *must* evolve to $\mathbf{T}(S_i)$ according to L. We will not consider this new concept because it is only relevant in quantum contexts.

individually asymmetric[15]. Huw Price (1996, p.88) refers to this fact as a **"loophole which allows a symmetric physical theory to have asymmetric consequences"**; he illustrates this point with the familiar analogy of a factory which produces equal numbers of left-handed and right-handed corkscrews: the production as a whole is completely unbiased, but each individual corkscrew is spatially asymmetric. As Steven Savitt (1996) says, the point may be more clearly posed in terms of the models of a theory: a theory *T* is time-reversal invariant if the time-reverse **T**(*m*) of every dynamically possible model *m* of *T* is also a model of *T*; the loophole consists in the fact that a time-reversal invariant theory may have temporally asymmetric models.

Of course, this loophole is not helpful for solving the problem of the arrow of time when we are dealing with a multiplicity of systems. Even if all the dynamically possible evolutions –relative to the time-reversal invariant law *L*– were asymmetric, for each system which evolves according to $S_i \rightarrow S_j$ there might be a system which evolves according to $\mathbf{T}(S_j) \rightarrow \mathbf{T}(S_i)$: this situation would restore the overall symmetry. But when we are studying the whole universe, such a situation is not yet possible. The universe is a single object: there is not another system which restores the symmetry when coupled with it. Therefore, the time-reversal invariance of the global laws which govern the universe on the large scale is not an obstacle to the possibility of describing a temporally asymmetric universe.

It is quite clear that these considerations are not applicable to the field equations as originally stated. However, the existence of a cosmic time allows us to formulate the issue in more familiar terms. Under this condition, Einstein's field equations are time-reversal invariant in the sense that if $h_{ij}(t, x^1, x^2, x^3)$ is a solution, $h_{ij}(-t, x^1, x^2, x^3)$ is also a solution, where, h_{ij} is the three-metric of each spacelike hypersurface of simultaneity. But the time-reversal invariance of the field equations does not prevent us from describing a temporally asymmetric universe with them: each temporally asymmetric solution of the equations represents a universe whose space-time is asymmetric in its geometrical properties along the cosmic time. This idea can also be

[15] This fact has been suggested by other authors; for example, Castagnino and Gunzig (1997, p.2545) point out that **"There are two causes for asymmetry in nature: either the laws of nature are asymmetric or the solutions of the equations of the theory are asymmetric […] If the laws of nature are time-symmetric, essentially the only way we have to explain the time-asymmetry of the universe is to postulate that the space of solutions is not time-symmetric"**.

formulated in terms of the concept of temporal isotropy. A temporally orientable space-time (*M*, *g*) is *temporally isotropic* if there is a diffeomorphism *d* of *M* onto itself which reverses the temporal orientations but preserves the metric *g*. However, when we want to express the temporal symmetry of a space-time having a cosmic time, it is necessary to strengthen this definition: a temporally orientable space-time which admits a cosmic time *t* will be *temporally symmetric* with respect to some spacelike hypersurface $t=\alpha$, where α is a constant, if it is temporally isotropic and the diffeomorphism *d* leaves fixed the hypersurface $t=\alpha$. Intuitively this means that, from the hypersurface $t=\alpha$, the space-time looks the same in both temporal senses. Therefore, if a temporally orientable space-time having a cosmic time is temporally asymmetric, we will not find a spacelike hypersurface $t=\alpha$ which splits the space-time in two "halves", one the temporal mirror image of the other regarding their intrinsic geometrical properties.

When we turn our attention to the standard models of present day cosmology, we find that it is not difficult to apply these concepts. In FLRW models, the temporal asymmetry of space-time may manifest itself in two different ways according to whether the universe has singular points in one or in both temporal extremities: this depends on the values of the factor k and of the cosmological constant Λ. Models of the first type are usually called "Big Bang-Big Chill universes". They are manifestly asymmetric in time: since the scale factor $a(t)$ increases with the cosmic time *t*, there is no hypersurface $t=\alpha$ from which the space-time looks the same in both temporal senses. In the so called "Big Bang-Big Crunch universes", on the contrary, $a(t)$ has a maximum value: therefore, the space-time might be temporally symmetric about the time of maximum expansion regarding its "radius" *a*. This is the case of FLRW models with densities of classical matter-radiation; however, this is just a particular case. When the cosmologist wants to study more complete models (*e. g*. inflationary models), he must add one or many fields to the classical densities. In contemporary cosmology many interesting results are obtained by adding, for instance, scalar fields $\phi_k(t)$ to the FLRW models. In this case, the models are still of the FLRW type because homogeneity is retained, and the time-reversal invariance of the Einstein's field equations manifests itself in the fact that if $[h_{ij}(t, x^1, x^2, x^3), \phi_k(t)]$ is a solution, then $[h_{ij}(-t, x^1, x^2, x^3), \phi_k(-t)]$ is also a solution (*cf*. Halliwell, 1994). When scalar fields are added to a Big Bang-Big Crunch FLRW model, the resulting universes are usually temporally asymmetric because both halves of the space-time about the time of maximum expansion differ in their geometrical properties. In other words, if we call the time of maximum expansion t_{ME}, the scalar factor $a(t)$ may be such that $a(t_{ME}+t) \neq a(t_{ME}-t)$ (*cf.*, for example, the models in Castagnino *et al.*, 2000; 2001). This means that a Big Bang-Big Crunch universe may be a temporally asymmetric object regarding the metric of the space-time: this geometrical asymmetry, only grounded on physical considerations, allows us to distinguish between both senses of the cosmic time, independently of any entropic consideration.

In his survey article about the direction of time, Savitt (1996) points out the difference between two questions involved in the general problem:

- The "*how possible*" question: how is it possible to formulate a temporally asymmetric model of the universe which satisfies the time-reversal invariant laws of physics?
- The "*why probable*" question: what reason is there to suppose that a temporally asymmetric universe is probable?

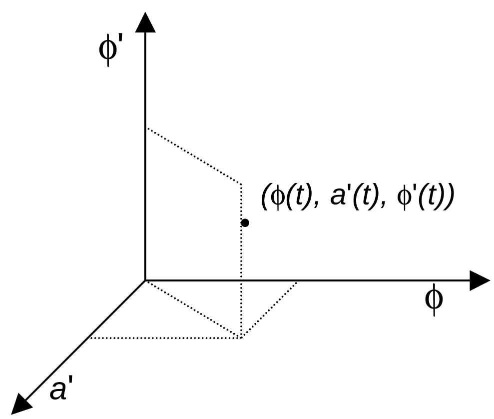
Up to this point, it is quite clear that the time-reversal invariance of the physical laws is not an obstacle to the construction of temporally asymmetric models. However, the "why probable" question remains: it is necessary to supply an argument for the high probability of temporal asymmetry. Of course, such an argument would be superfluous in the case of Big Bang-Big Chill universes, where the temporal asymmetry is manifest. But it is relevant in the case of Big Bang-Big Crunch models: how probable is a universe that is temporally asymmetric regarding the geometrical properties of space-time? By means of a very simple argument, the vanishing probability of perfect geometrical symmetry can be proved. Let us consider a FLRW model coupled with a scalar field $\phi(t)$. The Lagrangian L of the system will be a function $L(a, a', \phi, \phi')$, where a', ϕ' denote the corresponding time derivatives with respect to the cosmic time t. The action S can be computed as the integral of L:

$$S = \int L(a, a', \phi, \phi')\, dt\, dx^i$$

Analogously to the case of classical mechanics, where the Hamilton's principle leads to the Euler-Lagrange equations, in this case $\delta S=0$ leads to second order –in general, non-linear– differential equations whose precise form is not relevant here: we are interested in the functional dependence among the dynamical variables involved in these equations. Such dependence can be expressed:

$$a'' = f_1\,(a, a', \phi, \phi')$$
$$\phi'' = f_2\,(a, a', \phi, \phi')$$

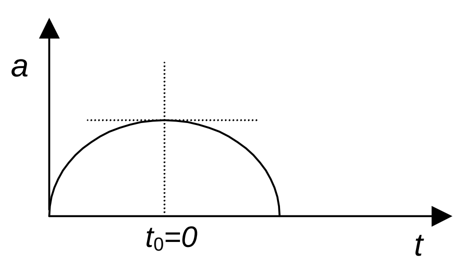
This means that the state of the universe is defined by four variables, a, a', ϕ, ϕ', all of them dependent of the cosmic time t. The Hamiltonian constraint (that is, the Einstein's equation corresponding to $\mu=\nu=0$) $H(a, \phi, a', \phi')=0$, permits to express one of the variables in function of the other three; for instance, $a = F(\phi, a', \phi')$. Therefore, the state of the universe can be represented by a point in a three dimensional phase space with coordinates ϕ, a', ϕ':



Since the equations involved are invariant under time translation, $t \to t+T$ (*cf*., for example, the equations in Castagnino *et al*., 2000; 2001), we can fix the time origin $t_0=0$ in any arbitrary point. We will call the condition at $t_0=0$ "initial condition" –where the term "initial" is used in a conventional sense–. Let us consider a Big Bang-Big Crunch model, and let us choose the time of maximum expansion as $t_0=0$. Then:

$$a'(0) = 0$$

If we want to construct a temporally symmetric model, the scale factor $a(t)$ must be symmetric about the time of maximum expansion:



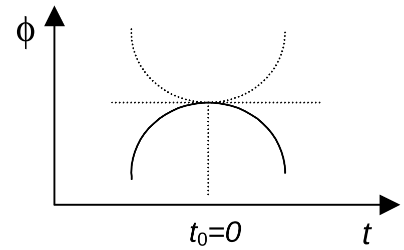
$$a'(0) = 0$$
$$a(t) = a(-t)$$

The field ϕ must also be symmetric about $t_0=0$; there are only two possible cases:

(a) Even symmetry:

$\phi(t) = \phi(-t)$

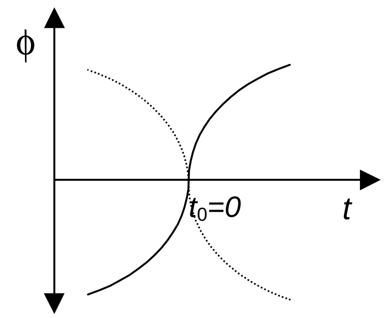
$\phi'(0) = 0$



(b) Odd symmetry

$\phi(t) = -\phi(-t)$

$\phi(0) = 0$



In order to find the probability of temporal symmetry, we will consider the space of the initial conditions that lead to symmetric universes. All the possible initial conditions are $(\phi(0), a'(0), \phi'(0))$: they are represented by the points belonging to the whole phase space. The initial conditions that lead to symmetric universes are: in case (a), $(\phi(0), 0, 0)$, represented by the points belonging to the ϕ–axis; in case (b), $(0, 0, \phi'(0))$, represented by the points belonging to the ϕ'–axis. Therefore, the space of all possible initial conditions has dimension *3* and the space of the initial conditions that lead to symmetric universes has dimension *1*. If we now consider the complete solution corresponding to each initial condition that leads to a symmetric universe, we find that the space of symmetric solutions has dimension *2*. In fact, if we construct the symmetric solutions passing through the points of the set $(\phi(0), 0, 0) \cup (0, 0, \phi'(0))$ of initial conditions previously obtained, the result will be a *2*-dimensional surface embedded in the *3*-dimensional phase space. This argument can be generalized to the case where ϕ is an *n*-components field of matter-radiation or to the case of many *n*-components fields. If we conceive the natural Lebesgue measure on the phase space –or any measure absolutely continuous with respect to it[16]– as a measure of probability, then our argument proves that temporally symmetric universes have probability zero regarding the reference class of all possible universes compatible with a Big Bang-Big Crunch FLRW model with scalar fields. In other words, geometrical time-symmetry is a very specific feature which demands an overwhelmingly improbable fine-tuning of all the state variables of the universe.

In summary, the widespread interest in entropy for solving the problem of the arrow of time is usually based on the implicit assumption that only a time-reversal non-invariant law may explain the difference between the two senses of time. The discussion in the present section has shown that this assumption is misguided: when we consider the whole universe, temporal asymmetry can be explained in terms of time-reversal invariant laws, as an intrinsic feature of space-time. In the standard models of present day cosmology it is possible to obtain this asymmetry, geometrical in nature, with no reference to the asymmetric behavior of the entropy of the universe.

9.- The ontological status of time

As we have seen, the traditional discussions on the problem of the arrow of time usually focus on seeking a physical criterion by which past and future can be distinguished; such a criterion is supposed to be found in a time-reversal non-invariant law or in some *de facto* temporal asymmetry of physical processes. Both approaches need the assumption that we have access to the difference between past and future independently of physical considerations and on the basis of our intuitive experience of the "flow" of time. But when we want to find an arrow of time only grounded in physical arguments, we must avoid any non-conventional distinction between past and future based on our inner perception of time. From an atemporal standpoint, the problem of the arrow of time becomes the problem of distinguishing between the two senses of time by means of exclusively physical arguments.

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[16] A measure μ' is *absolutely continuous* with respect to a measure μ in the phase space X iff for any measurable set $A \subseteq X$, $\mu(A)=0$ implies $\mu'(A)=0$.

We have also pointed out that the emphasis on the second law of thermodynamics owes its origin to the assumption that only a time-reversal non-invariant law can provide the appropriate starting point for solving the problem of the arrow of time. The lawlike and the *de facto* approaches share this assumption; the difference between them lies in the fact that the latter one attempts to reduce the second law to the fundamental laws of mechanics –classical on quantum– plus some restriction on initial conditions. However, as we have argued, when we are concerned with the description of the whole universe, time-reversal invariant laws are not an obstacle to the solution to the extent that –under the condition of existence of a cosmic time– they allow us to describe a temporally asymmetric universe where the two senses of time can be distinguished.

However, we know that, if t is the cosmic time, then if $[h_{ij}(t, x^1, x^2, x^3), \phi(t)]$ is a solution of the Einstein's field equations, $[h_{ij}(-t, x^1, x^2, x^3), \phi(-t)]$ is also a solution. In other words, we obtain two solutions, each one of which is the temporal mirror image of the other, that are both possible relative to the laws of general relativity. At this point, the ghost of symmetry seems to threaten again. Now we are committed to supplying a non-conventional criterion for picking out one of both nomologically admissible solutions. However, the threaten may be surmounted when the implicit assumptions involved in the problem are explicitly stated.

When we accept the need of choosing one between both solutions, we are assuming that each one of them describes a different possible universe: there are two possible objects, and we must decide which one of them corresponds to our actual universe. But why the two possible universes are different? The obvious answer is: they are different because they are arranged in time in opposed senses. This answer would be acceptable if we were trying to describe a system immersed in an environment: this background would allow us to distinguish between both solutions and to decide which one of them adequately describes the system of interest. But when the system is the whole universe, there is no environment. The question is, then: what does it mean that both possible universes are temporally opposed?

The very statements "two possible universes are *temporally opposed*" or "two possible universes are arranged *in time in opposed senses*" presuppose that there is a single time, common to both possible universes, with regard to which we can say that one is opposed to the other. But to suppose that we can meaningfully talk about a time common to two possible universes, as if there were time "outside" the universe itself, is completely contrary to the standard interpretation of general relativity. According to this interpretation, time –or better, space-time– coexists with the universe: it is not yet admissible to distinguish between the universe and the space-time in the same way as we divide between contained and container. In other words, each universe has its own space-time, and there is not a *temporal* viewpoint, external to both possible universes, from where we can compare them. If this point is accepted, it makes no sense to say that, when we obtain two solutions of the field equations, one the temporal mirror image of the other, they describe two possible universes whose difference consists in being opposed in time: on the contrary, such solutions are different but *equivalent descriptions of one and the same possible universe*[17]. Therefore, the ghost of symmetry disappears: it is not yet necessary to find a non-conventional criterion for selecting one of two nomologically admissible solutions since both are descriptions of a single possible universe.

Notwithstanding its plausibility, this equivalence thesis has been vigorously challenged by Earman (1974), who puts forward a number of arguments for discrediting it. Let us present the arguments in Earman's terms. He defines time-reversal invariance by means of the formal concept of model: a consistent theory T is *time-reversal invariant* just in case $\mathbf{T}(M_T)=M_T$, where M_T is the set of all dynamically possible models of T, and the time-reversal transformation $\mathbf{T}$ is a biyective map $\mathbf{T}: M_T \rightarrow M_T$ such that $\forall m \in M_T$, $\mathbf{TT}(m)=m$. On this basis, Earman rejects the claim that, if T is time-reversal invariant, then m and $\mathbf{T}(m)$ are equivalent descriptions of one and the same universe.

Earman's attack on the equivalence thesis runs along two main lines of argument. The first of them relies on comparing time-reversal invariance with invariance under charge conjugation **C** and mirror image reflection **P**. Earman argues that, since the *CPT* theorem applies in quantum field theory, who accepts the equivalence thesis must also accept that m and $\mathbf{CPT}(m)$ are equivalent descriptions of the same universe; for him, this is a clear argument against the equivalence thesis. The question is: why is the non-equivalence between m and $\mathbf{CPT}(m)$ so obvious? The answer is that it is not obvious at all. The same reasons for accepting that m and $\mathbf{T}(m)$ are equivalent descriptions of the same universe operate in admitting that m and $\mathbf{CPT}(m)$ are also equivalent descriptions: there is not a space-time background, common to different

[17] Of course, this is not an original position; it has been assumed by Reichenbach (1956), Gold (1966) ,

universes, where they can be compared regarding temporal orientation and spatial handedness, or where it is possible to compare charge signs through interaction. For Earman, from accepting the equivalence between *m* and **CPT**(*m*) it follows **"that the predicates 'having a positive (negative) charge' and 'having a righthanded (lefthanded) orientation' do not correspond to any objective feature of reality"** (Earman, 1974, p.24). But this is a *non sequitur*: having a positive (negative) charge and having a righthanded (lefthanded) orientation are objective properties within each universe; what the equivalence thesis denies is the existence of a "supra-universal" reality where objectivity is defined and where different universes can be meaningfully compared.

Earman's argument seems to be based on the distinction between continuous and discrete symmetry transformations: spatial translation, spatial rotation and temporal translation are continuous transformations; on the contrary, spatial reflection and time reversal are discrete transformations. Under the active interpretation, a transformation corresponds to a change from one system to another one; under the passive interpretation, a transformation consists in a change of the point of view from which the system is described. The usual position about symmetries assumes that, in the case of discrete transformations, only the active interpretation makes sense: **"The passive interpretation of continuous symmetries, like spatial rotation, is meaningful since one can suppose at least in principle that an idealized observer can rotate himself in space in correspondence with the given spatial rotation [...] But how is an observer, even an idealized one, supposed to 'rotate himself in time'?"** (Earman, 1974, pp.26-27; the same position is presented in Sklar, 1974). This is true when the idealized observer is immersed in the same space-time as the observed system. But, when the system is the universe as a whole, how is the observer supposed to rotate himself *in space* in order to describe the system from a different spatial perspective? What does this mean? Since there is not space outside the space-time, we cannot change our spatial position with respect to the universe: it is so impossible to rotate in space as to rotate in time. But this does not imply that the active interpretation is the correct one: what is the conceptual meaning of the idea of two identical universes, one translated *in space* or *in time* regarding the other? These observations point to the fact that both interpretations of

and Price (1996) among others.

symmetry transformations, when applied to the universe as a whole, collapse into conceptual nonsense. In contemporary cosmology, symmetry transformations are neither interpreted in terms of a change from one system to another one, nor in terms of a change of the observer's point of view. In fact, two mathematical models for the universe, defined by (M, g) and (M', g'), are taken to be equivalent if they are *isometric*, that is, if there is a diffeomorphism θ: $M \rightarrow M'$ which carries the metric g into the metric g' (Hawking and Ellis, 1973). In particular, symmetry transformations are isometries: models related by a symmetry transformation are isometric. Therefore, two models m and $\mathbf{S}(m)$, where $\mathbf{S}$ is any symmetry transformation, are considered as equivalent descriptions of one and the same universe[18]. In short, this discussion emphasizes the importance of proceeding with great caution when conclusions about finite portions of the universe are extrapolated to the universe as a whole: our philosophical convictions must be revised in moving from a local context to the cosmological level.

In his second line of attack on the equivalence thesis, Earman points to the fact that time-reversal invariance plays no role in accepting the equivalence between m and $\mathbf{T}(m)$. His argument goes as follows. Let us consider a theory T which is strictly non-invariant under time reversal, that is, for every kinematically possible model m of T, at most one of the pair m, $\mathbf{T}(m)$ is in the set M_T of dynamically possible models –models that satisfy the laws of T–. In this case we can construct a theory $T^*=\mathbf{T}(T)$ which is just like T except that T^* reverses the symmetry of T with respect to $\mathbf{T}$: in this case, if $m\in M_T$, then $\mathbf{T}(m)\in M_T{}^*$, and if $m\in M_T{}^*$, then $\mathbf{T}(m)\in M_T$. Therefore, who wants to maintain the equivalence thesis must accept that T and T^* provide equivalent descriptions, that is, descriptions which describe in different terms the very same physical situation. Since Earman rejects the equivalence between T and T^*, he concludes that **"considerations which would lead us to say that T is true while $\mathbf{T}(T)$ is false or vice versa are paralleled by considerations which in the case of a time reversal invariant theory T would lead us to say that for some $m\in M_T$, m but not $\mathbf{T}(m)$ or vice versa is the true description of the actual world"** (Earman, 1974, p.25).

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[18] It is worth noting that Earman himself seems to have changed his mind when, in the context of the "hole argument", formulates what he calls the "*Leibniz equivalence*", according to which diffeomorphic models represent the same physical situation. The denial of this principle is **"at odds with standard modern texts in general relativity, in which this equivalence is accepted unquestioningly in the specific case of manifolds with metrics"** (Earman and Norton, 1987, p.522).

In a nutshell, Earman is claiming that the equivalence thesis is too powerful: the equivalence between *m* and **T**(*m*) could be maintained whether or not the laws of physics were time-reversal invariant. What can we say to respond to this objection? The answer is: nothing. Earman's observation is completely correct, but it does not undermine the equivalence thesis. Let us suppose that the theory *T* consists of a time-reversal non-invariant law *L*, according to which some magnitude α monotonically *increases* with time; this means that there is a model $m \in M_T$ of the universe such that $\mathbf{T}(m) \notin M_T$. Now we can construct a theory $T^* = \mathbf{T}(T)$ consisting of a time-reversal non-invariant law $L^* = \mathbf{T}(L)$ which states that the magnitude α monotonically *decreases* with time; of course, if $m \in M_T$, then $\mathbf{T}(m) \in M_T^*$. The question is: how to decide whether *T* or **T**(*T*) is, in Earman's terms, "true"? We will consider that, say, *T* is true and **T**(*T*) is false because we know, by means of our observation of the processes in the world, that α increases toward the *future*. But this conclusion is based on our perception of the "sense" of time, perception independent of any theoretical consideration: only this previous intuition of the asymmetry between past and future allows us to decide between *T* and **T**(*T*). However, when the problem of the arrow of time is the question at issue, we cannot appeal to our inner perceptions: we must stand at a neutral point with regard to time. As Price (1996) claims, from this atemporal standpoint we are not entitled to state that α increases or decreases with time; we can only say that there exists an unidirectional gradient of α between the two extremities of the universe, and such a gradient is adequately explained by both *T* and **T**(*T*). Therefore, when we do not independently presuppose the "correct" sense of time, there is no difference between the physical situations described by *T* and **T**(*T*). In summary, the acceptance of the equivalence thesis does not depend on the time-reversal invariance of the physical laws, but rather on the fact that we are describing the universe as a whole without any appeal to a privileged sense of time: *T* and **T**(*T*) are equivalent theories whether or not *T* is time-reversal invariant.

Let us summarize the argument of this section from the start. As we have seen, the time-reversal invariance of the physical laws is not an obstacle to the possibility of describing a temporally asymmetric universe where the two senses of time can be distinguished. However, under the condition of the existence of a cosmic time, there are two solutions of the field equations, each one of which is the temporal mirror image of the other; therefore, now the problem is to supply a non-conventional criterion for choosing one of the two solutions without appealing to our inner perception of time. The way out of this problem is to accept the equivalence thesis, according to which the two solutions are different but equivalent descriptions of one and the same universe. Then we considered Earman's arguments against the equivalence thesis. In his first argument, Earman compares time-reversal invariance with CPT-invariance, and appeals to the usual rejection of the passive interpretation of discrete symmetries. With regard to this argument, we pointed out that both active and passive interpretations of symmetry transformations are nonsense when applied to the universe as a whole, because there is not a "supra-universal" space-time from where an observer can change his point of view for describing a single universe or two possible universes can be compared. In his second argument, Earman claims that the acceptance of the equivalence thesis would lead to the unacceptable conclusion that a theory T and its time-reversed theory $\mathbf{T}(T)$ provide equivalent descriptions of the same physical situation. In this case we accepted Earman's inference but rejected the unacceptable character of the conclusion: even if the theory T is not time-reversal invariant, the only way of deciding whether T or $\mathbf{T}(T)$ is "true" is by appealing to our pre-theoretical perception of past and future, but this perception must play no role when the problem of the arrow of time is the question at issue. In short, Earman's arguments are not enough for undermining the equivalence thesis, which is necessary for dissolving the problem of the choice between two nomologically admissible solutions of the cosmological equations.

Finally, it is worth while to emphasize that our acceptance of the equivalence thesis is not based on the reductionistic position criticized in previous sections: we are not claiming that m and $\mathbf{T}(m)$ are equivalent descriptions because temporal relations are grounded in non-temporal features of the universe; on the contrary, we have argued that the arrow of time, when it exists, arises from the very structure of space-time. The only element which leads us to the equivalence thesis is an assumption about the ontological status of space-time –and, *a fortiori*, of time–: space-time is not ontologically prior to the universe; the universe is not "placed" in space-time because there is not space-time outside or independent of the universe. This assumption, accepted unquestioningly by the cosmological community, is quite weak; it does not imply the adoption of a relationalist account of space-time: the substantivalist-relationalist debate involve not only questions about the ontological status of space-time, but also issues regarding the nature of motion. Moreover, to deny the ontological priority of space-time over the universe does not mean to assert the ontological priority of the events of the universe over space-time. In this sense, we agree with Earman (1989) when he concludes his careful study of the substantivalist-relationalist controversy declaring that general relativity will surely lead us to a conception of space-time that fits neither traditional relationalism nor traditional substantivalism.

10.- The relevance of time-reversal invariance

The arguments developed in the previous sections are useful for disentangling some questions frequently identified in the traditional discussions. In particular, they allow us to realize that, in addressing the issue of the arrow of time on the cosmological level, we must face two conceptually different problems, usually not distinguished enough in the literature. The "symmetry problem" consists in obtaining a temporally asymmetric model of the universe where the two senses of time can be distinguished (Section 8). Everybody agree that there is no difficulty in constructing temporally asymmetric models by means of time-reversal non-invariant laws; the problem seems to be how it is possible to describe a temporally asymmetric model by means of time-reversal invariant laws. However, this is a pseudoproblem that arises from confusing an equation with its solutions: time-reversal invariance is a property of *equations* –or *laws*–, temporal symmetry is a property of *solutions* –or *models*–, and nothing prevents from obtaining a temporally asymmetric solution by means of a time-reversal invariant equation. On the other hand, the "choice problem" consists in providing a non-conventional criterion for deciding between the two solutions, each one of which is the temporal mirror image of the other (Section 9). As we have argued, this problem vanishes when we realize that we are not committed to supplying such a criterion to the extent that both solutions are equivalent descriptions of the same possible universe.

In turn, these considerations show that the time-reversal invariance of the physical laws is not so essential to the problem of the arrow of time in cosmology as it has been traditionally assumed. With respect to the "symmetry problem", time-reversal invariance does not prevent from constructing temporally asymmetric models. On the other hand, the "choice problem" arises whether or not the laws describing the two temporally "opposed" models are time-reversal invariant. In fact, even a time-reversal non-invariant law or theory will have two models, one the temporal mirror image of the other: the need of deciding between them is independent of the features of the theory by means of which they were constructed. In other words, the "choice problem" is the result of the peculiar feature of a pair of models of a theory, and not of the property of time-reversal invariance of the theory. In summary, the problem of the arrow of time in cosmology requires that we focus our attention on the properties of the *models* of a physical theory: the properties of the theory itself –in particular, its time-reversal invariance or non-invariance– are rather less crucial to the problem than the traditional discussions suggest.

11.- Conclusions

In this paper we have addressed the possibility of defining an arrow of time for the whole universe, only based on the intrinsic features of space-time. As we have seen, when we approach the issue from an atemporal standpoint, the problem consists in establishing whether and under what conditions it is possible to construct a model of the universe which results temporally asymmetric regarding the geometrical properties of space-time; in such a model, we can distinguish between the two senses of time exclusively on the basis of geometrical considerations, with no reference to entropic arguments or to the distinction between past and future grounded in our subjective perception of time. Our inquiry has led us to conclude that, contrary to what is usually supposed, the time-reversal invariance of physical laws is not an obstacle to the definition of a cosmological arrow of time when a minimal assumption about the ontological status of space-time is accepted. However, this does not mean that this arrow of time will emerge in all possible universes described by general relativity: the existence of a cosmological arrow is constrained by the orientability of space-time and the possibility of defining a cosmic time. As we have seen, standard models of contemporary cosmology fulfill both requirements; if we accept that these models describe, even approximately, the actual universe, then we can conclude that the structure of the universe we inhabit introduces an intrinsic difference between the two senses of the cosmic time.

Nevertheless, this particular feature of our universe does not cancel the fact that the cosmological arrow of time is not an inevitable result of the laws of general relativity. We will not enter here into the traditional discussion about the nomological or the *de facto* nature of the arrow of time, but we only want to stress a conceptual point: even if the cosmological arrow of time is considered as an emergent property of space-time, its emergent nature does not undermine its objectivity (for a discussion about the objective character of emergent properties, *cf*. Lombardi, 2002). In other words, the fact that the cosmological arrow of time arises as a consequence of the particular features of our universe does not imply that the difference between the two senses of time is a non-objective illusion: even though it is not an inevitable consequence of the laws of physics, the cosmological arrow is a result of the objective properties of our universe, that is, properties that does not depend on our means of empirical access to reality or on our particular perspective as observers.

Of course, our conclusions leave the question of relating the cosmological arrow of time with other physical arrows as an open issue. The general strategy for approaching this problem should be based on inquiring how the different kinds of physical processes are described in terms of a temporally asymmetric space-time. This is a task which largely exceeds the purposes of the present paper (for a treatment of the subject in physical terms, *cf*. Castagnino and Gunzig, 1997, 1999). However, a basic element of our proposal should still be valid: whatever arrow we are interested in defining, the problem will consist in finding a difference between the two senses of time only grounded on physical arguments.

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