## The Athena Temple at Paestum and Pythagorean Theory

## Ned Nabers and Susan Ford Wiltshire

HE LACK of contemporary evidence for Pythagorean activity in southern Italy during Pythagoras' own time there, roughly 532/1 to 494/3 B.C.,<sup>1</sup> causes us to examine with special interest the case for Pythagorean qualities in the design of the temple of Athena at Paestum, which is commonly dated to around 510 B.C. In a study published in 1958 H. Kayser attempted to demonstrate that the number theories of Pythagorean philosophy influenced the design of the temple.<sup>2</sup> Although the results of his study were questionable at best,<sup>3</sup> the idea that this temple incorporated tenets of Pythagoreanism was taken up again by R. Ross Holloway in 1966.<sup>4</sup> Basing his arguments on the precise measurements of the temple published by F. Krauss,<sup>5</sup> Holloway clearly demonstrated that the number four, the creative principle in Pythagorean thought, and the numbers ten and twenty-four permeate the fundamental design of the temple. We wish to offer here further evidence corroborating this thesis.

The crucial importance of the numbers four, ten, and twentyfour to Pythagorean philosophy derives from the discovery by Pythagoras that intervals in the musical scale could be expressed by mathematical ratios involving the first four integers.<sup>6</sup> Here was empirical evidence, it was felt, for an inherent order in the uni-

<sup>4</sup> Holloway (supra n.3) 60-64; cf. his A View of Greek Art (Providence 1973) 64-68.

<sup>5</sup> Die Tempel von Paestum I.1 (Berlin 1959).

<sup>&</sup>lt;sup>1</sup>G. S. Kirk and J. E. Raven, *The Presocratic Philosophers* (Cambridge 1957) 217ff, review the biographical evidence, most of which is late and obscure. The tradition was that Pythagoras left his native Samos and settled in Croton in 532/1. E. L. Minar, Jr., *Early Pythagorean Politics* (Baltimore 1942) 133–35, suggests 494/3 for his death.

<sup>&</sup>lt;sup>2</sup> Paestum: Die Nomoi der drei altgriechischen Tempel zu Paestum (Heidelberg 1959) 49–60.

<sup>&</sup>lt;sup>3</sup> See R. Ross Holloway, "Architettura sacra e matematica pitagorica a Paestum," *ParPass* 106 (1966) 64 n.10.

<sup>&</sup>lt;sup>6</sup> W. K. C. Guthrie, A History of Greek Philosophy I (Cambridge 1962) 220–22, discusses the attribution of the discovery to Pythagoras and generally accepts its authenticity. The ratios are 1:2 (octave), 3:2 (fifth), and 4:3 (fourth) (Guthrie 223).

## 208 THE ATHENA TEMPLE AT PAESTUM

verse.<sup>7</sup> Ten, as the sum of the first four integers, was considered by the Pythagoreans to be the perfect  $(\tau \epsilon \lambda \epsilon \iota o \nu)$  number, comprising, in Aristotle's terms, the whole nature of numbers  $(\pi a \sigma a \nu \ldots \tau \eta \nu \tau \hat{\omega} \nu \ d\rho \iota \theta \mu \hat{\omega} \nu \ \varphi \upsilon \sigma \iota \nu$ , Metaph. 986a8–9). The four integers were represented graphically as an equilateral triangle with one point at the top, followed by two points and three more under them, then four at the bottom. This figure, called the  $\tau \epsilon \tau \rho a \kappa \tau \upsilon \varsigma$ , was a sacred emblem for the Pythagoreans, who were said to swear by their leader in the following formula: "by him who handed down to us the *tetractys*, source and root of everlasting nature."<sup>8</sup> Further, as the sum of the four numbers of the *tetractys* is ten, so their product is twenty-four.

We begin with the matter of the length of the stylobate of the Athena temple, determined by Krauss to be 100 Doric feet (each measuring 0.328 m.).<sup>9</sup> This of course is not remarkable, because many Greek temples incorporated the measurement of 100 units in their plans.<sup>10</sup> It does raise the problem, however, of the length of the foot used in the construction of the Athena temple, especially since W. B. Dinsmoor has asserted that an Ionic foot of *ca* 0.294 m. was employed at Paestum.<sup>11</sup>

<sup>7</sup> Pythagoras is said to have been the first to attach the term  $\kappa \delta \sigma \mu o \varsigma$  to the natural world, thus affirming the orderliness and arrangement perceptible within it; see Kirk and Raven (*supra* n.1) 229 n.3 for an assessment of the evidence. The implications of this discovery were for the Pythagoreans ethical and religious as well as scientific—by studying and conforming ourselves to the orderly structure of the universe, we ourselves can become  $\kappa \delta \sigma \mu o i$ ; cf. W. K. C. Guthrie, The Greek Philosophers (London 1950) 37–38.

<sup>8</sup> Porph. Vit.Pyth. 20, οὕ, μὰ τὸν ἀμετέρα γενεά παραδόντα τετρακτύν, παγὰν ἀενάου φύσιος ῥιζώματ' ἔχουσαν; cf. Iambl. Vit.Pyth. 150.

<sup>9</sup> Krauss (*supra* n.5) 2. Cf. J. J. Coulton, "Towards Understanding Doric Design: the Stylobate and Intercolumniations," BSA 69 (1974) 81.

<sup>10</sup> Cf. Holloway (supra n.3) 63. One might also point as an example of this practice to the other great sixth-century temple at Paestum, the first temple of Hera, which stands a few hundred meters to the south of the Athena temple. According to Gruben in H. Berve, G. Gruben, and M. Hirmer (Greek Temples, Theatres, and Shrines [New York 1962] 409), that temple measures 100 Ionian ells or cubits (=  $1\frac{1}{2}$  feet) from the axis of one corner column to the axis of the other corner column along its longer sides. This would show that the practice of calculating distances in temples from the axis of one corner column to the axis of another corner column was established at Paestum long before the erection of the temple of Athena.

<sup>11</sup> "The Basis of Greek Temple Design: Asia Minor, Greece, Italy," Atti del Settimo Congresso Internazionale di Archeologia Classica (Rome 1961) 358. If this is so, the principle of 100 units would be negated in the length of the earlier Hera temple (supra n.10) as measured from one corner column axis to another. In order for that idea to be valid a foot of ca 0.351 m. would have to have been used, which corresponds to Dinsmoor's (360) Samian, Ptolemaic, or Philetaeric foot of ca 0.350 m.

Dinsmoor did not give his evidence for such a foot at Paestum but did state that among nine examples of such an Ionic foot in southern Italy averaging 0.29395 m., four came from Paestum and Foce del Sele. He gave some of the dimensions of the three major temples at Paestum in terms of feet ca 0.294 m., including two from the temple of Athena: the diameter of the columns  $(4^{5}/_{16})$ Ionic feet) and the height of the columns (20% Ionic feet).<sup>12</sup> But Dinsmoor's case for such an Ionic foot in the temple of Athena is not argued. His thesis that in the Greek world there were only two widely used feet—an Ionic foot of *ca* 0.294 m. and a Doric foot of ca 0.3265 m.<sup>13</sup>—does not seem to have won wide acceptance. It is prudent here to recall Broneer's statement that "it is essential . . . to bear in mind that in a Doric temple the one basic dimension with immediate bearing on the foot measure is the length of the stylobate."<sup>14</sup> In the Athena temple the stylobate is 32.883 m. long, or 0.083 m. longer than precisely one hundred feet of 0.328 m. The error amounts to one-quarter of one percent and may be compared with the error of up to 0.15% which Dinsmoor found in mechanically manufactured wooden meter sticks sold in modern Athens.<sup>15</sup>

Krauss has demonstrated, furthermore, that a Doric foot of 0.328 m. fits so nicely into some of the other measurements of the building that the burden of proof, we believe, now rests on anyone who would deny that this was the unit of measurement used: the height of the columns of the peristyle is 6.122 m. (or  $18\frac{2}{3}$  Doric feet), the height of the entablature from the top of the abacus to the top of the sima is one-half the column height of 3.062 m. (or  $9\frac{1}{3}$  feet). The width of the triglyph is 0.550 m. (or  $1\frac{2}{3}$  feet), the width of a normal metope is 0.7625 m. (or 3 feet). In the Ionic porch in front of the cella the columns have an interaxial of half that of the Doric columns of the peristyle ( $2.626 \div 2 = 1.313$  m. or 4 feet); their lower diameter is 0.820 m. (or  $2\frac{1}{2}$  feet) and their upper diameter is 0.656 m. (or 2 feet). The height of the architrave is 0.574 m. (or  $1\frac{3}{4}$  feet).<sup>16</sup> Furthermore, as Holloway has ob-

<sup>&</sup>lt;sup>12</sup> Dinsmoor (*supra* n.11) 367.

<sup>&</sup>lt;sup>13</sup> Dinsmoor (*supra* n.11) 355–60.

<sup>&</sup>lt;sup>14</sup> Oscar Broneer, Isthmia I: The Temple of Poseidon (Princeton 1971) 177.

<sup>&</sup>lt;sup>15</sup> Dinsmoor (*supra* n.11) 357. For a different view of the stylobate of this temple see Malcolm Bell, "Stylobate and Roof in the Olympieion at Akragas," AJA 84 (1980) 364.

<sup>&</sup>lt;sup>16</sup> Krauss (supra n.5) 2, 3, 5, 17, and Abb. 43.

served, a foot of 0.328 m. is hardly extraordinary in the Greek world.<sup>17</sup>

The Athena temple has a peculiar characteristic which it shares with only one other Doric temple: all of the interaxials of the peristyle are uniform, on the fronts as well as on the flanks, with no corner contraction for the solution of the Doric angle conflict.<sup>18</sup> While it is certainly normal for a sixth-century Doric temple in southern Italy or Sicily to have no corner contraction, it remains that the Athena temple at Paestum is unique in the archaic period in having uniform interaxials all around the peristyle. And although at the end of the sixth century B.C. it was becoming common practice to have the front and flank interaxials the same, the Athena temple was possibly the earliest example of this practice and is certainly the only Doric temple at this time to combine uniform interaxials with the lack of corner contraction. The only other Doric temple to display this same uniformity of interaxials is the peripteral temple of Apollo on Delos which was begun in the fifth century B.C. In the case of the Apollo temple there is clear explanation for its uniform interaxials: the temple was begun in the Ionic order. Work on it seems to have stopped in 454 B.C. when the treasury of the Delian League was transferred to Athens; when the work was resumed *ca* 315, the design of the peristyle was changed to Doric. Since the lower steps of the krepis had already been laid, however, the joints of the stylobate and the placement of the columns of the peristyle followed the uniform spacing of the joints in the lower steps of the krepis which had anticipated an Ionic peristyle.<sup>19</sup>

Holloway's contribution is to show that the peristyle of the

<sup>17</sup> Holloway (*supra* n.3) 62 n.7.

<sup>19</sup> Dinsmoor (*supra* n.18) 184 and 221.

<sup>&</sup>lt;sup>18</sup> It should be noted here that the interaxials vary slightly but were all obviously intended to be the same. Krauss (*supra* n.5) 3 (*cf.* 17) states: "Die Säulenjoche sind ringsum gleich und betragen im Mittel 2,626 m. Die Säulen stehen senkrecht, der Tempel hat keine Kurvatur." The difference between the widest and the narrowest interaxial is only 0.072 m. (Krauss, Tafel 11). These very small variations between one interaxial and another are quite normal in a limestone temple such as this and occur even in classical marble temples like the Parthenon (*cf.* A. W. Lawrence, *Greek Architecture* [Baltimore 1957] 173 fig. 98). The average interaxial of the Athena temple, as determined by Krauss, is 0.002 m. longer than precisely 8 Doric feet of 0.328 m. E. Lorenzen's tables (*Along the Line Where Columns Are Set* [Copenhagen 1970] 147) show a difference of 0.004 m. between the interaxials on the fronts and those on the façades, but this is not supported by Krauss' careful measurements or by the figures in W. B. Dinsmoor's list (*The Architecture of Ancient Greece* [New York 1950] following p.340).

Athena temple at Paestum is based on a module of four Doric feet, each interaxial being uniformly eight Doric feet (2.626 m.) or twice the basic module, and that the rectangle of the peristyle, measuring from the axes of the corner columns, is 40 (or  $4 \times 10$ ) Doric feet by 96 (or  $4 \times 24$ ) Doric feet.<sup>20</sup> With his analysis Holloway has demonstrated persuasively the Pythagorean basis of the temple design.

The importance of the numbers four, ten, and twenty-four to the fundamental plan of the temple, however, goes even further. On the plan of the temple, and again measuring from column axis to column axis, a triangle formed from any two adjacent sides of the peristyle and a line extending from one corner column to its diagonal opposite (*Figure* 1)<sup>21</sup> will demonstrate Pythagorean qualities. One side of the triangle measures 40 (or  $4 \times 10$ ) Doric feet, another side 96 (or  $4 \times 24$ ), while the hypotenuse will be the square root of  $40^2 + 96^2 = 1600 + 9216 = 10,816$ , or 104 Doric feet or  $10 \times 10 + 4$ —each factor a Pythagorean number.

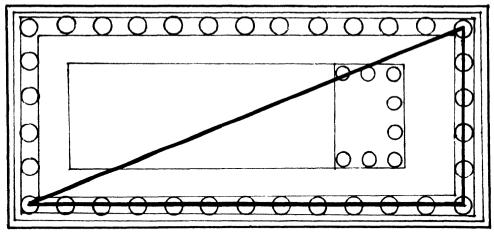


Figure 1: PLAN

R. A. Baldwin

211

Another way of looking at such a triangle is to observe that while one side measures  $4 \times 10$  Doric feet, the other two sides measure  $10^2 - 4$  and  $10^2 + 4$  respectively. It is important to remember that while for every right triangle the square of the hypotenuse will equal the sum of the squares of the other two sides, only an occasional right triangle will have the length of all three

<sup>&</sup>lt;sup>20</sup> Holloway (supra n.3) 63.

<sup>&</sup>lt;sup>21</sup> We gratefully acknowledge that an undergraduate at Vanderbilt University, Mr Joel Upchurch, first pointed out the existence of such a triangle in the plan of the temple and that the length of its hypotenuse was 104 Doric feet.

sides expressed in whole numbers. Such a right triangle is termed a 'perfect' or 'pythagorean' triangle in mathematics, and there are only sixteen basic pythagorean triangles with an hypotenuse less than one hundred.<sup>22</sup> The pythagorean triangle present in the plan of the Athena temple at Paestum is a version of the basic or primitive pythagorean triangle 5, 12, 13, enlarged by a factor of 8. Furthermore, as 40 + 96 + 104 equal 240, the perimeter of this triangle measures  $10 \times 24$  Doric feet.

Yet another pythagorean triangle inheres in the design of the temple (*Figure 2*). On the flank elevation such a triangle is formed

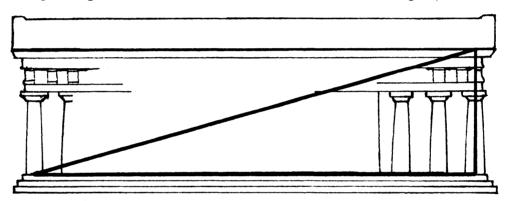


Figure 2: SOUTH SIDE ELEVATION

with its base stretching along the top of the stylobate from the axis of one corner column to the axis of the column at the other end of the line, a distance of 96 Doric feet. The vertical side of the triangle is formed by the lateral height of the temple from the top of the stylobate to the top of the sima. The height of the columns of the peristyle is 6.122 m. while the height of the entablature from the top of the abacus to the top of the sima is one-half this distance (3.062 m.).<sup>23</sup> The sum of these two measurements is 9.184 m., precisely 28 Doric feet of 0.328 m. The precision of this figure is a persuasive argument that the architect intended the height of the temple from the top of the stylobate to the top of the sima to be 28 Doric feet, since Krauss' careful measurements come to exactly this figure even though the top of the sima does not stand vertically over the axis of the peristyle columns. Seen from the side, the flank of a Greek temple is essentially a two-dimensional figure, and the top of the sima is the leading edge of its upper side. One can also

R. A. Baldwin

<sup>&</sup>lt;sup>22</sup> Cf. W. Sierpinski, Pythagorean Triangles, trans. A. Sharma (New York 1962) 1, 11-14.

<sup>&</sup>lt;sup>23</sup> Krauss (supra n.5) 2, 17, and Abb. 43.

note that architects elsewhere have been concerned with expressing the height of temples either in whole numbers or as a multiple of the standard interaxial (although admittedly not to the top of the horizontal sima). In Temple A at Agrigentum the height of the temple to the corona is three times the basic interaxial, and in Temple D on that same site the same ratio exists between the standard interaxial and the height of the temple to the roof-edge.<sup>24</sup> In addition, as Dinsmoor pointed out, the height of the Parthenon from its stylobate to the top of the geison is precisely 42 Doric feet.<sup>25</sup> The Parthenon, of course, has no horizontal sima.

The number 28 may seem unexpected, but it is the equivalent of 24 + 4, and it is also a 'perfect' number, the sum of all of its aliquot divisors or integral factors (1 + 2 + 4 + 7 + 14 = 28).<sup>26</sup> Even more noteworthy is the fact that the hypotenuse of such a triangle is the square root of 10,000 (96<sup>2</sup> + 28<sup>2</sup> or 9216 + 784 = 10,000), which is 100. The perimeter of this triangle is 28 + 96 + 100, or 224, which can also be resolved to 100 + 100 + 24. A triangle with the sides 28, 96, 100 is a version of the primitive pythagorean triangle 7, 24, 25, enlarged by a factor of 4.

The occurrence of the crucial Pythagorean numbers four, ten, and twenty-four, repeated in the basic dimensions of the building, *i.e.*, its height, breadth, and length (measured from column axis to column axis), leaves little room for doubt that their existence is intentional. Furthermore, it can hardly be sheer accident that the architect of the Athena temple created two pythagorean triangles in two major perspectives of its design using the three basic dimensions of the temple.<sup>27</sup>

<sup>26</sup> There is no evidence for such 'perfect' numbers in Greek theory before Euclid, although Iamblichus (*In Nic.* p.35.1–7 Pistelli) attributes the discovery of 'friendly' (or 'amicable') numbers to Pythagoras. Two numbers are 'friendly' if each is the sum of the aliquot divisors of the other; *cf.* Thomas Heath, *A History of Greek Mathematics* I (Oxford 1921) 74–75. We are grateful to our colleague R. Dale Sweeney for his observation of 28 as a 'perfect' number.

<sup>27</sup> Our attempts to identify other Greek temples, western or mainland, which show the same sort of planning around Pythagorean concepts have been completely unsuccessful. To take the nearly contemporary Tavole Paladine at Metapontum as an example, using the figures from Lorenzen's tables (*supra* n.18) 146, and employing the Ionic foot of 0.295 m. established for this temple by Dinsmoor (*supra* n.11) 358, we find the distances from the axis of one corner column to the axis of the other corner column 109.09 Ionic feet on the longer sides and 50.10 on the shorter. The hypotenuse would then equal 120.04 Ionic feet. Calculations from the dimensions of other temples yield similar fractional numbers. Even

213

<sup>&</sup>lt;sup>24</sup> Gruben (*supra* n.10) 437, 440–41.

<sup>&</sup>lt;sup>25</sup> Dinsmoor (*supra* n.11) 264.

## 214 THE ATHENA TEMPLE AT PAESTUM

We emphasize once again that we are using the precise measurements of the temple independently established by Krauss and that it was he who suggested the Doric foot of 0.328 m. Nor have we rounded off figures more than to the nearest one-hundredth of a Doric foot.<sup>28</sup> We do not claim that Pythagorean ideas governed the design of the temple in every part, from the dimensions of its stylobate down to the proportions of its guttae, but rather that its three basic dimensions and the triangles formed quite naturally from them display essential Pythagorean concepts of number theory. With this extension of the work of Kayser and especially of Holloway, we may now be even more confident that the teachings of Pythagoras in southern Italy affected the design of the Athena temple.<sup>29</sup>

It is impossible to reconstruct the relationship between philosophical and political history in southern Italy during this period. We know that Croton devastated Sybaris in 510 B.C.,<sup>30</sup> marking the expansion of Crotonian and thus probably of Pythagorean influence. Numismatic evidence for a 'Crotonian Alliance', examined by Kahrstedt<sup>31</sup> and others, does not provide a reliable chronology, although von Fritz accepts Kahrstedt's main point, that Croton exercised considerable economic and political influence through much of southern Italy from the latter part of the sixth to about

<sup>28</sup> For instance, when we state that the width of a corner triglyph (0.9885 m.) is 3 Doric feet, it will be found that  $0.9855 \div 0.328 = 3.004573171$ .

the peripteral temple of Apollo on Delos, which also has uniform interaxials all the way around its 6 by 13 peristyle, fails to achieve a pythagorean triangle. Since the interaxials are uniform, there does exist a triangle (measured from corner column axis to corner column axis again) in the plan which has the *proportions* of the 5, 12, 13 pythagorean triangle, but the sides of the triangle are not expressed in terms of even feet. The Pythagoreans considered only whole numbers to be numbers (Morris Kline, *Mathematical Thought from Ancient to Modern Times* [Oxford 1972] 32). The excavators determined that a foot of 0.297 m. was used in the construction, with 7<sup>11</sup>/<sub>16</sub> feet to each interaxial: Fernand Courby, *Les temples d'Apollon* (*Délos* 12 [Paris 1931]) 93–95. This yields a length from corner column axis to corner column axis of  $12 \times 7^{11}/_{16}$  or  $92^{1/4}$  feet and a width of  $5 \times 7^{11}/_{16}$  or  $38^7/_{16}$  feet. The hypotenuse of such a triangle would be equal to the square root of 9987.504 or 99.938 feet. This supports our contention that pythagorean triangles were not created by accident.

<sup>&</sup>lt;sup>29</sup> Even though he is using other arguments, many of which are questionable, Lorenzen (*supra* n.18) 80 claims that the planning arrangement of temples in southern Italy was altered after the arrival of Pythagoras in Croton. In his careful study Bell (*supra* n.15) 368–72 has argued persuasively that Pythagorean number theory influenced the design of the Olympieion at Akragas.

<sup>&</sup>lt;sup>30</sup> Diod. 11.90. See T. J. Dunbabin, The Western Greeks (Oxford 1948) 362-65.

<sup>&</sup>lt;sup>31</sup> U. Kahrstedt, "Zur Geschichte Grossgriechenlands im 5ten Jahrhundert," *Hermes* 53 (1918) 180–87.

the middle of the fifth century.<sup>32</sup> Iamblichus (*Vit.Pyth.* 85) lists seven Pythagoreans active at Poseidonia (Paestum), although their dates cannot be determined. Pythagoreans must have been active at nearby Elea by the early fifth century, for their influence on Parmenides is well-attested.<sup>33</sup> Even so, it is by no means certain that Pythagoreans had political control of any Italian city at any time; von Fritz contends rather that leaders in various cities were attracted to Pythagorean principles and were influenced by them in their political activities.<sup>34</sup>

Given these questions about the nature and extent of Pythagoreanism in southern Italy in the late sixth century, the evidence of the temple of Athena at Paestum takes on added significance. Here we have a structure of fairly certain date, contemporary with Pythagoras himself, which at least attests the Pythagorean consciousness of its architect and may reflect broader philosophical and political conditions at Paestum as well. Finally, as a physical monument, it manifests in an empirical way the fundamental Pythagorean proposition that "things are numbers" and suggests that the cosmic order apparent to the Pythagoreans in the musical scale may also be expressed in architectural form.

VANDERBILT UNIVERSITY August, 1980

<sup>&</sup>lt;sup>32</sup> Kurt von Fritz, Pythagorean Politics in Southern Italy (New York 1950) 85.

<sup>&</sup>lt;sup>33</sup> For example, Diog.Laer. 9.21. See Minar (*supra* n.1) 40.