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The balance of energy in earthquake faulting

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Summary. An earthquake is modelled kinematically by specifying the tangential slip history on a fault surface which expands within a uniformly rotating, self-gravitating, slightly anelastic earth model. The total amount of energy released by such an idealized earthquake is the sum of three distinct quantities: kinetic energy of rotation, gravitational potential energy and thermodynamic elastic internal energy. The first two of these quantities may also be interpreted as the work done throughout the earth model against the action of the apparent centrifugal and real gravitational body forces respectively. The total energy released by an earthquake fault is in general considerably smaller than any of its three individual constituents, since the work performed against body forces is very nearly balanced by the work performed against the initial hydrostatic pressure in the earth model. The smallest individual constituent is the change in the kinetic energy of rotation of the earth model, which may be as much as two orders of magnitude larger than the total energy released, even though the corresponding change in the angular velocity of rotation due to the redistribution of mass is extremely small. The total energy released by an earthquake fault may also be expressed in terms only of the final static displacement and the initial and final static traction on the fault surface itself. This alternative representation of the energy change is explicitly independent of both the rotation and the self-gravitation of the earth model. All of the energy released by an earthquake fault must be dissipated somewhere within the earth model. Energy may be dissipated during faulting either in heating on the walls of the fault surface, where work must generally be done against the action of the frictional traction acting to resist slip, or at the instantaneously expanding boundary of the fault surface, where some energy may be required to overcome cohesion and where there may be additional heating. The remainder of the energy released, which is generally referred to as the seismic energy, is dissipated both during and subsequent to faulting by the slight bodily friction which must be assumed to exist throughout the entire volume of any physically realizable earth model. The seismic energy may also be expressed in terms only of the displacement and incremental traction histories on the instantaneous fault surface during the course of faulting. This alternative representation of the seismic energy is explicitly independent of both the rotation and the self-gravitation of the earth model, and so therefore is the seismic efficiency, which is defined to be the ratio of the seismic energy to the total energy released. Classical formulae for the total energy released by an earthquake fault, the seismic energy and the seismic efficiency are based not only upon the neglect of rotation and self-gravitation, but also upon the assumption that the initial hydrostatic pressure and deviatoric stress are infinitesimal quantities; those classical formulae, upon which many seismological applications depend, are justified if the initial deviatoric stress at the hypocentre is small compared to the hypocentral rigidity.

1 Introduction

This paper is intended to present a general formalism which may be used as a basis for any calculations involving the concept of energy in the theory of seismic faulting. We model an earthquake fault as a surface across which a jump discontinuity in tangential displacement may develop, as a result of some stress relaxation process. We suppose this faulting to occur in a finite earth model which is both rotating and self-gravitating. We allow for the possibility of a large and not necessarily isotropic initial static stress field and, in addition, an intrinsic elastic anistropy. The results obtained will be valid for earth models which have fluid as well as solid portions, in which case there will be internal fluid–solid interfaces where tangential slip may occur. In the interest of brevity, we shall generally argue as if there are no internal fluid–solid interfaces, and occasionally the modifications required to extend certain results to that more general case will be briefly indicated.

Faulting is, of course, presumed to occur in the solid portion of the earth model. Specific forms for the history of slip on the fault surface are not proposed, nor do we seek to determine which forms may have certain properties which are deemed to be physically realistic. We do give a complete and systematic discussion of the release and consequent dissipation of energy in an earthquake faulting episode, under the presumption that the history of slip on the fault surface happens to be known. This is a necessary first step in an overall program which seeks to develop realistic models of seismic faulting based on an energy balance fracture criterion.

2 Formulation of a model for earthquake faulting

Prior to the onset of faulting, we consider a model of the Earth which is in mechanical equilibrium while in a state of uniform diurnal rigid body rotation about its own centre of mass. We suppose this earth model to be composed of a self-gravitating, perfectly elastic continuum occupying a finite simply connected volume V with surface ∂V. Let $\mathbf{\Omega}$ denote the angular velocity of rotation of this earth model about its centre of mass $\mathbf{0}$, and let t denote the time. We shall, unless otherwise noted, adopt the point of view of an observer situated in a non-inertial frame of reference which for all times t maintains a state of uniform rotation with angular velocity $\mathbf{\Omega}$. Let $\hat{\mathbf{x}}_1$, $\hat{\mathbf{x}}_2$, $\hat{\mathbf{x}}_3$ be a Cartesian axis system in this uniformly rotating frame of reference, with its origin at the centre of mass $\mathbf{0}$, and let $\hat{\mathbf{x}}_3$ be aligned along the axis of rotation, so that $\mathbf{\Omega} = \Omega\hat{\mathbf{x}}_3$. A point or material particle in V will be denoted by its position vector $\mathbf{x}$, measured in the uniformly rotating frame of reference. The unit outward normal to ∂V at the point $\mathbf{x}$ will be denoted by $\hat{\mathbf{n}}(\mathbf{x})$. We shall use E to denote all of space, as viewed in the uniformly rotating frame of reference.

Let $\rho_0(\mathbf{x})$ denote the initial mass density, and let $\mathbf{T}_0(\mathbf{x})$ denote the initial equilibrium static stress field, which need not be isotropic, throughout the volume V. Let $p_0(\mathbf{x})$ denote the initial hydrostatic pressure field, and let $\boldsymbol{\tau}_0(\mathbf{x})$ be the deviatoric part of the stress field $\mathbf{T}_0(\mathbf{x})$, i.e.

$$p_0 = -\tfrac{1}{3}\,tr\mathbf{T}_0,$$
$$\mathbf{T}_0 = -p_0\mathbf{I} + \boldsymbol{\tau}_0. \qquad (1)$$

Let $\phi_0(\mathbf{x})$ denote the initial gravitational potential produced throughout E by the mass density field $\rho_0(\mathbf{x})$, i.e.

$$\phi_0(\mathbf{x}) = -G\int_V \frac{\rho_0(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,dV' \qquad (2)$$

where G is Newton's constant of universal gravitation. The mechanical equilibrium of this uniformly rotating earth model is guaranteed by the condition

$$\rho_0\boldsymbol{\nabla}(\phi_0 + \psi) = \boldsymbol{\nabla}\cdot\mathbf{T}_0, \qquad (3)$$

which must be satisfied throughout the volume V, and by the free surface boundary condition

$$\hat{\mathbf{n}}\cdot\mathbf{T}_0 = \mathbf{0}, \qquad (4)$$

which must be satisfied on the boundary ∂V. Here $\psi(\mathbf{x})$ is the rotational potential due to the apparent centripetal acceleration, i.e.

$$\psi(\mathbf{x}) = -\tfrac{1}{2}\,[\Omega^2x^2 - (\boldsymbol{\Omega}\cdot\mathbf{x})^2]. \qquad (5)$$

We shall use $\mathbf{C}$ to denote the inertia tensor of this earth model, as viewed in the uniformly rotating frame of reference, i.e.

$$\mathbf{C} = \int_V \rho_0(\mathbf{x})\,[(\mathbf{x}\cdot\mathbf{x})\mathbf{I} - \mathbf{x}\mathbf{x}]\,dV. \qquad (6)$$

In order to insure the secular stability of this equilibrium configuration, we shall take it for granted that the rotation axis $\hat{\mathbf{x}}_3$ is the principal axis of greatest inertia of the earth model, and we shall use $C = \hat{\mathbf{x}}_3\cdot\mathbf{C}\cdot\hat{\mathbf{x}}_3$ to denote the greatest principal moment of inertia.

Suppose that, at time $t = 0$, earthquake faulting begins to occur somewhere within this earth model. This faulting, which is assumed to have a finite duration in time, will excite the natural elastic-gravitational modes of oscillation of the earth model. In any physically realizable earth model, there will be a dissipative mechanism present which must necessarily lead to the eventual decay of every non-secular mode, including the Eulerian free nutation or Chandler wobble, as well as any very long-period modes which owe their existence to the presence of fluid regions in the earth model. The final equilibrium configuration after the elapse of an arbitrarily long time must again be a state of uniform rigid body rotation about the centre of mass. Conservation of angular momentum requires that the axis of rotation in this final configuration be aligned along the initial axis of rotation $\hat{\mathbf{x}}_3$. The rate of angular rotation will in general be slightly different, since the redistribution of the mass of the earth model will give rise to a change in the magnitude of the principal moment of inertia.

Suppose that Ω is changed to $\Omega+\delta\Omega$, where $\delta\Omega$ may be of either sign, but is assumed to be infinitesimal, i.e. $|\delta\Omega/\Omega| \ll 1$. Let $\mathbf{r}(\mathbf{x}, t)$ denote the position vector of the material particle $\mathbf{x}$ at time $t \geq 0$, measured in the frame of reference which is rotating uniformly with angular velocity $\mathbf{\Omega}$. A convenient way to accommodate the change $\delta\Omega$ in the angular rate of rotation is to decompose $\mathbf{r}(\mathbf{x}, t)$ in the form (Dahlen & Smith 1975)

$$\mathbf{r}(\mathbf{x},t) = \mathbf{Q}(t)\cdot[\mathbf{x} + \mathbf{s}(\mathbf{x},t)], \tag{7}$$

where $\mathbf{Q}(t)$ is the proper orthogonal tensor whose component matrix $Q(t)$ relative to $\hat{\mathbf{x}}_1$, $\hat{\mathbf{x}}_2$, $\hat{\mathbf{x}}_3$ is

$$Q(t) = \begin{pmatrix} \cos\delta\Omega t & \sin\delta\Omega t & 0 \\ -\sin\delta\Omega t & \cos\delta\Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

The quantity $\mathbf{s}(\mathbf{x}, t)$, which is also assumed to be infinitesimal, is then the displacement of the material particle $\mathbf{x}$ at time $t \geq 0$, as viewed by an auxiliary observer situated in the reference frame which is rotating uniformly with the new angular velocity $\mathbf{\Omega}+\delta\mathbf{\Omega} = (\Omega+\delta\Omega)\,\hat{\mathbf{x}}_3$. We shall use $\mathbf{s}(\mathbf{x})$ to denote the ultimate static material particle displacement as viewed by that observer, i.e. the limit of $\mathbf{s}(\mathbf{x}, t)$ as $t\to\infty$. Relative to the original frame of reference which is rotating uniformly with angular velocity $\mathbf{\Omega}$, the Lagrangian particle position vector $\mathbf{r}(\mathbf{x}, t)$ in the new static equilibrium configuration is then

$$\mathbf{r}(\mathbf{x},t) = \mathbf{Q}(t)\cdot[\mathbf{x} + \mathbf{s}(\mathbf{x})]. \tag{9}$$

A fault surface is by definition a simply connected, open surface in the volume V across which the material particle displacement suffers a jump discontinuity. Let T denote the finite duration of the faulting episode, and let Σ denote the final fault surface. A point on the surface Σ will be denoted by its position vector $\boldsymbol{\xi}$, and the unit normal to Σ at the point $\boldsymbol{\xi}$ will be denoted by $\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})$. According to the usual convention, $\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})$ is taken to point out of the positive side of Σ. Faulting is assumed to initiate at time $t=0$ at a single point on the surface Σ; the zone of faulting then spreads continuously away from that point until at time $t=T$ it occupies all of Σ. Let $\Sigma(t)$ denote the instantaneous fault surface at time $t \geq 0$; for $0 \leq t \leq T$, $\Sigma(t)$ is then a simply connected subset of Σ, and for $t \geq T$, $\Sigma(t)$ is the constant domain Σ. We shall denote the instantaneous edge or boundary of the surface $\Sigma(t)$ by $\partial\Sigma(t)$. The two material particles which, prior to faulting, are located across from each other on the positive and negative sides of Σ at the point $\boldsymbol{\xi}$ will be denoted by $\boldsymbol{\xi}^{\pm}$, respectively, and the jump discontinuity $\mathbf{s}(\boldsymbol{\xi}^{+}, t) - \mathbf{s}(\boldsymbol{\xi}^{-}, t)$ will be denoted by $[\mathbf{s}(\boldsymbol{\xi}, t)]_-^+$. We shall make the quite reasonable assumption that $[\mathbf{s}(\boldsymbol{\xi}, t)]_-^+ = \mathbf{0}$ on the edge of the fault $\partial\Sigma(t)$, except where that edge might coincide with the outer surface ∂V, i.e. for all $\boldsymbol{\xi}$ on $\partial\Sigma(t) - \partial\Sigma(t)\cap\partial V$. We shall also restrict consideration to faulting in which the slip is purely tangential, i.e. we shall not allow the material on either side of the fault either to separate or interpenetrate. The linearized condition which guarantees this is evidently just that $[\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi}, t)]_-^+ = 0$ for all points $\boldsymbol{\xi}$ on $\Sigma(t)$. Second-order accuracy is however essential in calculations of energy, and we shall require for that purpose a tangential slip condition which is correct to second order in $\mathbf{s}(\mathbf{x}, t)$. The requisite condition may be shown to be

$$[\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi},t) - \mathbf{s}(\boldsymbol{\xi},t)\cdot\boldsymbol{\nabla}_{\xi}(\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi},t)) + \tfrac{1}{2}\mathbf{s}(\boldsymbol{\xi},t)\cdot\boldsymbol{\nabla}_{\xi}\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi},t)]_-^+ = 0, \tag{10}$$

where $\boldsymbol{\nabla}_{\xi}$ has been used to denote the surface gradient operator on $\Sigma(t)$, i.e. $\boldsymbol{\nabla}_{\xi} = \boldsymbol{\nabla} - \hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\,[\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\boldsymbol{\nabla}]$. On the final fault surface Σ, for $t \geq T$, $[\mathbf{s}(\boldsymbol{\xi}, t)]_-^+$ will be independent of the time t, and we shall denote this final static value of the slip on the fault surface Σ by $[s(\xi)]_-^+$. This model of a jump discontinuity in the tangential material particle displacement across a growing fault surface $\Sigma(t)$ in V will be assumed to be a complete kinematical description of the seismic faulting which accompanies an earthquake.

Apart from the presence of the already imagined slight bodily dissipation, the behaviour of the material comprising the earth model is assumed to be perfectly and linearly elastic, as well as isentropic, everywhere except on the instantaneous fault surface $\Sigma(t)$. We shall use $\tilde{\mathbf{T}}(\mathbf{x}, t)$ and $\mathbf{T}(\mathbf{x}, t)$, respectively, to denote the incremental non-symmetric Piola–Kirchhoff stress and the incremental Cauchy stress at the material particle $\mathbf{x}$ at time t. Both $\tilde{\mathbf{T}}(\mathbf{x}, t)$ and $\mathbf{T}(\mathbf{x}, t)$ are presumed to depend linearly upon the displacement gradient tensor $\nabla \mathbf{s}(\mathbf{x}, t)$, i.e.

$$\begin{aligned} \tilde{\mathbf{T}} &= \mathbf{\Lambda} : \nabla \mathbf{s}, \\ \mathbf{T} &= \mathbf{\Gamma} : \nabla \mathbf{s}, \end{aligned} \tag{11}$$

or, written in terms of Cartesian components relative to $\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3$,

$$\begin{aligned} \tilde{T}_{ij} &= \Lambda_{ijkl}\, \partial_k s_l, \\ T_{ij} &= \Gamma_{ijkl}\, \partial_k s_l. \end{aligned} \tag{12}$$

The most general form of the two fourth-order, isentropic elastic tensors $\Lambda(\mathbf{x})$ and $\Gamma(\mathbf{x})$, consistent with both the first and second laws of thermodynamics and with the principle of material frame-indifference, is (Dahlen 1972)

$$\Lambda_{ijkl} = C_{ijkl} + \tfrac{1}{2}(T_{ij}^0\,\delta_{kl} + T_{kl}^0\,\delta_{ij} + T_{ik}^0\,\delta_{jl} - T_{il}^0\,\delta_{jk} - T_{jk}^0\,\delta_{il} - T_{jl}^0\,\delta_{ik}) \tag{13}$$

and

$$\Gamma_{ijkl} = C_{ijkl} + \tfrac{1}{2}(-T_{ij}^0\delta_{kl} + T_{kl}^0\delta_{ij} + T_{ik}^0\,\delta_{jl} - T_{il}^0\,\delta_{jk} + T_{jk}^0\,\delta_{il} - T_{jl}^0\,\delta_{ik}), \tag{14}$$

where the components C_{ijkl} possess the familiar symmetry relations of an elastic tensor $\mathbf{C}(\mathbf{x})$, i.e.

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}. \tag{15}$$

In general, the only symmetry relations which the components Λ_{ijkl} and Γ_{ijkl} are required to satisfy are

$$\begin{aligned} \Lambda_{ijkl} &= \Lambda_{klij}, \\ \Gamma_{ijkl} &= \Gamma_{jikl}. \end{aligned} \tag{16}$$

The two incremental stress tensors $\tilde{\mathbf{T}}(\mathbf{x}, t)$ and $\mathbf{T}(\mathbf{x}, t)$ are related, correct to first order in $\mathbf{s}(\mathbf{x}, t)$, by

$$\tilde{T}_{ij} = T_{ij} + T_{ij}^0\,\partial_k s_k - T_{jk}^0\,\partial_k s_i, \tag{17}$$

and the two fourth-order isentropic elastic tensors $\Lambda(\mathbf{x})$ and $\Gamma(\mathbf{x})$ are therefore related by

$$\Lambda_{ijkl} = \Gamma_{ijkl} + T_{ij}^0\,\delta_{kl} - T_{jk}^0\,\delta_{il}. \tag{18}$$

In general, in order to specify completely a particular self-gravitating perfectly elastic earth model, we must prescribe, as well as Ω, $\rho_0(\mathbf{x})$ and $\mathbf{T}_0(\mathbf{x})$, all of the 21 independent components of the *in situ* isentropic elastic tensor $\mathbf{C}(\mathbf{x})$. If the initial stress is everywhere hydrostatic, i.e. $\mathbf{T}_0(\mathbf{x}) = -p_0(\mathbf{x})\mathbf{I}$ where $p_0(\mathbf{x})$ is the initial pressure, and if the material which comprises the earth model is everywhere elastically isotropic, then the components of the tensor $\mathbf{C}(\mathbf{x})$ may be written in the form

$$C_{ijkl} = (\kappa - {}^2\!/\!_3\mu)\,\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \tag{19}$$

where $\kappa(\mathbf{x})$ and $\mu(\mathbf{x})$ are the isentropic *in situ* bulk modulus and rigidity. More generally, we may always decompose the tensor $\mathbf{C}(\mathbf{x})$ into an isotropic and an anisotropic part, namely

$$C_{ijkl} = (\kappa - {}^2\!/\!_3\mu)\,\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \gamma_{ijkl}, \tag{20}$$

where $\kappa(\mathbf{x})$ and $\mu(\mathbf{x})$ are simply defined by

$$\begin{aligned} \kappa &= {}^1\!/\!_9\, C_{iijj}, \\ \mu &= {}^1\!/\!_{10}(C_{ijij} - {}^1\!/\!_3\, C_{iijj}), \end{aligned} \tag{21}$$

and where

$$\gamma_{iijj} = \gamma_{ijij} = 0. \tag{22}$$

Fluid portions of the earth model are characterized by the vanishing of all of the quantities $\tau_0(\mathbf{x}), \mu(\mathbf{x})$ and $\gamma(\mathbf{x})$, so that $\mathbf{C}(\mathbf{x})$, $\Lambda(\mathbf{x})$ and $\Gamma(\mathbf{x})$ take the forms

$$\begin{aligned} C_{ijkl} &= \kappa\,\delta_{ij}\delta_{kl}, \\ \Lambda_{ijkl} &= \kappa\,\delta_{ij}\delta_{kl} - p_0(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk}), \\ \Gamma_{ijkl} &= \kappa\,\delta_{ij}\delta_{kl}. \end{aligned} \tag{23}$$

In any reasonably realistic model of the Earth, the anisotropy will be relatively slight, i.e. we shall have $|\gamma_{ijkl}(\mathbf{x})| \ll \mu(\mathbf{x})$ in the solid portions of the earth model; we shall not however be obliged to make this assumption in what follows.

The outer surface ∂V of the earth model must remain free of traction for all times; the boundary condition which guarantees this is that

$$\hat{\mathbf{n}}\cdot\tilde{\mathbf{T}} = \mathbf{0}. \tag{24}$$

We require also a condition which guarantees that the traction across the instantaneous fault surface $\Sigma(t)$ will always be continuous. Dahlen (1972) has derived such a condition in terms of the incremental Piola–Kirchhoff stress tensor $\tilde{\mathbf{T}}(\mathbf{x}, t)$. Correct to first order in $\mathbf{s}(\mathbf{x}, t)$, we must have

$$[\boldsymbol{\nu}(\boldsymbol{\xi})\cdot\tilde{\mathbf{T}}(\boldsymbol{\xi}, t) - \boldsymbol{\nabla}_{\boldsymbol{\xi}}\cdot(\mathbf{s}(\boldsymbol{\xi}, t)\,\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{T}_0(\boldsymbol{\xi}))]_-^+ = \mathbf{0} \tag{25}$$

at all points $\boldsymbol{\xi}$ on $\Sigma(t)$. The continuity condition (25) can also be written in terms of the incremental Cauchy stress tensor $\mathbf{T}(\mathbf{x}, t)$ by making use of equation (18); this alternative version

$$[\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot(\mathbf{T}(\boldsymbol{\xi}, t) - \mathbf{s}(\boldsymbol{\xi}, t)\cdot\boldsymbol{\nabla}_{\boldsymbol{\xi}}\mathbf{T}_0(\boldsymbol{\xi})) - \mathbf{T}_0(\boldsymbol{\xi})\cdot\boldsymbol{\nabla}_{\boldsymbol{\xi}}(\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi}, t))]_-^+ = \mathbf{0} \tag{26}$$

is also correct only to first order in $\mathbf{s}(\mathbf{x}, t)$. The condition (26) may also be obtained by a somewhat more direct argument, without ever introducing the incremental Piola–Kirchhoff stress tensor $\tilde{\mathbf{T}}(\mathbf{x}, t)$. Continuity across Σ of $p_0(\mathbf{x})$, $\tau_0(\mathbf{x})$ or $\mathbf{T}_0(\mathbf{x})$ has not been assumed, nor shall it be in what follows; it is of course required that $[\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{T}_0(\boldsymbol{\xi})]_-^+ = \mathbf{0}$.

We shall use $\rho_1(\mathbf{x}, t)$ and $\phi_1(\mathbf{x}, t)$, respectively, to denote the incremental perturbations in the Eulerian mass density and gravitational potential associated with the elastic-gravitational displacement $\mathbf{s}(\mathbf{x}, t)$. If $\mathbf{s}(\mathbf{x}, t)$ is known, then $\rho_1(\mathbf{x}, t)$ may be determined, correct to first order in $\mathbf{s}(\mathbf{x}, t)$, from the linearized continuity equation

$$\rho_1 = -\boldsymbol{\nabla}\cdot(\rho_0\mathbf{s}). \tag{27}$$

The corresponding gravitational potential perturbation $\phi_1(\mathbf{x}, t)$ may then also be determined throughout all of space E, and again correct only to first order in $\mathbf{s}(\mathbf{x}, t)$, by solving the incremental Poisson equation

$$\begin{aligned} \nabla^2\phi_1 &= 4\pi G\rho_1 \text{ in } V, \\ \nabla^2\phi_1 &= 0 \text{ in } E - V, \end{aligned} \tag{28}$$

subject to the conditions that

$$\begin{aligned} &[\phi_1]_-^+ = 0, \\ &[\hat{\mathbf{n}}\cdot\boldsymbol{\nabla}\phi_1 + 4\pi G\rho_0\,\hat{\mathbf{n}}\cdot\mathbf{s}]_-^+ = 0 \end{aligned} \tag{29}$$

on the outer surface ∂V, and

$$\begin{aligned} &[\phi_1(\boldsymbol{\xi}, t)]_-^+ = 0, \\ &[\hat{\nu}(\boldsymbol{\xi})\cdot\boldsymbol{\nabla}\phi_1(\boldsymbol{\xi}, t) + 4\pi G\rho_0(\boldsymbol{\xi})\hat{\nu}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi}, t)]_-^+ = 0 \end{aligned} \tag{30}$$

on the fault surface $\Sigma(t)$. The solution for $\phi_1(\mathbf{x}, t)$ in terms of $\mathbf{s}(\mathbf{x}, t)$ may be written immediately in a version which is analogous to the expression (2) for $\phi_0(\mathbf{x})$, namely

$$\begin{aligned} \phi_1(\mathbf{x}, t) = &-G\int_V \frac{\rho_1(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}\,dV' - G\int_{\partial V}\frac{\rho_0(\mathbf{x}')\,\hat{\mathbf{n}}(\mathbf{x}')\cdot\mathbf{s}(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|}\,dA' \\ &+ G\int_{\Sigma(t)}\frac{[\rho_0(\boldsymbol{\xi})\hat{\nu}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi}, t)]_-^+}{|\mathbf{x} - \boldsymbol{\xi}|}\,dA. \end{aligned} \tag{31}$$

The two surface integral contributions to $\phi_1(\mathbf{x}, t)$ in equation (31) have the form of a gravitational potential due to two surface mass distributions $\rho_0(\mathbf{x})\hat{\mathbf{n}}(\mathbf{x})\cdot\mathbf{s}(\mathbf{x}, t)$ on the surface ∂V and $-[\rho_0(\boldsymbol{\xi})\hat{\nu}(\boldsymbol{\xi})\cdot\mathbf{s}(\boldsymbol{\xi}, t)]_-^+$ on the surface $\Sigma(t)$; this is to be expected on physical grounds. By inserting equation (27) into (31) and making use of Gauss' theorem, we may obtain a somewhat simpler form for $\phi_1(\mathbf{x}, t)$, namely

$$\phi_1(\mathbf{x}, t) = -G\int_V \rho_0(\mathbf{x}')\,\mathbf{s}(\mathbf{x}', t)\cdot\boldsymbol{\nabla}'\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|}\right)dV'. \tag{32}$$

It should be pointed out that in writing equation (31), it has been implicitly assumed that the elastic-gravitational particle displacement $\mathbf{s}(\mathbf{x}, t)$ is continuous throughout the volume V, except on the fault surface $\Sigma(t)$. Equation (31) is therefore inappropriate for any earth model which may have internal fluid-solid interfaces; it is on the other hand readily demonstrated that the alternative version (32) is not subject to this restriction.

The principal dynamical equation which governs the elastic-gravitational response $\mathbf{s}(\mathbf{x}, t)$ of a rotating earth model to an earthquake faulting episode is the law of conservation of momentum. The appropriate form of this law has been obtained by Dahlen & Smith (1975). For $t \geqslant 0$, $\mathbf{s}(\mathbf{x}, t)$ must satisfy

$$\rho_0\,\partial_t^2\mathbf{s} + 2\rho_0\boldsymbol{\Omega}\times\partial_t\mathbf{s} = -\rho_0\boldsymbol{\nabla}[\phi_1 + 2(\delta\Omega/\Omega)\,\psi] - \rho_0\mathbf{s}\cdot\boldsymbol{\nabla}[\boldsymbol{\nabla}(\phi_0 + \psi)] + \boldsymbol{\nabla}\cdot\tilde{\mathbf{T}}, \tag{33}$$

which must be solved throughout the volume V, subject to the initial conditions

$$\begin{aligned} &\mathbf{s}(\mathbf{x}, 0) = \mathbf{0}, \\ &\partial_t\mathbf{s}(\mathbf{x}, 0) = -\delta\boldsymbol{\Omega}\times\mathbf{x}. \end{aligned} \tag{34}$$

Equations (33) and (34) are correct to first order in $\mathbf{s}(\mathbf{x}, t)$ and in the change $\delta\Omega$ in the rate of angular rotation. Not unexpectedly, (33) and (34) are precisely the dynamical equations which would govern the response of the earth model, as viewed by the auxiliary observer who is rotating uniformly with the new angular velocity $\mathbf{\Omega}+\delta\mathbf{\Omega}$. We emphasize however that the perturbation $\delta\Omega$ is as yet unknown, and that these equations have been derived strictly from of the point of view of an observer in the original frame of reference rotating uniformly with angular velocity $\mathbf{\Omega}$, by making use of the representation (7). Alternatively, we may write the dynamical equation (33) in terms of the incremental Cauchy stress tensor $\mathbf{T}(\mathbf{x}, t)$ instead of the incremental Piola–Kirchhoff stress tensor $\widetilde{\mathbf{T}}(\mathbf{x}, t)$, by making use of relation (18); this alternative version is

$$\rho_0\, \partial_t^2 \mathbf{s}+2\rho_0 \mathbf{\Omega}\times \partial_t \mathbf{s} = -\rho_0 \boldsymbol{\nabla}[\phi_1+2(\delta\Omega/\Omega)\,\psi]-\rho_1 \boldsymbol{\nabla}(\phi_0+\psi)-\boldsymbol{\nabla}\cdot(\mathbf{s}\cdot\boldsymbol{\nabla}\mathbf{T}_0)+\boldsymbol{\nabla}\cdot\mathbf{T}, \tag{35}$$

which must also be solved subject to the initial conditions (34). Equations (33) and (35) are equivalent; with $\widetilde{\mathbf{T}}(\mathbf{x}, t)$ and $\mathbf{T}(\mathbf{x}, t)$ given in terms of $\boldsymbol{\nabla}\mathbf{s}(\mathbf{x}, t)$ by equations (11), and with $\rho_1(\mathbf{x}, t)$ and $\phi_1(\mathbf{x}, t)$ given in terms of $\mathbf{s}(\mathbf{x}, t)$ by equations (27) and (32) respectively, either of the two dynamical equations (33) or (35) represents an integro-differential equation for the elastic-gravitational displacement $\mathbf{s}(\mathbf{x}, t)$ throughout the volume V.

Suppose now that $\Sigma(t)$ and the slip $[\mathbf{s}(\boldsymbol{\xi}, t)]_-^+$ on $\Sigma(t)$ are known, and that we seek to determine the change $\delta\Omega$ in the angular rate of rotation and the dynamical displacement response $\mathbf{s}(\mathbf{x}, t)$, including the ultimate static displacement $\mathbf{s}(\mathbf{x})$, throughout the volume V of the earth model. A convenient approach to this problem is to introduce the concept of the equivalent body and surface force distributions which must be applied to the rotating earth model in the absence of any specified faulting in order to produce the same response $\delta\Omega$ and $\mathbf{s}(\mathbf{x}, t)$ as that produced by the faulting process. The form of the required equivalent body force distribution throughout the volume V has been derived by Dahlen (1972). The need to include an equivalent surface force distribution on the outer surface ∂V, in the event that $\Sigma(t)$ intersects ∂V, has been noted by Backus & Mulcahy (1976). The viewpoint of the present paper differs slightly from that of both Dahlen (1972) and Backus & Mulcahy (1976), in that Dahlen (1972) did not make use of the representation (7) to separate explicitly the change $\delta\Omega$ from the rest of the dynamical displacement $\mathbf{s}(\mathbf{x}, t)$, whereas Backus & Mulcahy (1976) neglected rotation completely. The arguments in both are however easily extended, and the results are substantially the same. We shall denote the equivalent body force density, measured per unit volume in the uniformly rotating frame of reference with angular velocity $\mathbf{\Omega}$, by $\mathbf{Q}(t)\cdot\rho_0(\mathbf{x})\,\mathbf{f}(\mathbf{x}, t)$, and we shall denote the equivalent surface force density, measured per unit area in the same frame of reference, by $\mathbf{Q}(t)\cdot\mathbf{t}(\mathbf{x},t)$. The quantities $\rho_0(\mathbf{x})\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{t}(\mathbf{x}, t)$ are then, respectively, the equivalent body and surface force densities, as viewed by the auxiliary observer who is rotating uniformly with the new angular velocity $\mathbf{\Omega}+\delta\mathbf{\Omega}$. At every point $\boldsymbol{\xi}$ on the fault surface $\Sigma(t)$, we shall define a symmetric second order tensor field $\mathbf{m}(\boldsymbol{\xi}, t)$, called the seismic moment density tensor, by the relation

$$m_{ij}(\boldsymbol{\xi}, t) = \Gamma_{ijkl}(\boldsymbol{\xi})\,[\nu_k(\boldsymbol{\xi})\, s_l(\boldsymbol{\xi}, t)]_-^+. \tag{36}$$

The equivalent body force density $\rho_0(\mathbf{x})\,\mathbf{f}(\mathbf{x}, t)$ is then given by

$$\rho_0(\mathbf{x})\,\mathbf{f}(\mathbf{x}, t) = -\int_{\Sigma(t)} \mathbf{m}(\boldsymbol{\xi}, t)\cdot\boldsymbol{\nabla}\delta(\mathbf{x}-\boldsymbol{\xi})\,dA, \tag{37}$$

where $\delta(\mathbf{x}-\boldsymbol{\xi})$ is the Dirac delta function, and where the gradient $\boldsymbol{\nabla}$ is to be taken with respect to the coordinates $\mathbf{x}$, and the equivalent surface force density $\mathbf{t}(\mathbf{x}, t)$ is given by

$$\mathbf{t}(\mathbf{x}, t) = \int_{\Sigma(t)} \hat{\mathbf{n}}(\mathbf{x})\cdot\mathbf{m}(\boldsymbol{\xi}, t)\,\delta(\mathbf{x}-\boldsymbol{\xi})\,dA. \tag{38}$$

It has been tacitly assumed in writing (36), (37) and (38) that the fourth-order isentropic elastic tensor $\Gamma(\mathbf{x})$ is continuous across the fault surface $\Sigma(t)$. The equivalent body force density $\rho_0(\mathbf{x})\mathbf{f}(\mathbf{x}, t)$ is evidently identically zero at every material particle $\mathbf{x}$ in the volume V, except on the fault surface $\Sigma(t)$, and the equivalent surface force density $\mathbf{t}(\mathbf{x}, t)$ is likewise identically zero at every material particle $\mathbf{x}$ on the surface ∂V, except on $\partial\Sigma(t)\cap\partial V$. The model of the earth we are considering is one which is completely isolated from interaction with any external bodies. Since an earthquake is a process which occurs wholly within this isolated system, the net force and torque exerted upon the earth model by the body and surface force distributions which are equivalent to the earthquake must necessarily be zero. It is easily verified that this is so, i.e.

$$\int_V \rho_0 \mathbf{f}\, dV + \int_{\partial V} \mathbf{t}\, dA = \mathbf{0},$$
$$\int_V \mathbf{x}\times\rho_0 \mathbf{f}\, dV + \int_{\partial V} \mathbf{x}\times\mathbf{t}\, dA = \mathbf{0}. \tag{39}$$

The latter condition, that there be no net torque, is guaranteed by the symmetry $\mathbf{m}(\boldsymbol{\xi}, t) = \mathbf{m}^T(\boldsymbol{\xi}, t)$ of the seismic moment density tensor. For $t \geq T$, each of the quantities $\mathbf{m}(\boldsymbol{\xi}, t)$, $\rho_0(\mathbf{x})\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{t}(\mathbf{x}, t)$ assumes a final time-independent value, which we shall denote, respectively, by $\mathbf{m}(\boldsymbol{\xi})$, $\rho_0(\mathbf{x})\mathbf{f}(\mathbf{x})$ and $\mathbf{t}(\mathbf{x})$; these are defined in terms of the final static slip $[\mathbf{s}(\boldsymbol{\xi})]_-^+$ on the fault surface Σ by equations (36), (37) and (38), respectively.

The problem of determining the response of a rotating earth model to the action of an imposed body force distribution has been considered in some detail by Dahlen & Smith (1975); the extension required in the more general case that there is also an imposed surface force distribution is straightforward. Let $\mathbf{s}_n(x)$, $1 \leq n \leq \infty$, be the set of all the complex normal mode displacement eigenfunctions of the earth model, with associated real non-secular normal mode eigenfrequencies $\omega_n \neq 0, \pm\Omega$, $1 \leq n \leq \infty$. Let the eigenfunctions be normalized in such a way that

$$\int_V \rho_0 \mathbf{s}_n \cdot \mathbf{s}_n^*\, dV = 1, \tag{40}$$

where an asterisk denotes the complex conjugate, and assume that they form a complete linear space of complex vector-valued functions over the volume V. Following Dahlen & Smith (1975), we shall write $\mathbf{s}(\mathbf{x}, t)$ in the form

$$\mathbf{s}(\mathbf{x}, t) = \left[\mathbf{s}(\mathbf{x}) + \sum_{n=1}^{\infty} a_n \mathbf{s}_n(\mathbf{x}) \exp(i\omega_n t)\right] H(t), \tag{41}$$

where $H(t)$ is the Heaviside step function. Upon defining the coefficients f_n, $1 \leq n \leq \infty$, and g_n, $1 \leq n \leq \infty$, by

$$f_n = \int_0^T dt\, \exp(-i\omega_n t)\left[\int_V \rho_0(\mathbf{x})\, \mathbf{f}(\mathbf{x}, t)\cdot\mathbf{s}_n^*(\mathbf{x})\, dV + \int_{\partial V} \mathbf{t}(\mathbf{x}, t)\cdot\mathbf{s}_n^*(\mathbf{x})\, dA\right] \tag{42}$$

and

$$g_n = \int_V \rho_0(\mathbf{x})[\boldsymbol{\Omega}\times\mathbf{x} + 2(i\omega_n)^{-1}\nabla\psi(\mathbf{x})]\cdot\mathbf{s}_n^*(\mathbf{x})\, dV, \tag{43}$$

the dynamical complex normal mode excitation amplitudes a_n, $1 \leqslant n \leqslant \infty$, may be shown to be given by

$$a_n = [f_n - (\delta\Omega/\Omega) g_n] \Big/ \left[2i\omega_n - 2i \int_V \rho_0 \mathbf{s}_n \cdot (i\boldsymbol{\Omega} \times \mathbf{s}_n)^* \, dV\right]. \tag{44}$$

A misprinted sign in the denominator of the corresponding formula of Dahlen & Smith (1975) has here been repaired.

The change $\delta\Omega$ in the rate of angular rotation, which appears explicitly in the expression (44) for the excitation amplitudes a_n, $1 \leqslant n \leqslant \infty$, cannot be determined without simultaneously solving for the final static displacement field $\mathbf{s}(\mathbf{x})$. Conservation of angular momentum requires that the angular momentum of the new uniformly rotating static equilibrium configuration of the earth model, subsequent to faulting and after all the normal modes of oscillation have decayed, be equal to that of the original equilibrium configuration. Correct to first order in $\mathbf{s}(\mathbf{x})$, the relation expressing this invariance of the angular momentum of the earth model is

$$C(\delta\Omega/\Omega) + 2 \int_V \rho_0(\mathbf{x}) \, [\mathbf{x} \cdot \mathbf{s}(\mathbf{x}) - (\hat{\mathbf{x}}_3 \cdot \mathbf{x}) (\hat{\mathbf{x}}_3 \cdot \mathbf{s}(\mathbf{x}))] \, dV = 0. \tag{45}$$

To determine $\delta\Omega$ and $\mathbf{s}(\mathbf{x})$, we must in general solve the static version of the conservation of momentum law, in either of the two forms

$$-\rho_0 \boldsymbol{\nabla}[\phi_1 + 2(\delta\Omega/\Omega)\psi] - \rho_0 \mathbf{s} \cdot \boldsymbol{\nabla}[\boldsymbol{\nabla}(\phi_0 + \psi)] + \boldsymbol{\nabla} \cdot \tilde{\mathbf{T}} + \rho_0 \mathbf{f} = \mathbf{0} \tag{46}$$

or

$$-\rho_0 \boldsymbol{\nabla}[\phi_1 + 2(\delta\Omega/\Omega)\psi] - \rho_1 \boldsymbol{\nabla}(\phi_0 + \psi) - \boldsymbol{\nabla} \cdot (\mathbf{s} \cdot \boldsymbol{\nabla} \mathbf{T}_0) + \boldsymbol{\nabla} \cdot \mathbf{T} + \rho_0 \mathbf{f} = \mathbf{0}, \tag{47}$$

subject to the constraint (45), as well as to the boundary condition

$$\hat{\mathbf{n}} \cdot \tilde{\mathbf{T}} = \mathbf{t} \tag{48}$$

on the outer surface ∂V. All of the first-order quantities which appear in equations (46), (47) and (48) are assumed to have attained their ultimate static values.

This problem of determining $\delta\Omega$ and $\mathbf{s}(\mathbf{x})$ can be converted into an infinite system of simultaneous linear algebraic equations by expanding $\mathbf{s}(\mathbf{x})$ in the form

$$\mathbf{s}(\mathbf{x}) = \sum_{n=1}^{\infty} a_n^{\text{final}} \mathbf{s}_n(\mathbf{x}), \tag{49}$$

so that $\mathbf{s}(\mathbf{x}, t)$ is of the form

$$\mathbf{s}(\mathbf{x}, t) = \left[\sum_{n=1}^{\infty} (a_n^{\text{final}} + a_n \exp(i\omega_n t)) \, \mathbf{s}_n(\mathbf{x})\right] H(t). \tag{50}$$

We define also the coefficients f_n^{final}, $1 \leqslant n \leqslant \infty$, and g_n^{final}, $1 \leqslant n \leqslant \infty$, by

$$f_n^{\text{final}} = \int_V \rho_0(\mathbf{x}) \, \mathbf{f}(\mathbf{x}) \cdot \mathbf{s}_n^*(\mathbf{x}) \, dV + \int_{\partial V} \mathbf{t}(\mathbf{x}) \cdot \mathbf{s}_n^*(\mathbf{x}) \, dA,$$
$$g_n^{\text{final}} = 2 \int_V \rho_0(\mathbf{x}) \, \boldsymbol{\nabla}\psi(\mathbf{x}) \cdot \mathbf{s}_n^*(\mathbf{x}) \, dV. \tag{51}$$

The quantity $\delta\Omega$ and the coefficients a_n^{final}, $1 \leqslant n \leqslant \infty$, can then, in principle, be determined simultaneously by solving the coupled system of equations (Dahlen & Smith 1975)

$$(\delta\Omega/\Omega)\, g_n^{\text{final}} + a_n^{\text{final}}\left[\omega_n^2 - 2\omega_n \int_V \rho_0 \mathbf{s}_n\cdot(i\mathbf{\Omega}\times\mathbf{s}_n)^*\, dV\right] + \sum_{\substack{m=1\\ m\neq n}}^{\infty} a_m^{\text{final}}\left[-\omega_m\omega_n\int_V \rho_0\mathbf{s}_m\cdot\mathbf{s}_n^*\, dV\right] = f_n^{\text{final}} \tag{52}$$

and

$$C(\delta\Omega/\Omega) + 2\sum_{m=1}^{\infty} a_m^{\text{final}}\int_V \rho_0[\mathbf{x}\cdot\mathbf{s}_m - (\hat{\mathbf{x}}_3\cdot\mathbf{x})(\hat{\mathbf{x}}_3\cdot\mathbf{s}_m)]\, dV = 0. \tag{53}$$

The dynamical displacement $\mathbf{s}(\mathbf{x}, t)$, as determined by the above procedure, will necessarily be real for all times t. This is guaranteed by the fact that all of the normal mode displacement eigenfunctions $\mathbf{s}_n(\mathbf{x})$, $1\leqslant n\leqslant\infty$, are either real or occur in complex conjugate pairs, as demonstrated by Dahlen & Smith (1975). To express the response $\delta\Omega$ and $\mathbf{s}(\mathbf{x}, t)$ in terms of the seismic moment density tensor $\mathbf{m}(\boldsymbol{\xi}, t)$, we need only to substitute (37) and (38) into the expressions (42) and (51) for the coefficients f_n and f_n^{final}, $1\leqslant n\leqslant\infty$. Upon interchanging the orders of integration and applying Gauss' theorem, we obtain

$$f_n = \int_0^T dt\, \exp(-i\omega_n t)\int_{\Sigma(t)} tr\,[\mathbf{m}(\boldsymbol{\xi}, t)\cdot\nabla\mathbf{s}_n^*(\boldsymbol{\xi})]\, dA \tag{54}$$

and

$$f_n^{\text{final}} = \int_\Sigma tr[\mathbf{m}(\boldsymbol{\xi})\cdot\nabla\mathbf{s}_n^*(\boldsymbol{\xi})]\, dA. \tag{55}$$

It might be noted that the equivalent body and surface forces $\rho_0(\mathbf{x})\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{t}(\mathbf{x}, t)$ have an advantage over the seismic moment density tensor $\mathbf{m}(\boldsymbol{\xi}, t)$ in that they are uniquely defined by the prescribed faulting $[\mathbf{s}(\boldsymbol{\xi}, t)]_-^+$ on $\Sigma(t)$. That is, there is only one possible choice of the quantities $\rho_0(\mathbf{x})\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{t}(\mathbf{x}, t)$ which, when inserted into (42) and (51), yields the same response as that produced by the prescribed faulting; there is, on the other hand, more than one choice of the tensor $\mathbf{m}(\boldsymbol{\xi}, t)$ which, when inserted into (54) and (55), has this property. This situation is discussed in some detail by Backus & Mulcahy (1976).

We have solved the problem of determining the response of a rotating earth model to a kinematically prescribed episode of earthquake faulting under the supposition that there is a dissipative mechanism present throughout the volume V of the earth model which is sufficiently slight that it does not appreciably alter the perfectly elastic dynamical equations (33) or (35) which govern the response. In essence, this amounts to the supposition that each of the normal mode displacement eigenfunctions $\mathbf{s}_n(\mathbf{x})$, $1\leqslant n\leqslant\infty$, of the earth model is unaffected by the dissipation, but that each of the associated non-secular eigenfrequencies ω_n, $1\leqslant n\leqslant\infty$, which would be purely real for a perfectly elastic earth model, is actually of the form $\omega_n(1+i/2Q_n)$ where $0<Q_n^{-1}\ll 1$. When this substitution is made in either of the expressions (41) or (50) for $\mathbf{s}(\mathbf{x}, t)$, it is clear that $\mathbf{s}(\mathbf{x}, t)\to\mathbf{s}(\mathbf{x})$ when $t\to\infty$, as has already been stipulated.

3 *The net release of energy*

We consider now the change in the energy of the earth model due to the faulting process. To determine the total amount of energy which is released by an earthquake, it is not necessary to know any of the details of the temporal history of the faulting. Making use of a standard thermodynamical procedure, we only need to compare the total energy in the final uniformly rotating equilibrium configuration, after the decay of the non-secular natural modes of oscillation, to the corresponding total energy in the initial uniformly rotating equilibrium configuration. Three types of energy must be taken into account: kinetic energy of rotation, gravitational potential energy and thermodynamic elastic internal energy.

We consider first the net change in the kinetic energy of uniform rotation of the earth model. We shall show that this change may be surprisingly large in comparison with the total energy which is released. Suppose that in the final uniformly rotating state, the principal moment of greatest inertia of the earth model has been changed from its initial value C to the value $C+\delta C$. The change in the kinetic energy of rotation, which we shall denote by $\mathscr{T}$ is then given by

$$\mathscr{T}=\tfrac{1}{2}(C+\delta C)(\Omega+\delta\Omega)^2-\tfrac{1}{2}C\Omega^2. \tag{56}$$

We may eliminate δC from the expression (56) by making use of the law of conservation of angular momentum, which requires that

$$(C+\delta C)(\Omega+\delta\Omega)=C\Omega. \tag{57}$$

The change $\mathscr{T}$ in the kinetic energy of rotation of the Earth model is therefore, simply,

$$\mathscr{T}=\tfrac{1}{2}C\Omega^2(\delta\Omega/\Omega), \tag{58}$$

and this expression is exact. Note that $\mathscr{T}$ is linear in the change $\delta\Omega$, so that an individual earthquake may act either to increase or decrease the rotational kinetic energy of the model, depending upon the sign of $\delta\Omega$.

The rotational kinetic energy $\tfrac{1}{2}C\Omega^2$ of the earth is about 2×10^{29} J. According to Ben Menahem & Israel (1970), a typical large earthquake ($M=8.5$) can give rise to a change Δ(lod) in the length of the day of only a few μs, i.e. $|\delta\Omega/\Omega|\simeq10^{-10}$. It should be emphasized that such a change is essentially negligible in comparison with changes in the length of the day which are actually observed. Changes of order $\Delta(\text{lod})\simeq1$ ms, i.e. $|\delta\Omega/\Omega|\simeq10^{-8}$, are observed to occur on a wide variety of time scales, and are known to be generated by a wide variety of geophysical phenomena other than earthquakes. The essentially unobservable change $|\delta\Omega/\Omega|=10^{-10}$ in the angular velocity of rotation which might be produced by a large earthquake is nevertheless associated with a substantial change $\mathscr{T}=2\times10^{19}$ J in the kinetic energy of rotation of the Earth. We may compare this with the total energy E_{GR} which is assigned to an earthquake of magnitude $M=8.5$ by the well-known empirical formula of Gutenburg & Richter (Richter 1958); when E_{GR} is measured in J, this formula, as corrected by Kanamori & Anderson (1975), reads

$$\log_{10}E_{GR}=4.8+1.5M. \tag{59}$$

For $M=8.5$, we obtain $E_{GR}=3.5\times10^{17}$ J, which is about two orders of magnitude smaller than $\mathscr{T}$. Even if we allow for the uncertainties which are inherent in any simple empirical formula such as (59), it is clear that there is a serious discrepancy between the relative magnitudes of $\mathscr{T}$ and E_{GR} and the strong intuitive notion that the slight change in angular velocity of rotation of the Earth must play a fairly negligible role in the mechanics of an earthquake. We shall now show how this discrepancy may be resolved.

We first seek an alternative characterization of the quantity $\mathscr{T}$; we begin by noting that $\mathscr{T}$ may be rewritten exactly in the form

$$\mathscr{T}=\tfrac{1}{2}\int_V\rho_0[\boldsymbol{\Omega}+\delta\boldsymbol{\Omega})\times(\mathbf{x}+\mathbf{s})]\cdot[(\boldsymbol{\Omega}+\delta\boldsymbol{\Omega})\times(\mathbf{x}+\mathbf{s})]\,dV-\tfrac{1}{2}\int_V\rho_0[\boldsymbol{\Omega}\times\mathbf{x}]\cdot[\boldsymbol{\Omega}\times\mathbf{x}]\,dV. \tag{60}$$

The condition that the angular momentum of the earth model must remain constant may be rewritten, also exactly, in the form

$$\int_V \rho_0(\mathbf{x}+\mathbf{s}) \times [(\boldsymbol{\Omega}+\delta\boldsymbol{\Omega}) \times (\mathbf{x}+\mathbf{s})]\,dV - \int_V \rho_0 \mathbf{x} \times [\boldsymbol{\Omega} \times \mathbf{x}]\,dV = \mathbf{0}. \tag{61}$$

We now form the dot product of equation (61) with the vector $\boldsymbol{\Omega}+\delta\boldsymbol{\Omega}$, and subtract the result from equation (60). Upon making use of equation (45), as well as the identities

$$\begin{aligned}(\boldsymbol{\Omega} \times \mathbf{x})\cdot(\boldsymbol{\Omega} \times \mathbf{s}) &= -s_j\,\partial_j \psi,\\ (\boldsymbol{\Omega} \times \mathbf{s})\cdot(\boldsymbol{\Omega} \times \mathbf{s}) &= -s_i s_j\,\partial_i \partial_j \psi,\end{aligned} \tag{62}$$

we can finally write $\mathscr{T}$ in the form

$$\mathscr{T} = \int_V \rho_0 s_j\,\partial_j \psi\,dV + ½\int_V [2(\delta\Omega/\Omega)\,\rho_0 s_j\,\partial_j\psi + \rho_0 s_i s_j\,\partial_i\partial_j\psi]\,dV, \tag{63}$$

where terms of third and fourth order in the infinitesimal quantities $\delta\Omega$ and $\mathbf{s}(\mathbf{x})$ have been neglected, so that (63) is correct to second order in those quantities. It is of interest to compare the form (63) for the change $\mathscr{T}$ with the corresponding expression which has been obtained by Dahlen (1973) for the change in the gravitational potential energy of the earth model. That change, which we shall denote by $\mathscr{M}$, may be written in the form

$$\mathscr{M} = \int_V \rho_0 s_j\,\partial_j\phi_0\,dV + ½\int_V [\rho_0 s_j\,\partial_j\phi_1 + \rho_0 s_i s_j\,\partial_i\partial_j\phi_0]\,dV, \tag{64}$$

which also is correct to second order in the infinitesimal quantity $\mathbf{s}(\mathbf{x})$. The correspondence between equations (63) and (64) is quite striking, and it allows an immediate alternative interpretation of the quantity $\mathscr{T}$. The change $\mathscr{M}$ in the gravitational potential energy of the earth model is the same as the total work which has been performed against the action of the gravitational body force. The change $\mathscr{T}$ in the rotational kinetic energy is therefore evidently the same as the total work which has been performed against the action of the apparent centrifugal force arising from the rotation of the earth model. The sum of the two changes $\mathscr{T}$ and $\mathscr{M}$, i.e.

$$\mathscr{T}+\mathscr{M} = \int_V \rho_0 s_j\,\partial_j(\phi_0+\psi)\,dV + ½\int_V \left[\rho_0 s_j\,\partial_j\left(\phi_1 + 2\,\frac{\delta\Omega}{\Omega}\,\psi\right) + \rho_0 s_i s_j\,\partial_i\partial_j(\phi_0+\psi)\right] dV, \tag{65}$$

is then the total work which has been performed against the action of both real and apparent body forces. The combination $\phi_0(\mathbf{x})+\psi(\mathbf{x})$ of the initial gravitational potential and the initial rotational potential is the initial geopotential, and the final change in the geopotential due to the change $\phi_1(\mathbf{x})$ in the gravitational potential and the change $\delta\Omega$ in the rate of angular rotation of the earth model is correct to first order in the latter quantity, $\phi_1(\mathbf{x}) + 2(\delta\Omega/\Omega)\,\psi(\mathbf{x})$.

It is well known that in the Earth, the geopotential $\phi_0(\mathbf{x})+\psi(\mathbf{x})$ differs from the purely gravitational potential $\phi_0(\mathbf{x})$ by only about one part in 290 (Jeffreys 1959). In general, we might therefore expect that the total change $\mathscr{T}+\mathscr{M}$ in the rotational kinetic energy and the gravitational potential energy of the earth produced by an earthquake will be about 290 times larger than the change $\mathscr{T}$ alone. It has already been pointed out that the change $\mathscr{T}$ due to a typical magnitude $M = 8.5$ earthquake is about two orders of magnitude larger than the energy E_{GR} which has been empirically assigned to such an earthquake. The total change $\mathscr{T}+\mathscr{M}$, which consists largely of work which has been done against the action of the earth's gravitational field, will in that case be about three to four orders of magnitude larger than E_{GR}. The original discrepancy between $\mathscr{T}$ and E_{GR}, after identification of $\mathscr{T}$ as the work done against the apparent centrifugal force in the rotating Earth, has therefore led to an even larger discrepancy, which is also contrary to intuition, between the sum $\mathscr{T}+\mathscr{M}$ and E_{GR}.

Let $\mathscr{U}$ denote the change in the stored thermodynamic elastic internal energy of the earth model, which is the only form of energy remaining to be considered. The situation with regard to the determination of this change $\mathscr{U}$ is rather different than that with regard to the two previously considered changes $\mathscr{T}$ and $\mathscr{M}$, since the absolute amount of elastic energy which is stored in either the initial or the final equilibrium configuration cannot be specified. The change $\mathscr{U}$ in the amount of stored elastic energy is however a perfectly well-defined concept (Dahlen 1973); it is defined by

$$\mathscr{U}=\int_V T_{ij}^0\,\tfrac{1}{2}(\partial_i s_j+\partial_j s_i)\,dV+\tfrac{1}{2}\int_V \Lambda_{ijkl}\,\partial_i s_j\,\partial_k s_l\,dV. \tag{66}$$

The first term in equation (66) is linear in the final static strain tensor $\tfrac{1}{2}[\nabla\mathbf{s}(\mathbf{x})+\nabla\mathbf{s}^T(\mathbf{x})]$, and is just the first order work which has been performed against the initial static stress field $\mathbf{T}_0(\mathbf{x})$. This may be further decomposed into first order work performed against the initial hydrostatic pressure $p_0(\mathbf{x})$ and first order work performed against the initial deviatoric stress $\boldsymbol{\tau}_0(\mathbf{x})$, by making use of equations (1). It might be noted that there are second order contributions to the work done against $\mathbf{T}_0(\mathbf{x})$ as well, arising from the explicit dependence displayed in equation (13) of the fourth order isentropic elastic tensor $\Lambda(\mathbf{x})$ on $\mathbf{T}_0(\mathbf{x})$. Both of the integrals in equation (66) are improper, because of the fact that $\mathbf{s}(\mathbf{x})$, which appears differentiated in the integrands, suffers a jump discontinuity across the final fault surface Σ. The remedy for this situation is obvious. Let V_0, with surface ∂V_0, be a volume element which completely surrounds the fault surface Σ, so that $V-V_0$ is a possibly punctured volume which does not contain Σ. The change in the elastic energy which is stored within this volume $V-V_0$ is well-defined, and is given by a formula identical to (66) except that the domain of integration V is replaced by $V-V_0$. The change $\mathscr{U}$ in the stored elastic energy of the entire earth model can then be accurately defined to be the limit of this quantity as the volume V_0 collapses to zero, in such a way that the surface ∂V_0 exactly envelops both sides of the final fault surface Σ. We shall assume implicitly in what follows that the integrals in equation (66) are to be evaluated by carrying out this limiting process.

The total change in the energy of the earth model, which we shall denote by ΔE, is given by

$$\Delta E=\mathscr{T}+\mathscr{M}+\mathscr{U}, \tag{67}$$

where $\mathscr{T}+\mathscr{M}$ is defined by equation (65) and $\mathscr{U}$ is defined by equation (66). This quantity ΔE must necessarily be negative, since an earthquake is a phenomenon which occurs spontaneously; the corresponding positive quantity $-\Delta E$ is then the total amount of energy released by the earthquake. An application of Gauss' theorem before passing to the limit $V_0\to 0$ in equation (66) can be used to write ΔE in the form

$$\Delta E=\int_V s_j[\rho_0\,\partial_j(\phi_0+\psi)-\partial_i T_{ij}^0]\,dV+\tfrac{1}{2}\int_V s_j\left[\rho_0\partial_j\left(\phi_1+2\frac{\delta\Omega}{\Omega}\psi\right)+\rho_0 s_i\,\partial_i\,\partial_j(\phi_0+\psi)-\partial_i\tilde{T}_{ij}\right]$$
$$\times\,dV+\tfrac{1}{2}\int_{\partial V} n_i(2T_{ij}^0+\tilde{T}_{ij})\,s_j\,dA-\tfrac{1}{2}\int_\Sigma [\nu_i(2T_{ij}^0+\tilde{T}_{ij})\,s_j]_-^+\,dA. \tag{68}$$

The first volume integral in equation (68) vanishes identically by virtue of the static equilibrium condition (3) in the initial uniformly rotating configuration of the earth model. The second volume integral in equation (68) vanishes identically by virtue of the corresponding first order static equilibrium condition (46) in the final uniformly rotating configuration of the earth model; the equivalent body force $\rho_0(\mathbf{x})\mathbf{f}(\mathbf{x})$ in equation (46) does not appear in this context. Finally, the surface integral over the outer surface ∂V of the earth model in equation (68) also vanishes identically, by virtue of the boundary conditions (4) and (24) which guarantee that ∂V is a traction-free surface. We have therefore succeeded in expressing the total change ΔE in the energy of the earth model in terms only of the initial and final static tractions and the final static displacement on the final fault surface Σ; written in invariant notation

$$\Delta E = -\tfrac{1}{2}\int_\Sigma [\hat{\nu}\cdot(2\mathbf{T}_0+\tilde{\mathbf{T}})\cdot\mathbf{s}]_-^+\,dA. \tag{69}$$

The result (69) is valid also for an earth model which has internal fluid–solid discontinuities. Gauss' theorem must in that case be applied separately to each fluid or solid portion, and additional surface integrals of the quantity $-\tfrac{1}{2}[\hat{\nu}(\xi)\cdot(2\mathbf{T}_0(\xi)+\tilde{\mathbf{T}}(\xi,t))\cdot\mathbf{s}(\xi,t)]_-^+$ then arise in (68) over all the interfaces where slip may occur. These additional surface integrals may all be shown to vanish, essentially because the traction at a fluid–solid interface must be normal to the interface, whereas the slip is necessarily tangential. To demonstrate that these additional surface integrals vanish, correct to second order in the displacement $\mathbf{s}(\mathbf{x},t)$, requires the use of the second-order tangential slip condition (10) on fluid–solid interfaces.

The final formula (69) for the total change in energy ΔE due to an earthquake fault is remarkable in that it is explicitly independent of both the rotation and the self-gravitation of the earth model. The absence of rotation and self-gravitation correspond, respectively, to the limits $\Omega\to 0$ and $G\to 0$. Since equation (69) does not explicitly contain either Ω or G, it is clear that exactly the same expression would have been obtained if both rotation and self-gravitation had never even been considered. Both $\mathscr{T}$ and $\mathscr{M}$ would in that case have been identically zero, and only elastic internal energy would have been available for release by an earthquake fault. It should be mentioned that Dahlen (1973), in a note added in proof, has given a formula for ΔE which is similar to (69), but which is incorrect because of a failure to take into account the perturbation $2(\delta\Omega/\Omega)\psi(\mathbf{x})$ in the rotational potential in the final uniformly rotating configuration of the earth model.

We are now able to resolve the discrepancy that the work $\mathscr{T}+\mathscr{M}$ performed against the action of body forces appears to be so much larger than the traditional empirical estimate E_{GR} of the total energy involved in earthquake faulting. We shall assume implicitly that E_{GR} provides at least a rough estimate of the total energy change ΔE, as it is intended to do. If that is so, then the work $\mathscr{T}+\mathscr{M}$ performed against the action of body forces must evidently be very nearly equal and opposite to the change $\mathscr{U}$ in stored elastic energy. The discrepancy will be resolved if we are able to understand the reason for this balance between $\mathscr{T}+\mathscr{M}$ and $\mathscr{U}$. The clue lies in the fact that both $\mathscr{T}+\mathscr{M}$ and $\mathscr{U}$ are quantities of first order in the displacement $\mathbf{s}(\mathbf{x})$. Both may evidently therefore be of either sign; the sign of $\mathscr{T}+\mathscr{M}$ for any given earthquake will depend in an obvious way upon the nature of the permanent elevation changes in the vicinity of the earthquake epicentre. The dominant contribution to $\mathscr{U}$ is clearly the work which has been performed against the initial hydrostatic pressure $p_0(\mathbf{x})$. We now investigate the extent to which the total energy change ΔE depends upon $p_0(\mathbf{x})$. Making use of equations (10), (17) and (26), and applying Gauss' theorem on curved surfaces, the formula (69) for ΔE can be transformed, correct to second order in $\mathbf{s}(\mathbf{x})$, into the form

$$\Delta E = -\tfrac{1}{2}\int_\Sigma (\mathbf{t}_0+\langle \mathbf{t}_f\rangle)\cdot[\mathbf{s}]_-^+\,dA + \tfrac{1}{2}\int_\Sigma [(\hat{\nu}\cdot\boldsymbol{\tau}_0-\hat{\nu}(\hat{\nu}\cdot\boldsymbol{\tau}_0\cdot\hat{\nu}))\cdot(\mathbf{s}\cdot\nabla_\xi\,\mathbf{s})]_-^+\,dA$$

$$-\tfrac{1}{2}\int_\Sigma \langle(\hat{\nu}\cdot\boldsymbol{\tau}_0\cdot\hat{\nu})\nabla_\xi(\hat{\nu}\cdot\mathbf{s})-\hat{\nu}\cdot(\mathbf{s}\cdot\nabla_\xi\boldsymbol{\tau}_0)-\boldsymbol{\tau}_0\cdot\nabla_\xi(\hat{\nu}\cdot\mathbf{s})\rangle\cdot[\mathbf{s}]_-^+\,dA, \tag{70}$$

where $\mathbf{t}_0(\xi)$ and $\mathbf{t}_f(\xi)$ are tangent vectors on Σ defined by

$$\mathbf{t}_0=\hat{\nu}\cdot\boldsymbol{\tau}_0-\hat{\nu}(\hat{\nu}\cdot\boldsymbol{\tau}_0\cdot\hat{\nu})$$
$$\mathbf{t}_f=\mathbf{t}_0+\hat{\nu}\cdot(\mathbf{T}-\tfrac{1}{3}\,tr\mathbf{T}\mathbf{I})-\hat{\nu}(\hat{\nu}\cdot\mathbf{T}\cdot\hat{\nu}-\tfrac{1}{3}\,tr\mathbf{T}), \tag{71}$$

and where an angle bracket has been used to denote the average value of a quantity on both sides of Σ, i.e. $\langle\mathbf{t}_f(\xi)\rangle=\tfrac{1}{2}[\mathbf{t}_f(\xi^+)+\mathbf{t}_f(\xi^-)]$. The quantity $\mathbf{t}_0(\xi)$ is just the tangential component of the initial traction on the fault surface Σ. Equation (70) is considerably less elegant and tidy than (69), but in conjunction with (12) and (14), it reveals that ΔE is explicitly independent of the initial hydrostatic pressure $p_0(\mathbf{x})$. This makes it clear that the balance between $\mathscr{T}+\mathscr{M}$ and $\mathscr{U}$ is in essence a balance between the work performed against body forces and that concomitantly performed against the initial hydrostatic pressure $p_0(\mathbf{x})$. This is not surprising, since the origin of the field $p_0(\mathbf{x})$ in the earth model may be attributed directly to the presence of the body forces. The initial deviatoric stress $\tau_0(\mathbf{x})$, which does make both a first and second order explicit contribution to the sum ΔE, is, in contrast, tectonic in origin. The quantities $\mathbf{t}_0(\xi)$ and $\langle\mathbf{t}_f(\xi)\rangle$ can, in general, be expected to be of roughly the same magnitude, so that ΔE, although strictly of first order, is virtually of second order in $\mathbf{s}(\mathbf{x})$.

Equation (70) bears a strong resemblance to the classical expression which is conventionally used in seismological investigations to determine the total energy released by an earthquake fault. Savage (1969) has given a particularly clear discussion of the assumptions which are embodied in deriving this classical expression. Both rotation and self-gravitation are ignored, and the initial static stress $\mathbf{T}_0(\mathbf{x})$ is assumed not only to be infinitesimally small, but also to be the result of an infinitesimal, purely elastic internal strain away from some more natural unstrained, unstressed state. The total change $\Delta E_{\text{classical}}$ in the elastic strain energy of the earth model upon introducing an earthquake fault may be shown under these circumstances to be

$$\Delta E_{\text{classical}}=-\tfrac{1}{2}\int_\Sigma(\mathbf{t}_0+\mathbf{t}_f)\cdot[\mathbf{s}]_-^+\,dA. \tag{72}$$

In this approximation, the quantity $\mathbf{t}_f(\xi)$ can be interpreted physically as just the tangential component of the final traction on Σ; it therefore satisfies $[\mathbf{t}_f(\xi)]_-^+=\mathbf{0}$. In any realistic earth model, the magnitude of the initial deviatoric stress will be such that $|\tau_{ij}^0(\mathbf{x})|\ll\mu(\mathbf{x})$. Equation (70) shows that in that case $\Delta E_{\text{classical}}$ will provide a very good approximation to ΔE; in fact,

$$\Delta E_{\text{classical}}=\Delta E[1+O(\tau_0/\mu)]. \tag{73}$$

The conclusion (73) was reached earlier by Dahlen (1973), but the argument given there is fallacious. The conclusion is correct only because of offsetting algebraic errors, including a failure to utilize the tangential slip condition (10), which is correct to second order in $\mathbf{s}(\mathbf{x})$.

4 The dissipation of released energy

The ultimate fate of all the energy released by an earthquake fault is to be dissipated somewhere within the volume of the earth model; this is a requirement of the law of conservation of energy. Three regions of energy dissipation may be distinguished, both spatially and temporally. First, energy may be dissipated during faulting, i.e. for $0 \leq t \leq T$, in heating on the walls of the instantaneous fault surface $\Sigma(t)$, where work must generally be performed against the frictional traction which acts there to resist slip. Second, energy may also be dissipated during this interval $0 \leq t \leq T$, either in overcoming material cohesion and thereby creating fresh fault surface area, or simply in additional heating, along the instantaneously expanding edge $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$ of the fault. Third and finally, the remainder of the released energy must constitute the energy of oscillation of the various non-secular normal modes of the earth model, and this energy of oscillation will be dissipated by the slight bodily anelasticity which has already been assumed to exist throughout the entire volume V; bodily dissipation will commence at the onset of faulting, i.e. at $t = 0$, and will continue until every non-secular normal mode of oscillation has thoroughly decayed. We shall denote the energy dissipated by bodily friction, which is generally referred to as the seismic energy, by E_S. We examine now the partition of released energy ΔE into seismic energy E_S and energy $\Delta E - E_S$ which is dissipated during faulting either on the walls or immediately at the edge of the expanding fault surface. We begin by considering the rates of change with time of the kinetic energy, the gravitational potential energy and the elastic internal energy of the earth model, during and subsequent to faulting; attention is fixed for the moment on a particular instant $t \geq 0$.

The instantaneous rate of change of the overall kinetic energy of the earth model, which we shall denote by $\dot{\mathscr{T}}(t)$, is given exactly by

$$\dot{\mathscr{T}}(t) = \frac{d}{dt}\int_V \tfrac{1}{2}\rho_0[\partial_t \mathbf{s} + (\boldsymbol{\Omega} + \delta\boldsymbol{\Omega}) \times (\mathbf{x} + \mathbf{s})] \cdot [\partial_t \mathbf{s} + (\boldsymbol{\Omega} + \delta\boldsymbol{\Omega}) \times (\mathbf{x} + \mathbf{s})]\, dV, \tag{74}$$

and the condition that the instantaneous rate of change of the angular momentum of the earth model must be zero may be written, also exactly, in the form

$$\frac{d}{dt}\int_V \rho_0(\mathbf{x} + \mathbf{s}) \times [\partial_t \mathbf{s} + (\boldsymbol{\Omega} + \delta\boldsymbol{\Omega}) \times (\mathbf{x} + \mathbf{s})]\, dV = \mathbf{0}. \tag{75}$$

The instantaneous rate of change of the gravitational potential energy of the earth model, which we shall denote by $\dot{\mathscr{M}}(t)$, is given correct to second order in $\mathbf{s}(\mathbf{x}, t)$ by

$$\dot{\mathscr{M}}(t) = \frac{d}{dt}\left\{\int_V \rho_0 s_j\, \partial_j \phi_0\, dV + \tfrac{1}{2}\int_V [\rho_0 s_j\, \partial_j \phi_1 + \rho_0 s_i s_j\, \partial_i \partial_j \phi_0]\, dV\right\}, \tag{76}$$

and the instantaneous rate of change of the stored elastic internal energy of the earth model, which we shall denote by $\dot{\mathscr{U}}(t)$, is defined as

$$\dot{\mathscr{U}}(t) = \frac{d}{dt}\left\{\tfrac{1}{2}\int_V T^0_{ij}(\partial_i s_j + \partial_j s_i)\, dV + \tfrac{1}{2}\int_V \Lambda_{ijkl}\, \partial_i s_j\, \partial_k s_l\, dV\right\}. \tag{77}$$

The sum $\dot{\mathscr{T}}(t) + \dot{\mathscr{M}}(t) + \dot{\mathscr{U}}(t)$ is then the instantaneous rate of change of the total energy content of the earth model.

During the time interval $0 \leq t \leq T$ that active faulting is occurring, we must devote special attention to the propagating edge or tip of the fault. Let $\epsilon(\boldsymbol{\xi})$ denote distance measured from a point $\boldsymbol{\xi}$ on this instantaneous fault tip $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$. Experience indicates that the macroscopic manifestation of either a non-zero material cohesion or fault tip heating will be $\epsilon^{-1/2}(\boldsymbol{\xi})$ singularities in each of the incremental variables $\partial_t \mathbf{s}(\mathbf{x}, t)$, $\boldsymbol{\nabla}\mathbf{s}(\mathbf{x}, t)$, and therefore $\tilde{\mathbf{T}}(\mathbf{x}, t)$ and $\mathbf{T}(\mathbf{x}, t)$, all along the curve $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$. Following the procedure employed by Kostrov (1974), we can allow in a very general way for the possible existence of either or both sorts of fault tip dissipation by simply recognizing that such $\epsilon^{-1/2}(\xi)$ singularities may be present. The volume integrals in each of equations (74), (75), (76) and (77) are rendered improper by the presence of these singularities, but as long as no singularity is stronger than $\epsilon^{-1/2}(\xi)$, every such improper integral will exist, provided they are suitably defined in the obvious way. We let $V_0(t)$, with surface $\partial V_0(t)$, be a closed interior volume element which, at the instant t, surrounds the instantaneous fault surface $\Sigma(t)$, and we define each of the volume integrals in equations (74), (75), (76) and (77) to be the limit of the corresponding volume integral over the possibly punctured volume $V-V_0(t)$, as $V_0(t)\to 0$ in such a way that $V_0(t)$ completely envelops $\Sigma(t)$. The principal consequence of recognizing that there may be $\epsilon^{-1/2}(\xi)$ singularities is then that in carrying out the differentiation of any integral defined over the volume $V-V_0(t)$, we must remember to take the time variation of the volume of integration into account. We may for this purpose suppose that $V_0(t)$ has almost entirely collapsed onto the fault surface $\Sigma(t)$, so that only a cylinder or torus surrounding the edge $\partial\Sigma(t)-\partial\Sigma(t)\cap\partial V$ of the fault surface remains. Let $V_\epsilon(t)$, with surface $\partial V_\epsilon(t)$, denote the volume of this cylinder or torus, and let $\Sigma^-(t)$ be a subset of Σ which at the instant t lies just inside $\Sigma(t)$, so that the surface $\partial V_0(t)$ consists of $\partial V_\epsilon(t)$ plus both sides of $\Sigma^-(t)$. Both $V_\epsilon(t)$ and $\partial V_\epsilon(t)$ will be time-dependent during the interval of faulting $0\leqslant t\leqslant T$, since both are assumed to move with the edge of the fault $\partial\Sigma(t)-\partial\Sigma(t)\cap\partial V$. Let $\mathbf{v}(\xi)$ denote the rupture velocity of the fault tip $\partial\Sigma(t)$ as it passes through the point ξ on Σ; at any point ξ on the final fault boundary $\partial\Sigma$, it must be the case that $\mathbf{v}(\xi)=\mathbf{0}$. Let $\hat{\mathbf{n}}(\mathbf{x},t)$ be the unit inward normal to $V_\epsilon(t)$ at the material particle $\mathbf{x}$ on $\partial V_\epsilon(t)$, and let $q(\mathbf{x},t)$ be the inward normal velocity of $\partial V_\epsilon(t)$ as it passes through the material particle $\mathbf{x}$. If $\mathbf{x}$ on $\partial V_\epsilon(t)$ tends to ξ on $\partial\Sigma(t)-\partial\Sigma(t)\cap\partial V$ as $V_\epsilon(t)\to 0$, then the condition which guarantees that $V_\epsilon(t)$ and $\partial V_\epsilon(t)$ move with $\partial\Sigma(t)-\partial\Sigma(t)\cap\partial V$ is

$$q(\mathbf{x},t)=\hat{\mathbf{n}}(\mathbf{x},t)\cdot\mathbf{v}(\xi). \tag{78}$$

Upon applying the standard theorem for differentiating an integral defined over a moving volume, as well as Gauss' theorem, and making use of (75) to help reduce (74), we can finally write the sum $\dot{\mathscr{T}}(t)+\dot{\mathscr{M}}(t)+\dot{\mathscr{U}}(t)$ in the form

$$\begin{aligned}\dot{\mathscr{T}}(t)+\dot{\mathscr{M}}(t)+\dot{\mathscr{U}}(t)=&\int_V \partial_t s_j[\rho_0\,\partial_j(\phi_0+\psi)-\partial_i T^0_{ij}]\,dV\\&+\int_V \partial_t s_j\Big[\rho_0\,\partial_t^2 s_j+2\rho_0\epsilon_{jkl}\Omega_k\,\partial_t s_l+\rho_0\,\partial_j\Big(\phi_1+2\frac{\delta\Omega}{\Omega}\psi\Big)\\&+\rho_0 s_i\,\partial_i\,\partial_j(\phi_0+\psi)-\partial_i\tilde{T}_{ij}\Big]\,dV+\int_{\partial V} n_i(T^0_{ij}+\tilde{T}_{ij})\,\partial_t s_j\,dA\\&-\lim_{\Sigma^-(t)\to\Sigma(t)}\int_{\Sigma^-(t)}[\nu_i(T^0_{ij}+\tilde{T}_{ij})\,\partial_t s_j]^+_-\,dA\\&+\lim_{V_\epsilon(t)\to 0}\int_{\partial V_\epsilon(t)}[qw+n_i(T^0_{ij}+\tilde{T}_{ij})\,\partial_t s_j]\,dA,\end{aligned}\tag{79}$$

where

$$\begin{aligned}w=&\tfrac12\rho_0\,|\partial_t\mathbf{s}+(\boldsymbol{\Omega}+\delta\boldsymbol{\Omega})\times(\mathbf{x}+\mathbf{s})|^2+\rho_0 s_j\,\partial_j\phi_0+T^0_{ij}\tfrac12(\partial_i s_j+\partial_j s_i)\\&+\tfrac12[\rho_0 s_j\,\partial_j\phi_1+\rho_0 s_i s_j\,\partial_i\,\partial_j\phi_0+\Lambda_{ijkl}\,\partial_i s_j\,\partial_k s_l]\end{aligned}\tag{80}$$

is, correct to second order in $\delta\Omega$ and $\mathbf{s}(\mathbf{x},t)$, the density of total energy in the earth model, measured per unit volume at the material particle $\mathbf{x}$ at time t.

In deriving the result (79), we have also made use of the symmetry (16) of the isentropic elastic tensor $\Lambda(\mathbf{x})$ and of the fact that $\phi_1(\mathbf{x}, t)$ is a linear functional of $\mathbf{s}(\mathbf{x}, t)$, as expressed by equation (32). The first two volume integrals in (79), as well as the surface integral over the outer surface ∂V, all vanish identically, since they contain replicas of (3), (33), (4) and (24). Upon defining a quantity $\dot{\mathscr{E}}(t)$ by the relation

$$\dot{\mathscr{E}}(t) = -\lim_{V_\epsilon(t)\to 0} \int_{\partial V_\epsilon(t)} [qw + \hat{\mathbf{n}}\cdot(\mathbf{T}_0 + \tilde{\mathbf{T}})\cdot\partial_t \mathbf{s}]\, dA, \tag{81}$$

equation (79) is therefore reduced to the simple final result

$$\dot{\mathscr{T}}(t) + \dot{\mathscr{M}}(t) + \dot{\mathscr{U}}(t) + \dot{\mathscr{E}}(t) = -\lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} [\hat{\nu}\cdot(\mathbf{T}_0 + \tilde{\mathbf{T}})\cdot\partial_t \mathbf{s}]_-^+\, dA. \tag{82}$$

The proof of (82) fails if the earth model contains any internal fluid–solid interfaces, but the result itself remains valid. The additional considerations needed to verify this are similar to those used to verify that (68) also remains valid in that case.

Equation (81) states precisely that $\dot{\mathscr{E}}(t)$ is the instantaneous flux of macroscopic total energy into the propagating fault tip $\partial\Sigma(t) - \partial\Sigma(t)\cap\partial V$; this flux is seen to arise quite naturally as a macroscopic measure of the total energy dissipation at the fault tip. There are two distinct contributions to this energy flux, both of which have a simple physical interpretation. The first is an advective contribution due to the physical motion of the surface $\partial V_\epsilon(t)$ as it moves with the fault tip $\partial\Sigma(t) - \partial\Sigma(t)\cap\partial V$, while the second arises from the work which is being instantaneously performed by the traction exerted on the surface $\partial V_\epsilon(t)$. The interpretation of equation (82) is perhaps most elegant if there is no fault tip heating; it is then the statement that the instantaneous rate of change of the total energy of the earth model, including the surface energy associated with the formation of fresh fault surface, is equal to the instantaneous rate of working of the traction on the fault surface $\Sigma(t)$. The outer surface ∂V of the earth model is traction-free, and no work is performed there. The fact that we must in general allow for heating as well as cohesion at the fault tip in the energy balance (82) has been pointed out by Richards (1976). In any model of seismic faulting based on a macroscopic energy balance fracture criterion, it is the quantity $\dot{\mathscr{E}}(t)$, which incorporates both possibilities, that would generally be independently prescribed; how much of the total flux $\dot{\mathscr{E}}(t)$ goes into doing work against cohesion and how much goes simply into heating cannot be decided without making additional essentially microscopic assumptions about the cohesive strength of the material comprising the earth model.

The flux of energy into the tip of an expanding crack in an elastic solid body which is neither rotating nor self-gravitating and which is not subject to a large initial static stress, has been discussed previously and in some detail by Freund (1972). By a simple extension of an argument in that paper, the expression (81) for the quantity $\dot{\mathscr{E}}(t)$ can be simplified, and equation (82) can be given a slightly different physical interpretation. Suppose that at a point $\boldsymbol{\xi}$ on $\partial\Sigma(t) - \partial\Sigma(t)\cap\partial V$, the cross section of the volume $V_\epsilon(t)$ is chosen to be a rectangle with sides of dimension ϵ_1 perpendicular to the fault surface Σ and ϵ_2 parallel to the fault surface Σ. Note that in this instance $q(\mathbf{x}, t) = 0$ if $\mathbf{x}$ is on a face of $\partial V_\epsilon(t)$ which is parallel to Σ. If now $V_\epsilon(t)$ is allowed to vanish by first letting $\epsilon_1 \to 0$ and then letting $\epsilon_2 \to 0$, and if the integrals over the two faces of $\partial V_\epsilon(t)$ which are perpendicular to Σ are assumed to vanish as $\epsilon_1 \to 0$, then $\dot{\mathscr{E}}(t)$ may be reduced to

$$\dot{\mathscr{E}}(t) = \lim_{\Sigma^\pm(t)\to\Sigma(t)} \int_{\Sigma^+(t)-\Sigma^-(t)} [\hat{\nu}\cdot(\mathbf{T}_0 + \tilde{\mathbf{T}})\cdot\partial_t \mathbf{s}]_-^+\, dA, \tag{83}$$

where $\Sigma^+(t)$ is a subset of Σ which at the instant t lies just outside the instantaneous fault surface $\Sigma(t)$. According to equation (83), the quantity $\mathscr{C}(t)$ may be alternatively viewed as minus the rate of working of the macroscopic traction which prevails immediately at the instantaneous position of the fault tip $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$; there is no contribution to $\dot{\mathscr{C}}(t)$ from $\partial\Sigma(t) \cap \partial V$, since the integrand in equation (83) is not singular along that curve. The fundamental energy rate balance (82) can now be rewritten in the form

$$\dot{\mathscr{T}}(t) + \dot{\mathscr{M}}(t) + \dot{\mathscr{U}}(t) = -\lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} [\hat{\nu}\cdot(\mathbf{T}_0+\tilde{\mathbf{T}})\cdot\partial_t \mathbf{s}]_-^+ \, dA$$
$$- \lim_{\Sigma^\pm(t)\to\Sigma(t)} \int_{\Sigma^+(t)-\Sigma^-(t)} [\hat{\nu}\cdot(\mathbf{T}_0+\tilde{\mathbf{T}})\cdot\partial_t \mathbf{s}]_-^+ \, dA. \tag{84}$$

This alternative version states that the instantaneous rate of change of the sum of the kinetic and gravitational-elastic potential energies of the earth model is equal to the rate of working of the tractions on the fault surface $\Sigma(t)$, including the working at the instantaneous fault tip $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$, where if there is either cohesion or heating both the tractions and the material particle velocities may be singular. In general the rate of working of the tractions on both $\Sigma(t)$ and $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$ will be negative (or possibly zero), corresponding to a dissipation of the total energy of the earth model.

We now integrate either equation (82) or (84) with respect to time from $t = 0$ to $t = \infty$. Thus far the slight bodily dissipation of the earth model, which must be present if the non-secular normal modes of oscillation are ever to decay, has not been explicitly introduced. We now assume that the only effect of this slight bodily dissipation is to add to the sum $\dot{\mathscr{T}}(t) + \dot{\mathscr{M}}(t) + \dot{\mathscr{U}}(t)$ throughout the interval $0 \leqslant t \leqslant \infty$ a small negative quantity, representing the instantaneous rate of working against the bodily friction. The total seismic energy E_s which is dissipated by bodily friction is then evidently given by

$$\int_0^\infty [\dot{\mathscr{T}}(t) + \dot{\mathscr{M}}(t) + \dot{\mathscr{U}}(t)]\, dt = \mathscr{T} + \mathscr{M} + \mathscr{U} + E_s, \tag{85}$$

where, as before, $\mathscr{T} + \mathscr{M}$ is the net work which has been performed against the action of body forces, and $\mathscr{U}$ is the net change in the stored elastic energy of the earth model. Combining equations (82) or (84) with (67) and (85), we obtain

$$-\Delta E = E_s + \int_0^\infty \dot{\mathscr{C}}(t)\, dt + \int_0^\infty dt \lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} [\hat{\nu}\cdot(\mathbf{T}_0+\tilde{\mathbf{T}})\cdot\partial_t \mathbf{s}]_-^+ \, dA. \tag{86}$$

This equation, which might well have been written down from first principles, is precisely a statement of the already mentioned tripartite partition of the total energy released by an earthquake fault. The total energy released, which must be positive, is $-\Delta E$. Each of the three terms on the right-hand side of equation (87) will in general also be positive (or possibly zero). The first term E_s is the seismic energy dissipated by bodily friction, the second term is the total energy which has been dissipated in the generally irreversible creation of fresh fault surface at the propagating tip fault $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$, and the third term is the total energy which has been dissipated in doing work on the walls of the fault, where the traction is evidently frictional in nature. Generally, energy will be dissipated on the fault surface $\Sigma(t)$ and at the fault tip $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$ only during faulting, $0 \leqslant t \leqslant T$, and both of the infinite upper limits of integration in equation (86) can therefore be replaced by T.

Since $[\mathbf{s}(\boldsymbol{\xi})]_-^+ = \mathbf{0}$ on $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$, it is clear that

$$\frac{d}{dt}\left\{\lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} [\hat{\boldsymbol{\nu}}\cdot(\mathbf{T}_0+\tilde{\mathbf{T}})\cdot\mathbf{s}]_-^+\,dA\right\} = \lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} [\hat{\boldsymbol{\nu}}\cdot(\mathbf{T}_0+\tilde{\mathbf{T}})\cdot\partial_t\mathbf{s}]_-^+\,dA + \lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} [\hat{\boldsymbol{\nu}}\cdot\partial_t\tilde{\mathbf{T}}\cdot\mathbf{s}]_-^+\,dA. \qquad (87)$$

Substituting the identity (87) into (86), and making use of the form (69) for the energy change ΔE, we obtain for the seismic energy E_s the final form

$$E_s = -\tfrac{1}{2}\int_\Sigma [\hat{\boldsymbol{\nu}}\cdot\tilde{\mathbf{T}}\cdot\mathbf{s}]_-^+\,dA + \int_0^\infty dt \lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} [\hat{\boldsymbol{\nu}}\cdot\partial_t\tilde{\mathbf{T}}\cdot\mathbf{s}]_-^+\,dA - \int_0^\infty \dot{\mathscr{E}}(t)\,dt \qquad (88)$$

The seismic energy dissipated throughout the volume V of the earth model has here been expressed in terms only of quantities on the fault surface $\Sigma(t)$ and at the fault tip $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$. The first term in equation (88) depends only upon the final displacement and incremental traction on the final surface of faulting Σ, but the second term depends intimately on the entire history of the displacement and incremental traction on the growing fault surface $\Sigma(t)$. It is apparent that there can in general be no simple equation which relates E_s only to the final static changes in the earth model. It should be noted that the infinite upper limit of integration in the second term of equation (88) cannot in general be replaced by T, since the quantity $\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\partial_t\tilde{\mathbf{T}}(\boldsymbol{\xi}, t)$ need not vanish on either side of Σ for $t \geqslant T$, even though $[\mathbf{s}(\boldsymbol{\xi}, t)]_-^+$ has attained its final static value $[\mathbf{s}(\boldsymbol{\xi})]_-^+$. The seismic energy E_s, by its definition, must be strictly quadratic in the infinitesimal particle displacement $\mathbf{s}(\mathbf{x}, t)$. This is true for the first two terms in equation (88) by inspection, and it is easily shown to be true for the third term as well.

By far the most important point we wish to make regarding the seismic energy E_s is that it is explicitly independent of both the rotation and the self-gravitation of the earth model, i.e. exactly the same expression for E_s would have been obtained if neither rotation nor self-gravitation had ever been considered. This is evident from the fact that equation (88) does not contain explicitly either the rate of rotation Ω or the gravitational constant G. It has been pointed out already that the same remark is valid for the total energy change ΔE, given by equation (69). The seismic efficiency of an earthquake, which we shall denote by η, is customarily defined as

$$\eta = -E_s/\Delta E, \qquad (89)$$

i.e. it is the fraction of the net released energy which is not dissipated immediately at the earthquake source, and which is therefore available for exciting the various non-secular normal modes of the earth model. Since both E_s and ΔE are explicitly independent of the rotation and self-gravitation of the earth model, it is clear that the seismic efficiency η must be as well.

Kostrov (1974) has derived an expression for the seismic energy which is similar to (88), using an argument in which rotation is neglected, and, more seriously, the initial static stress field $\mathbf{T}_0(\mathbf{x})$ and the initial gravitational potential $\phi_0(\mathbf{x})$ are assumed to be infinitesimal quantities. The expression he obtains may be written in the form

$$E_s^{\text{classical}} = -\tfrac{1}{2}\int_\Sigma \mathbf{t}\cdot[\mathbf{s}]_-^+\,dA + \int_0^\infty dt \lim_{\Sigma^-(t)\to\Sigma(t)} \int_{\Sigma^-(t)} \partial_t\mathbf{t}\cdot[\mathbf{s}]_-^+\,dA - \int_0 \dot{\mathscr{E}}_{\text{classical}}(t)\,dt, \qquad (90)$$

where $\mathbf{t}(\boldsymbol{\xi},t) = \hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot[\mathbf{T}(\boldsymbol{\xi},t) - (1/3)tr\mathbf{T}(\boldsymbol{\xi},t)\mathbf{I}] - \hat{\boldsymbol{\nu}}(\boldsymbol{\xi})[\hat{\boldsymbol{\nu}}(\boldsymbol{\xi})\cdot\mathbf{T}(\boldsymbol{\xi},t)\cdot\hat{\boldsymbol{\nu}}(\boldsymbol{\xi}) - (1/3)\,tr\mathbf{T}(\boldsymbol{\xi},t)]$ is the tangential component of the incremental traction acting on Σ, and

$$\dot{\mathscr{E}}_{\text{classical}}(t) = \underset{\Sigma^{\pm}(t)\to\Sigma(t)}{\text{limit}} \int_{\Sigma^{+}(t)\,-\,\Sigma^{-}(t)} (\tau_0 + \boldsymbol{\tau})\cdot[\partial_t \mathbf{s}]_-^+ \, dA \tag{91}$$

is the corresponding classical formula for the macroscopic rate of energy dissipation at the fault tip $\partial\Sigma(t) - \partial\Sigma(t) \cap \partial V$. By differentiating the second order tangential slip condition (10) with respect to time and inserting the result into (83) and (86), it is straightforward to show, by an argument similar to that which led from (69) to (70) and (73), that

$$\dot{\mathscr{E}}_{\text{classical}}(t) = \dot{\mathscr{E}}(t)[1 + 0(\tau_0/\mu)] \tag{92}$$

and

$$E^{\text{s}}_{\text{classical}} = E_{\text{s}}[1 + 0(\tau_0/\mu)]. \tag{93}$$

This, together with (73), justifies the continued usage of the classical formulae (72), (90) and (91) in seismological applications.

Conclusion

The possible influence of the rotation of the Earth upon the energy release of an earthquake does not appear to have been considered previously. In particular, the seemingly paradoxical fact that the change in the rotational kinetic energy of the Earth associated with the change in the angular velocity of rotation caused by a fairly large earthquake can exceed by as much as two orders of magnitude the conventional empirical estimate of the total energy released by such an earthquake has not, to my knowledge, been noted, although that deduction is remarkably easy. The need to consider a gravitational contribution to the energy release, especially if permanent elevation changes in the vicinity of the earthquake focus have occurred, has on the other hand often been noted, although seldom quantitatively explored. A not uncommon misconception appears to be that the gravitational energy contribution can be adequately accounted for by adding the first-order work done against gravitational body forces to the classical expression $\Delta E_{\text{classical}}$ for the energy change in the absence of gravity. We have shown this *ad hoc* procedure to be completely unfounded, as it amounts essentially to having counted the relatively enormous change in gravitational potential energy twice. Both the total energy change ΔE and the seismic energy E_{s} can be expressed in terms of quantities only on the fault surface of an earthquake, and in this form both ΔE and E_{s}, and therefore also the seismic efficiency η, are explicitly independent of the rotation and the self-gravitation of the Earth.

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