The basic structure of neoclassical general equilibrium theory

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THE BASIC STRUCTURE OF NEOCLASSICAL GENERAL EQUILIBRIUM THEORY*

0. INTRODUCTION

It is the aim of this paper to present a structuralist reconstruction of a type of neoclassical economic theory that is predominantly used as point of departure in the neoclassical analysis, especially in that of international trade. We have chosen to call it "general equilibrium of a closed economy" (GECE).

We have tried to bring out as clearly as possible the meaning of that structure for the particular research-strategy employed by scientists in the field: the structure of the theory's models already indicates how economists will proceed in order to find the models.[1]

To avoid misconceptions, we shall always concisely illustrate the concept to be reconstructed *in the economist's way*. This will clarify to the reader what exactly we want to describe in the language of structuralism, and thus will enable him to check whether or not we are right. Also, this will reveal the reader something that structuralism *pur sang* cannot reveal: the spirit of the discipline.

Only minor modifications will be necessary to make the structure fit other forms of neoclassical general equilibrium analysis.[2]

1. THE PARTIAL POTENTIAL MODEL

In economics, a "model" is given by a set of assumptions. A "two by two model", for instance, is an imaginary economic region ("country") where there are two goods (γ_1, γ_2) to be produced and two factors ("means of production") to produce with (ϕ_1, ϕ_2).

Primary concepts are therefore

(1) kind of goods: $\gamma \in \Gamma$, and a function q_{output} such that

$$q_{\text{output}}: \Gamma \to R^+$$

assigns a nonnegative[3] real number y_γ to goods γ ($y_\gamma = q_{\text{output}}(\gamma)$).

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(2) kinds of factors: $\phi \in \Phi$, used in the production of each of the goods γ, and a function q_{input} such that

$$q_{\text{input}}: \Phi \times \Gamma \rightarrow R^+$$

assigns a nonnegative real number $\alpha_{\phi,\gamma}$ to every combination of a factor with a good in the production of which the factor is used, $\alpha_{\phi,\gamma} = q_{\text{input}}(\phi, \gamma)$ representing the amount of factor ϕ used in the production of good γ.

We shall use the expression "industry γ" to refer to the production of good γ. So, in a two by two model, there are two industries in the country: industry$_{\gamma_1}$, and industry$_{\gamma_2}$. We shall use "n" to denote the number of industries and "m" to denote the number of factors. "With each commodity . . . is associated a real number, its price."[4] The second type of nontheoretical concepts in the economists' model are the prices of the ϕ's and γ's. There are therefore

(3) the prices of goods $\gamma \in \Gamma$, a function p such that

$$p: \Gamma \rightarrow R^+$$

assigns a nonnegative[5] real number p_γ to goods γ.

(4) the prices of factors $\phi \in \Phi$, a function w[6] such that

$$w: \Phi \rightarrow R^+$$

assigns a nonnegative[5] real number w_ϕ to factors ϕ.

This is what constitutes the theory's partial potential model:

D1 x is a partial potential model of GECE ($x \in M_{pp}$) if there exist Γ, Φ, q_{input}, q_{output}, w, p such that

(1) $x = \langle \Gamma, \Phi, q_{\text{input}}, q_{\text{output}}, w, p \rangle$
(2) Γ is a finite, nonempty set $\Gamma = \{\gamma_1, \ldots, \gamma_n\}$
(3) Φ is a finite, nonempty set $\Phi = \{\phi_1, \ldots, \phi_m\}$
(4) q_{input}, q_{output}, p and w are functions as defined above.

It is about these "things" that GECE analysts speak. It is important here to understand the verb "exist" in D1 in the right way. A naive understanding here might allow only for those items which can be "read off" from "real" countries, and according to such a narrow interpretation of "existence" it might be objected that, for instance, there "exist" no countries with $m = n = 2$ (i.e., there "exist" no two by two models). But this conception of existence is clearly besides the point. "Existence" in theoretical context always goes beyond "observable" or "directly conceivable" existence.

It should be stressed from the outset that in economics, as well as in any theoretical field, "*existence*" (as in "existence theorems") is synonymous with *conceivability*. It is the mathematician's perfectly legitimate use of the verb: there "exist" real numbers that are integer and odd, but there "exist" no real numbers that are positive and negative. The applications of the structures we deal with in this paper are therefore *mental constructions*, and not necessarily objects that are empirically observed, though this is by no means excluded. It cannot be overemphasized that it is the goal of the endeavours[7] of General Equilibrium-analysts to *say true things about abstract systems*, that is, about *mental constructions*.

2. THE POTENTIAL MODEL: THE THEORETICAL FUNCTIONS

The theoretical functions in the potential models of GECE serve the purpose of allowing the formulation of restrictions on the class of partial potential models, restrictions to the effect that "equilibrium" exists and is – preferably – unique. More precisely, we may introduce in each partial potential model $x = \langle \Gamma, \Phi, q_{\text{input}}, q_{\text{output}}, w, p \rangle$ the *state space* $S(x)$ *of* x as the set of all possible states, where a possible state consists of a four-tuple, containing a matrix and three vectors: $s = \langle [\alpha], [y], [w], [p] \rangle$, denoting a certain combination of quantities of factors, goods, wages of factors and prices of goods, respectively. If we denote by "$R(u)$" the *range* of a variable[8] u, i.e., the set of all possible values of u, then $S(x)$ can be written as the Cartesian product

$$R([\alpha]) \times R([y]) \times R([w]) \times R([p]).$$

Note that $S(x)$ does not really depend on x, so that we can simply talk about "the" state space.

With the help of theoretical functions this space is narrowed down to a subset of "equilibrium states". The procedure is to add theoretical functions plus further requirements (called "special conditions" in Hamminga (1983)) to the functions occurring in the partial potential model. In this way one defines a class of potential models: partial potential models to which theoretical functions are added; and a class of models: those potential models satisfying the special conditions. The class of models then can be used to narrow down the class of partial potential models to the class of all "theoretically admitted" partial potential models, namely those which are "parts of" proper models. The latter class will contain only partial potential models the state space of which is restricted to a set of equilibrium states.

"Equilibrium" simply is given by the solution of a certain set of equations. We shall first characterize this set of equations, in which the theoretical functions play a crucial role. For purposes of graphical illustration in the economist's way (figure 1), we assume $m = n = 2$.

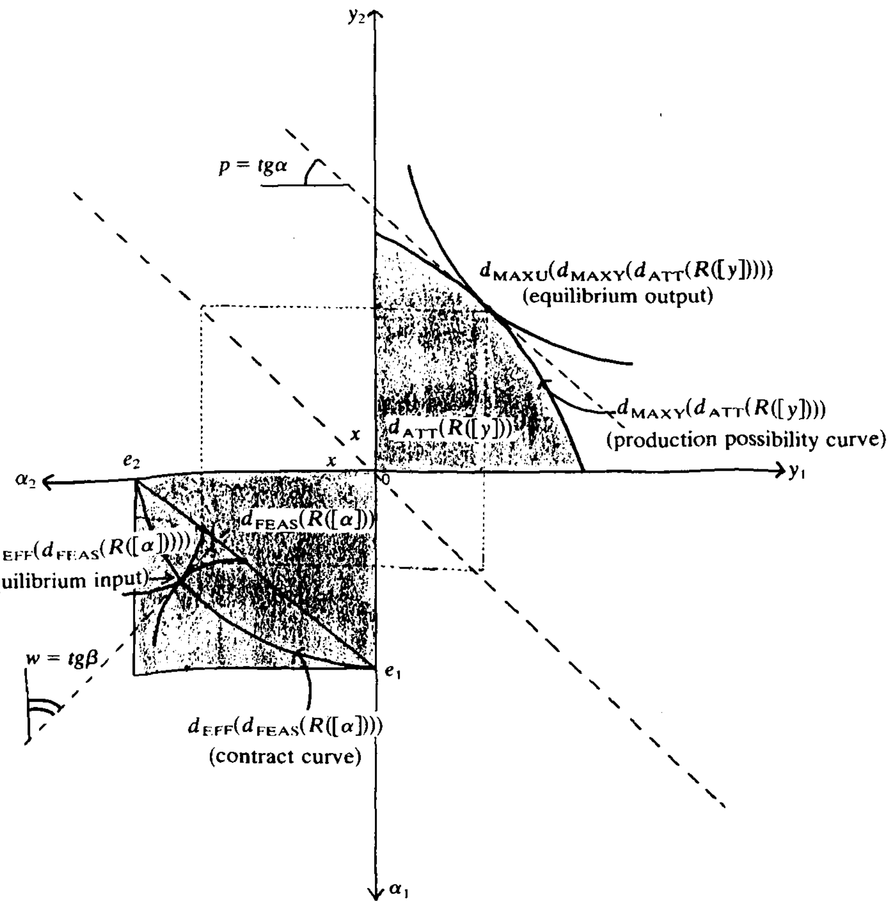

Fig. 1. A graphical model of GECE ($m = n = 2$).

The first quadrant represents $R([y])$, the "range of $[y]$", where $[y]$ is a vector of amounts of γ_1 and γ_2. Similarly $R([\alpha])$ is represented by the third quadrant, the axes of which measure α_1, the amount of ϕ_1, and α_2, the amount of ϕ_2 *positively*. Any point in this quadrant represents the sum total of the input of both factors in the two industries, and, given a certain total amount of each of the factors available in the country, such a point can also be taken to measure factor input in one of the industries, thereby also uniquely specifying factor input in the other (being the unused rest of the available factors).[9] For $m = n = 2$ the state space $S(x)$ of an $x \in M_{pp}$ therefore is represented by the set of all quadruples $\langle[\alpha], [y], [w], [p]\rangle$, where $[\alpha]$ is a point in the third quadrant, $[y]$ is a point in the first quadrant, $[w]$, being an exchange ratio of factors can be represented by the slope of a line in the third quadrant, and, similarly, some value of $[p]$ can be represented by a line with a certain slope in the first quadrant. Theoretical functions allow the formulation of restrictions such that, given a specific form of these functions, a solution $\langle[\alpha], [y], [w], [p]\rangle$ exists and is unique.

There are three theoretical functions used for that purpose:

(1) the factor endowment function of the country, a function $q_{endowment}$ such that

$$q_{endowment}\colon \Phi \rightarrow R^+$$

assigns a nonnegative real number e_ϕ to each kind of factor ϕ. The numbers represent the total amount of factors of production that are available in the country. In figure 1, they are represented by e_1 and e_2.

(2) the production functions of the industries, a function G such that

$$G\colon\ R([\alpha]) \rightarrow R([y])$$

assigns a nonnegative real vector $[y]$ to factor input matrices $[\alpha]$, where $\alpha_{\phi\gamma}$ is the amount of factor ϕ used in the production of γ ($\phi = 1, \ldots, m$; $\gamma = 1, \ldots, n$).

The production function G gives the amounts y_1 and y_2 that will be produced, if the factors are distributed in a certain way over the industries of the country.

In the case of $m = n = 2$, $[\alpha]$ is just a two by two matrix.[10]

$$[\alpha] = \begin{bmatrix} \alpha_{11} & \alpha_{21} \\ \alpha_{12} & \alpha_{22} \end{bmatrix}$$

(3) the aggregate utility function of the country, a function U such that

$$U: R([y]) \to R^+$$

assigns a nonnegative real number to every output vector $[y]$ in the first quadrant of figure 1.

So, we arrive at our potential model.

D2 x is a *potential model* of GECE ($x \in M_p$) if there exist Γ, Φ, q_{input}, q_{output}, w, p, $q_{\text{endowment}}$, G, U such that
(1) $x = \langle \Gamma, \Phi, q_{\text{input}}, q_{\text{output}}, w, p, q_{\text{endowment}}, G, U \rangle$
(2) $\langle \Gamma, \Phi, q_{\text{input}}, w, p \rangle \in M_{pp}$
(3) $q_{\text{endowment}}: \Phi \to R^+$
(4) $G: R([\alpha]) \to R([y])$ is smooth
(5) $U: R([y]) \to R^+$.

3. EQUILIBRIUM CONDITIONS ON QUANTITIES

Equilibrium conditions on the state set $S(x) = R([\alpha]) \times R([y]) \times R([w]) \times R([p])$ are introduced with the help of theoretical functions. We shall first deal with the basic[11] restrictions on all possible *quantities* ($R([\alpha]) \times R([y])$) in the state set.

The restrictions are formulated with the help of six definitions $\Delta 1, \ldots, \Delta 6$, defining, in interaction, two sequences of subsequently narrower sets:

$\Delta 1$ A factor input matrix $[\alpha]$ is *feasible* given a factor endowment vector $[e]$ (write: *feas* $([\alpha] \mid [e])$) iff

$$[\alpha] \in d_{\text{FEAS}}(R([\alpha])) := \left\{ [\alpha] \,\middle|\, \overset{m}{\underset{\phi=1}{\forall}} \left(\sum_{\gamma=1}^{n} \alpha_{\phi\gamma} \leq e_\phi \right) \right\}$$

In figure 1 the set $d_{\text{FEAS}}(R([\alpha]))$ is represented by the rectangular shaded area in the third quadrant. Its size is determined by the value of e_1 and e_2, that is, by the amounts of ϕ_1 and ϕ_2 that are available in the country. The terminology is evident: factor input matrices outside this area are not feasible in the country.

$\Delta 2$ An output vector is *attainable* given a factor endowment vector and a production function (write: *att* ([y] | [e], G)), iff:

$[y] \in d_{ATT}(R([y])) := \{[y] \mid$ there is $[\alpha]$ such that
(1) $[y] = G([\alpha])$
(2) *feas* $([\alpha] \mid [e])\}$.

The set $d_{ATT}(R([y]))$ is in figure 1 the shaded area in the first quadrant. Its shape is determined by [e] and G, and we have drawn the one that economists are accustomed to use for illustrative purposes. The terminology reflects the intuition that factor endowments and production functions determine what combinations of output quantities can be *attained* in a country.

Δ3 An output vector [y] is on the *maximum boundary* or *production possibility curve* given [e] and G (write: *maxy* ([y] | [e], G)), iff

$[y] \in d_{MAXY}(d_{ATT}(R([y]))) :=$
$:= \{[y] \mid$ (1) *att* $([y] \mid [e], G)$
(2) for all $[y]'$, if $[y] \leq [y]'$ and *att* $([y]' \mid [e], G)$ then $[y]' = [y]\}$

The set $d_{MAXY}(d_{ATT}(R([y])))$ is the curve indicated by the arrow in the first quadrant.

Δ4 A factor input matrix [α] is on the *contract curve*, or is *efficient* given [e] and G (write: *eff* ([α] | [e], G)), iff

$[\alpha] \in d_{EFF}(d_{FEAS}(R([\alpha]))) :=$
$:= \{[\alpha] \mid$ there is $[y]$ such that
(1) $G([\alpha]) = [y]$
(2) *feas* $([\alpha] \mid [e])$
(3) *maxy* $([y] \mid [e], G)\}$.

The set $d_{EFF}(d_{FEAS}(R([\alpha])))$ is the curve indicated by the arrow in the third quadrant. The explication of this graph is standard economics (see Henderson and Quandt, 1980, p. 288 etc., and Lancaster, 1957). We measure factor input in industry γ_1 taking point e_2 as the origin, and factor input in industry γ_2 by taking e_1 as the origin, so that $\alpha_{11} + \alpha_{12} = e_1$ and $\alpha_{21} + \alpha_{22} = e_2$.

If the country "chooses" a factor input matrix on the contract curve, it will produce an output that is on the production possibility curve.

Δ5 An output vector $[y]$ is a *maximum utility output vector* or an *equilibrium output vector* given $[e]$, G and U (write: *maxu* $([y] \mid [e], G, U)$) iff

$$[y] \in d_{\text{MAXU}}(d_{\text{MAXY}}(d_{\text{ATT}}(R([y])))) :=$$
$$:= \{[y] \mid (1)\ maxy([y] \mid [e], G)$$
$$(2) \text{ for all } [y]', \text{ if } maxy([y]' \mid [e], G) \text{ and if } U([y]) \leq U([y]') \text{ then } [y] = [y]'\}.$$

The set $d_{\text{MAXU}}(d_{\text{MAXY}}(d_{\text{ATT}}(R([y]))))$ is hoped to be represented by one and only one point in the first quadrant (why and how this "hope", is explained below).

Δ6 A factor input matrix $[\alpha]$ is *optimal* or an *equilibrium input matrix* given $[e]$, G and U (write: *opt* $([\alpha] \mid [e], G, U)$) iff

$$[\alpha] \in d_{\text{OPT}}(d_{\text{EFF}}(d_{\text{FEAS}}(R([\alpha])))) :=$$
$$:= \{[\alpha] \mid \text{there is } [y] \text{ such that}$$
$$(1)\ eff([\alpha] \mid [e], G)$$
$$(2)\ G([\alpha]) = [y]$$
$$(3)\ maxu([y] \mid [e], G, U)\}.$$

The set $d_{\text{OPT}}(d_{\text{EFF}}(d_{\text{FEAS}}(R([\alpha]))))$ is hoped to be represented by one and only one point in the third quadrant (why and how this "hope", is explained below).

4. EQUILIBRIUM CONDITIONS ON PRICES

In this section, we shall treat the restrictions on all possible *prices* in the state set $S(x)$, that is, on $R([w]) \times R([p])$. It is important to note that by means of Δ1, . . . , Δ6 we have been able to fix the quantities in GECE without introducing a price system! This is characteristic of Walrasian structures in economics: in *statics*, the study of timeless equilibrium, prices are defined with the help of equilibrium quantities $[\alpha]^0$ and $[y]^0$. (When it comes to *dynamics*, the study of the time paths towards equilibrium, price changes (tâtonnement) constitute, of course, the very mechanism of equilibrium. Dynamics will not be reconstructed in this paper.)

Dynamically inspired assumptions of economists are that, in equilibrium, ratios of factor prices w will be proportional to the ratio of marginal productivity of the factors, which can usually be proven to be equal in all industries. Prices p_γ of goods γ to be produced are restricted by the condition that in all industries, total cost must be equal to total revenue.[12] Graphically, this means that, in figure 1, the price ratio of γ_1 to γ_2 is represented by $tg\alpha$ and the price ratio of ϕ_1 to ϕ_2 by $tg\beta$. This results from Δ7, where $G([\alpha])_\gamma$ is the γth component of $G([\alpha])$, i.e., the amount of the γth good produced with factor input matrix $[\alpha]$.

Δ7 A pair of price vectors $([w],[p])$ is an *equilibrium price vector* given $[e]$, G and U (write: $eq(([w],[p]) \mid [e], G, U)$) iff

$$([w],[p]) \in d_{\mathrm{EQ}}(R([w]) \times R([p])) :=$$
$$:= \{([w],[p]) \mid \text{there are } [\alpha] \text{ and } [y] \text{ such that}$$
$$(1)\ opt([\alpha] \mid [e], G, U) \text{ and } maxu([y] \mid [e], G, U)$$
$$(2)\ y_\gamma p_\gamma = \alpha_{1\gamma} w_1 + \cdots + \alpha_{n\gamma} w_n (\gamma = 1, \ldots, m)$$
$$(3)\ w_\phi = p_\gamma \cdot (\partial G([\alpha])_\gamma)/(\partial \alpha_{\phi\gamma})(\gamma = 1, \ldots, m; \phi = 1, \ldots, n)\}$$

In requirement 2, the expression $y_\gamma p_\gamma$ represents the total revenue of industry γ as a result of the selling of an amount y_γ at a market-price p_γ. This total revenue is set equal to $\alpha_{1\gamma} w_1 + \cdots + \alpha_{n\gamma} w_n$, i.e., total cost of industry γ.

Requirement 3 states that the factor price w_ϕ equals its marginal productivity in producing γ, times the value of a unit of the commodity produced (p_γ). If w_ϕ would be larger than this expression on the right hand side of requirement 3, then the costs of the last units of ϕ bought by industry γ would exceed their contribution to the value of production, if it would be smaller, industry γ could gain by increasing the input of ϕ.

It is hoped for that $d_{\mathrm{EQ}}(R([w]) \times R([p]))$ has one and only one element, that is, that equilibrium prices w_ϕ, $\phi = 1, \ldots, n$; p_γ, $\gamma = 1, \ldots, m$ exist and are unique. To hopes like these, the next section is devoted. All concepts introduced by Δ1, . . . , Δ7 are mere tools, defined upon those included in M_p, and therefore need no separate mention in D2. In fact, Δ1, . . . , Δ7 define the concept of "equilibrium".

5. *M*, THE BATTLE FOR EXISTENCE AND UNIQUENESS

Summarizing our results with respect to Δ1, . . . , Δ7, we have constructed the following tools:

(1) $\{[y] \mid maxu([y] \mid [e], G, U)\}$
(2) $\{[\alpha] \mid opt([\alpha] \mid [e], G, U)\}$
(3) $\{([w],[p]) \mid eq(([w],[p]) \mid [e], G, U)\}$.

Let us denote their elements with an upper zero:

$$[y]^0 \in \{[y] \mid maxu([y] \mid [e], G, U)\}$$
$$[\alpha]^0 \in \{[\alpha] \mid opt([\alpha] \mid [e], G, U)\}$$
$$([w],[p])^0 \in \{([w],[p]) \mid eq(([w],[p]) \mid [e], G, U)\}.$$

By considering, for any given $\langle [e], G, U\rangle$ corresponding tuples $\langle [\alpha]^0, [y]^0, ([w],[p])^0\rangle$ we obtain the meta-economic relation Δ, defined by $\Delta 1, \ldots, \Delta 7$. Δ relates elements from $R(\langle [e], G, U\rangle)$ and elements of the state space $R([\alpha]) \times R([y]) \times R([w]) \times R([p])$.[8] We now can express the "hope" for existence and uniqueness as the hope for Δ to be (1) defined on its whole domain $R(\langle [e], G, U\rangle)$ (existence) and to be (2) unique to the right, i.e.,

$$\Delta\colon R(\langle [e], G, U\rangle) \to R([\alpha]) \times R([y]) \times R([w]) \times R([p]).$$

How can we come to know whether Δ, as defined by $\Delta 1, \ldots, \Delta 7$, has these two properties? The only way by which we can answer this question is mathematical analysis. And that is what general equilibrium economists do: the battle for existence and uniqueness is fought in a field constructed out of real numbers by means of M_{pp}, M_p and Δ.

To understand the nature of this battle, we should first come to know which sets of reals are involved. We have

$$R([y]) = (R^+)^m \qquad R([p]) = (R^+)^m$$
$$R([\alpha]) = (R^+)^{m\times n} \qquad R([w]) = (R^+)^n$$
$$R([e]) = (R^+)^n$$
$$R(G) = \mathrm{Pot}\,[(R^+)^{m\times n} \times (R^+)^m]$$
$$R(U) = \mathrm{Pot}\,[(R^+)^{m+1}]$$

where "$(R^+)^m$" denotes a Cartesian product of m times the set of nonnegative reals.

The domain of Δ satisfies

$$R(\langle [e], G, U\rangle) = (R^+)^n \times \mathrm{Pot}[(R^+)^{m\times n} \times (R^+)^m] \times \mathrm{Pot}[(R^+)^{m+1}].$$

The range of Δ satisfies

$$R([\alpha]) \times R([y]) \times R([w]) \times R([p]) = (R^+)^{n\times m} \times (R^+)^m \times (R^+)^n \times (R^+)^m.$$

It is not difficult to find triples $\langle [e], G, U\rangle$ which via Δ correspond to many more than one tuple $\langle [\alpha]^0, [y]^0, ([w],[p])^0\rangle$, that is, the definitions

$\Delta 1, \ldots, \Delta 7$ alone cannot guarantee that Δ will be a function. What do economists do? They *partition* $R(\langle [e], G, U \rangle)$ and start to work piece for piece, i.e., they show that Δ, when restricted to subsets H of $R(\langle [e], G, U \rangle)$ is a function. The mathematics involved is that of finding $([\alpha]^0, [y]^0, ([w], [p])^0)$ as the solution of the constrained extremum problem that is characterized by $\Delta 1, \ldots, \Delta 7$. But to say something more interesting about equilibrium than what is already contained in these definitions, we need some specifying assumptions about $[e]$, G, and U that give existing mathematical knowledge (that is: the respectable and well-explored parametric functions and properties) a grip on the problem. Such *special conditions*[13] sometimes introduce parametric descriptions of G and U, allowing the derivation of existence and uniqueness theorems, but economists strive for weaker, in fact, the weakest possible special conditions for existence and uniqueness; they introduce restrictions on signs of first and second order derivatives of the functions G and U,[14] their degree of homogeneity, the signs of elasticities, and even restrictions so weak as to be formulated in terms of *combinations* of certain types of $[e]$, G, and U.[15]

If multiple equilibria cannot be ruled out in some of the sets of the partitioning of $R(\langle [e], G, U \rangle)$, then the neighbourhoods of uniqueness are mathematically specified and distinguished from each other.

All these efforts can be described as finding subsets H of $R(\langle [e], G, U \rangle)$ such that

$$\Delta\colon H \rightarrow R([\alpha]) \times R([y]) \times R([w]) \times R([p])$$

is a function defined for every element of H. Such sets H we call *strategically admissible*.[16]

D3 If $H \subseteq R(\langle [e], G, U \rangle)$ then H is *strategically admissible* iff Δ, restricted to H, is a function.

If one is a Platonist with respect to mathematics, one also believes in the existence of the *union E of all strategically admitted H's*, though the puzzle might be so difficult that it takes some more centuries of undisturbed civilization to find E, that is, to acquire a complete mathematical description of E, in the general case of m goods and n factors of production. This problem situation induces an obvious research strategy aiming at a full specification of E.

Economists start with some strategically admissible H (such as the two by two model with G and U satisfying rather strong special conditions such as linear homogeneity and other restrictions on slopes and curvatures), and subsequently try to enlarge H by the strategies of theory development described in Hamminga (1983): (1) Field extension, that is, adding more factors and commodities to the model. This strategy aims at arriving at types of sets H that contain any number of ϕ's and γ's. (2) Weakening of special conditions, that is, finding a set H_1 in which the original set H_0 is contained. (3) Finding alternative special conditions, that is, finding a set H_2 that has excess content over the original H_0, where H_0 also has excess content over H_2. Such sets H_2 and H_0 may be – but usually are not – completely disjoint. This kind of development can be illustrated as in figure 2.

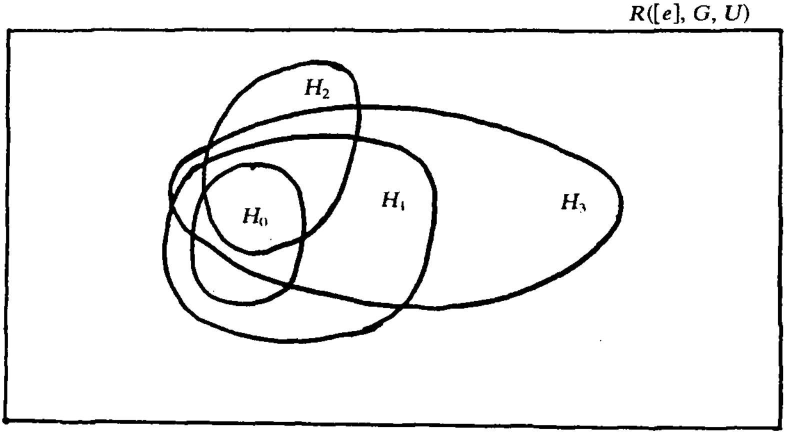

Fig. 2. Domain of relation Δ and the exploration of those parts of it for which Δ is a function defined everywhere.

In any potential model $x = \langle \Gamma, \Phi, q_{\text{input}}, q_{\text{output}}, w, p, q_{\text{endowment}}, G, U\rangle$ for which $\langle [e], G, U\rangle$ belongs to some admissible H, the corresponding state $\langle [\alpha]^0, [y]^0, ([w], [p])^0\rangle$ is uniquely determined. Any such potential model therefore is accepted as a *proper* model of GECE.

D4 x is a *model* of GECE ($x \in M$) if there exist Γ, Φ, q_{input}, q_{output}, w, p, $q_{\text{endowment}}$, G, U such that
(1) $x = \langle \Gamma, \Phi, q_{\text{input}}, q_{\text{output}}, w, p, q_{\text{endowment}}, G, U\rangle$
(2) $x \in M_p$
(3) $\langle [e], G, U\rangle \in E$, where E is the union of all strategically admissible H's.

Note that if some x is in two different strategically admissible H's, say in $H_1 \cap H_2$, then the Δ-images "with respect to" H_1 and H_2 can be proved to be identical. So taking the union of strategically admissible

H's cannot conflict with Δ being a function. Note, further, that economist's proofs of existence and uniqueness amount to showing that for some particular specified H, H is strategically admissible. In the light of D4, this, of course, implies that all potential models x for which $\langle [e], G, U\rangle$ is in H are proper models of GECE.

It should be stressed again that existence in the phrase "if there exist" of D4 should be read the mathematician's way.

Economist's methods are questioned every now and then. They are said to behave quasi-empirical: to fake empirical science. We can conclude that, for the analysis of theory structure and theory development, it is not relevant whether or not GECE-like structures are "empirical" in the various meanings of this term that are introduced in the philosophical literature.

6. DISPLACEMENT OF EQUILIBRIUM: COMPARATIVE STATICS

Strictly speaking, $\Delta 3$ and $\Delta 4$ are redundant in defining equilibrium with the help of Δ: maximizing utility on the set $d_{\mathrm{ATT}}(R([y]))$ would yield an $[y]^0$ directly, and thereby an $[\alpha]^0$ as $G^{-1}([y]^0)$.

However, $\Delta 3$ and $\Delta 4$ treat important concepts, not of statics, but of *comparative statics*, the study of the effects of *changes* of $[e]$, G and U on the equilibrium values $([\alpha]^0, [y]^0, ([w], [p])^0)$. Comparative statics is the economist's aim for which statics is the means: to study shifts of equilibria as a result of exogenous causes, it is expedient to have a static system with existent and unique equilibrium.

An overwhelming proportion of the famous theorems of economics are comparative statical theorems. They are, understandably, more interesting and provoking than statical theorems because they deal with *changes* in prices and quantities of factors and goods as results of changes in technique, preferences of the public, availability of factors, international trade (tariffs, protection), etc. All these causes of change are, in general equilibrium theory, ultimately formulated in terms of shifts of $[e]$, G, U or a combination of these.

Many of these shifts are analysed in terms of shifts of the contract curve $d_{\mathrm{EFF}}(d_{\mathrm{FEAS}}(R([\alpha])))$ and the production possibility curve $d_{\mathrm{MAXY}}(d_{\mathrm{ATT}}(R([y])))$.

An example that needs no introduction of additional terminology is the *Rybczynski theorem*: in his (1955) Rybczynski (using graphical and verbal means only!) proves for the case $m = n = 2$ that if the availability of one of the factors of production increases, the other one being constant, the output of the good using the accumulating factor intensively will increase and the output of the other good will decrease in absolute amount, provided that commodity and factor prices are kept constant. This is an example of an interesting comparative statical theorem.

7. THE SCOPE OF GECE

There are some more structures in economics, slightly different from each other and from GECE, which are more or less loosely subsumed under the heading "general equilibrium theory". It's always hard to destil a logically unambiguous concept from the practical name-giving habits of a group of working people, like theoretical economists. The class would certainly be chosen too wide if we would call a theory a "general equilibrium theory" as soon as it is phrased in terms of a constrained extremum problem. But structures similar to GECE, using disaggregate utility functions (one function for each "individual" or household) should certainly be included. The same holds for structures of the theory of international trade, where there is more than one country. Then, other complications are introduced, even in the – embryonic – statical phase of theorizing: joint production and intermediate goods are examples.

But by far the most of such complications can be phrased in terms of the slopes, curvatures and configurations of $[e]$, G, and U, and therefore leave the basic structure, presented in this paper, undamaged.

We believe, therefore, that GECE can serve as a solid basis for reconstructing the different types of general equilibrium theories of economics.

NOTES

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[1] This work may be contrasted with Händler's. In Händler (1980), a very rich structure is taken as the point of departure and various kinds of theories can be obtained as special cases. In contrast to this and much in the spirit of Balzer (1982a), we shall concentrate here on the bare essentials, trying to keep things as simple as possible. It is tempting to regard GECE as an extension of pure exchange economics (PEE) as treated in the latter paper, for, in addition to PEE, GECE includes production. On the other hand, GECE does not contain features of individual consumers with their respective utility functions and commodity endowments. The theories are intended to describe quite different aspects of reality: while PEE deals with exchange of commodities among individuals, GECE

treats the "aggregate" phenomenon of the "most efficient production" in a country. Therefore, a comparison of GECE with PEE is not straightforward, and we do not attempt to investigate their intertheoretic relation in this paper. The most striking difference between both theories expressing GECE's superiority is that in GECE it is possible to determine states of equilibrium uniquely in the proper models of the theory – which is impossible in PEE.

[2] We assume that the reader is acquainted with the structuralist meta-theory. The original reference is Sneed (1971). A brief introduction with special attention to equilibrium theory is found in Balzer (1982b). We use set-theoretic notation. Especially, R^+ will denote the set of nonnegative real numbers, and vectors and matrices will be indicated by sharp brackets, like $[y]$.

[3] Some types of analysis also allow for negative real numbers. These types can always be represented by an isomorphic structure that only uses nonnegative reals.

[4] Debreu (1959), p. 32.

[5] See note 3.

[6] Many economists are accustomed to call w the "wage" of a factor, regardless whether the factor is labour, capital or land.

[7] Popper (1934), p. 55.

[8] This notation must not be confused with the "range of a function" as used in set theory. What we denote here by the *range of a function* f, $R(f)$, is the set of all possible functions f.

[9] A point in the third quadrant represents therefore, at first sight, a *vector*, and not a factor input *matrix*. It will, below, uniquely correspond to such a matrix, after we introduced in Δ1, as fixed and given, the total amount of factors ϕ available in the country (e_ϕ). Then, $\alpha_{\phi\gamma_2} = e_\phi - \alpha_{\phi\gamma_1}$. Thus we may say that any point in the third quadrant "represents a complete distribution $[\alpha]$ of factors". This way of talking will always be used in connection with figure 1.

[10] Definition Δ1, below, will be seen to imply $\alpha_{\phi 1} + \alpha_{\phi 2} \leq e_\phi$ $(\phi = 1, 2)$, where Δ2, . . . , Δ6 will be such that inequality can (usually) be cancelled. In that case, one of the rows of $[\alpha]$ will of course provide all the information represented by the matrix. See also note 9.

[11] The following restrictions under Δ belong to the neoclassical FEA (Hamminga, 1983).

[12] Henderson and Quandt (1958), Ch. 9.

[13] Hamminga (1983), pp. 41–44, pp. 49–50, and pp. 66–70.

[14] The first requirement, in Δ7, of the set d_{EQ} already contains some restriction on G: that infinitesimal calculus applies, at least in some neighbourhood of $[\alpha]^0$. This is why we had to require smoothness of G in D2.

[15] Consider, for instance, the following very characteristic quotation from Hicks' *Value and Capital* (1939, pp. 319–20): "(It should be observed that the function f is arbitrary, in the same way as the utility function u was arbitrary. Any function $\phi(f)$, which is 0 when f is 0, would serve.)". Or take this one, from Södersten (1971, p. 48): "A production function shows the relationship between input of factors of production and output of a good (or output of several goods if we assume joint production). For many reasons it would be advantageous if we could use unspecified production functions. This means that to derive the results we wanted, to prove certain theorems, we would need to assume only that a relationship exists between inputs and outputs, but we would not have to assume anything specific about the nature of this relationship.

As a matter of fact, when we come to the effects of technical progress on international trade we will refer to results that have been derived using only this weak assumption. But for most of the theorems of trade theory a more specific relationship between inputs and outputs has to be assumed."

[16] *H* can be regarded as a generalization of what Papandreou (1958) called a "generic function" (Ch. 3).

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