

THE BEHAVIOR OF U.S. PUBLIC DEBT AND DEFICITS*

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Abstract

How do governments react to the accumulation of debt? Do they take corrective measures or do let the debt grow? Whereas standard time series test cannot reject a unit root in the U.S. debt-GDP ratio, this paper provides evidence of corrective action: The U.S. primary surplus is an increasing function of the debt-GDP ratio. The debt-GDP ratio displays mean-reversion if one controls for war-time spending and for cyclical fluctuations. The positive response of the primary surplus to changes in debt also shows that U.S. fiscal policy is satisfying an intertemporal budget constraint.

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I. Introduction

How do governments react to the accumulation of debt? Do they take corrective measures when the debt-GDP ratio starts rising, or do let the debt grow? Empirically, it is difficult to reject a unit root in real debt and in the debt-GDP ratio.¹ Theoretically, a non-stationary debt-income ratio is implied by some models of optimal government finance. In the public policy debate, on the other hand, a high and growing debt-GDP ratio is widely viewed as worrisome. Such concerns find support in macroeconomic models with limited taxation (Blanchard [1984], Bohn [1991b]). Even if the debt-GDP ratio is declining—as currently in the U.S.—the difficulty of rejecting a unit root makes one wonder if the decline is due to luck (e.g., high economic growth) or policy design.

This paper shows that one can find direct evidence for corrective actions by examining the response of the primary (non-interest) budget surplus to changes in the debt-income ratio. A positive response shows that the government is taking actions—reducing non-interest outlays or raising revenue—that counteract the changes in debt. This approach is more promising than a univariate time series analysis of the debt-income ratio, because the debt-income ratio is in practice bounced around by various shocks (e.g., fluctuations in income growth, in interest rates, and in government spending) that make mean reversion difficult to detect.

For U.S. data, I find significant evidence that the primary surplus is an increasing function of the debt-GDP ratio for 1916-1995 and various subperiods. I also show that the link between debt and primary surpluses can easily be obscured by war-time spending and by cyclical fluctuations. A univariate regression of primary surpluses on debt would fail to find a significant correlation between the two. But a more fully articulated equation for primary surpluses motivated by Barro's [1979] tax smoothing model shows a significant conditional impact of debt on primary surpluses.

Given the estimated positive response of primary surpluses to the debt-GDP ratio, the government budget identity implies that the debt-GDP ratio should be mean-reverting.

This can be confirmed in an autoregression, provided one controls for war-time spending and for cyclical fluctuations. Under fairly weak conditions, a positive (at least linear) response of primary surpluses to the debt-income ratio also implies that government policy is sustainable in the sense of satisfying an intertemporal budget constraint. The paper shows that U.S. fiscal policy has historically been sustainable despite extended periods of primary deficits. (This has been controversial; see Hamilton and Flavin [1986], Kremers [1989], Trehan and Walsh [1988] and [1991], and Wilcox [1989].)

The paper is organized as follows. Section II examines the relationship of U.S. government debt and primary surpluses. Section III examines the dynamics of government debt. Section IV explores non-linearities in the surplus-debt relationship. Section V comments on the implications for the intertemporal budget constraint. Section VI concludes.

II. Evidence on U.S. Government Debt and Primary Surpluses

The starting point for the analysis of government finance is the period-by-period budget equation $D_{t+1} = (D_t - S_t) \cdot (1 + R_{t+1})$. Next period's debt is given by this period's debt (D_t) minus the primary surplus (taxes minus non-interest spending, S_t) times the gross interest factor $1+R_{t+1}$. In a growing economy with a growing tax base and growing government spending, it is instructive to write this budget equation in ratio form as

$$(1) \quad d_{t+1} = x_{t+1} \cdot [d_t - s_t],$$

where $d_t = D_t/Y_t$ is the ratio of (start of period) debt to aggregate income (GDP), $s_t = S_t/Y_t$ is the ratio of the primary surplus to income, and $x_{t+1} = (1+R_{t+1}) \cdot Y_t/Y_{t+1} \approx 1+r_{t+1}-y_{t+1}$ is the ratio of the gross return on government debt to the gross growth rate of income (both of which can be measured in either real or nominal terms since inflation cancels out in the ratio). The variables r_{t+1} and y_{t+1} denote the real interest rate and the real growth rate, respectively.

As explained in the introduction, the idea of the paper is to search for a systematic relationship between the debt-income ratio and the primary surplus, and specifically, to estimate a regression of the form

$$(2) \quad s_t = \rho \cdot d_t + \alpha \cdot Z_t + \varepsilon_t = \rho \cdot d_t + \mu_t$$

where Z_t is a set of other determinants of the primary surplus, ε_t an error term, and $\mu_t = \alpha \cdot Z_t + \varepsilon_t$. This agenda raises questions about the time series properties of the debt and surplus series and about what one should assume about μ_t . These questions are interdependent. If debt and the primary surplus are both non-stationary while μ_t is stationary, one could interpret a simple regression of s_t on d_t as a cointegrating regression without having to model the μ_t process explicitly. But if s_t and d_t do not have unit roots, as I will argue below, a regression of s_t on d_t that omits other determinants of the primary surplus will produce inconsistent estimates due to omitted variables bias.

Because of the potential omitted variables problems, the empirical analysis is based on an explicit theoretical model of fiscal policy, Barro's [1979] tax-smoothing model. Barro considers an optimizing government that minimizes the cost of tax collection by smoothing marginal tax rates over time. Key features of the optimal policy are that tax rates should depend only on permanent government spending and on the level of debt.

The tax-smoothing model yields an equation for the primary surplus, if one subtracts non-interest government spending from tax revenues (all relative to GDP). The model implies that the non-debt determinants of the primary surplus (the Z_t) are the level of temporary government spending, GVAR, and a business cycle indicator, YVAR. The model for the surplus-GNP ratio is then

$$(3) \quad s_t = \rho \cdot d_t + \alpha_0 + \alpha_G \cdot GVAR_t + \alpha_Y \cdot YVAR_t + \varepsilon_t.$$

The variables GVAR and YVAR are taken from Barro [1986a, 1986b] for 1916-83 and updated to 1995.

Estimates of equation (3) are in Table I. All regressions use Ordinary Least Squares (OLS) estimation. White's [1980] robust standard errors are provided to address potential concerns about heteroskedasticity and (with a Newey-West [1987] lag window) about autocorrelation. Regression 1 shows the results for the full sample period 1916-1995. Regression 2 excludes the two world wars. Regressions 3-4 show results for the period 1916-

83, for which Barro's original regressors are available; no. 3 has results for the entire period, while no. 4 focuses on the sample period 1920-82 excluding 1941-47, which is the period used by Barro [1986a]. Regression 5 shows results for the post-war period 1948-95, and no.6 shows results for 1960-84, the sample period examined by Hamilton and Flavin [1986] and by Wilcox [1989].

The coefficients on d_t are significantly positive in all regressions and they are quantitatively reasonable. For example, the ρ -value of 0.054 in Regression 1 means that a marginal increase in government debt by \$100 increases the primary surplus in the following year by \$5.40. The estimates for ρ are all between 2.8 and 5.4 percent. These results show that the U.S. government is systematically responding to changes in the debt-GDP ratio. Even if one argued that Dickey-Fuller distributions should be used to evaluate the significance of ρ (because of the near-unit root behavior of d_t), the ρ -values in lines 1, 3, and 5 would be significant at the 1 percent level (3.51 critical value). As the Barro model predicts, the variables GVAR and YVAR enter negatively in all regressions.²

The conceptual importance of GVAR and YVAR is best illustrated in comparison to regressions that omit these variables. A univariate OLS regression of the primary surplus on initial debt for 1916-1995 yields an insignificant and slightly negative slope coefficient of -0.01 ($t = -0.28$). To see why a univariate regression yields such different results, consider Figure I. Figure Ia shows a scatter plot of the primary surplus against initial debt. The war years are clearly visible (and labeled) as negative outliers. Apart from the outliers, there appears to be a positive relationship between s_t and d_t . But this relationship is sufficiently obscured by the outliers that a univariate regression yields an insignificant point estimate. In Figure 1b, the primary surplus is adjusted for the estimated other determinants of the primary surplus, GVAR and YVAR. The graph plots the estimated non-cyclical, non-war related component of the primary surplus, $s_t - (\alpha_0 + \alpha_G \cdot \text{GVAR}_t + \alpha_Y \cdot \text{YVAR}_t)$, against d_t . In contrast to Figure Ia, the World War II observations do not show up as outliers—except that the debt was high. After adjusting for cyclical factors and fluctuations in government

spending, there is a clear positive conditional correlation between debt and primary surplus.

III. Implications for the Debt-GDP Ratio

The above results have strong implications for the time series properties of government debt. The budget identity (1) and the surplus equation (2) imply that the change in the debt-GDP ratio depends on the lagged level and on the non-debt components of the primary surplus,

$$(4) \quad \Delta d_{t+1} = d_{t+1} - d_t = -[1 - \bar{x}_{t+1} \cdot (1 - \rho)] \cdot d_t - x_{t+1} \cdot \mu_t.$$

Assuming stationary x_{t+1} and μ_t , the debt-GDP ratio should be a stationary, mean-reverting process, if $\bar{x} \cdot (1 - \rho) < 1$. This condition is clearly satisfied for U.S. data, since ρ is significantly positive and U.S. interest rates have been near or below the growth rate of GDP. For the 1916-95 sample, the average real return on government debt was $\bar{r} = 0.1$ percent while the average real GDP-growth rate was $\bar{y} = 3.3$ percent; hence, $\bar{x} \approx 1 + \bar{r} - \bar{y} \approx 0.97 < 1$.

In standard Dickey-Fuller and Phillips-Perron unit root regressions, however, one cannot reject a unit root in d_t .³ This has led much of the fiscal policy literature to treat the debt-GDP ratio—and similarly, real debt—as a non-stationary variable (see Trehan and Walsh [1988, 1991], Hakkio and Rush [1986], Ahmed and Rogers [1995]). Complicating the puzzle, a unit root in s_t is strongly rejected.⁴ This is difficult to reconcile with eq.(2) if d_t is non-stationary and $\rho > 0$; and if $\bar{x} < 1$, a stationary s_t in eq.(1) should make d_t stationary. This section will show that debt-GDP ratio is in fact stationary and the unit root regressions are misleading.

The key problem with the unit root regressions is that they ignore the systematic components in μ_t . If one uses the tax-smoothing model to represent μ_t , eq.(4) implies that the change in the debt-GDP ratio should be a declining function its level and an increasing function of YVAR and GVAR. Table II reports regressions of Δd_{t+1} on these variables. The coefficient on the lagged debt-GDP ratio d_t is significantly negative in all cases, as suggested by $\bar{x} \cdot (1 - \rho) < 1$, using critical values from either the Normal or the Dickey-Fuller distributions. The YVAR and the GVAR variables enter positively, as predicted, though not always significantly.

To understand why standard unit root regressions fail to detect this mean-reversion, recall that such regression are consistent only under the null hypothesis of a unit root and that they have low power against autoregressive alternatives if the AR-coefficient is close to one. For plausible values of \bar{x} and ρ , eq.(4) implies an AR-coefficient strictly below one, but not much below one, so that d_t should be stationary and highly autocorrelated. Hence, the unit root regressions are misspecified and inconsistent because they omit GVAR and YVAR. Intuitively, GVAR is important because war-time government spending will tend to drive up the debt-GDP ratio and then, as long as the war continues, trigger high deficits (Δd_{t+1}) at a time when the debt-GDP ratio is already high. This positive comovement of Δd_{t+1} and d_t during wartimes obscures the underlying mean-reversion effect. Table II shows that there is strong evidence for mean reversion, provided one accounts for fluctuations in GVAR and YVAR.

The evidence for mean reversion is also important because it differs from previous findings in the tax-smoothing literature, notably Barro [1979] and Kremers [1989]. In his original tax-smoothing paper, Barro [1979] finds no evidence for mean reversion in the debt-income ratio. Kremers [1989] argues that the government indirectly stabilizes the debt-income ratio through a response to changes in the cost of debt service, but he cannot reject a unit root in the debt-income ratio itself. The main difference between Table II and Barro's and Kremers' specification is that both Barro and Kremers use (scaled) changes in nominal government debt as the dependent variable and add a proxy for expected inflation as regressor. This suggests that their inability to find mean reversion is due to problems associated with estimating inflation. All the data in regressions 3 and 4 are taken directly from Barro [1986a] so that data differences cannot explain the different results.

Overall, the estimated policy rules suggest that U.S. fiscal policy is sufficient to keep the debt-GDP ratio stationary in the future unless interest rates and growth rates move very unfavorably. With $\rho=0.054$ and a long-run real growth rate of, say, 2.5 percent, real interest rates would have to rise above 8 percent to make d_t non-stationary (using the criterion $1+\bar{r}$ -

$\bar{y} \approx \bar{x} < 1/(1-\rho)$ from eq.(4)). Evaluated at the sample means of $\bar{\mu}=-0.03$ and $\bar{x}=0.9668$ (for 1916-95), eq.(4) also implies that the steady-state distribution of d_t should have a mean of 0.351. This is remarkably close to the actual sample mean of $\bar{d}=0.343$, suggesting that eq.(1)-(4) characterize the process of debt accumulation quite well.⁵

IV. Non-Linearities in the Surplus-Debt Relationship

Going beyond the linear model, this section examines potential non-linearities in the relation between the primary surplus and the debt-GDP ratio. Do governments respond more to primary deficits when the debt is high? Or is the surplus-to-debt relationship perhaps “flattening out” at high debt levels? To explore non-linearities, I generalize eq.(3) by adding powers of d_t and functions of the form $\max(0, d_t-d^*)$ that pick out periods with debt above d^* . Because of the obvious problems of over-fitting (e.g., the war-time observations can be perfectly matched with a few “max” functions that pick out sample points with high debt) and multicollinearity, such regressions should be interpreted cautiously. But the results with simple non-linear functions are intriguing because they suggest an increasing marginal response of surpluses to changes in debt.

In Table III, line 1 reproduces the linear specification from Table I for comparison. Line 2 adds a quadratic term and shows that its coefficient is positive. It suggests that the marginal response of primary surpluses to changes in debt is increasing in the debt-GDP ratio, rising from insignificantly negative values for $d_t \leq 0.21$ to a significantly positive 0.063 at $d_t=0.5$, and rising further to 0.167 at $d_t=1.0$. Line 3 shows that a cubic term is insignificant; the same is true for higher order polynomial terms (not reported). Line 4 replaces the quadratic term by a piecewise linear function with a breakpoint at $d^* = \bar{d} = 0.343$, the average debt level. As in the quadratic regression, the estimated marginal response is insignificant at low levels of the debt-GDP ratio but significantly positive at higher levels. Overall, the limited range of observed d_t values makes it difficult to pin down the exact form of the non-linearity (e.g., note the virtually identical R^2 values in the quadratic and the piecewise linear form), but all non-linear regressions indicate that the marginal impact of

debt on the primary surplus is increasing rather than decreasing with higher debt. The finding that the marginal response of primary surpluses to debt is insignificant at low debt-GDP levels may be of independent interest, e.g., in the context of political economy models of fiscal policy.

The above results are remarkably uniform across different sample periods—the ρ coefficients are always positive. Given the public policy debate about the post-1980 deficit problems, one may wonder whether U.S. fiscal policy since 1980 has differed from the previous pattern. An examination of regression residuals shows that actual primary surpluses have indeed been below the predicted values for much of the 1984-95 period, especially for 1985-86 (by 0.8-1.3 percent of GDP) and again for 1992-93 (by 1.0-1.6 percent). But these underpredictions are far from statistically significant, given standard errors of 1.3-1.4 percent of GDP.⁶

V. Implications for the Intertemporal Budget Constraint

The question whether U.S. fiscal policy has been sustainable in the sense of being consistent with an intertemporal budget constraint has long been controversial (see Hamilton and Flavin [1986], Hakkio and Rush [1986], Kremers [1989], Trehan and Walsh [1988] and [1991], and Wilcox [1989]). Bohn [1995] suggests that the existing sustainability tests do not adequately deal with the interest rate implications of uncertainty and risk aversion. This section explains why eq.(2) provides a new sustainability test that does not require interest rate assumptions and why Tables I and III provide strong evidence in favor of sustainability.

Intuitively, the intertemporal budget constraint is difficult to test in a stochastic environment, because even plausible indicators such as average deficits and the realized path of the debt-GDP ratio can be quite misleading—especially if interest rates on government bonds are on average below the rate of economic growth, as they have been in the U.S. Then frequent primary budget deficits do not provide convincing evidence against sustainability, because at low interest rates, a variety of sustainable policies will display primary deficits on average and potentially for long periods. For example, a policy that stabilizes the debt-

GDP ratio at some level d implies an average primary deficit of $\bar{s} = -(1-\bar{x}) \cdot d < 0$ if $\bar{x} < 1$. For the U.S., the actual surplus-GDP ratio has averaged to a negative $\bar{s} = -1.2$ percent for 1916-95, suggesting that the U.S. government has indeed exploited low interest rates to run primary deficits. In the other direction, a stable debt-GDP ratio does not provide convincing evidence for sustainability, because non-sustainable policies do not necessarily display a explosive debt-income ratio. For example, if the government simply set the primary surplus to zero at all times and rolled-over the existing debt with interest, the debt-GDP ratio would decline in expectation. But this policy clearly violates the intertemporal budget constraint and it is unsustainable if future growth may fall below the interest rate with positive probability.⁷

A positive response of primary surpluses to changes in the debt-income ratio does, in contrast, provide reliable information about sustainability, regardless of how interest rates and growth rates compare. Intuitively, even if the government can run primary budget deficits on average and in most states of nature because of low interest rates, permanent primary deficits will lead to excessive debt accumulation in at least some “bad” states of nature. If debt keeps growing relative to aggregate income, a sustainable policy must eventually respond by moving towards primary surpluses. A strictly positive and at least linear response of the primary surplus to changes in the debt-income ratio turns out to be sufficient for sustainability.

Specifically, suppose one can write $s_t = f(d_t) + \mu_t$ as a function of the debt-GDP ratio and other determinants μ_t , generalizing eq.(2). Also assume that μ_t is bounded and the present value of future GDP is finite. Then fiscal policy satisfies the intertemporal budget constraint if there is a debt-GDP ratio d^* such that $f'(d_t) \geq \rho > 0$ for all $d_t \geq d^*$.⁸ The key requirement here is that the primary surplus increases at least linearly with d_t at high debt-income ratios ($\geq d^*$). This ensures that any upward movement in the debt-GDP ratio due to negative shocks (e.g., low growth, wars, high interest rates) is eventually reversed through primary surpluses.

Since the linear policy rule (2) is covered as the special case $f(d_t) = \rho \cdot d_t$, Table I can be

interpreted as a test for the sustainability of U.S. fiscal policy. The significantly positive ρ coefficients show that U.S. fiscal policy has been on a path consistent with the intertemporal budget constraint.⁹ Table III reinforces this conclusion, because all the non-linear estimates indicate an increasing derivative $f'(d_t)$, rebutting concerns that the response of surpluses to the debt-GDP ratio might be less than linear.

A strength of this sustainability test is that it does not require any assumptions about interest rates. It is valid in economies with uncertainty and risk aversion and for arbitrary debt management policies, whether or not government bond rates are above or below the growth rate. The main alternatives are sustainability tests based on estimating a “transversality condition” that involves government debt discounted at some interest rate (e.g., Wilcox [1989]) and cointegration tests (e.g., Trehan and Walsh [1988], Ahmed and Rogers [1995]). The transversality condition tests depend sensitively on the choice of discount rates; Bohn [1995] has shown that the commonly used government bond rates are inappropriate as discount rates.¹⁰ The cointegration literature generally uses real levels of fiscal variables to test for cointegration and finds unit roots in real government spending, debt, and taxes. Given the stationarity of the debt-GDP ratio found above, it appears that the unit root in debt levels is either not really there or due to a unit root in GDP.¹¹

VI. Conclusions

The paper shows that the U.S. government has historically responded to increases in the debt-GDP ratio by raising the primary surplus, or equivalently, by reducing the primary deficit. In univariate regressions, this positive response is obscured by war-time spending and by cyclical fluctuations, but it is highly significant if one corrects for fluctuations in government spending and in aggregate income.

The positive response of primary surpluses to the debt-GDP ratio also implies that debt-GDP ratio should be mean-reverting, despite the fact that a unit root is virtually impossible to reject in standard univariate time series tests. The paper shows that the unit root tests are inconsistent and misleading because they do not properly adjust for

fluctuations in GDP and in government spending.

Finally, I show that an estimated positive response of primary surpluses to the debt-GDP ratio can be interpreted as a new test for the sustainability of U.S. fiscal policy. It provides strong evidence that U.S. fiscal policy has been sustainable in the sense of satisfying an intertemporal budget constraint for the sample period 1916-95 and various subperiods, despite the rather frequent primary budget deficits.

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Footnotes

1. For U.S. data, see, e.g., Trehan and Walsh [1988, 1991], Bohn [1991a], Kremers [1991]. See Corsetti and Roubini [1991] for international evidence.
2. The YVAR coefficients are above one in absolute value (as in Barro [1986a]), which suggests that tax revenues fall by more than GDP in a recession. This is not quite consistent with the basic tax smoothing model, but it might be consistent with optimal policy if tax distortions vary over the business cycle. Note that there is no logical inconsistency in using a tax-smoothing model to show $\rho > 0$; though Barro's deterministic model implies $\rho = 0$, one can construct stochastic versions of Barro's model in which the welfare-maximizing policy implies $\rho > 0$. (An example is available from the author.)
3. An augmented Dickey-Fuller test on d_t (with two lags, 1916-95 data) yields a t-value of 2.28 and a Phillips-Perron test (with lag window of size 3) yields a t-value of 2.30—both well below the critical 1 and 5 percent values of 3.51 and 2.89, respectively. Tests with different assumptions about the lag structure yield similar results.
4. In the 1916-95 sample, augmented Dickey-Fuller and Phillips-Perron tests on s_t yield t-values of 3.74 and 4.53, respectively, which are significant at 1 percent. The test specifications and critical values are as in the previous footnote about d_t .
5. These estimates also allow predictions about the path of future debt conditional, say, on real interest rates staying at the higher $\bar{r} = 3.7$ percent level observed for 1983-95. For $\bar{x} = 1.007$ (using 1983-95 data), the debt-GDP ratio should converge to a new steady-state distribution with a mean of 0.67. A natural question in this context is if the surplus-debt relationship is invariant with respect to real interest rates. I examined this issue by including real interest rate measures in regression (3) and found that the surplus-debt relationship does not seem to depend on the level of interest rates.
6. Predictions from both the linear and the quadratic model were examined (Table III, lines 1 and 2). While the time path of the prediction errors is fairly robust, the numerical values depend somewhat on the specification. Using Andrews' [1993] testing algorithm,

no significant structural breaks were found in the 1980-1993 period, neither for the intercept nor for the slope coefficients.

7. The technical aspects of this example are explained in Bohn [1994, 1995]. Intuitively, debt grows at the rate of interest. If GDP growth is above the rate of interest with probability one, the economy is dynamically inefficient and the intertemporal budget constraint is irrelevant. But if there is a positive probability that future interest rates are above the growth rate, a zero primary surplus is not sustainable.
8. A formal proof is available from the author. The technical assumptions of boundedness and a finite present of GDP are not explicitly tested. But they are fairly weak restrictions, because GVAR and YVAR are bounded by construction and because a finite present value of GDP is implied, e.g., by dynamic efficiency.
9. Note that the OLS estimates in Table I are unbiased without requiring assumptions about ergodicity, provided the error ε_t is independent of the explanatory variables (Hamilton [1994, p.208]). This robustness with respect to ergodicity is reassuring in the context of testing for sustainability, because one can construct examples in which some fiscal variables are not ergodic; see Bohn [1994]. Ergodicity is required, however, for all asymptotic inferences.
10. This problem is especially serious given the historically low interest rates. If $\bar{r} < \bar{y}$, a discounting of future debt at the rate \bar{r} has the absurd implication that all policies with stationary debt-income ratios fail the sustainability test. In a dynamically efficient economy, an asymptotic discount rate above the growth rate must be applied to any variable growing at the same rate as income. Given a stationary debt-GDP ratio, this applies to government debt, showing that the correct discount rate must be above \bar{y} .
11. The cointegration literature also requires assumptions about interest rates; or alternatively, restrictive assumptions about the stochastic processes driving fiscal policy and marginal utility are needed (see Ahmed and Rogers [1995]).