The Beliefs about Mathematics, its Teaching and Learning of those Involved in Secondary Mathematics Pre-Service Teacher Education

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Secondary mathematics pre-service teachers often have different experiences of mathematics and its teaching and learning during their initial teacher education. This paper documents the beliefs about mathematics, its teaching, and its learning, of mathematicians and mathematics educators who teach secondary mathematics pre-service teachers. The beliefs of the surveyed sample of eighty-two academics and differences between groups were characterised using descriptive statistics and one-way comparisons between groups ANOVA. Generally, respondents had a Problem-solving view of mathematics and those with education backgrounds were more in agreement with that method of teaching.

Within secondary pre-service mathematics teaching programs, mathematicians typically teach the mathematical content courses whilst mathematics educators teach the mathematics pedagogy courses. However, the practice of mathematics in university is often different from mathematics teaching and learning practices in secondary schools. The preliminary research reported in this paper is ultimately concerned with how pre-service teachers reconcile the different pespectives of mathematics that might be communicated to them by mathematicians and mathematics educators. The research began by surveying mathematicians and mathematics educators who teach secondary mathematics pre-service teachers about their beliefs about mathematics and mathematics teaching and learning.

Theoretical Background

Australian Curriculum: Mathematics (ACM) identifies mathematics as an inquiry discipline requiring a problem-solving pedagogy. The problem-solving proficiency strand states, "Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively" (ACARA, n.d.). Teachers' beliefs strongly influence their classroom practices (McLeod, 1992; Mosvold & Fauskanger, 2014), which in turn influence their students' beliefs about mathematics and their ability to learn it. Beswick's (2005, 2012) study of twenty-five secondary mathematics teachers showed that individual teachers can hold differing beliefs about mathematics in schools, and as a discipline, and can bring these conflicting beliefs about mathematics learning to the classroom, thus influencing the classroom environment.

Ernest's (1989) study of mathematical beliefs describes three different conceptions of mathematics: Problem-solving, Platonist, and Instrumentalist views. In the Problem-solving view mathematics is a dynamic field of human invention where the teacher is a facilitator helping students become confident in posing and solving problems. The Platonist view sees mathematics as a structured, unchanging body of knowledge where the teacher is an explainer helping students towards conceptual understanding. The Instrumentalist view takes mathematics to be a collection of procedures, facts and skills with the teacher an instructor supporting students to master skills and procedures.

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A limited number of studies has investigated university mathematicians' beliefs about their teaching and their impact on student beliefs and practice. Carlson and Bloom (2005) studied how mathematicians solve problems and their associated emotional perspectives. Dreyfus and Eisenberg (1986) investigated the aesthetic value of mathematics and recommended that teaching include the "aha" of problem solving and that "considerations of two or more solution paths could bring practical benefits, developing a familiarity with different solution methods and deeper conceptual understanding" (p. 9). However, Burton's (1999) study of 70 mathematicians found they believed students needed to learn mathematics before they could begin mathematising, which Burton (1999) described as "objective mathematics they, as teachers, thrust towards reluctant learners" (p. 20).

The preparation of secondary mathematics teachers is shared between discipline experts (mathematicians) and education experts (mathematics educators), who teach separate courses in initial teacher education (ITE) programs that may unintentionally communicate different visions of mathematics. Therefore this paper addresses the following research questions: (1) What are the beliefs about mathematics and mathematics teaching and learning espoused by mathematicians and mathematics educators who teach pre-service teachers? (2) Are there differences in the beliefs of these two groups?

Research Design and Data Collection Methods

A survey was developed in two sections using a five-point Likert scale to elicit responses (strongly disagree to strongly agree). Twenty six items from Beswick's (2005) survey of teacher beliefs were used in the first section, *Beliefs about mathematics, its teaching and its learning*. The second section comprised seven items developed from Ernest's (1989) three conceptions of mathematics.

An invitation to complete the survey online was sent to Australian mathematicians, statisticians, and mathematics educators involved in ITE programs via the Mathematics Education Research Group of Australasia, Australian Mathematics Society, and the Heads of School/Faculty of Education and Mathematics or Science at all Australian universities for forwarding to the relevant staff. There were 82 (from 120) respondents who completed all items in the survey. They represented 35 different Australian universities and five international universities, while three where retired and three looking for work. Forty-nine (60%) were male, 33 (40%) were female, and the median age was 46. Sixty respondents (73%) taught mathematics content subjects only, eight (10%) taught mathematics pedagogy only and 14 (17%) taught both discipline and pedagogy. There was a wide range of qualifications amongst the respondents which included: PhD in mathematics 44 (54%); PhD in education 12 (15%); PhD in mathematics and a Graduate Diploma in Education (GDE) 11 (13%); and no PhD 15 (18%). Descriptive statistics and one-way between groups ANOVA with Bonferroni post-hoc tests were used to analyse data using SPSS.

Analysis and Discussion of Responses for the Whole Group

The results are organised around participants' conceptions of mathematics, beliefs about learning mathematics, and beliefs about teaching mathematics.

Conceptions of Mathematics

The survey items 1-7 in Table 1 are linked to Ernest's (1989) three conceptions of mathematics: Problem-solving (items 1, 2, 7); Platonist (items 3, 4); and Instrumental (items 5, 6). Considering first the Problem-solving view of mathematics: all respondents identified mathematics as a “continually expanding field of human inquiry” (1) and most 71 (87%) agreed (or strongly agreed) with it “remaining open for revision,” (2) and “interrelated and sharing methods of inquiry with other areas of knowledge” (7). Interestingly, 57 (70%) disagreed with the Platonist view that mathematics was a “static but unified body of knowledge,” (3) whilst they were reasonably evenly spread in their views as to whether mathematics was “discovered not created” (4) (29 or 35% agreed, 29 or 35% undecided, 24 or 30% disagreed). No one agreed with the Instrumental view that mathematics is an “unrelated collection of facts, rules and skills” (5) but 8 (10%) agreed that mathematics is “entirely distinct from other fields” (6) and is “computation” (item 20 from Table 2). These responses indicate that respondents generally have a Problem-solving view of mathematics as a discipline with some aspects of Platonist views.

Table 1
Survey Responses about Conceptions of Mathematics

	Item Number	D	U	A
1	Mathematics is a continually expanding field of human inquiry.	0	0	82
2	Mathematics is not a finished product, and its results remain open to revision.	6	5	71
3	Mathematics is a static but unified body of knowledge, consisting of interconnecting structures and truths.	57	13	12
4	Mathematics is discovered, not created.	24	29	29
5	Mathematics is a useful but unrelated collection of facts, rules and skills.	78	4	0
6	Mathematics is entirely distinct from other disciplines.	68	6	8
7	Mathematics and other areas of knowledge are interrelated or partly integrated, sharing concepts and methods of inquiry.	3	8	71

Note: D = Strongly disagree or Disagree; U = Undecided; A = Strongly Agree or Agree

Beliefs about Learning Mathematics

Beswick (2005) identified items 1-3 and 5-8 in Table 2 as identifying a Problem-solving view of learning mathematics. Most academics (at least 90%) agreed with items 1-3, 5 and 6. However, they were less in agreement about being “fascinated with how students think” (7) (10, 12% undecided; 67, 82% agreed) and “providing interesting problems to be investigated in small groups” (8) (23, 28% undecided; 55, 67% agreed). The Platonist view of mathematics learning differs from the Instrumental view as in the Platonist view the learner is actively constructing their knowledge wheras in the Instrumental view the learner is passively receiving the knowledge. Items 16-17 were difficult to distinguish between the Platonist and Instrumental views: just under half the academics agreed with this Platonist/Instrumental view while about one quarter disagreed. Thirty-eight (46%) agreed mathematics “should be presented in the correct sequence” (18), a more Platonist view, while 21 (26%) disagreed. Items 19 and 21 represented an Instrumental view. Thirty-four (41%) agreed the best way to learn was an “expository style” (19) and 25 (30%) disagreed. However for item 21 only 2 (2%) believed that “telling students the answer was an efficient way of facilitating mathematics learning” and 61 (74%) disagreed. This indicates a general belief that mathematics learning is best achieved with problem solving but the content needs to be structured so content and skills are related.

Table 2

Survey Responses about Beliefs about Mathematics and Beliefs about Learning and Teaching Mathematics (Beswick, 2005).

	Item Number	D	U	A	
1	A vital task for the teacher is motivating students to solve their own mathematical problems.	4	1	77	PS L
2	Ignoring the mathematical ideas that students generate themselves can seriously limit their learning.	4	4	74	PS L
3	It is important for students to be given opportunities to reflect on and evaluate their own mathematical understanding.	2	1	79	PS L
4	It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.	1	2	79	PL
5	Effective mathematics teachers enjoy learning and “doing” mathematics themselves.	0	5	77	PS L
6	Knowing how to solve a mathematics problem is as important as getting the correct solution.	1	1	80	PS L
7	Teachers of mathematics should be fascinated with how students think and intrigued by alternative ideas.	5	10	67	PS L
8	Providing students with interesting problems to investigate in small groups is an effective way to teach mathematics.	4	23	55	PS L
9	Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.	0	3	79	
10	Allowing a student to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.	1	3	78	PS PT
11	Students always benefit by discussing their solutions to mathematical problems with each other.	7	18	57	PS PT
12	Persistent questioning has a significant effect on students’ mathematical learning.	5	22	55	PS PT
13	Justifying the mathematical statements that a person makes is an extremely important part of mathematics.	1	2	79	PS PT
14	As a result of my experience in mathematics classes, I have developed an attitude of inquiry.	8	20	54	PS PT
15	Teachers can create, for all students, a non-threatening environment for learning mathematics.	5	13	64	PS PT
16	It is the teacher’s responsibility to provide students with clear and concise solution methods for mathematical problems.	19	24	39	PI L
17	There is an established amount of mathematical content that should be covered at each grade level.	20	22	40	PI L
18	It is important that mathematics content be presented to students in the correct sequence.	21	23	38	PI L
19	Mathematical material is best presented in an expository style: demonstrating, explaining and describing concepts and skills.	25	23	34	IL

20	Mathematics is computation.	68	6	8	
21	Telling the students the answer is an efficient way of facilitating their mathematics learning.	61	19	2	IL
22	I would feel uncomfortable if a student suggested a solution to a mathematical problem that I hadn't thought of previously.	75	1	6	IT
23	It is not necessary for teachers to understand the source of students' errors; follow-up instruction will correct their difficulties.	75	3	4	IT
24	Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics.	54	21	7	PI T
25	It is important to cover all the topics in the mathematics curriculum in the textbook sequence.	68	7	7	IT
26	If a students' explanation of a mathematical solution doesn't make sense to the teacher it is best to ignore it.	76	5	1	IT

Note: D = Strongly disagree or Disagree; U = Undecided; A = Strongly Agree or Agree; PS=Problem-solving; P=Platonist; I=Instrumental; L=learning; T=teaching

Beliefs about Teaching Mathematics

Ernest (1989) identified three different roles the teacher can take: an instructor (Instrumentalist), an explainer (Platonist), or a facilitator (Problem-solving). The instructor role aligns with traditional teaching methods (items 22, 23, 25, 26 in Table 2) which most academics disagreed with (at least 83%). A Platonist view of the teacher is an "explainer" helping students develop conceptual understanding and integrate knowledge (Ernest, 1989). "Listening to explanations as the most effective way to learn" (24) could be either an Instrumental or Platonist view: whilst 54 (66%) disagreed, 21 (26%) were undecided. A Platonist view requires teaching strategies that will support students constructing knowledge. Items 10-15 ask about these methods, though most could also be interpreted as Problem-solving. Academics were supportive of these, particularly "allowing students to struggle" (10) (78, or 95%, agreed) and "the importance of justifying statements" (13) (79, or 96%, agreed). With the other items, there were reasonable numbers who were undecided. For example, 13 (16%) were unsure of the value of a "nonthreatening environment" (15) and 22 (27%) were unsure of the "value of persistent questioning in learning" (12), while 54 (66%) had "developed an attitude of inquiry because of classroom experiences" (14). These responses indicate that, overall, respondents generally believed in a Problem-solving/Platonist view of teaching mathematics but there were aspects with which not all were comfortable, for example, the use of questioning, and developing an attitude of inquiry in the classroom.

Analysis and Discussion of Differences between Groups

There were differences in the responses related to participants' teaching responsibility and qualifications.

Differences Related to Teaching Responsibility

There were 60 (73%) respondents who only taught mathematics or statistics content courses (mathematicians), while 8 (10%) only taught pedagogy courses only (mathematics educators) and 14 (17%) taught both discipline and pedagogy. Table 3 shows four beliefs with significant differences ($p<0.05$) in responses based on teaching responsibilities.

Those who taught both content and pedagogy had a larger mean agreement rating than mathematicians that investigating interesting problems in small groups was effective teaching (8), indicating their more Problem-solving view of learning. Mathematics educators had a lower mean agreement rating than mathematicians and those who teach both content and pedagogy for the importance of the "correct sequence" (18). Mathematics educators had a lower mean agreement rating than mathematicians about using an "expository style" (19), associated with an Instrumental view of learning. There was no significant difference between those who taught pedagogy only and those who taught content and pedagogy. Each group agreed that "mathematics is beautiful, creative, useful and a way of knowing and way of thinking," (9) but those who taught both mathematics content and pedagogy had a lower mean agreement rating than both the other groups.

These results indicate that those teaching mathematics pedagogy tend to have a more Problem-solving belief about learning whilst those teaching mathematics content have a stronger belief that some mathematical ideas need to be understood before other concepts can be learnt/understood, a more Platonist belief. Mathematicians tended to believe more in the value of expository teaching, an Instrumental view, than those who teach pedagogy.

Table 3
Differences in Beliefs Related to Teaching Responsibility

Abbreviated item and number	F (p-value)	Statistic	Teach content	Teach pedagogy	Teach content & pedagogy
Students need interesting problems to investigate in small groups. (8)	4.857	Mean	3.68^{a}	4.25^{ab}	4.43^{b}
	(0.010)	SE	0.122	0.250	0.173
Beautiful, creative, useful; a way of knowing & thinking. (9)	5.151	Mean	4.77^{b}	5.00^{b}	4.36^{a}
	(0.008)	SE	0.060	0.000	0.199
Content should be presented in the correct sequence. (18)	5.350	Mean	3.50^{b}	2.25^{a}	3.43^{b}
	(0.007)	SE	0.136	0.250	0.272
Mathematics should be presented in an expository style. (19)	5.030	Mean	3.37^{a}	2.38^{b}	2.79^{ab}
	(0.009)	SE	0.128	0.324	0.239

[a, b] Mean values within a row with unlike superscript letters are significantly different ($p<0.05$). For example, for item 9 the (Bonferroni-adjusted) t-test results show a small p comparing "teaching both content and pedagogy" with "teaching content", and with "teaching pedagogy", but not between "teaching content", and "teaching pedagogy."

Differences Related to Qualifications

As numbers in some individual qualification categories were small, the following groupings were formed: no PhD 15 (18%), PhD in education 12 (15%), PhD in mathematics 44 (54%) and PhD in mathematics and a GDE 11 (13%). The ANOVA indicated six beliefs that showed a statistically significant difference ($p<0.05$), see Table 4.

Mathematicians with a PhD in mathematics had a lower mean agreement rating than those with a GDE for "ignoring students' mathematical ideas can limit their learning" (2). Mathematicians had a lower mean agreement rating than each of the other groups that teachers should be "fascinated with how students think" (7), indicating less of a Problem-solving view of learning. Mathematics educators with a PhD in education had a higher mean agreement rating than mathematicians and those with no PhD in "providing students with interesting problems to investigate in small groups" (8), indicating a stronger Problem-solving view of learning. Those with no PhD had a higher mean agreement rating for there being a "set amount of mathematical content to cover at each level" (17), a Platonist – Instrumental view. Mathematics educators with a PhD in education had a lower mean agreement rating than those with no PhD that "mathematics must be presented in the correct sequence" (18), less of a Platonist view. Mathematicians and those with no PhD had a stronger mean agreement rating than mathematics educators that "mathematics should be presented in an expository style" (19), the Instrumental view of teaching. This result suggests that gaining postgraduate education qualifications may lead people away from an Instrumental view of mathematics and traditional teaching strategies towards developing a more Problem-solving view of mathematics and mathematics teaching.

Table 4
Differences in Beliefs Related to Qualifications

Abbreviated item and number	F (p-value)	Statistic	PhD M, S	PhD Ed	PhD M & GDE	No PhD
Ignoring students' mathematical ideas can limit their learning. (2)	3.228	Mean	4.02^{a}	4.58^{ab}	4.82^{b}	4.20^{ab}
	(0.027)	SE	0.144	0.149	0.122	0.262
Teachers should be fascinated with students' thinking. (7)	8.868	Mean	3.68^{a}	4.50^{b}	4.55^{b}	4.73^{b}
	(0.000)	SE	0.152	0.151	0.157	0.118
Students with interesting problems to investigate in small groups. (8)	4.729	Mean	3.68^{b}	4.50^{a}	4.36^{ab}	3.53^{b}
	(0.004)	SE	0.121	0.195	0.203	0.322
An established amount of content to be covered at each level. (17)	3.090	Mean	3.41^{ab}	2.75^{a}	3.00^{ab}	3.87^{b}
	(0.032)	SE	0.157	0.329	0.270	0.256
Content should be presented in the correct sequence. (18)	3.451	Mean	3.57^{a}	2.50^{ab}	3.45^{ab}	3.40^{b}
	(0.020)	SE	0.154	0.314	0.312	0.254
Mathematics is best presented in an expository style. (19)	5.130	Mean	3.39^{b}	2.50^{a}	2.55^{ab}	3.53^{b}
	(0.003)	SE	0.135	0.359	0.247	0.236

$^{a, b}$ Mean values within a row with unlike superscript letters are significantly different ($p<0.05$). For example, for item 19 the (Bonferroni-adjusted) t-test results show a small p comparing "a PhD in mathematics or statistics" with "a PhD in education", but not between the others. M=mathematics, S=statistics, Ed=education.

Conclusions

Teachers' practices directly influence the beliefs about mathematics of the students in their classes (McLeod, 1992; Mosvold & Fauskanger, 2014). Hence it seems reasonable to assume that in ITE programs the practices of teacher educators – whether they are teaching content or teaching pedagogy – would influence the beliefs about mathematics teaching and learning held by pre-service teachers. It is important that pre-service teachers have experiences that will support the Problem-solving beliefs about mathematics in the *ACM*.

Generally the survey sent to mathematicians, statisticians and mathematics educators involved in ITE programs in Australia revealed those teaching future secondary mathematics teachers hold a Problem-solving view of mathematics as a discipline, but with some aspects of Platonist views (Ernest, 1989). However, 10% agreed with statements of beliefs aligned with the Instrumental view of mathematics (Ernest, 1989), that is, that mathematics was "entirely distinct from other disciplines". Such beliefs are contrary to the aim of the *ACM*"recognis[ing] connections between the areas of mathematics and other disciplines" (ACARA, n.d.). Largely respondents believed in a Problem-solving /Platonist view of teaching mathematics, particularly "allowing students to struggle" (95% agreement) and "the importance of justifying statements" (96% agreement); but there were aspects with which not all were comfortable, for example, the use of questioning, and developing an attitude of inquiry in the classroom. These are teaching strategies commonly associated with problem solving (Van der Walle, Karp & Bay-Williams, 2016).

Some interesting differences were identified between respondents with/without educational qualifications, and with/without teaching responsibilities in pedagogy. While the analysis can say nothing about causality, it may be that postgraduate exposure to educational theories and perspectives supports the development of a Problem-solving view of mathematics learning, while specialisation in mathematics at the postgraduate level supports beliefs about correct content and sequencing. We are exploring these tentative claims in semi-structured interviews with a sample of survey respondents to better understand their beliefs about mathematics, and how mathematics is taught and learned. As a result we hope to support boundary dialogues (Goos & Bennison, 2018) between mathematicians and mathematics educators, aiming for greater coherence for the pre-service teachers who learn from mathematicians and mathematics educators.

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