The Binary Cascade

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Abstract

Geant4 is a toolkit for the simulation of the passage of particles through matter. Amongst its applications are hadronic calorimeters of LHC detectors and simulation of radiation environments. For these types of simulation, a good description of secondaries generated by inelastic interactions of primary nucleons and pions is particularly important.

The Geant4 Binary Cascade is a hybrid between a classical intra-nuclear cascade and a QMD [1] model, for the simulation of inelastic scattering of pions, protons and neutrons, and light ions of intermediate energies off nuclei. The nucleus is modeled by individual nucleons bound in the nuclear potential. Binary collisions of projectiles or projectile constituants and secondaries with single nucleons, resonance production, and decay are simulated according to measured, parametrised or calculated cross sections. Pauli's exclusion principle, i.e. blocking of interactions due to Fermi statistics, reduces the free cross section to an effective intra-nuclear cross section. Secondary particles are allowed to further interact with remaining nucleons.

We will describe the modeling, and give an overview of the components of the model, their object oriented design, and implementation.

INTRODUCTION

The Binary Cascade models interactions of nucleons, pions, and light ions with nuclei for incident particle energies in the energy range starting from few MeV up to few GeV. A new approach to cascade calculations is used in Binary Cascade. The nucleus is described as a detailed 3-dimensional nucleus, and the primary particles interact with nucleons of the nucleus in binary collisions. The secondary particles re-scatter with nucleons, creating the cascade. The model is implemented in the framework of the GEANT4 toolkit [2], more specifically as a final-state generator, called a model, in the context of the GHAD [3] frameworks.

MODELING OVERVIEW

The Binary Cascade is an intra-nuclear cascade propagating primary and secondary particles within a nucleus. Interactions are one on one reactions between a primary or secondary particle and a nucleon of the nucleus. Measured cross-sections are used where available to calculate interaction probabilities. Strong resonances decay using PDG branching ratios.

In the following we give an overview of the modeling components, full details of the model are published in [4].

The nucleus model

The nucleus is modeled as 3-dimensional spheric and isotropic nucleus. Nucleons are explicitly placed using nuclear density distributions. For light nuclei with mass number $A \leq 16$, we use a harmonic-oscillator shell model density distribution [6]:

$$\rho(r) = (\pi R^2)^{-3/2} \exp\left(-r^2/R^2\right) \quad (1)$$

where $R \propto A^{1/3}$. For heavier nuclei we use a Wood-Saxon form [7] for the density distribution:

$$\rho(r) = \frac{\rho_0}{1 + \exp\left[(r-R)/a\right]} \quad (2)$$

where is $a = 0.545$fm, and $R = r_0 A^{1/3}$, with the radius parameter r_0 varying with the mass number A, $r_0 = 1.16(1 - 1.16A^{2/3})$ fm.
To be consistent with Pauli's exclusion principle, the minimal inter-nucleon distance is 0.8 fm.

Nucleons carry momentum randomly chosen between 0 and the Fermi momentum $p_F(r)$. In the local Thomas-Fermi approximation, this is a function of the local density, and hence a function of the radius:

$$p_F(r) = \hbar c (3\pi^2 \rho(r))^{1/3} \quad (3)$$

The momenta are chosen such that the vector sum of all momenta is 0.

Nuclear potential

The effect of the collective nuclear interaction upon primary and secondary particles is included in the model through a time-invariant potential. For nucleons this potential is determined by the Fermi momentum $p_F(r)$

$$V(r) = -\frac{p_F^2(r)}{2m} \quad (4)$$

where m is the mass of the proton or neutron.

For pions we use a simple approximation given by the lowest order optical potential

$$V(r) = -\frac{2\pi(\hbar c)^2 A}{\overline{m}_\pi}(1 + \frac{m_\pi}{M}) b_0 \rho(r) \quad (5)$$

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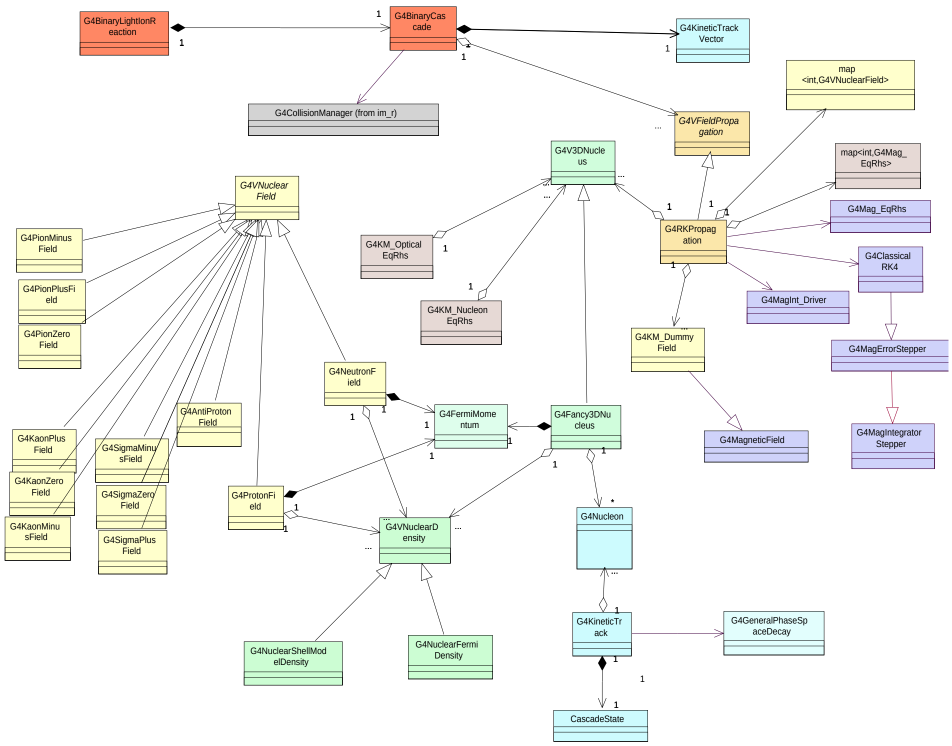

Figure 1: Object model diagram of the cascade component of the Binary Cascade.

where A is the nuclear mass number, m_π and M are the mass of the pion and the nucleon, $\overline{m}_\pi$ is the reduced pion mass, $\overline{m}_\pi = (m_\pi m_N)/(m_\pi + m_N)$, m_N is the mass of the nucleus, and $\rho(r)$ is the local density as given by equation (1) or (2).

The transport algorithm

The primary and all secondary particles are propagating in the nuclear potential. The nucleons of the nucleus are invariant in space. The propagation is performed by integrating the equations of motion in time steps. This integration re-uses a Runge-Kutta integration method available in Geant4 for propagation of particles in a magnetic field. A time step is determined by the time to the first interaction, decay, or particles entering or leaving the nucleus. For the calculation of the time steps nuclear fields are ignored and particle trajectories are straight line trajectories. At the end of each time step the action limiting the time step is performed. Either an interaction occurs, a strong resonance decays, or a particle enters or leaves the nucleus.

The scattering term

The imaginary part of the R-matrix acts like a scattering term. We use free two-body cross-sections from experimental data and parameterizations thereof. For resonance re-scattering we use the solution of an in-medium Boltzmann-Uehling-Uhlenbeck (BUU) equation [9]. The Binary Cascade takes an extensive set of strong resonances into account:

- The delta resonances with masses of 1232, 1600, 1620, 1700, 1900, 1905, 1910, 1920, 1930, and 1950 MeV,
- the excited nucleons with masses of 1440, 1520, 1535, 1650, 1675, 1680, 1700, 1710, 1720, 1900, 1990, 2090, 2190, 2220, and 2250 MeV.

Nucleon-nucleon scattering in the t-channel is described via resonance excitation. Cross-sections are derived from the measured proton-proton inelastic scattering cross-section using isospin invariance and applying the corresponding Clebsch-Gordon coefficients.

Non-elastic meson-nucleon scattering, excluding true absorption, is modeled as s-channel resonance excitation. The cross-section can be written in in the form of a Breit-Wigner function:

$$\sigma(\sqrt{s}) = \sum_{FS} \frac{2J+1}{(2S_1+1)(2S_2+1)} \times \frac{\pi}{k^2} \frac{\Gamma_{IS}\Gamma_{FS}}{(\sqrt{s}-M)^2+\Gamma^2/4} \quad (6)$$

$S1$ and $S2$ are the spins of the primary particles, k is the momentum of the primary particles in the center-of-mass system, J and M is the spin and the nominal mass of the resonance, Γ_{IS} and Γ_{FS} are the partial widths of the resonance for the initial and final state, and Γ is the total stochastic width of the resonance.

True pion absorption is modeled as S-wave absorption of pions on quasi-deuterons.

Elastic nucleon-nucleon scattering cross-sections and angular distribution are taken from a phase shift analysis of experimental data derived from the SAID database [5].

Decay branching ratios are taken from the PDG, and are case by case corrected for stochastic width, and in-medium effects. Nominal resonance masses are chosen within the error band given by PDG, such that the total 2-particle scattering cross-section predicted is consistent with the measured values.

For all interactions we apply the classical form of Pauli Blocking, i.e. a reaction is allowed only if all nucleons in the final state have momenta larger than the local Fermi momentum $p_F(r)$

Transition to pre-equilibrium decay

The cascade stops propagating particles when the mean energy of all particles in the system is below cut-off. The properties of the residual exciton system and of the nucleus are evaluated and are passed to a pre-equilibrium decay code [8] for further treatment.

Light ion reactions

The scattering of light ions on nuclei is modeled by propagating the nucleons of the ion as free particles through the nucleus. The secondary ion fragment is formed of the primary nucleons not undergoing any interaction. The excitation energy of the fragment is derived from the properties of the interacting nucleons and the initial ion. The excited fragment is passed to equilibrium decay code for further decay or de-excitation.

OBJECT ORIENTED DESIGN

The GEANT4 Binary Cascade was designed using Object Oriented Analysis and Design techniques leading to a clear domain decomposition and component structure. The two main components for the Binary Cascade are the cascade part and the scatterer. The scatterer component was designed to be independent of the cascade and contains all classes relevant to the modeling of collisions of particles. The scatterer component contains all classes related to physical cross-sections and angular distributions. Template Meta Programming is used for compile time code generation, dramatically reducing the code size for resonance decay, and, at the same time, improving reliability and code readability. Design patterns like Composite, Bridge and Hidden Adapter are used to achieve decoupling of logic and algorithm.

The cascade component is responsible for the propagation of all particles through the nucleus. It uses the scatterer through an abstract interface. Re-use of other parts of GEANT4 is essential. The model of the nucleus and its implementation as used by the Binary Cascade is also used by other hadronic models, e.g. high energy strings models. The integration of the equations of motion in the nuclear field uses a Runge-Kutta integration methods from the geometry/magnetic field category in GEANT4. The object model of the cascade component in shown in figure 1.

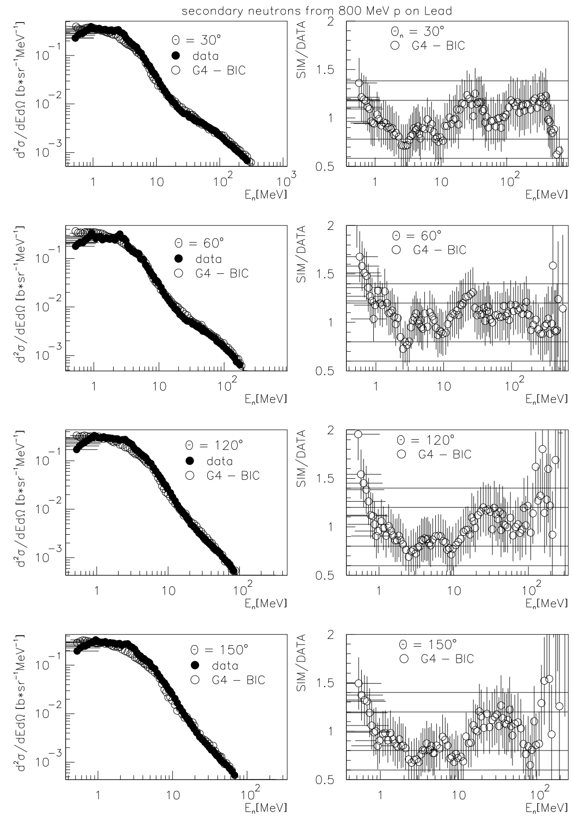

Figure 2: Cross sections for production of secondary neutrons at four different scattering angles (30°, 60°, 120°, 150°) in inelastic scattering of 800 MeV/c protons off lead are shown on the left side. Full dots are data [12], open circles show preliminary results from simulation. The right side shows the ratio of simulation to experimental data.

COMPARISON WITH EXPERIMENT

A verification suite [10] has been developed to evaluate and optimize hadronic models in the cascade energy range in GEANT4. The verification is done by comparing simulation results with experimental data from thin target measurements, mainly as collected in the EXFOR database [11].

Up to now, only data from measurements of absolute differential cross sections are utilized in the suite. The modular structure of GEANT4 allows the generation of single events with a known incident particle energy

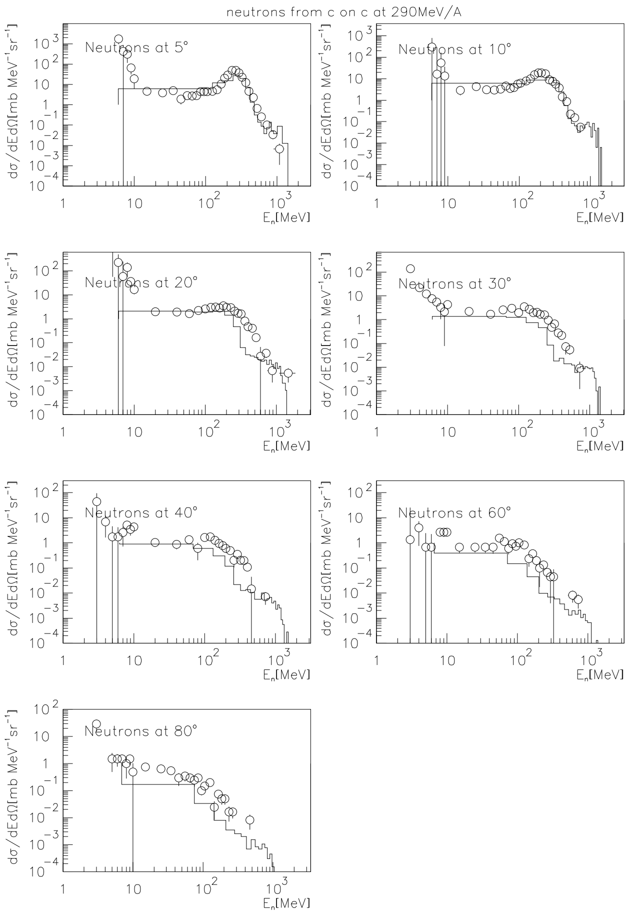

Figure 3: Cross sections for production of secondary neutrons at seven different scattering angles (5°, 10°, 20°, 30°, 40°, 60°, and 80°) in inelastic scattering of 290 MeV/nucleon carbon ions off carbon. Open circles are data [13], and the histogram is the result from simulation.

and any explicitly defined hadronic final-state generator. The kinematics of secondaries produced in the interaction are then analyzed and the resulting angular, momentum, energy, and baryon number spectra are stored in histograms. The histograms are compared to published measurements of the differential and double differential cross-sections, $d\sigma/dE$, $d\sigma/d\Omega$, $d^2\sigma/dEd\Omega$, and the invariant cross-section, $Ed^3\sigma/d^3p$. The test suite also allows to verify that the model conserves e.g. energy-momentum or baryon number.

In the following we concentrate on comparisons of measured double differential cross-sections for neutron production with results obtained from simulation.

Figure 2 shows a comparison of measured neutron production cross-sections for neutrons produced in inelastic scattering of protons with momentum of 800 MeV/c off lead nuclei [12] with simulation. The simulation results are from an ongoing study of the transition from the cascade model to pre-equilibrium decay. This study demonstrates that the GEANT4 models can reproduce the experimental data to within the experimental error.

In figure 3 we show first results from the modeling of light ion reactions. We compare scattering of Carbon ions with a momentum of 290 MeV/c per nucleon on Carbon against experimental data [13]. The agreement of Binary Cascade with this specific set of data is quite satisfactory. However, more data are available, and simulation and experiment must be compared over a wider range of projectile energies using also different ion projectiles and target materials before we can draw final conclusions.

SUMMARY

We have given an overview of the Binary Cascade model and its implementation in GEANT4. For proton induced reactions, the model describes the available experimental data on neutron production very well. Also our comparisons with experimental data for light ion scattering are quite satisfactory.

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