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# The Black Hole Horizon as a Quantum Surface*

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**Abstract**

In previous work it was argued why it is practically inevitable to assume the existence of an $S$-matrix that describes particle absorption and production by a black hole, and the relationship between this $S$-matrix and string theory was derived. The physical interpretation of the corresponding mathematical expressions however is quite different from string theory. We have an algebra of operators now defined on a two-dimensional Euclidean "world sheet". The algebra simplifies if one restricts it to the self dual projection of the fundamental surface elements $W^{\mu\nu}$.

Our two dimensional functional integrals correspond to a rather unusual field theory. The long distance structure of this theory follows directly from the long distance structure of the standard model at the GeV scale. We emphasize the rather delicate physical interpretation of this approach.

## 1. Introduction

Since the discovery of Hawking radiation [1] by black holes it has become clear that these objects are in no fundamental way different from ordinary heavy particles and other extended objects: particles of any kind can be absorbed or emitted by them just as if they were heavy radio-active nuclei.

Yet the derivation makes use of a unique principle: general relativity. The way in which quantum mechanics should be combined with general relativity is still largely problematic, so some skepticism concerning our knowledge how to apply the rules is not misguided‡. At first it was thought [3] that, since the radiation is thermal, black holes can only occur in mixed quantum mechanical states, to be described by a density matrix. Even if one starts with a single quantum mechanical state for a collapsing object it was said that the black hole should end up in such a mixed state. But later it was realized that this could be merely an optical effect. Whenever we have a system for which the Hamiltonian is not infinitely precisely known, but given as a probability distribution, pure states will automatically turn into mixed states as they evolve in time.

Out of this probability distribution of Hamiltonians only one is the True Hamiltonian of our world. We should be interested in obtaining this single Hamiltonian, either by experimentation, or by extending our theories beyond the one that gave us the Hawking radiation alone.

A better theory can indeed by constructed [4], and it turns out to have very important implications, not only for black holes but for quantum field theory as a whole, and in particular the question how to incorporate the gravitational force. In this author's opinion this may be an extremely important alley towards a much more comprehensive theory, which has not at all enjoyed the attention it deserves.

The idea is to make one basic assumption, namely that the evolution of a system containing one or more black holes should be in total agreement with all that is known about quantum mechanics. There should exist a Hilbert space of states that either evolve according to a Schrödinger equation or, equivalently, in which operators evolve as in a Heisenberg picture. Giving up such an assumption would be tantamount to giving up the hope to derive any physical Law at all for these things, and this author finds that unacceptable.

In return, one obtains rewarding insights in what Nature could be like at Planck scales. No direct contradictions are seen, but we do seem to obtain severe constraints on the possible forms interactions can take at these scales. It is of crucial importance to observe that these constraints follow for the field theoretical interactions outside the black hole itself, so that they should be equally valid if no black hole is present at all.

Stated differently, if we want to write down the complete spectrum of all states in Hilbert space then only those with small amounts of energy per unit of length are free of black holes. They form the infrared end of the spectrum that is now described by the "Standard Model". The "generic state" automatically does contain black holes. Paradoxically, given the total volume, the majority of all states are describing a single black hole that just barely fits inside this volume.

The physical degrees of freedom will have to be remarkably similar to those of string theory [5], but our theory is not exactly string theory in the usual sense. The similarity comes about because the horizon is a two-dimensional surface in Minkowski space, and it is governed by the same two-dimensional mathematics as a string world sheet. As a consequence the spectrum of massless particle states seems to emerge just as in string theory. The massive states are different because, if one drives the analogy further, the string constant will turn out to be imaginary rather than real.

More to the point, the physics is different. Here we are dealing with a two dimensional Euclidean world on which the Rindler Hamiltonian acts. In Section 2 we explain why it should be suspected that in some sense the spectrum of physical states is discrete, and why the asymptotic plane waves at spatial infinity can to some extent be ignored. In an ideal theory all this should become evident from the representations of the operator algebra.

## 2. The spectrum of states in Rindler space

The total entropy $S$ of a black hole with mass $M$ residing inside a volume $V$ in natural units is [1, 3]

$$S = 4\pi M^2 + CV T^3, \tag{2.1}$$

where $C$ is a constant depending on the details of our favorite

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‡ See Ref [2], where an unconventional interpretation of quantum mechanics at a black hole horizon gives a radiation temperature that differs from the one usually obtained by a factor 2. This alternative derivation is not "wrong" as is often claimed, but starts with an axiom that, though impossible to refute, is in conflict with most field theoretical models for the black hole.

quantum field theory (it may vary slightly with $T$ but stays of order one), and $T$ is the temperature, which is usually taken to be

$$T = 1/(8\pi M). \tag{2.2}$$

Now let us take $V$ to be a multiple of $R^3$, the volume of the hole itself, but not very much more than that. Since

$$R = 2M \tag{2.3}$$

this implies that the vacuum contribution to $S$ (the second term in (2.1) is negligible compared to the contribution of the hole itself, as soon as the hole has a mass much larger than the Planck mass. Now note that in a first approximation exp ($S$) is the number of states with total energy $M$. Clearly, this number is dominated by the black hole in such a way that there is roughly one $\mathbb{Z}(2)$ degree of freedom for each unit $\delta\Sigma$ of surface area of the horizon, where*

$$\delta\Sigma = (4 \ln 2)M_{\rm Pl}^{-2}. \tag{2.4}$$

It may be remarkable that in comparison the contribution of the surrounding vacuum to the degrees of freedom of the system is small (only about one per unit of volume equal to the volume of the hole itself).

Now consider the limit $M \Rightarrow \infty$. The black hole horizon is then neatly described by Rindler space. The contibution of the volume $V$ in (2.1) becomes altogether invisible then (because there the temperature $T$ drops to zero), so we leave it out. Essentially, the entire universe in Rindler space describes a region close to the black hole horizon, and we remember that if we took $V$ that small its contribution in (2.1) vanishes. This is why we believe that the total number of physical states in Rindler space cannot exceed exp ($\Sigma/4$), where $\Sigma$ is the total surface area in the two transverse dimensions. This means we have one physical $\mathbb{Z}(2)$ degree of freedom per unit $\delta\Sigma$ of surface area in the transverse direction.

This result may seem surprising and disturbing at first, but can be understood in a simple model. Take a brick wall separating the true horizon in Rindler space from the outside region [7]. The brick wall is approximately one Planck distance away from the true horizon. Inside the brick wall all field theoretical degrees of freedom are constrained to be zero (so also on the horizon the fields are zero). Outside the wall we just take standard field theory in the Rindler coordinate frame. Let us further insist that we only consider all those states in our field theory for which the total Rindler energy is approximately one per unit of transverse surface element. (This puts the total Rindler energy for the entire Black hole proportional to its surface $R^2 = 4M^2$, and since the unit of time in Rindler space is $2M$ times the unit of time for an external observer of the black hole, this constraint corresponds to limiting the total energy of the field quanta to be proportional to the black hole mass itself; the proportionality constant is not exactly one [7].)

Then one obtains the spectrum described above (apart from constants of order unity, which should not concern us much here). If we would take periodic boundary conditions in the two transverse directions only, this spectrum of the Rindler vacuum would turn out to be discrete. If the reader finds this strange, just realize that most quanta with a finite amount of energy cannot get very far from the horizon and its wall because they are pulled back by the infinite gravitational potential. Only those particles that are massless and heading directly outwards could escape, but their spectrum is of measure zero; most particles have some amount of transverse momentum which is sufficient to pull them back in.*

In reality of course there is no brick wall, but this argument shows that gravitational effects at the Planck length could in principle be responsible for this discrete spectrum. If we now realize that the only assumption made in deriving this curious spectrum of states was the Hawking expression (2.1) for the black hole entropy, and its relation to the number of states in Hilbert space, we see that it must be a rather solid result that would be extremely difficult to avoid.

Yet this discreteness is not directly obtained when we try to do perturbative quantum field theory directly in the Rindler coordinates. One gets a singularity at the horizon, but such a result should not be believed because it hinges upon neglecting any non-linear effects, whereas we know very well that the gravitational force stays by no means small when we approach the horizon closer than the Planck distance.

Our aim is to derive the dynamics of this discrete set of degrees of freedom at the horizon. But because of our complete ignorance of the field theoretical interactions at that scale we will not succeed in doing that. Instead, we obtain properties of these variables smeared over much larger distances. They are string-like, as was derived in [4, 5].

## 3. The degrees of freedom in the S-matrix

Let $(x^1, x^2) = \tilde{x}$, $x^3 = z$ and $x^0 = t$ be ordinary Cartesian coordinates in Minkowski space. The Rindler coordinates $\zeta$ and $\tau$ are defined by

$$z = \zeta \cosh \tau; \quad t = \zeta \sinh \tau. \tag{3.1}$$

Following [5] we characterize the particles that enter the black hole crossing the future event horizon ($z - t = 0$), by giving the distribution of the + component of their momentum, along with possible other detectable quantum numbers such as the electric charge distribution $\rho_{\rm in}$ (which we'll determine later):

$$|{\rm in}\rangle = |\{p_+(\tilde{x}), \rho_{\rm in}(\tilde{x}), \ldots\}\rangle_{\rm in}, \tag{3.2}$$

$$p_+ = p_0 + p_3, \tag{3.3}$$

and for the particles that leave the hole by crossing the past horizon ($z + t = 0$) we give the following distributions:

$$|{\rm out}\rangle = |\{p_-(\tilde{x}), \rho_{\rm out}(\tilde{x}), \ldots\}\rangle_{\rm out}, \tag{3.4}$$

$$p_- = p_0 - p_3. \tag{3.5}$$

The essential thing observed in [4, 5] is that if we know only one element of the matrix $\langle{\rm out}|{\rm in}\rangle$, we can derive all others, after which the normalization can also be fixed by requiring unitarity. This means that if we assume the mere existence of an $S$-matrix, we can actually derive the form it must have, using ordinary quantum field theory (this will

* The quantity $\delta\Sigma$ was interpreted by Mukhanov [6] as a quantization of the horizon. He suggests that the black hole mass-squared is quantized accordingly. However, in our picture these quantized states should still be enormously degenerate, and it is not clear whether or not the levels will be split to the extent that Mukhanov's quantization becomes obliterated.

* The author thanks S. Colman for raising the question concerning the role of the infinite exterior space when we say that the black hole spectrum including exterior space is discrete. What is really meant is that exterior space is taken to be larger than the hole itself, but certainly not infinite.

limit our derivations to those energy scales where we think we know all interactions, i.e., the GeV or TeV range).

Consider namely any small change in the momentum distribution and/or charge distribution of the in-state:

$$p_+ \Rightarrow p_+ + \delta p_+(\tilde{x}); \quad \rho_{\text{in}}(\tilde{x}) \Rightarrow \rho_{\text{in}} + \delta\rho_{\text{in}}(\tilde{x}), \tag{3.6}$$

where $\delta p_+$ and $\delta\rho_{\text{in}}$ are arbitrary and small. Then $\delta p_+$ produces a tiny gravitational field and $\delta\rho_{\text{in}}$ a tiny electromagnetic field that affect the outgoing particles.

To be precise, $\delta p_+$ causes a gravitational shock wave [8] that brings about a shift in the coordinates $x^- = \frac{1}{2}(x^0 - x^3)$ of the outgoing particles:

$$\delta x^-(\tilde{x}) = 4\pi G \int d^2\tilde{x}' f(\tilde{x}, \tilde{x}')\delta p_+(\tilde{x}'), \tag{3.7}$$

with the Green function $f$ obeying

$$\tilde{\partial}^2 f(\tilde{x}, \tilde{x}') = -\delta^2(\tilde{x} - \tilde{x}'), \tag{3.8}$$

and $G$ is the gravitational constant. This shift rotates any outgoing wave with momentum $p_-$ by multiplying it with a factor

$$\exp(\mathrm{i}\delta x^- p_-) = \exp\left(4\pi\mathrm{i}G \int d^2\tilde{x}\, d^2\tilde{x}' f(\tilde{x}, \tilde{x}')p_-(\tilde{x})\delta p_+(\tilde{x}')\right). \tag{3.9}$$

Furthermore, $\delta\rho_{\text{in}}$ causes an electromagnetic shock wave (Čerenkov radiation) that brings about a phase rotation

$$\exp(\mathrm{i}\Lambda(\tilde{x})); \quad \Lambda(\tilde{x}) = \int d^2\tilde{x}' f(\tilde{x}, \tilde{x}')\delta\rho_{\text{in}}(\tilde{x}'), \tag{3.10}$$

so that waves of outgoing particles with charge distribution $\rho_{\text{out}}$ are rotated by

$$\exp\left(\mathrm{i}\int d^2\tilde{x}\rho_{\text{out}}(\tilde{x})\Lambda(\tilde{x})\right) = \exp\left(\mathrm{i}\int d^2\tilde{x}\, d^2\tilde{x}' f(\tilde{x}, \tilde{x}')\rho_{\text{out}}(\tilde{x})\delta\rho_{\text{in}}(\tilde{x}')\right). \tag{3.11}$$

Note that the normalization here differs somewhat from [5]. We included the uinit charge $e$ in our definition of $\rho$.

If $\rho$ and $p_+$ are all that can be detected of the ingoing particles then no other effect will be seen on the amplitudes for the outgoing particles than (3.9) and (3.11). So (3.9) and (3.11) then represent the change in the $S$-matrix elements. But then the entire $S$ matrix can be seen to be these same expressions (up to a common normalization factor) with $\delta p_+$ and $\delta\rho_{\text{in}}$ replaced by the total functions $p_+$ and $\rho_{\text{in}}$. This we can rewrite as a functional integral:

$$\langle\text{out}|\text{in}\rangle = \int \mathscr{D}x^+\mathscr{D}x^-\mathscr{D}\phi \exp\int d^2\tilde{x}\left\{\frac{\mathrm{i}}{4\pi G}\tilde{\partial}x^-\tilde{\partial}x^+ - \mathrm{i}p_+x^+ + \mathrm{i}p_-x^- - \tfrac{1}{2}\mathrm{i}(\tilde{\partial}\phi)^2 - \mathrm{i}\phi(\rho_{\text{out}} - \rho_{\text{in}})\right\}. \tag{3.12}$$

Here a factor depending only on $\rho^2_{\text{out}}$ and one depending only on $\rho^2_{\text{in}}$ were omitted as being probably irrelevant.

Writing

$$x^\pm = \tfrac{1}{2}(x^0 \pm x^3); \quad \rho = \rho_{\text{in}} - \rho_{\text{out}}; \quad \tilde{p} = 0,$$
$$p_0 = \tfrac{1}{2}(p_- - p_+); \quad p_3 = -\tfrac{1}{2}(p_- + p_+), \tag{3.13}$$

(so that $p_\mu$ represent the total ingoing momentum) this is seen to be

$$\int \mathscr{D}x^+\mathscr{D}x^-\mathscr{D}\phi \exp\int d^2\tilde{x}\left\{\frac{-\mathrm{i}}{16\pi G}\tilde{\partial}x^\mu\tilde{\partial}x^\mu + \mathrm{i}x^\mu p_\mu - \tfrac{1}{2}\mathrm{i}(\tilde{\partial}\phi)^2 + \mathrm{i}\phi\rho\right\}, \tag{3.14}$$

and we see that if besides gravity the only other long range force is electromagnetism then automaticlly a "fifth dimension" $\phi$ appears. Since the electric charges $Q = \int d^2\tilde{x}\rho$ are quantized in multiples of $e$ the variable $\phi$ must be taken to be periodic with period $2\pi/e$.

In many respects (3.14) resembles a string action; in particular we see that the external momenta are introduced as "vertex insertions" $\exp(\mathrm{i}x \cdot p)$. Since the transverse directions $\tilde{x}$ are at the GeV or TeV scale and the longitudinal variables $x^+$ and $x^-$ at the Planck scale it is legitimate to view (3.14) as a string acton in the approximation where only infinitesimal excitations from the transverse Euclidean world sheet were taken into account.

Note that the world sheet is Euclidean, so that the first i in the exponent would not be expected by string theorists. Our string constant $T$ is the imaginary number $\mathrm{i}/(8\pi G)$.

It is suspected that one can get beyond the infinitesimal excitation approximation by replacing (3.14) by the fully covariant Nambu action (indeed one can argue that taking also the transverse gravitational interactions between incoming and outgoing particles into account automatically leads to a Nambu action), but even then one does not (yet?) recover the discrete Rindler space spectrum of Section 2.

## 4. The physics of the displacement operators $p_\pm(\tilde{x})$

Our $S$ matrix could be called quantum deterministic, with which we mean to say that no information dissappears into the black hole. Now this of course is in conflict with usual wisdom. One consequence is that there cannot be any globally conserved additive quantum number such as baryon number (unless of course a local gauge field would be coupled to it). Information concerning baryon number of the ingoing particles is not transmitted to the outgoing particles so that no unitary $S$ matrix is then possible. We simply conclude that all such quantum conservation laws must be violated.

So far, so good. But closer analysis of the physics of our derivation of $S$ does force us to consider the problem of information drainage more precisely. It was postulated that ingoing particles cause a shift among the outgoing ones. But then there is a small region of space-time, possibly containing outgoing particles, that is sent to the other side of the horizon. Where did the information it contained go? Stated differently, since only half of the space of outgoing particles is at our side of the horizon so that it can be used to describe Hilbert space, it seems that the shift operators generated by $p_\pm(\tilde{x})$ have to be accompanied by projection operators that remove this unseen part of space.

We ignore these projection operators. This is because we had to postulate that no "quantum information" should get lost. The operators $p_+$ and $p_-$ we use are taken to be Hermitean. This apparent "mistake" implies that we alter the precise mathematics of field theory near the horizon, as follows.

If we ignore momentarily the electromagnetic contribution, then all information that specifies the states in our Hilbert space is contained in the displacements $\delta x^{\pm}$ of the lines $\tilde{x}$ = constant (or, equivalently, the conjugated operators $p_{\pm}(\tilde{x})$). This is as if on all these lines there were exactly one particle, that is never seen to escape behind the horizon.

In reality the number of particles on each of these lines should of course be variable. If we take our bundle of lines $\tilde{x}$ = constant to be sufficiently dense then it is not unreasonable to restrict ourselves to the case that particles with the same $\tilde{x}$ are far apart in Rindler time. Second quantization might make the particle density fairly dense again, but we are not yet in a stage that the order in which the various limits should be taken should concern us now.

Instead of one single particle on each line $\tilde{x}$ = constant we can also take an infinite series $x^+(i)$, $x^-(j)$, each having uncertainties $\delta x^+(i)$, $\delta x^-(j)$. Since as a function of Rindler time these $x$'s scale, we can take the uncertainties $\delta x^+(i)$ and $\delta x^-(j)$ to obey

$$0 < x^+(i) < \delta x^+(i-1) \quad \text{and} \quad 0 < x^-(j) < \delta x^-(j+1) \tag{4.1}$$

This way one can achieve that the total amount of information carried by all these particles is the same as the information carried by a single particle whose position is fixed infinitely precisely. More precisely, we choose some number $F$ (for instance an integer > 1), and two series $u^+(i)$ and $u^-(j)$ so that

$$0 < u^+(i) < F^{-i} \quad \text{and} \quad u^+(i)F^{i+1} \quad \text{is integer}, \tag{4.2}$$

and a similar distribution of $u^-$ values.

If now only the number

$$x^{\pm} = \sum_i u^{\pm}(i) \tag{4.3}$$

is given then (4.2) allows us to reobtain all entries $u^{\pm}(i)$ unambiguously. A particle with rank $i$ is taken to have the coordinates

$$x^+(i) = \sum_{i' \leqslant i} u^+(i'); \quad x^-(j) = -\sum_{j' \geqslant j} u^-(j).$$

This way three things were achieved:

(i) A single real number characterizes the locations of all particles at a given $\tilde{x}$;

(ii) All coordinates are at the right of the horizon regardless how we shift the total, (4.3).

(iii) The momentum operators, conjugated to the total $x^{\pm}$, correspond to the total momenta of all particles, so that our description of their gravitational field is correct.

The price paid, namely an unusual distribution of a hierarchy of particles along each longitudinal line, and an uncertainty in their precise locations, implies that non-trivial physics is assumed. It also makes any interpretation needed to match this description to the usual one in Rindler space very difficult.

Remarkably, the $S$-matrix obtained is close to the string $S$-matrix. At distance scales large compared to the Planck length this is the only thing one will see; the amplitudes are not strange at all.

## 5. Operator algebra on the horizon

Our starting point is that states in Hilbert space are uniquely determined by specifying any one of the following four functions: the distribution of ingoing momenta $p_+(\tilde{x})$, the outgoing momenta $p_-(\tilde{x})$, the conjugated operators $x^+(\tilde{x})$, or $x^-(\tilde{x})$. They obey the algebra

$$[p_+(\tilde{x}), p_+(\tilde{x}')] = 0; \quad [p_+(\tilde{x}), x^+(\tilde{x}')] = -\mathrm{i}\delta^2(\tilde{x}, \tilde{x}'); \tag{5.1}$$

$$[p_-(\tilde{x}), p_-(\tilde{x}')] = 0; \quad [p_-(\tilde{x}), x^-(\tilde{x}')] = -\mathrm{i}\delta^2(\tilde{x}, \tilde{x}'), \tag{5.2}$$

and we have the relation

$$x^-(\tilde{x}) = 4\pi G \int \mathrm{d}^2\tilde{x}' f(\tilde{x}, \tilde{x}')p_+(\tilde{x}'). \tag{5.3}$$

This implies

$$[x^-(\tilde{x}), x^+(\tilde{x}')] = -4\pi\mathrm{i}Gf(\tilde{x}, \tilde{x}'), \tag{5.4}$$

so that we have also

$$x^+(\tilde{x}) = -4\pi G \int \mathrm{d}^2\tilde{x}' f(\tilde{x}, \tilde{x}')p_-(\tilde{x}'). \tag{5.5}$$

The albebraic relations among $\rho(\tilde{x})$ and $\phi(\tilde{x}')$ are slightly more subtle because of the quantization of electric charge and the ensuing periodic boundary conditions on $\phi$. We will disregard these from here on.

The relations (5.1–5) are not infinitely accurate. This is because we neglected any gravitational curvature in the sideways directions. This is fine as long as transverse distance scales are kept considerably larger than the Planck scale. One may convince oneself that this implies neglecting higher orders in the derivatives $\partial x^{\pm}/\partial\tilde{x}$. Is there any way to obtain a more precise algebra? It is natural to search for an algebra that is invariant under Lorentz transformations. One might hope that such an algebra could generate the correct degrees of freedom at the Planck scale (in particular quantized degrees of freedom).

It was proposed in Ref [4], that Hilbert space on the horizon may be generated by the operator algebra of fundamental surface elements,

$$W^{\mu\nu}(\tilde{\sigma}) = \varepsilon^{ab}\frac{\partial x^{\mu}}{\partial\sigma^a}\frac{\partial x^{\nu}}{\partial\sigma^b}. \tag{5.6}$$

where the transverse coordinates $\tilde{x}$ were replaced by more arbitrary surface coordinates $\sigma^1$, $\sigma^2$. The relations (5.1–5) may be used in the case $\tilde{\sigma} = \tilde{x}$, when the derivatives are small. This means

$$W^{12} = 1; \quad W^{1\mu} = \frac{\partial x^{\mu}}{\partial\sigma^2}; \quad W^{2\mu} = -\frac{\partial x^{\mu}}{\partial\sigma^1}; \quad W^{34} = \mathcal{O}(\partial x^{\mu})^2. \tag{5.7}$$

The commutation rules can then be rewritten in the form

$$\sum_{\lambda}[W^{\lambda\mu}(\tilde{\sigma}), W^{\lambda\nu}(\tilde{\sigma}')] = \tfrac{1}{2}T\varepsilon^{\mu\nu\kappa\lambda}W^{\kappa\lambda}(\tilde{\sigma})\delta^2(\tilde{\sigma} - \tilde{\sigma}'), \tag{5.8}$$

which is written in such a way that it remains true in all coordinate frames. $T$ is a constant ("string constant") of order one in Planck units. Instead of (5.1–5) we can take this to be the equation that generalizes to arbitrary surfaces. It has the advantage of being linear in $W$.

Now (5.8) is not a closed algebra, because the left hand side still contains a summation. A complete algebra is obtained as follows.

Let $K$ be i times the self dual part of $W$:

$$K^{\mu\nu} = \mathrm{i}(W^{\mu\nu} + \tfrac{1}{2}\varepsilon^{\mu\nu\kappa\lambda}W^{\kappa\lambda}). \tag{5.9}$$

It has three independent components:

$$K_1 = \mathrm{i}(W^{23} + W^{14}); \quad K_2 = \mathrm{i}(W^{31} + W^{24});$$
$$K_3 = \mathrm{i}(W^{12} + W^{34}). \tag{5.10}$$

Now from (5.8) we derive that these obey a complete commutator algebra,

$$[K_a(\tilde{\sigma}), K_b(\sigma')] = \mathrm{i}T\varepsilon_{abc}K_c(\tilde{\sigma})\delta^2(\tilde{\sigma} - \tilde{\sigma}'). \tag{5.11}$$

Apart from a complication to be mentioned shortly, this is a local and complete algebra of the kind we were looking for. At first sight it seems to generate an infinite dimensional Hilbert space because the operators $K$, like the $W$, are distributions. But let us introduce test functions $f(\sigma)$, $g(\sigma)$ and define operators

$$L_a^{(f)} = T^{-1}\int K_a(\tilde{\sigma})f(\tilde{\sigma})\,\mathrm{d}^2\tilde{\sigma}, \tag{5.12}$$

then these satisfy commutation rules:

$$[L_a^{(f)}, L_b^{(g)}] = \mathrm{i}\varepsilon_{abc}L_c^{(fg)}. \tag{5.13}$$

Let us now restrict to test functions $f(\tilde{\sigma})$ that can only take the values 0 or 1. Then $L_a^{(f)}$ satisfy the commutation rules of ordinary angular momentum operators. Note that for such an $f$ the integral (5.12) is nothing but a boundary integral:

$$L_1^{(f)} = \mathrm{i}T^{-1}\oint_{\delta f}(x^2\,\mathrm{d}x^3 + x^1\,\mathrm{d}x^4), \quad \text{etc.}, \tag{5.14}$$

where $\delta f$ stands for the boundary of the support of $f$. We conclude that for every closed curve $\delta f$ on $\tilde{\sigma}$ space we have three "angular momentum" operators $L_a^{(f)}$ that satisfy the usual commutation rules and addition rules for angular momenta. Given such a bunch of closed curves $f_i$ we can characterize the contribution of that part of the horizon to Hilbert pace by the usual quantum numbers $l_i$ and $m_i$. These are discrete and so, in some sense, we seem to come close to our aim of realizing a discrete Hilbert space for black holes.

Unfortunately, there is a snag. The operators $L_a$ are not Hermitean. From the definition (5.6) we see that $W^{ij}$ are Hermitean and $W^{i4}$ anti-Hermitean. Therefore, $L_a^\dagger$ correspond to the anti-self dual parts of $W^{\mu\nu}$. The commutation rules between $L_a$ and $L_a^\dagger$ are non-local (they follow from (5.1–5)). The operators $L^2$ are Hermitean, but not necessarily positive (they are only nonnegative for time-like surface elements). If we may assume the smallest surface elements to be timelike we can still build our surface using quantum nembers $l_i$ and $m_i$ but the states we get are not properly normalized (it is for finding the norms of the states that we need Hermitean conjugation). If

$$\psi\{l_i, m_i\}$$

are the basis elements constructed using the self dual operators $L_i$, and

$$\phi\{l_i, m_i\}$$

the basis elements generated by the anti-self dual $L_i^\dagger$, then we have

$$\langle\phi\{l_i', m_i'\}|\psi\{l_i, m_i\}\rangle = \prod_i \delta_{l_i, l_i'}\delta_{m_i, m_i'}, \tag{5.15}$$

but the $\psi$ themselves, or the $\phi$ themselves, are not orthonormal. Therefore it is far from clear whether or not we actually obtained a complete representation of our Hilbert space.

## 6. Vector, spinor and scalar fields

It is clear that the Maxwell field brings about an interesting new degree of freedom in our "string": a periodic fifth dimension. But the Standard model in the GeV or TeV range has more light or massless fields. How should these affect our string dynamics? Naively, one would expect for instance that the non-Abelian fields give us more Kaluza Klein dimensions, with the topology of these non-Abelian group spaces.

In principle answering this question should be a straightforward exercise. The rules were given: see in which way the ingoing particles affect the outgoing ones by exchanging these new fields.

But in practice there is some disappointment. It would have been nice if all physical constants of the Standard model would somehow be reflected in corresponding constants for the string. This is not so. From now on we use the words "standard model" for any renormalizable field theory that one may assume to describe the interactions among ordinary particles in a certain energy range.

Take now for instance any light scalar field $\varphi$ in the standard model. What happens to our $S$ matrix when we include $\varphi$ exchange?

Imagine particles coupled to the scalar field by a Yukawa term in the Lagrangian. Such particles are surrounded by a scalar force field (protons and neutrons for instance are surrounded by a pion field). Now accelerate our particles to a speed very close to the speed of light. The, originally spherical, scalar field configuration gets Lorentz contracted taking the shape of a pancake, but its strength is not enhanced like the electromagnetic vector field, or the spin two gravitational field. In the limit of very high center-of-mass energy the effect of scalar particle exchange will be reduced to zero. Similarly, we expect the effects of spin $\frac{1}{2}$ exchange also not to survive.

This assumes the currents to be limited in strength; there are however circumstances where a scalar field does have an effect. This is when there is a Higgs mechanism. In that case the vector field becomes a short range field, decaying with a factor $\exp(-m|\tilde{x}|)$. This means that also the equation for the periodic dimension $\phi$ on our string must now satisfy a massive Klein–Gordon equation

$$(\tilde{\partial}^2 - m^2)\phi = 0. \tag{6.1}$$

To see that this is correct we consider the vacuum as being a super conductor. When an electrically charged particle enters it gives a current $j_\mu^{(1)}$, which in its own rest frame is

$$j_\mu^{(1)} = Qu_\mu\delta^3(x). \tag{6.2}$$

But because the super conductor screens this charge there is always a neutralizing current $j_\mu^{(2)}$,

$$j_\mu^{(2)} = -Qu_\mu\frac{\mathrm{e}^{-m|x|}}{4\pi|x|}, \tag{6.3}$$

so that

$$\int \mathrm{d}^3x(j_\mu^{(1)}(x) + j_\mu^{(2)}(x)) = 0. \tag{6.4}$$

The current $j_\mu^{(2)}$ also satisfies the Klein–Gordon equation, therefore a boost to a velocity limiting that of light also produces a current that satisfies the same equation. We see now where the mass term in (6.1) comes from. It is due tot he compensating current $j^{(2)}$. We conclude:

– If due to a Higgs mechanism a photon gets a mass $m$ then the correspondig "string coordinate" $\phi$ gets a mass term $-\frac{1}{2}\mathrm{i}m^2\phi^2$ in the Lagrangian (3.14), which explicitly breaks the global translation symmetry $\phi \Rightarrow \phi + C$!

So that is what happens. A spontaneous Higg breakdown* of a local symmetry corresponds to an explicit breakdown of a global symmetry in the string! It is remarkable that a scalar field variable in the standard model does not induce a new degree of freedom in the string but does give an extra term in its Lagrangian.

Suppose now that we have a non-Abelian gauge field in our standard model. Using the prescription given earlier turns out to be somewhat tricky. It is tempting to consider "colored particles" entering the black hole producing non-Abelian currents $j_\mu^a$; photon exchange diagrams give amplitudes proportional to $j_\mu^a(\tilde{x})C_{ab}j_\mu^b(\tilde{x}')$, where $C_{ab}$ is a Casimir operator. But if $\tilde{x} \neq \tilde{x}'$ this is not gauge invariant. Furthermore, what happens at the longer distance scale when there is a confinement mechanism? We propose the following approach.

Confinement can be understood precisely when we use what is called the Abelian projection [10]. Here one uses the Cartan subgroup of the gauge group. This is the largest subgroup of mutually commuting generators, and is Abelian by construction. There is a general argument that any non-Abelian gauge theory is topologically identical to an Abelian theory with this Cartan group as gauge group, with in addition magnetic monopoles. So there are electric and magnetic charges.

What the electric charges do in our string theory we already know. Their currents correspond to vertex insertions with momentum in the corresponding Kaluza–Klein dimension. We need to know how to incorporate magnetic currents.

Suppose therefore that particles with magnetic charge enter the black hole. What effect do they have on the electrically charged particles that leave the hole?

The answer to that is that an (extra) Dirac string will connect to the hole. The outgoing particle fields will all be gauge rotated by a factor exp $(i\Lambda(\tilde{x}))$, where $\Lambda(\tilde{x})$ has the Dirac string singularity. This we can account for at the level of the Langrangian (3.14) by postulating that at such a monopole source insertion the field $\phi(\tilde{x})$ makes a full twist. Remember that $\phi$ is periodic. We require that at the vertex of a single monopole the field $\phi$ turns into $\phi \pm 2\pi/e$ if we follow a closed curve around this vertex (the sign of course depends on the sign of the monopole). Indeed, this is precisely the action of the so-called "disorder operator" $\phi^{\text{Dual}}(\tilde{x})$.

It can now be understood what happens when we have confinement. Confinement corresponds to a Higgs mechanism with respect to the magnetic charges [11] (the dually opposite of the ordinary "electric" Higgs mechanism). Apparently we must now insert a mass term for the disorder parameter. It immediately implies that the order parameter $\phi$ will then be ill defined. At the long distance scale there is no observable gauge field in the standard model, hence no Kaluza–Klein scalar in the string!

This is why we prefer to retreat to the Cartan subgroup when dealing with a non-Abelian gauge field in the standard model.

One remark could be made. It is seen that the quantum numbers of all fields on the string world sheet are exactly those of the generators of local symmetry transformations in the standard model: $x_\mu$ are the generators of general coordinate transformations and $\phi$ corresponds to the generator $\Lambda$ of a local gauge transformation. Can we have spin $\frac{1}{2}$ fermions on the string? The answer may be yes, provided that we have a local symmetry generator with spin $\frac{1}{2}$. This, of course, is the generator of a local supersymmetry transformation. So, dynamical fermions on the string will occur only if there is some version of supergravity in our standard model interactions. The fermion mass on the string world sheet will be equal to the gravitino mass in the standard model! It is not obvious whether this implies the necessity of introducing local super symmetry. In principle it could be that although fermionic operators exist on the black hole horizon, (the oddness of) fermion number is simply not transmitted from one point to another on this horizon.

* We use these words because this is an often used description of the Higgs mechanism. However it is well known that on closer inspection a Higgs mechanism does not really break a local symmetry [9]. It is better to talk about a more subtle kind of phase transition.

## References


1. Hawking, S. W., Comm. Math. Phys. **43**, 199 (1975); Hartle, J. B. and Hawking, S. W., Phys. Rev. **D13**, 2188 (1976); Unruh, W. G., Phys. Rev. **D14**, 870 (1976); Hawking, S. W. and Gibbons, G., Phys. Rev. **D15**, 2738 (1977); Hawking, S. W., Comm. Math. Phys. **87**, 395 (1982).
2. 't Hooft, G., J. Geom. and Phys. **1**, 45 (1984); 't Hooft, G., "Black holes and the foundation of quantum mechanics", In: Niels Bohr: Physics and the World. Proc. of the Niels Bohr Centennial Symposium, Boston, MA, USA, Nov. 12–14, 1985 (Edited by H. Feshbach *et al.*), p. 171, Publ. by Harwood Academic Publ. GmbH (1988).
3. Hawking, S. W., Phys. Rev. **D14**, 2460 (1976); Comm. Math. Phys. **87**, 395 (1982); Hawking, S. W. and Laflamme, R., Phys. Lett. **B209**, 39 (1988).
4. 't Hooft, G., Nucl. Phys. **B335**, 138 (1990); 't Hooft, G., in "Proceedings of the 4th seminar on Quantum Gravity", May 25–29, 1987, Moscow, USSR, (Edited by M. A. Markov *et al.*), pp. 551–567, World Scientific (1988); "Quantum gravity and black holes", in: "Proceedings of a NATO Advanced Study Institute on Nonperturbative Quantum Field Theory", Cargèse, July (1987), (Edited by G. 't Hooft *et al.*), 201–226; Plenum Press, New York (1987); 't Hooft, G., lectures given at the Symposium of the China Center of Adv. Science and Technology on "Fields, Strings and Quantum Gravity", Beijing, May, June 1989, Gordon and Breach, to be publ.; "More on the Black Hole *S*-matrix", presented at the V Seminar on the Quantum Theory of Gravitation, Moscow, May 28–June 1 (1990).
5. 't Hooft, G., Physica Scripta **T15**, 143 (1987); "Gravitational collapse and quantum mechanics. In "Superstrings, Anomalies and Unification", 5th Adriatic meeting on particle physics, Dubrovnik, June 1986. (Edited by M. Martinis and I. Andric), p. 282. World Scientific, Singapore (1987).
6. Mukhanov, V. F., "The Entropy of Black Holes", in "Complexity, Entropy and the Physics of Information, SFI Studies in the Sciences of Complexity, vol IX (Edited by W. Zurek), Addison-Wesley, 1990; see also Schiffer, M., "Black Hole Spectroscopy", São Paolo preprint IFT/P – 38/89 (1989).
7. 't Hooft, G., Nucl. Phys. **B256**, 727 (1985).
8. Aichelburg, P. C. and Sexl, R. U., J. Gen. Grav. **2**, 303 (1971); Dray, T. and 't Hooft, G., Nucl. Phys. **B253**, 173 (1985).
9. 't Hooft, G., in: "Recent Developments in Gauge Theories," Cargèse, Aug., Sept. 1979 (Edited by G. 't Hooft *et al.*) pp. 117–134, Plenum Press, New York (1980).
10. 't Hooft, G., Nucl. Phys. **B190**, 455 (1981); Physica Scripta **25**, 1339 (1982).
11. 't Hooft, G., E. P. S. Int. Conf. on High Energy Physics, Palermo, June 1975; 't Hooft, G., Nucl. Phys. **B138**, 1 (1978); **B153**, 141 (1979); Acta Phys. Austriaca, Suppl. **23**, 531 (1980); Physica Scripta, **24**, 841 (1981).