

# The Black Hole Information Paradox

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## Abstract

A concise survey of the black hole information paradox and its current status is given. A summary is also given of recent arguments against remnants. The assumptions underlying remnants, namely unitarity and causality, would imply that Reissner Nordstrom black holes have infinite internal states. These can be argued to lead to an unacceptable infinite production rate of such black holes in background fields.

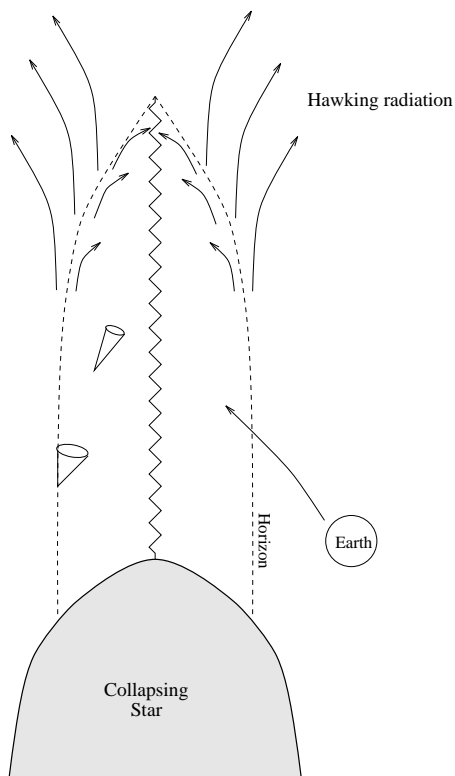
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Black holes exist—we have seen new experimental evidence for this claim in descriptions of Hubble telescope data at this conference. In my talk I’d like to explain to you how this simple statement gets us into a deep paradox in our understanding of nature. This paradox is pushing us to consider abandoning one of our cherished principles of energy conservation, locality and causality, or stability. It suggests that a revision of the fundamental underpinnings of physics may be necessary.

To see what the problem is, consider the formation of a large black hole from a collapsing star of mass  $M_0$ , as shown in fig. 1. Once it has formed, Hawking [1] has shown that, according to the basic principles of quantum field theory, it will radiate its mass away. This result can be understood by recalling that in quantum field theory the vacuum is populated by virtual pairs of particles and antiparticles. The gravitational field of the black hole can promote one member of a pair into a real outgoing particle, while the other partner falls into the black hole with a net negative energy as measured at infinity.



**Fig. 1:** A spacetime diagram of collapse of a star to form a black hole. Hawking radiation arises from escape of one of a pair of virtual particles, and leads to evaporation of the black hole. Also shown are examples of light cones.

Now let's ask ourselves a question: suppose that the Earth were to fall into this black hole, destroying our civilization. However, a sufficiently advanced civilization might do very careful measurements on the outgoing radiation from the black hole. Could they be reconstructed from these the history of our planet up to its annihilation—would we at least leave some legacy?

To illustrate further, suppose the Earth were blown to bits by a powerful bomb. In that case, we know that, at least in principle, an advanced civilization could decipher our history right up to the explosion—this would of course require a very careful measurement of the outgoing quantum state of the fragments, and then a backwards evolution of it to the initial state before the explosion.

But according to Hawking, black holes are different. Here it would be impossible even in principle to reconstruct the initial state. To see this, consider the idealization where the initial state of the black hole (including the Earth) was a quantum-mechanical pure state: in density-matrix language,

$$\rho = |\psi\rangle\langle\psi|. \quad (1)$$

This has zero entropy  $S$ , where

$$S = -\text{Tr} \rho \ell n \rho. \quad (2)$$

Hawking showed that the outgoing radiation is approximately thermal, and carries a large entropy

$$S \sim M_0^2 / M_{pl}^2, \quad (3)$$

where  $M_{pl}$  is the Planck mass. Correspondingly, once the black hole has evaporated away completely the final state is a mixed state of the form

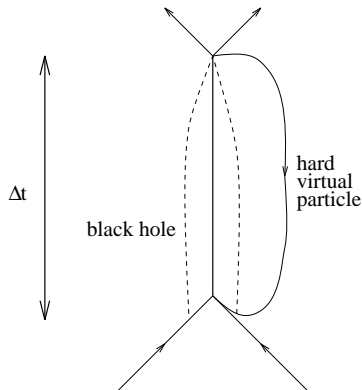
$$\rho = \sum_n p_n |n\rangle\langle n|. \quad (4)$$

(One can in fact derive such an expression directly[1].) Such evolution of a pure state to a mixed state implies a fundamental loss of information

$$\Delta I = -\Delta S \sim -M_0^2 / M_{pl}^2. \quad (5)$$

This loss of information ensures that not only we but also our history would be obliterated.

As Hawking also pointed out, this behavior violates quantum mechanics, which keeps pure states pure. Our state of uneasiness about this is significantly heightened when we realize that such information loss also apparently implies energy non-conservation [2-4].



**Fig. 2:** Shown is the contribution of a virtual black hole to two body scattering; information and energy loss are expected to contaminate arbitrary processes through such diagrams.

The basic point here is that information transmittal requires energy, so its loss should imply loss of energy. Indeed, suppose that the black hole forms and evaporates in a time  $\Delta t$ ; then one expects an uncertainty-principle like relation

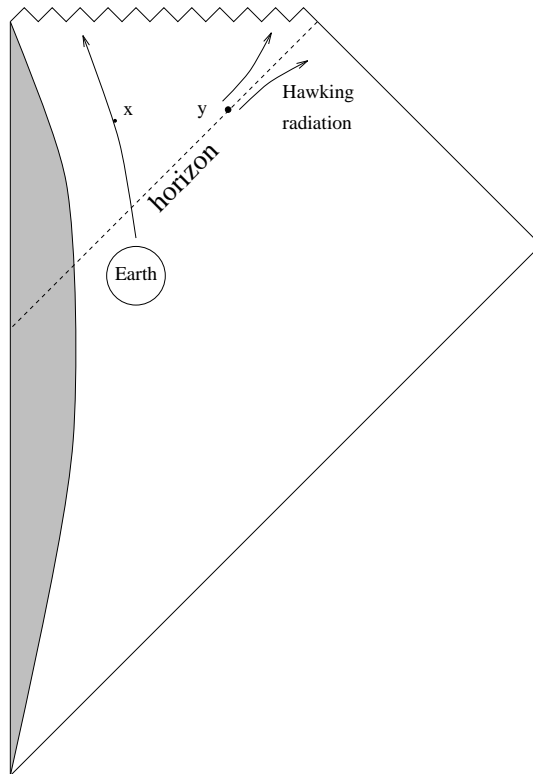
$$\Delta E \sim 1/\Delta t \tag{6}$$

governing the minimum amount of energy lost.<sup>1</sup> Basic quantum principles imply that such formation/evaporation should be taking place all the time in virtual processes, as illustrated in fig. 2. The amplitude for these processes should approach unity as the size of the loop approaches the Planck scale—there is no small dimensionless number to suppress it. According to (6), we would therefore expect Planck size energy violations with planckian characteristic time scale. This would give the world the appearance of a thermal bath at the Planck temperature, in clear contradiction with experiment. And that suggests we explore alternatives to Hawking’s picture.

Can the information come out in the Hawking radiation? To investigate this it is useful to redraw fig. 1 as a Penrose diagram, fig. 3. In such a diagram light rays are at  $45^\circ$  angles, which makes causality easy to study. From this diagram the difficulty is clear. The Hawking radiation is produced near the event horizon at point  $y$ . For an event at  $x$  in the history of the Earth to influence this, a signal would have to be transmitted beyond

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<sup>1</sup> There are examples of information loss without energy loss, as illustrated by spacetime wormholes [5,6]. However, such loss is not *repeatable* [4]—it will not persist as needed to describe a sequence of black hole formations and evaporations.



**Fig. 3:** By a conformal transformation, fig. 1 can be redrawn in the form of a Penrose diagram, in which light travels at  $45^\circ$  angles.

the speed of light, that is acausally or nonlocally. In field theory parlance, observables at  $x$  and  $y$  would have to fail to commute,

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)] \neq 0, \quad (7)$$

for spacelike separated  $x$  and  $y$ . Once we've allowed such violations of locality/causality, we have to explain why they don't imply paradoxes such as the possibility of killing one's grandmother or winning money at the racetrack.

This prompts consideration of other scenarios. Another apparent possibility is that all of the information is released in the final burst, as the black hole evaporates past the Planck mass. By this time locality/causality inside the black hole has broken down. However, here we confront another problem. The energy available is of order the Planck mass,  $E \sim M_{pl}$ . The information that is to be radiated is large, as we saw in eq. (5). Again, in accordance with uncertainty principle arguments, the only way to radiate a large amount of information with a small amount of available energy is to do it very slowly, for example by emitting extremely soft photons. An estimate [7,8] of the time required is

$$t \sim \left( \frac{M_0}{M_{pl}} \right)^4 t_{pl}, \quad (8)$$

which exceeds the age of the universe for black holes with initial masses comparable to that of an average building. Therefore this scenario implies long-lived remnants.

Long-lived black hole remnants thus remain as our final possibility. These should have masses of order  $M_{pl}$ . There should also be an infinite number of species of such objects, since they must be able to encode in their internal states the information from an arbitrarily large black hole. This leads to the final objection: in any process with total available energy greater than  $M_{pl}$  (e.g. in nuclear reactors), there is a tiny but non-zero amplitude to pair produce a given species of remnant. When multiplied by the infinite number of species, this gives an infinite rate: the Universe is unstable to instantaneous decay into remnants, again in clear contradiction with experiment.

We have now painted ourselves into the corner that is the black hole information paradox. Information loss implies energy non-conservation, information in Hawking radiation requires non-locality and/or acausality, and remnants lead to catastrophic instabilities. Beginning with the observation that black holes exist and applying basic physical principles has gotten us into serious trouble.

For a time it seemed that the most likely out was remnants—one might imagine that planckian physics would allow us to find a loophole by which they weren't infinitely produced, and proposals along these lines have been made by [9,10]. This now appears to me quite unlikely. Before concluding this talk with my views, I'll outline more recent reasoning that makes remnants look very unlikely to me. More details are in [11], and related objections are given in [12].

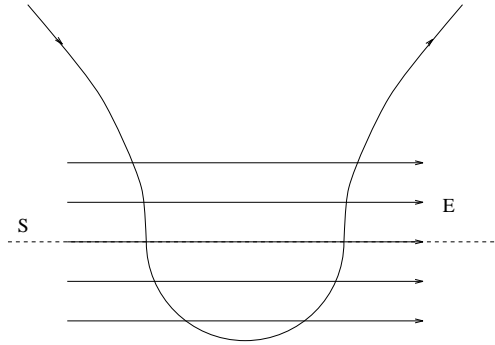
The objection arises from considering black hole pair production, a phenomenon of great interest in its own right. To see its relevance, consider two properties of Reissner-Nordstrom black holes, with charge  $Q$ , and with masses  $M \geq Q$ .

The first property is that quantum mechanics and locality/causality apparently imply an infinite number of internal states of such a black hole. (See *e.g.* [13].) To see this, note that although these black holes evaporate, one can argue that the radiation should shut off as the extremal mass,  $M = Q$ , is approached. (Here the temperature vanishes.) Now consider taking such an extremal black hole and throwing the Earth into it. This will raise it above extremality, but it subsequently evaporates back to  $M = Q$ . A causality argument like that given for neutral black holes implies that the information carried by the Earth can't escape. Validity of quantum mechanics implies it isn't destroyed. Therefore the resultant extremal black hole has augmented information. Repeating this experiment

we can load the black hole with arbitrarily large information, so it must have an infinite number of internal states.

The second property is that the Universe is not observed to be unstable to infinite production of these black holes.

If we can understand how these statements are reconciled for Reissner-Nordstrom black holes, then they could give us a prototype for a sensible theory of remnants. On the other hand, failure to reconcile these would suggest that either quantum mechanics or locality/causality fails. Pair production of Reissner-Nordstrom black holes is therefore a litmus test for remnants.



**Fig. 4:** Above the slice  $S$  (dotted) are shown the lorentzian trajectories of a pair of oppositely charged particles in an electric field. Below  $S$  is the euclidean continuation of this solution. Matching the euclidean and lorentzian solutions smoothly along  $S$  gives a picture of Schwinger production followed by subsequent evolution of the pair of created particles.

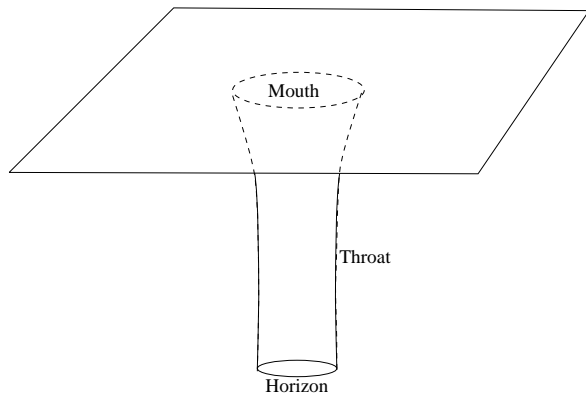
It is easy to investigate such pair production in a uniform background electromagnetic field. First, recall that Schwinger [14] calculated the rate of pair production of charged particles of mass  $m$  and charge  $q$  in a background electric field and found

$$\Gamma \propto e^{-\pi m^2/qE}. \quad (9)$$

This can be easily derived by instanton methods. The particles are produced at rest, and subsequently run off to opposite ends of the electric field following hyperbolic spacetime trajectories, as shown in fig. 4. (By energy conservation, they must be produced with a separation  $\ell = \frac{2m}{qE}$ .) The continuation to euclidean time,  $t \rightarrow i\tau$ , of such trajectories gives a circle, shown in the lower half of fig. 4. The action of this euclidean trajectory is

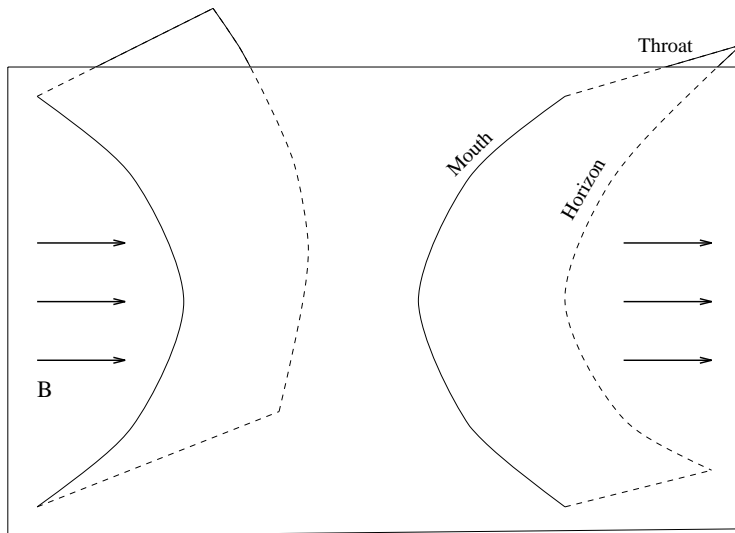
$$S_{euc} = -\frac{\pi m^2}{qE}, \quad (10)$$

which gives the exponent in (9).



**Fig. 5:** The spatial geometry of a charged black hole. As the black hole nears the extremal mass,  $M = Q$ , the length of the throat region diverges.

The analogous solution for charged black holes in a background field is known and is called the Ernst solution [15]. To avoid problems of black hole discharge we'll take the charge and field to be magnetic. The spatial geometry of such a black hole is sketched in fig. 5. There is a long “throat” that connects the outside world to the horizon. Fig. 6 shows a representation of the Ernst solution, with hyperbolic motion of the oppositely-charged pair of black holes.



**Fig. 6:** The Ernst solution, which corresponds to a pair of oppositely charged black holes accelerating in a background field. The “fins” represent the trajectories of the throats of fig. 5, and their cross-sections are therefore two-spheres.

As pointed out by Gibbons [16] and by Garfinkle and Strominger [17], the euclidean continuation of this solution describes pair production of black holes. Half of the euclidean



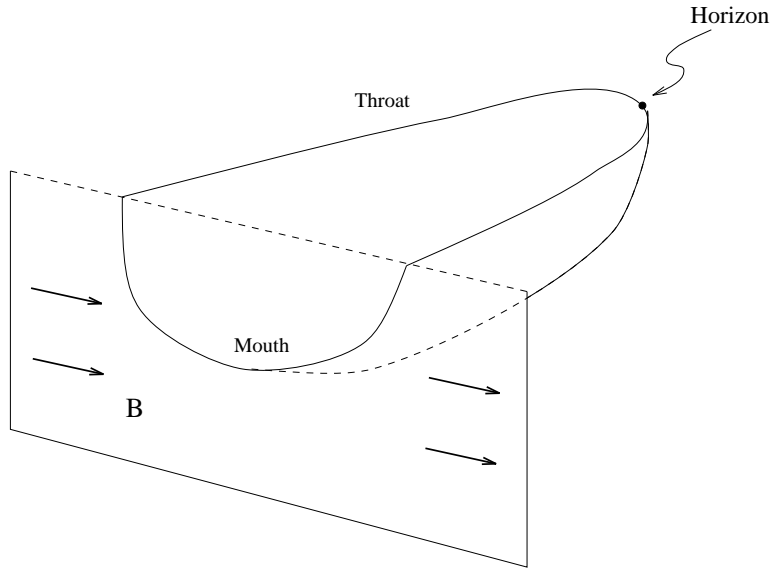
solution is pictured in fig. 7. Note that the throats appear to connect at the horizon. The production rate from this configuration is again

$$\Gamma \propto e^{-S_{euc}}, \quad (11)$$

and a calculation [17,18] shows that

$$S_{euc} \simeq -\pi Q/B \quad (12)$$

in agreement with (9).



**Fig. 7:** The euclidean version of the Ernst solution is the analog of the lower half of fig. 4. If one wishes to have a smooth geometry, the throat regions get identified at the horizon. This can however be substantially modified by quantum corrections.

One detail so far omitted is that in both the particle and black hole case, one should include quantum-mechanical configurations near to the classical configurations pictured. In the particle case, functional integration over these configurations gives the known fluctuation determinant, which contributes subleading quantum corrections to the rate. In the black hole case, one is instead integrating over nearby geometries as well as field configurations, and the result is hard to compute.

This matter brings us back to the infinite number of states. As we all know, an astronaut thrown into a black hole appears to get frozen at the horizon. Therefore, an

external observer might describe the infinite number of black hole states as frozen states at the horizon. When we integrate over the neighboring configurations, our integral should include these configurations in particular.

The states appear to have arbitrarily short wavelengths, and ordinarily one might expect their contribution to the integral to be suppressed through (11) because these wild fluctuations (at least in the frame of the external observer) would have large euclidean actions. But not so in gravity, where the negative gravitational energy leads to cancellations. In fact these configurations have actions of order (12). Integrating over this infinite number of contributions therefore gives a rate

$$\Gamma = \infty e^{-\pi Q/B} = \infty. \quad (13)$$

This reasoning can be checked by examining the limit  $QB \ll 1$ . In this limit, the length of the throat region in fig. 7 diverges,  $\ell \sim -Q \ln QB$ . If one examines the geometry far down this throat, it appears insensitive to the boundary conditions in the asymptotic region, aside from knowing about the acceleration due to the field. Indeed, in this region, the solution is closely approximated by a free euclidean black hole. The periodicity corresponds to the fact that it is thermally excited to a temperature,

$$T \sim B \quad (14)$$

which can be thought of as arising from the acceleration radiation.

The functional integral over the euclidean geometry of a free black hole has another interpretation, as argued by Gibbons and Hawking [19]: it gives the partition function for the black hole,

$$Z = Tr(e^{-\beta H}), \quad (15)$$

at the appropriate temperature. Thus we have what amounts to a “low-energy theorem:” the rate is proportional to the partition function for the black hole, although we don’t know how to explicitly calculate either. Despite this, we know that if there are an infinite number of black hole states with approximately equal energies, (15) must be infinite. This confirms the infinite production rate.

These general arguments, although not completely rigorous, indicate that the assumption of an infinite number of black hole states indeed implies infinite pair production. If true this must mean one of our original assumptions was wrong: either quantum mechanics or locality/causality fails. This removes the *raison d’être* for remnants.

I'll conclude by summarizing my views, although be forewarned that controversy exists. First, it seems to me fairly clear that there is a connection between repeatable information loss and energy non-conservation (although we are investigating this in more detail [20]). Furthermore, basic quantum principles would seemingly force this to occur on a massive scale if it occurs in black hole evaporation. Thus I see little hope for a resolution of the paradox here. Second, the naive expectation of infinite pair production of remnants seems to be borne out by study of production of charged black holes. So I don't see much hope here either.

Finally, people have for some time known that comparing observations inside a black hole to those outside requires comparing reference frames with ultrahigh relative boosts. This may open a loophole through which the reasoning behind the paradox fails in a way contrary to our low energy intuition. There have been suggestions [21-23] that it is locality that fails, and Susskind, Thorlacius, and Uglum have in particular advocated finding the requisite non-locality from string theory. Although considerable effort has been expended to confirm this, and there are suggestive hints, it has not yet been possible to really test this assertion [24,25]. The paradox remains, although it may have just begun to crumble.

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