The Bond Yield "Conundrum" from a Macro-Finance Perspective

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In 2004 and 2005, long-term interest rates remained remarkably low despite improving economic conditions and rising short-term interest rates, a situation that then-Federal Reserve Board Chairman Alan Greenspan dubbed a "conundrum." We document the extent and timing of this conundrum using two empirical no-arbitrage macro-finance models of the term structure of interest rates. These models confirm that the recent behavior of long-term yields has been unusual—that is, it cannot be explained within the framework of the models. Therefore, we consider other macroeconomic factors omitted from the models and find that some of these variables, particularly declines in long-term bond volatility, may explain a portion of the conundrum. Foreign official purchases of U.S. Treasuries appear to have played little or no role.

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[L]ong-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. This development contrasts with most experience, which suggests that, other things being equal, increasing short-term interest rates are normally accompanied by a rise in longer-term yields . . . For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.[1]

I. Introduction

As a broad empirical regularity, long-term interest rates tend to move month by month in the same direction as short-term rates, although by a lesser amount. In the United States, the simple correlation since 1984 between changes in short-term and long-term rates implies that a 1 percentage point increase in the monthly federal funds rate has been accompanied, on average, by about a 30-basis-point increase in the 10-year rate. However, in 2004 and 2005, long-term interest rates followed a very different pattern. For example, while the federal funds rate steadily rose from 1 percent in June 2004 to 4.2 percent in December 2005, the rate on 10-year U.S. Treasury notes *fell* from 4.7 percent to 4.5 percent over that same period.[2] This directional divergence between U.S. short and long rates appears even more unusual given other pressures at the time, such as a robust economic expansion, rising energy prices, and a deteriorating federal fiscal situation, all of which have tended to boost long-term interest rates in the past. In this paper, we investigate the seemingly odd behavior of long-term interest rates over this recent episode, a development which then-Fed Chairman Alan Greenspan labeled a "conundrum."[3]

Determining whether recent long-term interest rate movements truly represent a conundrum—as opposed to simply an extension of secular declines in long-term interest rates that have been taking place over the past 20 years—requires a rigorous theoretical framework which can take into account the important factors that affect long-term rates. Our choice of a theoretical framework for this examination is guided by much recent research that suggests a joint *macro-finance* modeling strategy provides the most comprehensive explanation of movements in long-term rates.[4] From a macroeconomic perspective, the short-term interest rate is a policy instrument under the direct control of the central bank, which adjusts that rate to achieve its economic stabilization goals. Therefore, financial market participants' understanding of central bank behavior will be an important element in the formation of their expectations of future short-term interest rates, which, in turn, will be a key component for the

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[1] Testimony of Federal Reserve Board Chairman Alan Greenspan to the U.S. Senate, February 16, 2005.

[2] In addition, given the upward shifts in money market futures rates during 2005, the extent of the policy tightening appears to have been unanticipated.

[3] Although Greenspan referred to "world" bond markets, our focus is exclusively on U.S. markets. For a discussion of recent bond rates in Japan from a macro-finance perspective, see Oda and Ueda (2005).

[4] The connection between the macroeconomic and finance views of the term structure has been a very fertile area for recent research, which includes, among many others, Ang and Piazzesi (2003), Wu (2002), Dewachter and Lyrio (2006), Evans and Marshall (2001), Diebold, Rudebusch, and Aruoba (2006), Hördahl, Tristani, and Vestin (2006), Rudebusch and Wu (2004, 2006), Kozicki and Tinsley (2001, 2005), and Diebold, Piazzesi, and Rudebusch (2005).

pricing of longer-term bonds. To illustrate this point, it is useful to contrast the recent behavior of U.S. interest rates with their behavior during the previous episode of extended monetary tightening, which occurred a decade earlier. Specifically, from January 1994 to February 1995, as the Fed raised the short-term federal funds rate by 3 percentage points, the 10-year rate also rose by 1.7 percentage points. This reaction was somewhat greater than the average response, and a common interpretation of this episode is that bond investors were especially worried that inflation pressures might not be counteracted by the Fed in a timely fashion. In contrast, there is a widespread view that long-term inflation expectations remained well contained in 2004 and 2005, which helped hold down long bond rates. Understanding the differential behavior of long-term interest rates during these two episodes likely requires an appreciation of the macroeconomic underpinnings of bond rate dynamics.

A finance perspective, which stresses the importance of changing investor perceptions of risk for bond pricing, is also likely to be a crucial element in assessing whether there is any bond rate conundrum. Indeed, many have suggested that a reduction in the risk premium is responsible for recent low levels of bond rates. Such a reduction may be attributable to changes in the amount of risk or to changes in the pricing of that risk, and numerous factors have been suggested that could have induced such changes. For example, a new appreciation of lower macroeconomic volatility or reduced monetary policy uncertainty could alter assessments of the amount of interest rate risk faced by investors. Alternatively, risk aversion may have been reduced as new global investors entered the market. Indeed, this is one interpretation of the explanation suggested by Bernanke (2005), who argued that "over the past decade a combination of diverse forces has created a significant increase in the global supply of saving—a global saving glut—which helps to explain both the increase in the U.S. current account deficit and the relatively low level of long-term real interest rates in the world today." In any case, it is likely important to allow for time variation in risk premiums in understanding the recent behavior of the bond rate.

To conduct our analysis of the conundrum, we use two macro-finance models of the term structure from the literature. The first is a vector autoregression (VAR)-based model developed by Bernanke, Reinhart, and Sack (2004), denoted BRS (2004), and the second is a New-Keynesian-based model developed by Rudebusch and Wu (2004), denoted RW (2004). The BRS and RW models share several basic features. First, they are both factor models, so only a small number of sources of variation underlie the pricing of the entire term structure of interest rates. Second, both models impose the standard no-arbitrage restriction from finance (e.g., Duffie and Kan [1996]), which ensures that, after accounting for risk, the dynamic evolution of yields over time and across states of nature is consistent with the cross-sectional shape of the yield curve at any one point in time. Finally, both models have important bidirectional linkages between interest rates and macroeconomic variables.

Despite their broad similarities, the BRS and RW models also have technical specifications that differ in important ways. Because these differences may lead to different results, we use both models in our analysis to provide us with additional perspective and robustness checks on our results. Specifically, we extend both models forward from their original samples to the 2004–05 "conundrum" period to determine whether the recent low level of long-term bond yields was unusual from the viewpoint of the models. Our analysis indicates that the level of long-term bond yields in 2004 and 2005 is, in fact, substantially lower than can be explained by either of these models—in other words, that this period does constitute a conundrum from a macro-finance perspective.

Having documented the existence of a conundrum within a rigorous econometric framework, we then turn to possible explanations—factors that necessarily lie outside of the two macro-finance models we have considered. We examine several popular explanations for the conundrum that have been promoted by financial analysts and find that one of them—lower volatility of long-term Treasury yields—seems to have substantial explanatory power. Interestingly, we find no support for the view that foreign official purchases of U.S. Treasuries have contributed to the low level of long-term yields, even though this factor is regarded by many financial market participants to have been the single most important factor holding down long-term U.S. Treasury yields.

Although there are numerous papers specifying finance and macro-finance models of the term structure, our paper fills a gap in the literature by applying these models to analyze the bond yield conundrum. The most closely related paper to the present one is Kim and Wright (2005), who use a pure finance three-factor model of the term structure to analyze the recent low level of long-term rates and find that a declining term premium is the key factor underlying those low rates. By contrast, we use two macro-finance models of the term structure and find that, within the framework of these models, there is in fact a conundrum—that the model-implied term premiums from our two models are *unable* to explain the low level of long-term yields observed in 2004–05, despite the fact that the models provide an otherwise excellent fit to the data over the previous 20 years.

The remainder of the paper proceeds as follows. In Section II, we describe the BRS and RW macro-finance models of the term structure. In Section III, we discuss the estimation of these models and use them to show that there was a bond yield conundrum in the 2004–05 period. In Section IV, we investigate what other factors outside of the two models could potentially help to explain the bond yield conundrum episode. Section V concludes.

II. Macro-Finance Models of the Term Structure

To investigate whether and to what extent the low level of U.S. long-term interest rates can be explained by macroeconomic conditions, we will use two macro-finance models of the term structure, broadly following Ang and Piazzesi (2003), Piazzesi (2005), and many others. For analyzing the behavior of the yield curve, these macro-finance models offer a number of advantages over both pure finance models and pure macroeconomic models.

First, in contrast to standard finance models of the term structure (e.g., Dai and Singleton [2000]), which relate the yield curve to current and past interest rates, macro-finance models recognize that interest rates and macroeconomic variables evolve jointly over time, with feedback running from interest rates to macroeconomic variables and also from macroeconomic variables back to interest rates. The latter channel in particular is crucial for the behavior of short-term interest rates, which are determined in many countries by the central bank as a function of the state of macroeconomic variables such as the output gap and inflation.[5]

A second advantage of macro-finance models is that they allow the behavior of risk premiums to depend explicitly on macroeconomic conditions. Standard consumption-based models of asset returns imply that risk premiums are determined by the covariance of an asset's return with the marginal utility of consumption (e.g., Cochrane [2001]). Moreover, empirical studies of excess returns in bond markets and shorter-duration interest rate derivatives (Cochrane and Piazzesi [2005] and Piazzesi and Swanson [2004]) find a strong countercyclical relationship between economic activity and excess returns on these securities, particularly during recessions. Macro-finance models of the term structure explicitly allow for a relationship between risk premiums and macroeconomic conditions.

A third advantage of macro-finance models is that, in contrast to standard macroeconomic models, a substantial component of observed bond yields is allowed to reflect term or risk premiums that may vary substantially over time. Indeed, as in a finance model, macro-finance models allow term premiums to evolve according to the estimated dynamics of the model in a way that is consistent with an absence of arbitrage opportunities in financial markets. Thus, while macro-finance models of the term structure retain the appealing macroeconometric features of a dynamic stochastic general equilibrium (DSGE) model or a VAR, they do not impose the expectations hypothesis and can allow for a rich behavior of term premiums.

To explore the recent behavior of long-term bond rates, we use two macro-finance models from the literature, the VAR-based model developed in BRS (2004) and the more structural New Keynesian-based model developed in RW (2004). We discuss each in turn.

A. The BRS Model

The BRS model uses five observable macroeconomic variables, specified below, as factors with which to fit the yield curve. The dynamics of the five factors are governed by a VAR with four lags, which can be stacked into the 20-element vector X_t and described by a VAR(1) model:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \epsilon_t, \tag{1}$$

where the stochastic shocks ϵ_t are *i.i.d.* over time and have a standard normal distribution. Consistent with the companion form specification in (1), only the uppermost blocks of Φ and Σ are nontrivial. For simplicity, the prices of risk in the

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[5] While a reduced-form relationship between past interest rates and the future behavior of short-term interest rates exists, the Lucas critique suggests that it may be unstable in the face of changing monetary policy. A more structural macro-finance approach that models monetary policy is arguably less subject to this criticism. Kim and Wright (2005), who examine the recent conundrum with a standard finance model, may alleviate this problem by using a fairly short estimation sample (since 1990).

model, specified below, are assumed to load only on the five current values of the macroeconomic variables (i.e., the top five elements of X_t).

Following Duffie and Kan (1996), Dai and Singleton (2000), and Ang and Piazzesi (2003), among others, we assume that the stochastic discount factor with which bonds in the model are priced is conditionally log-normal with functional form:[6]

$$m_{t+1} = \exp(-i_t - \lambda_t'\lambda_t/2 - \lambda_t'\epsilon_{t+1}), \tag{2}$$

where λ_t is a 20-dimensional vector of market prices of risk associated with the innovations to the VAR, ϵ_t. Of course, because only the first five elements of ϵ_t are nonzero, we assume that only the first five elements of λ_t are nonzero, which is essentially a normalization. Following the papers cited above, the market prices of risk λ_t are assumed to be affine functions of the state variables of the economy:

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{3}$$

where λ_0 is a 20-dimensional vector of constants and λ_1 is a 20-by-20 matrix of risk price factor loadings on the state variables X_t. As mentioned above, we assume for simplicity that the prices of risk depend only on the current values of the variables in X_t, thus only the upper-left five-by-five block of λ_1 is nonzero.

Bonds in the model are priced according to the standard relationship with the stochastic pricing kernel:

$$P_t^n = E_t m_{t+1} P_{t+1}^{n-1}, \tag{4}$$

where P_t^n denotes the price of an n-period zero-coupon bond at date t. Let y_t^n denote the corresponding continuously compounded yield on the same zero-coupon bond. Then it follows from equations (1)–(4) that

$$y_t^n = \frac{1}{n}(a_n + b_n' X_t), \tag{5}$$

where $a_0 = 0$, $b_0 = 0$, and a_n (a scalar) and b_n (a five-by-one vector) are computed recursively according to

$$a_{n+1} = \delta_0 + a_n + b_n'(\mu - \Sigma\lambda_0) - b_n'\Sigma\Sigma' b_n,$$

$$b_{n+1} = \delta_1 + (\Phi - \Sigma\lambda_1)' b_n, \tag{6}$$

and where δ_0 (a scalar) and δ_1 (a five-by-one vector) describe the one-period interest rate as a function of the state of the economy: $y_t^1 \equiv i_t = \delta_0 + \delta_1' X_t$.

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[6] It is a well-known and important result from finance that the assumption of no arbitrage (or even more weakly, the law of one price) implies that a stochastic discount factor must exist, even in an incomplete market—see, for example, Cochrane (2001) for discussion. The papers cited here place the additional functional form restriction on the pricing kernel for tractability.

B. The RW Model

The RW model shares many features with the BRS model, since both models price bonds using an affine no-arbitrage framework. Thus, for expositional simplicity and to conserve notation, when concepts in the two models are the same, we will take the corresponding variable name from the BRS model exposition above and recycle it in the description of the RW model below (even though the number of factors, and hence the number of dimensions, in the two models are different). Note that, despite the similarity in the bond-pricing framework, the dynamics of the underlying macroeconomic factors in the RW model differ in important ways from BRS, with RW using a more structural New Keynesian macroeconomic framework to model these dynamic relationships as opposed to a VAR.

Bond pricing in the RW model is governed by the same affine no-arbitrage framework in (2)–(6) described above. However, in contrast to the BRS model, the RW model defines the one-period interest rate to be a sum of two latent factors, L_t and S_t:

$$i_t = \delta_0 + L_t + S_t, \tag{7}$$

where L_t can be thought of as the "level" of the yield curve and S_t as (the negative of) the yield curve "slope," as discussed in RW (2004).[7] Intuitively, the Fed sets the one-period nominal interest rate in the model as the sum of a constant steady-state real interest rate (δ_0), a time-varying medium- to long-term inflation rate (L_t), and a cyclically responsive component S_t that is given by

$$S_t = \rho_S S_{t-1} + (1-\rho_S)[g_y y_t + g_\pi(\pi_t - L_t)] + u_{S,t}, \tag{8}$$

$$u_{S,t} = \rho_u u_{S,t-1} + \epsilon_{S,t}, \tag{9}$$

where y_t is the output gap and π_t the inflation rate. Equation (8) is essentially a Taylor (1993) rule for the short-term rate i_t with both inertia (through the lag S_{t-1}) and serially correlated errors (as described by Rudebusch [2002b]).

The dynamics of L_t are given by

$$L_t = \rho_L L_{t-1} + (1-\rho_L)\pi_t + \epsilon_{L,t}, \tag{10}$$

which says that the medium-term inflation goal is persistent but may depend also on the recent behavior of short-term inflation, as suggested by the evidence in Gürkaynak, Sack, and Swanson (2005).

The latent factors L_t and S_t are jointly determined with the macroeconomic variables y_t and π_t by a New Keynesian-type model (adjusted to apply to monthly data):

$$\pi_t = \mu_\pi L_t + (1-\mu_\pi)[\alpha_{\pi_1}\pi_{t-1} + \alpha_{\pi_2}\pi_{t-2}] + \alpha_y y_{t-1} + \epsilon_{\pi,t}, \tag{11}$$

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[7] RW (2004) present charts comparing their estimated values for L_t and S_t to traditional principal components decompositions of the yield curve into "level" and "slope" factors, and show that the two sets of series line up very closely, justifying the use of the terminology.

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y)[\beta_{y1} y_{t-1} + \beta_{y2} y_{t-2}] - \beta_r (i_{t-1} - L_{t-1}) + \epsilon_{y,t}. \qquad (12)$$

That is, inflation responds to the private sector's expectation of the medium-term inflation goal (L_t), two lags of inflation, and the output gap; the output gap, in turn, responds to the expected future output gap, two lags of the output gap, and a real interest rate. Equations (11) and (12) are "hybrid" New Keynesian equations in the sense that the expectational coefficients μ_π and μ_y are allowed to be less than unity when the model is fitted to the data.

III. Is There a Bond Yield "Conundrum"?

To investigate whether the recent low level of U.S. long-term bond yields is a conundrum from a macro-finance perspective, we use the BRS and RW models to fit the recent macroeconomic and interest rate data and examine the two model predictions for long-term bond yields.

A. Model Estimates

1. BRS model estimates

Our original inclination was to use the "off-the-shelf" BRS model parameter estimates for our analysis. However, after examining the data and model estimation code (kindly supplied by Brian Sack) and fine-tuning the nonlinear optimization procedure, we were able to obtain a better fit to the data than reported in BRS (2004), with root-mean-squared errors (RMSEs) that were about half as large even over the identical sample. Thus, we do not use the parameter values as estimated by BRS, since we have found that the model can fit the data better than originally reported. Moreover, because we are reestimating the parameters of the model, we take the opportunity to extend the BRS estimation period through the end of 2005, which has the added advantage of giving the model the best possible chance of fitting the recent low "conundrum" level of bond yields by fitting them in rather than out of the sample. In every other respect, however, we have followed BRS (2004) in their analysis.

We estimate the BRS model on monthly macroeconomic and bond yield data over the period from January 1984 to December 2005.[8] Ideally, one would like to perform the estimation over the whole postwar period, but the preponderance of empirical evidence suggests that the relationships between interest rates and macroeconomic variables have not been stable over the past 40 years. In particular, many authors (e.g., Fuhrer [1996], Bernanke and Mihov [1998], and Clarida, Galí, and Gertler [2000]) have found that there was an important structural break in the conduct of monetary policy around 1980, when Paul Volcker became Chairman of the Fed.[9] As a result, macro-finance models estimated over the entire postwar period would likely be subject to the Lucas critique, so we avoid this issue by beginning our empirical estimation in January 1984, after the Volcker disinflation, after the structural

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[8] Because the VAR has four lags, this requires using macroeconomic data back to September 1983.

[9] Evidence of a structural break in the conduct of monetary policy is also indirectly supported by the greater stability of inflation expectations that seem to have held in the United States in the 1990s and 2000s relative to the 1970s.

breaks in monetary policy found by the authors above, and after the structural break in U.S. GDP volatility found by McConnell and Perez-Quiros (2000) and others.

Following BRS (2004), we use five macroeconomic variables as state variables of the model, which are the affine factors for pricing bonds: the federal funds rate, the deviation of employment from trend (where trend is measured using an HP filter), the year-on-year percentage change in the core personal consumption expenditures (PCE) deflator, the Blue Chip survey of inflation expectations for the upcoming year, and the rate on the Eurodollar futures contract with four quarters to expiration. The last variable in particular is included to capture aspects of the stance of monetary policy that may not be adequately represented by the current and past federal funds rate, employment gap, and inflation alone, such as financial market expectations for the future path of policy over the next 12 months. For example, the "deflation scare" episode in the United States in the summer of 2003 led the Fed to include greater forward-looking language in its monetary policy announcements, such as the now-famous "considerable period" phrase,[10] which helped to shape financial market expectations for the future path of policy in a way that was perhaps different from what the past behavior of output, inflation, and interest rates would have suggested. The inclusion of the four-quarter-ahead Eurodollar futures rate thus helps to capture changes in the expected future path of policy that may not be adequately accounted for by the lags of the VAR. Similarly, the inclusion of the Blue Chip inflation expectations survey helps to capture changes in the expected future course of inflation that may not be adequately captured by the VAR. Of course, the inclusion of these forward-looking expectational variables in the VAR creates a tension or internal inconsistency between the forecast data series that are included as variables and the forecasts that would be implied by the model itself based on the VAR. Although this technical issue can be addressed by estimating the model in such a way as to ensure consistency between the model and survey forecasts to the greatest extent possible (see Swanson [2006]), doing so would require moving away from standard VAR estimation procedures, which is beyond the scope of the present paper. Thus, we follow BRS (2004) and simply estimate a VAR on all five of the macroeconomic time series above, and leave the issue of consistency between model-based forecasts and forecast data to subsequent work.

Although the BRS model (and also the RW model) is linear in the state variables, the model's predictions for bond yields are highly nonlinear in the parameters (as are the impulse response functions in a VAR, for example).[11] Estimation of the bond-pricing implications of the model is thus highly nonlinear and can be tricky in practice, with a great many local minima. (Indeed, as noted above, by experimenting and exploring the parameter space, we were able to obtain a better fit to the data than reported by BRS [2004].) To reduce the number of model parameters that must be estimated nonlinearly, we follow Ang, Piazzesi, and Wei (2006) and BRS (2004)

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[10] On August 12, 2003, the Federal Open Market Committee (FOMC) released a statement that, although it was maintaining its federal funds rate target at the level of 1 percent, it judged "that, on balance, the risk of inflation becoming undesirably low is likely to be the predominant concern for the foreseeable future. In these circumstances, the Committee believes that policy accommodation can be maintained for a considerable period."

[11] Indeed, to compute the model-implied 10-year yield, we must project the monthly VAR and prices of risk forward 120 months. Thus, when fitting the model to the data, we are essentially trying to minimize a 120th-degree polynomial.

and estimate the model in two stages: first, we estimate the VAR on the five macroeconomic time series above (with four lags) over the 1984–2005 period by ordinary least squares; and second, we take the VAR coefficients as given and estimate the stochastic pricing kernel factor loadings (which, between λ_0 and λ_1, comprise 30 parameters) using nonlinear least squares to fit our bond yield data over the same period, 1984–2005. As in BRS (2004), we estimate the model to match the six-month and one-, two-, three-, four-, five-, seven-, and 10-year yields as closely as possible, with equal weight on fitting each of those maturities. These yield curve data are continuously compounded zero-coupon yields estimated by the Fed from off-the-run Treasury coupon securities (for details, see Gürkaynak, Sack, and Wright [2006]); following BRS (2004), we use the monthly average values of these yields.[12]

Our results from estimating the BRS model are reported in Tables 1, 2, and 3. The VAR parameter estimates in Table 1 show that each variable in the VAR has a

Table 1 BRS VAR Parameter Estimates

VAR coefficient estimates (Φ)					
	y_t	π_t	$\hat{\pi}_t^{BC}$	i_t	$\hat{i}_t^{ed}$
y_{t-1}	1.078	0.078	–0.080	0.027	0.266
y_{t-2}	0.171	0.138	0.228	0.123	0.254
y_{t-3}	–0.102	0.062	–0.055	–0.038	–0.283
y_{t-4}	–0.180	–0.249	–0.094	–0.103	–0.278
π_{t-1}	0.022	0.883	0.059	0.170	0.311
π_{t-2}	–0.036	–0.067	–0.012	–0.141	–0.033
π_{t-3}	–0.071	0.187	0.001	–0.000	–0.234
π_{t-4}	0.073	–0.016	0.026	–0.012	–0.068
$\hat{\pi}_{t-1}^{BC}$	–0.094	0.083	1.027	0.078	–0.104
$\hat{\pi}_{t-2}^{BC}$	0.273	–0.086	–0.118	–0.174	0.208
$\hat{\pi}_{t-3}^{BC}$	–0.239	0.124	–0.021	0.341	0.222
$\hat{\pi}_{t-4}^{BC}$	0.063	–0.114	0.005	–0.263	–0.190
i_{t-1}	0.010	–0.107	0.089	1.311	0.105
i_{t-2}	0.070	0.120	–0.130	–0.413	–0.310
i_{t-3}	–0.092	–0.106	0.098	0.177	0.367
i_{t-4}	0.018	0.093	–0.054	–0.115	–0.073
$\hat{i}_{t-1}^{ed}$	0.029	–0.007	0.064	0.162	1.097
$\hat{i}_{t-2}^{ed}$	–0.005	0.007	–0.071	–0.065	–0.264
$\hat{i}_{t-3}^{ed}$	–0.020	–0.043	0.017	–0.029	0.139
$\hat{i}_{t-4}^{ed}$	–0.003	0.036	–0.004	–0.034	–0.119
Constant	–0.006	0.039	0.046	0.006	0.087
Cholesky-factored residual variance (Σ) (Var $= \Sigma\Sigma'$)					
	0.0867	0	0	0	0
	0.0034	0.1618	0	0	0
	–0.0001	0.0111	0.0808	0	0
	0.0047	–0.0105	0.0168	0.1539	0
	0.0800	0.0309	0.0283	0.1150	0.3826

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[12] The finance and macro-finance literatures typically use end-of-month yield data rather than month-average yields. However, the model is linear in the state variables, so it is completely consistent to view the model's

Table 2 BRS Model Risk Factor Loadings

λ_0	λ_1				
–1.998	0.473	0.290	–1.292	2.517	0.209
–2.349	0.712	0.666	6.036	–2.022	0.256
–20.822	13.846	–0.323	1.028	4.941	12.002
3.366	–1.814	0.019	–0.287	–0.939	–1.720
2.525	–1.064	–0.465	–1.174	–0.030	–1.186

Table 3 BRS Model Prediction Errors

Treasury maturity	Model prediction error (RMSE, basis points)
Six months	29.52
One year	33.35
Two years	26.98
Three years	26.58
Four years	27.78
Five years	30.03
Seven years	34.45
10 years	39.30
Average	31.00

coefficient near unity on its own first lag, with smaller coefficients on longer lags of itself and on lags of other variables. The sum of the coefficients on each variable’s own lags is near unity. For the federal funds rate, the coefficients suggest a hump-shaped impulse response and autocorrelation functions, but the other variables exhibit impulse response and autocorrelation functions that are closer to a geometric decay.

The BRS model’s estimated risk factor loadings are reported in Table 2. Although these estimated loadings show the greatest coefficients on the Blue Chip inflation expectations variable, that variable also has a relatively lower variance, as can be seen in Table 1, so the net effect on bond prices is not as great as the factor loadings would suggest by themselves.[13]

Table 3 reports the quality of the model’s fit, in terms of RMSEs for the eight points on the yield curve that the model was estimated against. Except at the shortest horizons, these RMSEs are less than half as large as those reported by BRS (2004) for their sample period. This difference is not simply due to the sample period: even when we restrict our sample to the one used by BRS (2004), we are able to obtain RMSEs less than half as large by experimenting and exploring the parameter space.

2. RW model estimates

For the RW model, we take the original parameter values as estimated by RW (2004) from January 1988 to December 2000 using monthly data. In that estimation, the

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equations as holding for the month-average data as well as the daily data and the end-of-month data. Moreover, our macro data are typically monthly averages, so using month-average yields is arguably more consistent with the macro data.

[13] It would be desirable to perform a variance decomposition of the 10-year Treasury term premium on the five stochastic shocks ϵ. However, the term premium at these longer horizons is highly nonlinear in the parameters of the model, so that a direct computation of the variance decomposition is impossible and the delta method is likely to be a very poor approximation.

inflation rate was measured by the year-on-year change in the overall PCE deflator, the output gap was measured by capacity utilization, and interest rates were end-of-month data on five U.S. Treasury zero-coupon yields that have maturities of one, three, 12, 36, and 60 months (yields are unsmoothed Fama-Bliss data expressed at an annual rate in percent).

Since there are two underlying latent factors but five observable yields, RW (2004) follow the usual strategy and assume that the three-, 12-, and 36-month yields are measured with *i.i.d.* error, as in Ang and Piazzesi (2003). (Note that this implies that, for a given set of parameter values, the latent factors L_t and S_t can be backed out perfectly from the observed one-month and 60-month bond yields.) The estimated size of the measurement error that is required to fit the other yields is a common metric for the quality of the model's fit. Also note that the 10-year rate is not used in the estimation, so examining the recent episode of low bond rates is an out-of-sample exercise in terms of both the estimation sample and bond maturity.

In contrast to the BRS model, all of the parameters of the RW model are estimated in a single step by maximum likelihood. Table 4 reports the RW model parameter estimates, from RW (2004). First, consider the dynamics of the factors. The factor L_t is very persistent, with a ρ_L estimate of 0.989, which implies a small but significant weight on actual inflation. The dynamics of S_t are also very persistent, but this persistence does not come from partial adjustment since the ρ_S estimate is a minuscule 0.026. Instead, S_t responds with only a very short lag to output and inflation shocks.

Table 4 RW Model Parameter Estimates

Factor dynamics			
ρ_L	0.989 (0.0068)	g_π	1.253 (0.0066)
ρ_S	0.026 (0.0111)	g_y	0.200 (0.0066)
ρ_u	0.975 (0.0062)		
Inflation dynamics			
μ_π	0.074 (0.0113)	$\alpha_{\pi 1}$	1.154 (0.0525)
α_y	0.014 (0.0074)	$\alpha_{\pi 2}$	–0.155 (0.0066)
Output dynamics			
μ_y	0.009 (0.0066)	β_{y1}	0.918 (0.0604)
β_r	0.089 (0.0067)	β_{y2}	0.078 (0.0066)
Risk price (λ_1)			
	L_t	S_t	
$\Lambda_{L,t}$	–0.0045 (0.0068)	0.0168 (0.0068)	
$\Lambda_{S,t}$	–0.0223 (0.0064)	0.0083 (0.0067)	
Standard deviations			
σ_L	0.342 (0.0089)	σ_π	0.238 (0.0110)
σ_S	0.559 (0.0313)	σ_y	0.603 (0.0128)
Standard deviations of measurement error			
Three-month	0.288 (0.0162)		
12-month	0.334 (0.0194)		
36-month	0.127 (0.0094)		

Note: Standard errors of the estimates are in parentheses.

The persistence in S_t reflects the fact that the Fed adjusts the short rate promptly to various determinants—output, inflation, and other influences in the residual u_t—that are themselves quite persistent (e.g., $\rho_u = 0.975$).[14] The monetary policy interpretation of the slope factor is supported by the values of the estimated inflation and output response coefficients, g_π and g_y, which are 1.253 and 0.200, respectively. These estimates are similar to the usual single-equation estimates of the Taylor rule during this sample period (e.g., Rudebusch [2002b]).

The estimated parameters describing the inflation dynamics also appear reasonable.[15] In particular, the estimated weight on explicit forward-looking expectations in determining inflation, μ_π, is 0.074. Since this estimate is based on monthly data, with time aggregation, it implies a weight of about 0.21 on the interim inflation objective at a quarterly frequency. This estimate appears consistent with many earlier estimates obtained using a variety of different methods and specifications. For example, using survey data on expectations, Rudebusch (2002a) obtains a broadly comparable μ_π estimate of 0.29, which is in the middle of the range of estimates in the literature. However, by using the yield curve to extract inflation expectations, our estimates bring new information to bear on this important macroeconomic question.

The estimated parameters describing the output dynamics also fall within reasonable ranges.[16] Specifically, the estimated value of $\mu_y = 0.009$ implies a negligible weight at a quarterly frequency on forward-looking output expectations in the determination of output behavior. This is very much in accord with the maximum likelihood estimation results reported by Fuhrer and Rudebusch (2004).

The risk price matrix (λ_1) appears significant, and the model fits the three-month, 12-month, and 36-month yields with measurement error standard deviations comparable to others in the literature. For instance, the standard deviation of the three-month yield is 29 basis points (annualized), compared with 30 basis points in the VAR-based "macro lag model" in Ang and Piazzesi (2003). The standard deviation of the 36-month yield is 13 basis points, the same as in Ang and Piazzesi (2003).

To consider the bond yield "conundrum" in 2004–05, we must extend the RW model forward to include this period. We do so as follows. As mentioned above, we take the parameter values as estimated from RW (2004). We then extend the output gap and inflation data forward to the end of 2005. For consistency with our BRS model analysis, we use the continuously compounded zero-coupon yield curve data from the Fed for the period January 1988 to December 2005 (although we use the month-end values of these data rather than the month average). From the one-month and five-year zero-coupon yields, we can back out the RW model's level and slope factors, L_t and S_t, over the extended sample 1988–2005, as noted above. (Over the period 1988–2000, these agree closely with the original values for L_t and S_t in RW

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[14] Thus, our estimate of ρ_S decisively dismisses the interest rate smoothing or monetary policy inertia interpretation of the persistence in the short rate.

[15] After taking into account time aggregation and the higher cyclical variability of capacity utilization compared with the output gap, the elasticity of inflation with respect to output ($\alpha_y = 0.014$) appears about half the size of estimates that use the entire postwar sample of quarterly data, for example, Rudebusch (2002a). The estimate does appear more in line with estimates obtained in recent shorter samples (Rudebusch [2001]).

[16] The interest rate sensitivity of output ($\beta_r = 0.089$), after taking into account the time aggregation and the use of capacity utilization rather than the output gap, appears broadly in line with estimates that use the entire postwar sample of quarterly data.

(2004), but they are not identical.) Having obtained L_t and S_t over the full sample, and given the parameters of the model as estimated by RW (2004), we can then compute the model's implications for bond yields of all maturities from 1988–2005.

B. Macro-Finance Model Analysis of the 10-Year Treasury Yield

Before analyzing the recent episode of bond yields within the framework of the macro-finance models above, it may be useful to summarize the salient differences between the two models. Our estimates of the BRS model use monthly average interest rate data and include the 2004–05 period in the estimation sample. The RW model estimates use end-of-month interest rate data and are based on a sample that ends in 2000. (The in-sample/out-of-sample distinction is reinforced because the BRS model includes the 10-year yield in the estimation, while the RW model does not.) The BRS model is a completely autoregressive, nonstructural specification (except for the no-arbitrage assumption), while the RW model is more tightly parameterized with identifiable, explicitly forward-looking aggregate demand, aggregate supply, and policy rule equations. As an indication of the differences in parameterization, the BRS model has 100 parameters describing the factor dynamics and 25 risk-pricing parameters, excluding constants, while the RW model has 13 and four, respectively. Again, we view these differences between the two models as useful to the extent they can illuminate the robustness of our results.

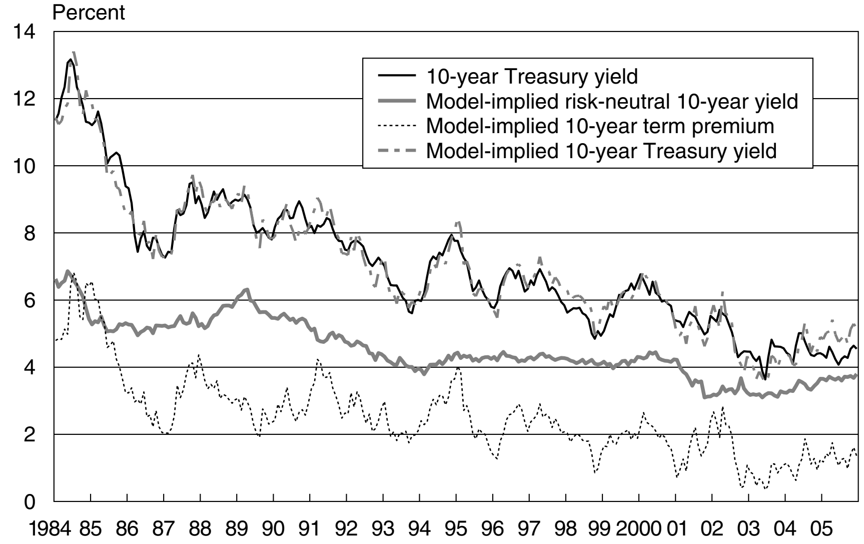
To start our analysis, we plot the 10-year zero-coupon U.S. Treasury yield together with the decomposition of that yield into its constituent components, as implied by the BRS and RW macro-finance models, in Figures 1 and 2, respectively. We focus on the 10-year yield in particular because that yield was the benchmark for long-term interest rates in the United States over the period of interest, and because discussions of the long-term bond yield "conundrum" by policymakers and in the popular press have often focused on the 10-year U.S. Treasury in particular.

In Figure 1 [1], we plot the zero-coupon U.S. Treasury yield from 1984 through 2005 together with the BRS model decomposition of that yield. The BRS model-implied risk-neutral rate (the gray solid line) is the model's estimated yield on a riskless 10-year zero-coupon bond at each date t in a hypothetical world in which the prices of risk λ_t are always equal to zero and the state variables of the economy are governed by the VAR in Table 1. The BRS model-implied 10-year Treasury yield (the gray dashed line in Figure 1) is the model's estimated value of the same 10-year zero-coupon bond when the prices of risk λ_t are no longer zero, but are instead the estimated affine function of the macroeconomic variables given in Table 2. The BRS model-implied term premium (the black dotted line) is the difference between the gray dashed line and the gray solid line, and can be interpreted as the model's estimate of the risk or term premium on the 10-year zero-coupon bond at each date t. Finally, the BRS model does not match the data perfectly, so the model's residuals—the difference between the model predictions (the gray dashed line) and the data (the black solid line)—are graphed in Figure 1 [2].

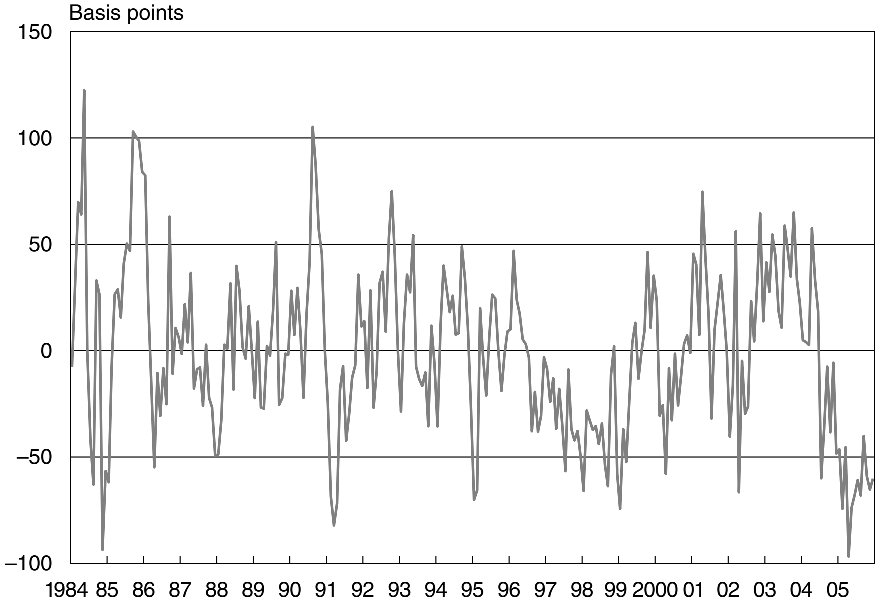
As can be seen in Figure 1 [1], the close fit of the BRS model to the data is striking. Not only does the model capture the general downward trend in the 10-year yield over this period, but even the high-frequency swings in this yield in the late 1980s and mid-1990s are matched extremely well. The fit is even more remarkable in light of

Figure 1 Ten-Year Treasury Yield and BRS Model Implied Values

[1] Ten-Year Treasury Yield and BRS Model Decomposition



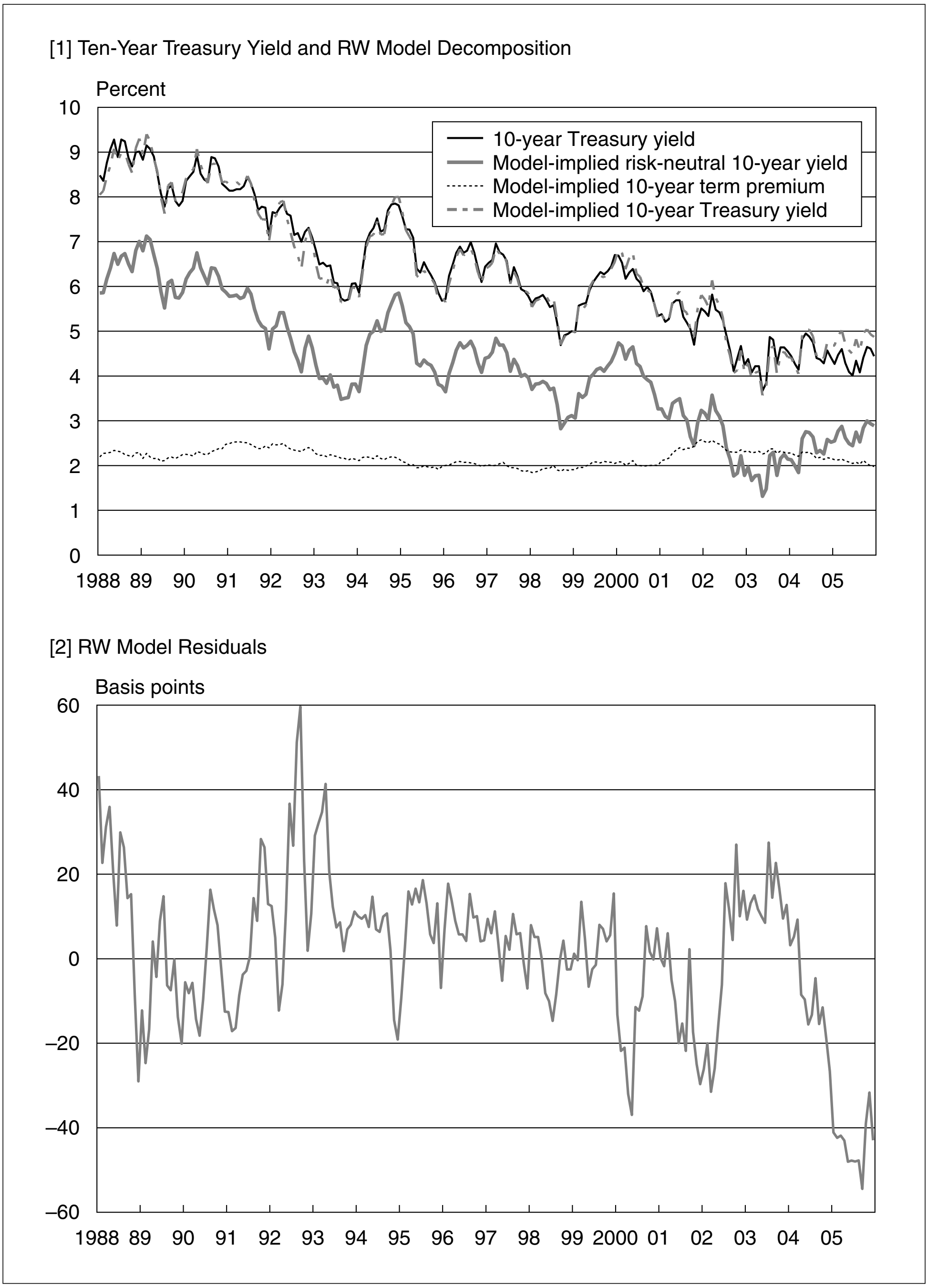
[2] BRS Model Residuals



the fact that the model was not optimized to fit the 10-year yield, but rather placed equal weight on eight maturities all along the term structure.

According to the model, both the risk-neutral 10-year yield and the term premium have fallen over our sample, with the fall in the term premium being somewhat more

Figure 2 Ten-Year Treasury Yield and RW Model Implied Values



important. According to the model, the risk-neutral yield has fallen about 250 basis points over this period while the term premium has fallen about 350 basis points, and the term premium has fallen from about one-half of the total 10-year U.S. Treasury yield in the 1980s to about one-third of that yield more recently.

However, despite the model's excellent fit to the data overall, the recent period of low 10-year yields is one episode that the model notably fails to fit. The model's residuals from mid-2004 through the end of 2005 have typically been around 50 basis points, and have been even greater over almost all of 2005. Although the model has failed to fit the data on a few other occasions as well—most notably at the end of 1984, in 1985, and from 1997 to 1999—these previous episodes were typically either much briefer (late 1984) or milder in size (1997 to 1999) than the latest episode. Indeed, the most recent episode's residuals of 50 to 75 basis points are even more remarkable in light of the very low level of long-term yields—thus, while the model's residuals amounted to about one-10th the level of the 10-year Treasury yield in 1984 and 1985, and about one-15th in 1997–98, the model's residuals have been about one-sixth the level of the 10-year Treasury yield in the most recent period.

In Figure 2, we present the analogous pair of graphs for the 10-year bond yield decomposition implied by the RW model. Again, the fit of the model to the data is excellent, and this is all the more remarkable given that the RW model was not optimized to fit the 10-year yield at all—indeed, RW (2004) chose the five-year yield as the longest maturity in the estimation.

A striking observation from Figures 1 and 2 is that, although both the RW and BRS macro-finance models produce very good fits to the 10-year Treasury yield, the two models' implied decomposition of that yield into expected short rate and term premium components is very different. In the RW model, the term premium is relatively constant over the 1988–2005 period, hovering around the 2 percent level with very little high-frequency variation; instead, the RW model attributes most of the variation in the 10-year bond yield over time to changes in the expected future path of short rates. By contrast, the BRS model attributes most of the high-frequency variation in the 10-year yield to changes in the term premium component, with the risk-neutral component generally trending smoothly downward over time.

Why is the decomposition implied by the two models so different? Recall that the BRS model is estimated in two stages, with the macroeconomic VAR estimated first. The smoothly downward-trending risk-neutral rate in the BRS model is essentially a projection of the future path of short rates based on the VAR, so the VAR in the BRS model is implying a smoother path for the risk-neutral 10-year yield than is the case for the actual 10-year yield in the data. By contrast, in the RW model, the future path of short-term interest rates is affected greatly by the inflation "level" factor L_t. Since L_t can vary at high frequency in response to exogenous shocks to itself and to inflation, the RW model's specification allows these shocks to pass essentially directly through to the risk-neutral 10-year yield, leading to significant high-frequency variation in that variable.

The high-frequency variation in the risk-neutral 10-year yield that is allowed for (but not assumed) by the RW model is arguably one of its strengths—indeed, Gürkaynak, Sack, and Swanson (2005) found significant systematic variation in far-ahead forward nominal interest rates in response to macroeconomic news in a way that suggested changes in inflation expectations rather than changes in term premiums. Similarly, Kozicki and Tinsley (2001) found that statistical models that allow for a "moving endpoint" are able to fit interest rate and inflation time series much better than standard stationary or difference-stationary VARs. A weakness of the BRS model and many other macroeconomic models, according to both of these papers, is that those models assume long-run features of the economy, such as the steady-state real interest rate and rate of inflation, are too well anchored in response to shocks. The RW model relaxes that restriction to a much greater extent than does the BRS model.[17]

Finally, as was the case for the BRS model, the recent episode of low 10-year yields is one that the RW model conspicuously fails to fit. Thus, from the perspective of both the BRS and RW models, the recent behavior of long-term Treasury yields does represent a conundrum.

IV. Factors That May Underlie the Conundrum

According to the BRS and RW macro-finance models, the recent behavior of long-term interest rates does present us with something of a conundrum. In this section, we investigate whether any additional factors lying outside of the two models could potentially explain this episode of unusually low long-term bond yields. One could of course search for correlations between our macro-finance model residuals and *nonlinear* functions of the variables that are already included in the model, such as the output gap, inflation, and the federal funds rate. We focus instead on searching for variables outside of the models because we do not have particularly strong priors that nonlinearities in the above variables have played an important role in the U.S. long-term Treasury market recently, while many plausible candidate variables have been omitted from the model (a number of which have been emphasized in the financial press).

We first briefly review various explanations for the conundrum that have been discussed in the literature, and then provide some empirical analysis based on the estimated macro-finance models to help assess the importance of the various factors.

A. Popular Explanations for Low Long-Term Bond Yields

A large number of possible candidate explanations for low bond yields have been discussed by financial analysts. A simple means to summarize this set of explanations (and to some extent limit consideration to the most important ones) is to examine a survey of bond traders, hedge fund managers, and business economists conducted by the firm Macroeconomic Advisers. Their survey was taken in early March 2005 and asked participants to provide their views on the importance of various factors that might explain the low level of the 10-year Treasury yield. Table 5 displays the seven most important factors that were identified as holding down bond yields as well as a rough estimate of how much each factor was judged by survey respondents to have reduced the bond yield (in basis points).[18] The largest effect by far was attributed to increased demand for U.S. longer-term securities from foreign central banks. Indeed,

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[17] The VAR in the BRS model is not restricted to be stationary, and thus the BRS model does allow for an unanchored steady state to the extent that the model estimates a unit root. Nonetheless, the estimation of the VAR is based on the high-frequency dynamics of the system—using four lags of monthly data—rather than on the behavior of long-term bond yields, and the high-frequency dynamics of the model may not speak so strongly in favor of a unit root as do long-term bond yields or other more direct measures of the long-term implications of the model.

[18] Several other factors (such as increased demand by holders of mortgage-backed securities) were often noted in the survey but were assessed to have an effect of only a few basis points and are excluded from the table.

Table 5 Survey Respondents' Assessments of Factors Holding Down the 10-Year Treasury Yield

Factor affecting yields	Effect in basis points
Demand by foreign central banks	21
Increased demand by pension funds	11
"Reaching for yield"	10
Minimal inflation risk	10
Greater transparency of the Fed	8
Excess global savings	8
Low economic growth volatility	7

Source: Macroeconomic Advisers (2005).

on average, the survey respondents thought that purchases by foreign central banks had lowered U.S. long-term rates by 21 basis points.[19]

After foreign official purchases of U.S. Treasuries, survey respondents assigned about equal importance to the other factors in Table 5, with each factor accounting for 7 to 11 basis points of lower long-term rates. For example, an increased demand for long bonds in light of the greater likelihood of future corporate pension fund reform in the United States, the United Kingdom, and elsewhere was assigned an importance of about 11 basis points. Such reforms would likely encourage pension funds to better match the duration of their assets to their liabilities, which is expected to boost demand for long-term bonds.[20] Although this factor has been widely cited (e.g., Bank for International Settlements [2005]), it is difficult to quantify, and we do not include it in our empirical analysis below.

Similarly, we do not quantify two factors that appear to relate to shifts in investor appetite for risk. These include the factor, "reaching for yield," which is shorthand for the view that high levels of liquidity had encouraged investors to reduce their aversion to risk (perhaps irrationally in an almost bubble-like manner). Also, a closely related factor is the view that excess global saving—the global saving glut hypothesis of Bernanke (2005) noted in Section I—stemming perhaps from less home bias among foreign investors or rapid economic growth in countries with high saving rates, had boosted foreign demand for bonds generally (e.g., Warnock and Warnock [2005]).

We do, however, employ some proxies that try to measure a possible decrease in the amount of risk that investors may feel they face. Indeed, the Macroeconomic Advisers survey respondents pointed to lower uncertainty as an important factor in three of their responses: minimal inflation risk, greater transparency of the Fed (which presumably would translate into lower short-term interest rate uncertainty), and low economic growth volatility. Taken together, these three factors suggest a very important role for reduced macroeconomic uncertainty in reducing the long-term bond rate.

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[19] In the academic literature, BRS (2004) and Warnock and Warnock (2005) have also stressed the importance of the recent increases in foreign central bank purchases, especially by East Asian countries. Wu (2005), however, notes that finding significant effects of such purchases on U.S. Treasury yields is sensitive to the sample period and can be quite elusive.

[20] These demands for duration would, of course, reach out much longer than a 10-year horizon, and indeed in 2005 there was strong demand for the revived 30-year U.S. Treasury bond, for a newly launched French 50-year bond issue, and for a 50-year British inflation-indexed bond.

B. An Empirical Assessment of Various Explanations

To examine some popular explanations for low long-term bond yields, we consider six variables excluded from the BRS and RW macro-finance models: three measures of financial market volatility, two measures of macroeconomic volatility, and one measure of international capital flows. For financial market volatility, we use the Merrill Lynch MOVE Index to measure the implied volatility in the longer-term U.S. Treasury market;[21] we use the implied volatility from Eurodollar options to measure uncertainty about the near-term path of monetary policy;[22] and we use the VIX measure of implied volatility from options on the S&P 500 index to measure uncertainty in the stock market. For macroeconomic uncertainty, we proxy for output uncertainty by using the eight-quarter trailing standard deviation of the growth rate of real GDP, interpolating between the resulting quarterly volatility measure to get a monthly series, and we proxy for inflation uncertainty by using the 24-month trailing standard deviation of core PCE deflator inflation. Finally, we proxy for foreign government and foreign central bank purchases of U.S. Treasury securities by using the 12-month change in the custodial holdings by the New York Fed for all foreign official institutions (and we normalize this series by the total stock of U.S. Treasury debt in the hands of the public).[23]

All of these series are natural candidates for omitted variables that could be impacting the level of long-term U.S. Treasury yields. For example, reduced volatility in long-term bond markets would tend to make those securities more attractive relative to other assets and drive long-term bond yields down, all else equal. Conversely, a reduced degree of uncertainty about stock returns would tend to drive the prices of stocks up and might require long-term interest rates to rise to keep fixed-income securities attractive by comparison. Reduced uncertainty about the future path of monetary policy may lower the riskiness of holding long-term bonds leading to a fall in long-term yields. Reduced uncertainty about inflation may make fixed income securities of all maturities more attractive, and reduced uncertainty about output may lower the risk premium and raise the prices of all risky assets in general. Finally, inelastic demand by foreign central banks and governments for long-term U.S. Treasury securities has often been cited in the popular press (and in Warnock and Warnock [2005]) as a reason why yields on those securities have been so low recently.

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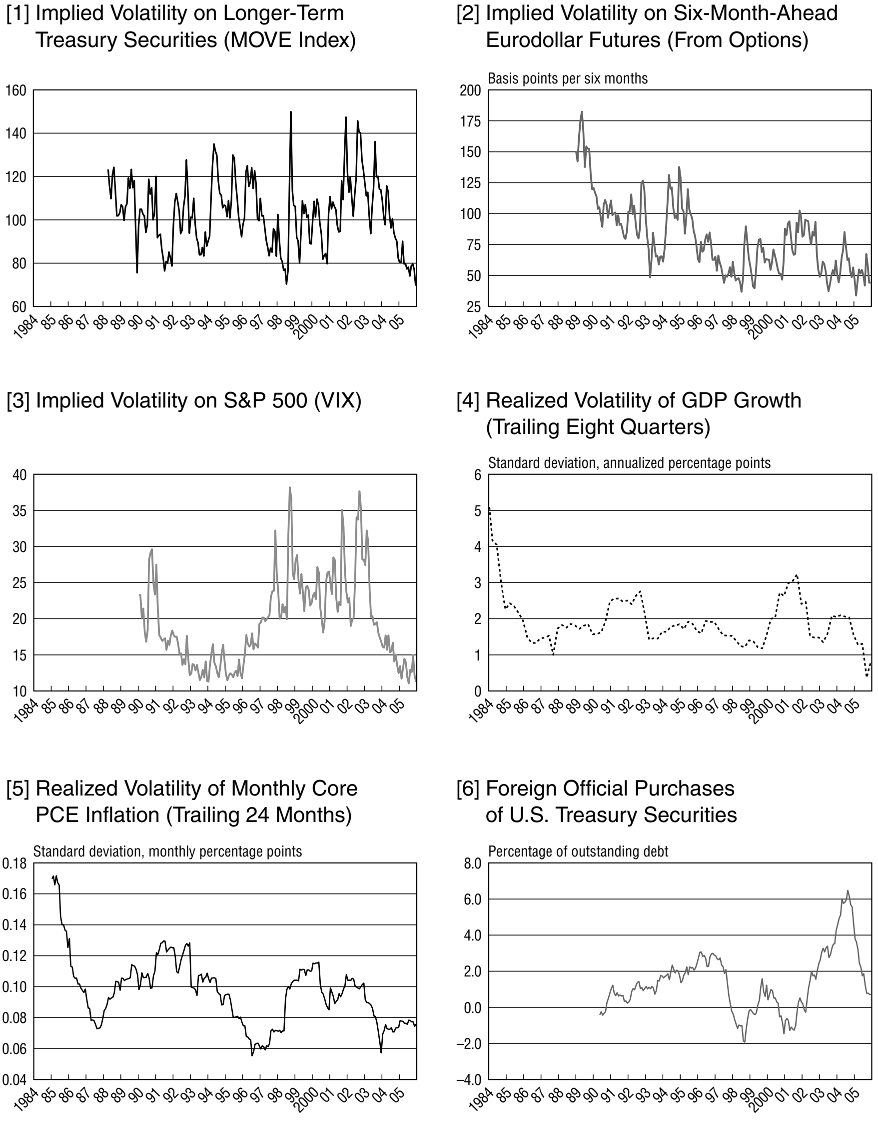
[21] The Merrill Option Volatility Estimate (MOVE) Index reports the average implied volatility across a wide range of outstanding options on the two-year, five-year, 10-year, and 30-year U.S. Treasury securities (with a total weight of 40 percent on the 10-year Treasury and total weights of 20 percent each on the other maturities). Note that the options underlying the MOVE Index have expiration dates of approximately one month; thus, the MOVE Index measures the implied volatility of long-term yields over a relatively short horizon. Such short-term uncertainty about long-maturity yields may be related but is not the same as uncertainty about the path of short-term interest rates in the distant future.

[22] Specifically, we use the closest to at-the-money Eurodollar option with expiration in six months' time to measure the implied volatility for the 90-day Eurodollar rate.

[23] The idea that the available quantity of long-term U.S. Treasury securities matters for long-term yields suggests that we should also look at total *issuance* of marketable U.S. Treasury securities by the U.S. government as well as the purchases of those securities by foreign governments. In preliminary results not reported here, we do find evidence that greater total issuance of marketable U.S. Treasuries does lead to higher long-term interest rates (Bikbov and Chernov [2006] also find a correlation between total issuance of U.S. Treasuries and their macro-finance model residuals). Even when we include total issuance as an explanatory variable in our regressions below, however, we find little evidence that foreign official purchases have had a significant impact on long-term Treasury yields.

Graphs of each of these six variables over the 1984–2005 sample are presented in Figure 3 (note that not all of the above series, particularly the implied volatility series for financial market data, are available for the early part of the sample). It is immediately clear from the figure that many of these series show a marked dip in the last few years—thus, as readers of the financial press are well aware, there is no shortage of possible explanations for the conundrum!

Figure 3 Explanatory Variables for BRS and RW Model Residuals



[1] Implied Volatility on Longer-Term Treasury Securities (MOVE Index)

[2] Implied Volatility on Six-Month-Ahead Eurodollar Futures (From Options)

[3] Implied Volatility on S&P 500 (VIX)

[4] Realized Volatility of GDP Growth (Trailing Eight Quarters)

[5] Realized Volatility of Monthly Core PCE Inflation (Trailing 24 Months)

[6] Foreign Official Purchases of U.S. Treasury Securities

Note: Not all data series going back to 1984 are available.

In Table 6, we conduct a preliminary statistical analysis of the importance of these variables by regressing the *residuals* from the BRS model and the RW model on each of the six independent variables described above, in a set of univariate regressions. A significant correlation between one of these variables and the models' residuals would suggest that that variable is perhaps an important determinant of long-term U.S. Treasury yields that has not been captured by the model.[24]

As can be seen in Table 6, and as was evident in Figure 3, many of the variables above are correlated with the models' residuals. Indeed, five of the six variables are statistically significant at the 10 percent level for at least one of the two models' residuals. Two of the six variables—implied volatility on long-term Treasuries and GDP volatility—are statistically significant at the 5 percent level or better for both models' residuals. Interestingly, foreign official purchases of U.S. Treasuries—the variable that is most emphasized by the financial press and by the Macroeconomic Advisers survey respondents—is the only one of the six variables that is not significant at even the 10 percent level for either of the two sets of model residuals (and has a positive sign, rather than the negative correlation hypothesized by the press). This result is very similar to that of Wu (2005), who finds that, after controlling for macroeconomic determinants of bond yields, the correlation between foreign official purchases and bond yields is significantly *positive* between 1987 and 2000, and negative only since 2002. Thus, the correlation that has been emphasized so much by the financial press is not one that has been consistent over recent history.

Table 6 Univariate Regressions of BRS and RW Model Residuals

Independent variable	Model	Sample	Coefficient	*t*-statistic	R^2
Implied volatility on longer-term Treasury securities (Merrill Lynch MOVE Index, in basis points)	BRS	4/88–12/05	1.10	7.83	0.23
	RW	4/88–12/05	0.38	4.94	0.10
Implied volatility on six-month-ahead Eurodollar futures (from options, in basis points)	BRS	1/89–12/05	0.278	3.17	0.05
	RW	1/89–12/05	0.042	0.95	0.00
Implied volatility on S&P 500 (VIX)	BRS	1/90–12/05	0.814	1.85	0.02
	RW	1/90–12/05	0.049	0.23	0.00
Realized volatility of GDP growth (trailing eight-quarter standard deviation, in percent)	BRS	1/88–12/05	20.3	4.25	0.08
	RW	1/88–12/05	5.7	2.26	0.02
Realized volatility of monthly core PCE inflation (trailing 24-month standard deviation, in percent)	BRS	1/88–12/05	316	2.33	0.02
	RW	1/88–12/05	99	1.41	0.01
Foreign official purchases of U.S. Treasury securities (trailing 12-month total, as percentage of U.S. debt in public hands)	BRS	5/90–12/05	149	0.89	0.00
	RW	5/90–12/05	47	0.58	0.00

Note: BRS and RW model residuals are in basis points. Sample period start dates differ owing to differences in data availability for each explanatory variable. Each regression also includes a constant term (not reported).

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[24] The purpose of these regressions is to help identify which variables might potentially be the most important to incorporate into the models going forward. Of course, just because a variable is significantly contemporaneously correlated with our macro-finance model residuals does not necessarily imply that it will remain significant when fully incorporated into a macro-finance framework, although we interpret a strong contemporaneous correlation as suggestive that it will be. Also, there is no guarantee that our macro-finance model residuals are orthogonal even to the variables included in the models because of the overidentifying restrictions those models impose. We interpret the regression results under the assumption that the overidentifying model restrictions are correct, although again, we consider both models in order to provide some assurance of robustness.

A primary problem with Table 6, of course, is that many of the variables show declines in 2004 and 2005, and thus univariate regression results may be double-counting the explanatory power of the variables for the conundrum. Table 7 thus reports results from multivariate regressions for the same variables over the May 1990 to December 2005 period (the longest sample for which we have data on all six explanatory variables).

The multivariate regression results in Table 7 are, for the most part, consistent with the univariate regression results in Table 6. The most significant and robust explanatory variable is the implied volatility on longer-term Treasuries. Macroeconomic volatility also seems to play a statistically significant role, with core PCE inflation volatility significant at the 5 percent level for both models' residuals, and GDP growth volatility significant for the BRS model residuals. The signs of these coefficients are also what one would typically expect, with lower implied volatility on Treasuries and lower realized volatility of output and inflation all being correlated with lower yields on long-term Treasury securities. Again, the series on foreign official purchases is the only one not statistically significant at the 10 percent level for at least one of the two models' residuals and continues to have a positive sign.

C. Decomposition of the Bond Yield Conundrum

How much of the bond yield conundrum can these additional variables explain? In Table 8, we perform a detailed decomposition of the decline in bond yields from June 2004 to June 2005, according to each of the BRS and RW models and the regression results from Table 7. We choose June 2004 as the starting point for the change in yields because that month, when the FOMC embarked on the removal of monetary accommodation, is most often cited as the beginning of the conundrum. During this period, long-term interest rates edged down as the Fed steadily tightened policy.

The first row of Table 8 reports the actual change in 10-year yields over the June 2004 to June 2005 period (which differs slightly across the BRS and RW models because the BRS model uses month-average yield data while the RW model uses

Table 7 Multivariate Regressions of BRS and RW Model Residuals

Independent variable	BRS model Coefficient (*t*-statistic)	RW model Coefficient (*t*-statistic)
Implied volatility on longer-term Treasury securities (Merrill Lynch MOVE Index, in basis points)	1.203 (5.47)	0.490 (4.11)
Implied volatility on six-month-ahead Eurodollar futures (from options, in basis points)	–0.229 (–1.35)	–0.168 (–1.83)
Implied volatility on S&P 500 (VIX)	–0.334 (–0.63)	–0.497 (–1.73)
Realized volatility of GDP growth (trailing eight-quarter standard deviation, in percent)	15.4 (3.10)	3.9 (1.45)
Realized volatility of monthly core PCE inflation (trailing 24-month standard deviation, in percent)	360 (2.18)	214 (2.39)
Foreign official purchases of U.S. Treasury securities (trailing 12-month total, as percentage of U.S. debt in public hands)	147 (0.76)	38 (0.04)
R^2	0.30	0.14

Note: BRS and RW model residuals are in basis points. Sample is May 1990 to December 2005. Each regression also includes a constant term (not reported).

Table 8 Decomposition of Long-Term Bond Yield Conundrum

		BRS model	RW model
1	Observed change in 10-year yield (basis points), June 2004–June 2005	–93.3	–87.0
Of which:			
2	Model-implied change in risk-neutral 10-year yield	13.1	–29.6
3	Model-implied change in term premium	–19.9	–25.2
4	Change in model residuals	–86.5	–32.2
Of which:			
5	Change in implied volatility on longer-term Treasuries	–29.9	–12.2
6	Change in realized volatility of core PCE inflation	1.1	0.7
7	Change in realized volatility of GDP growth	–11.6	–2.9
8	Change in implied volatility of Eurodollar rate	7.0	5.1
9	Change in implied volatility of S&P 500	1.2	1.7
10	Change in foreign official purchases	–6.0	–1.6
11	Unexplained by above	–48.6	–23.0

Note: Row 1 denotes the change in observed zero-coupon 10-year Treasury yields from June 2004 to June 2005; the values differ across the BRS and RW models because the BRS model uses month-average yield data while the RW model uses end-of-month data. Rows 2–4 are the decompositions implied by the BRS and RW macro-finance models, graphed in Figures 1 and 2. Rows 5–10 are the changes implied by the regression coefficients in Table 7 applied to the change in each independent variable from June 2004 to June 2005. Row 11 is the difference between row 4 and the sum of rows 5–10.

end-of-month yields), amounting to a fall of about 90 basis points. Both the BRS and RW models imply that only a small part of this change—about 20 to 25 basis points—can be attributed to changes in the term premiums (row 3). Moreover, the BRS model even predicts that risk-neutral long-term yields should have *risen* about 13 basis points (row 2), based on the improving outlooks for GDP and inflation. Thus, the decline in long-term yields from June 2004 to June 2005 is largely unexplained by both the BRS and RW models, with changes in the models' residuals (row 4) accounting for about 87 and 32 basis points of the roughly 90-basis-point fall in yields from June 2004 to June 2005.

Rows 5–10 of Table 8 decompose the BRS and RW model residuals into the parts that are explained by each of the six variables in the previous section, according to the regressions in Table 7 multiplied by the change in each independent variable over the June 2004 to June 2005 period. Not surprisingly, the fall in implied volatility on longer-term Treasuries from June 2004 to June 2005 accounts for the greatest fraction of the two models' residuals, explaining a little over one-third of the residuals of each model. More surprisingly, the change in core PCE deflator inflation volatility over this period accounts for essentially none of the conundrum, even though it was statistically significant in Table 7. Apparently, the change in inflation volatility from 2004 to 2005 was close to zero and even slightly positive, so that the conundrum period played little role in the statistical significance of the results for that variable in Table 7. The fall in GDP volatility from 2004 to 2005 seems to have played a more important role, accounting for about one-eighth (12 basis points) of the BRS model residuals and about one-10th (3 basis points) of the RW model residuals. The remaining three variables in Table 8 were not statistically significant in the previous section and account for relatively small changes in yields over the conundrum period.

As can be seen in the last row of Table 8, even the six omitted variables studied in the previous section can explain only about one-fourth to, at most, one-half of the BRS and RW model residuals. Thus, we have by no means found the missing link that explains the bond yield conundrum, but we have found evidence that reductions in longer-term Treasury volatility probably have played an important role, and that foreign official purchases of U.S. Treasuries probably have not.

V. Conclusions

We draw a number of conclusions from the above analysis. First, the low level of long-term bond yields in the United States during the 2004–05 period does appear to be a conundrum when viewed through a macro-finance lens. Specifically, neither of the two macro-finance empirical models we consider is able to explain the recent low level of, or the fall in, long-term bond yields. This finding is remarkable given that both models fit the earlier long-term yield data quite well. Therefore, the conundrum can likely only be explained with variables that lie outside of our baseline macro-finance models. Of the six such variables that we consider, declines in the volatility of long-term Treasury yields seem to have played the most important role. Even so, at best, almost two-thirds of the conundrum remains unexplained.

Interestingly, we find that the explanation for the conundrum emphasized by financial market participants—namely, large-scale purchases of long-term Treasuries by foreign central banks—has essentially *no* explanatory power for the conundrum episode. This discrepancy may reflect a difference between unconditional and conditional correlations. In particular, long-term Treasury yields have been declining steadily over time, and foreign official holdings and purchases of U.S. Treasuries have been rising steadily; thus, the unconditional correlation suggests a substantial negative effect. In contrast, a macro-finance econometric framework attributes the downward trend in long-term yields largely to declining current and future projected levels of inflation. After controlling for such factors, the residuals from our two baseline macro-finance models have no significant correlation with foreign official purchases of U.S. Treasuries.

Of course, this leaves us with about two-thirds of the bond yield conundrum yet to be explained, to say nothing of the similar or perhaps even more extreme behavior of long-term yields in other countries, such as Germany and Japan. The resolution of these "conundrum" episodes, in the United States and abroad, presents a rich frontier for future research.

References

Ang, Andrew, and Monika Piazzesi, "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables," *Journal of Monetary Economics*, 50 (4), 2003, pp. 745–787.

———, ———, and Min Wei, "What Does the Yield Curve Tell Us about GDP Growth?" *Journal of Econometrics*, 131 (1–2), 2006, pp. 359–403.

Bank for International Settlements, *75th Annual Report*, Basel: Bank for International Settlements, 2005.

Bernanke, Ben S., "The Global Saving Glut and the U.S. Current Account Deficit," speech at the Sandridge Lecture, Virginia Association of Economics, Richmond, Virginia, March 10, 2005.

———, and Illian Mihov, "Measuring Monetary Policy," *Quarterly Journal of Economics*, 113 (3), 1998, pp. 869–902.

———, Vincent R. Reinhart, and Brian P. Sack, "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment," *Brookings Papers on Economic Activity*, 2, 2004, pp. 1–78.

Bikbov, Ruslan, and Mikhail Chernov, "No-Arbitrage Macroeconomic Determinants of the Yield Curve," manuscript, Columbia Business School, 2006.

Clarida, Richard, Jordi Galí, and Mark Gertler, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115 (1), 2000, pp. 147–180.

Cochrane, John, *Asset Pricing,* Princeton: Princeton University Press, 2001.

———, and Monika Piazzesi, "Bond Risk Premia," *American Economic Review*, 95 (1), 2005, pp. 138–160.

Dai, Qiang, and Kenneth J. Singleton, "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, 55 (5), 2000, pp. 1943–1978.

Dewachter, Hans, and Marco Lyrio, "Macro Factors and the Term Structure of Interest Rates," *Journal of Money, Credit and Banking*, 38 (1), 2006, pp. 119–140.

Diebold, Francis X., Monika Piazzesi, and Glenn D. Rudebusch, "Modeling Bond Yields in Finance and Macroeconomics," *American Economic Review*, 95 (2), 2005, pp. 415–420.

———, Glenn D. Rudebusch, and S. Boragan Aruoba, "The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach," *Journal of Econometrics*, 127 (1–2), 2006, pp. 309–338.

Duffie, Darrell, and Rui Kan, "A Yield-Factor Model of Interest Rates," *Mathematical Finance*, 6 (4), 1996, pp. 379–406.

Evans, Charles L., and David A. Marshall, "Economic Determinants of the Term Structure of Nominal Interest Rates," manuscript, Federal Reserve Bank of Chicago, 2001.

Fuhrer, Jeffrey C., "Monetary Policy Shifts and Long-Term Interest Rates," *Quarterly Journal of Economics*, 111 (4), 1996, pp. 1183–1209.

———, and Glenn D. Rudebusch, "Estimating the Euler Equation for Output," *Journal of Monetary Economics*, 51 (6), 2004, pp. 1133–1153.

Gürkaynak, Refet, Brian P. Sack, and Eric T. Swanson, "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," *American Economic Review*, 95 (1), 2005, pp. 425–436.

———, ———, and Jonathan H. Wright, "The U.S. Treasury Yield Curve: 1961 to the Present," Finance and Economics Discussion Paper No. 2006-28, Board of Governors of the Federal Reserve System, 2006.

Hördahl, Peter, Oreste Tristani, and David Vestin, "A Joint Econometric Model of Macroeconomic and Term-Structure Dynamics," *Journal of Econometrics*, 131 (1–2), 2006, pp. 405–444.

Kim, Don H., and Jonathan H. Wright, "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates," Finance and Economics Discussion Paper No. 33, Board of Governors of the Federal Reserve System, 2005.

Kozicki, Sharon, and Peter A. Tinsley, "Shifting Endpoints in the Term Structure of Interest Rates," *Journal of Monetary Economics*, 47 (3), 2001, pp. 613–652.

———, and ———, "What Do You Expect? Imperfect Policy Credibility and Tests of the Expectations Hypothesis," *Journal of Monetary Economics*, 52 (2), 2005, pp. 421–447.

Macroeconomic Advisers, *Monetary Policy Insight's Survey on Long-Term Interest Rates*, March 8, 2005.

McConnell, Margaret, and Gabriel Perez-Quiros, "Output Fluctuations in the United States: What Has Changed since the Early 1980s?" *American Economic Review*, 90 (5), 2000, pp. 1464–1476.

Oda, Nobuyuki, and Kazuo Ueda, "The Effects of the Bank of Japan's Zero Interest Rate Commitment and Quantitative Monetary Easing on the Yield Curve: A Macro-Finance Approach," Bank of Japan Working Paper No. 05-E-6, Bank of Japan, 2005.

Piazzesi, Monika, "Bond Yields and the Federal Reserve," *Journal of Political Economy*, 113 (2), 2005, pp. 311–344.

———, and Eric T. Swanson, "Futures Prices as Risk-Adjusted Forecasts of Monetary Policy," NBER Working Paper No. 10547, National Bureau of Economic Research, 2004.

Rudebusch, Glenn D., "Is the Fed Too Timid? Monetary Policy in an Uncertain World," *The Review of Economics and Statistics*, 83 (2), 2001, pp. 203–217.

———, "Assessing Nominal Income Rules for Monetary Policy with Model and Data Uncertainty," *Economic Journal*, 112 (479), 2002a, pp. 402–432.

———, "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia," *Journal of Monetary Economics*, 49 (6), 2002b, pp. 1161–1187.

———, and Tao Wu, "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy," working paper, Federal Reserve Bank of San Francisco, 2004.

———, and ———, "Accounting for a Shift in Term Structure Behavior with No-Arbitrage and Macro-Finance Models," manuscript, 2006 (forthcoming in the *Journal of Money, Credit and Banking*).

Swanson, Eric T., "Econometric Estimation When the 'True' Model Forecasts or Errors Are Observed," manuscript, Federal Reserve Bank of San Francisco, 2006.

Taylor, John B., "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 1993, pp. 195–214.

Warnock, Francis E., and Veronica C. Warnock, "International Capital Flows and U.S. Interest Rates," International Finance Discussion Paper No. 840, Board of Governors of the Federal Reserve System, 2005.

Wu, Tao, "Macro Factors and the Affine Term Structure of Interest Rates," Working Paper in Applied Economic Theory No. 6, Federal Reserve Bank of San Francisco, 2002 (forthcoming in the *Journal of Money, Credit and Banking*).

———, "The Long-Term Interest Rate Conundrum: Not Unraveled Yet?" *FRBSF Economic Letter*, 8, Federal Reserve Bank of San Francisco, April 29, 2005.

Comment

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I. Introduction

I would first like to thank the organizers for inviting me to discuss this interesting and very topical paper by Glenn D. Rudebusch, Eric T. Swanson, and Tao Wu.

My comments are structured in four parts. In Section II, I would like to start with some general remarks on the macro-finance literature in general and on the Rudebusch and Wu (RW [2004]) model in particular. In Section III, I will use a very similar macro-finance model developed by my European Central Bank (ECB) colleagues Peter Hördahl (now at the Bank for International Settlements), Oreste Tristani, and David Vestin estimated for the euro area, and reveal that they also find that current bond yields—the German seven-year bond yield, to be precise—are too low compared to the prediction of their state-of-the-art, macro-finance, term structure model. In Section IV, I will then try to explain the possible euro-conundrum by the very same

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[25] The views expressed are solely my own and do not necessarily reflect those of the European Central Bank. I thank my colleagues Oreste Tristani and David Vestin for sharing valuable insights about the Höhrdahl-Tristani-Vestin model and Oreste Tristani for providing an updated residual series of their model. I also thank Ramon Adalid, Björn Fischer, and Marco Lo Duca for providing the other data used in this comment and Michaela Kloepfer for editorial support.

model-exogenous variables, as the authors of the current paper have done. The only variable able to account for the euro-conundrum is a measure of euro area "excess liquidity." Therefore, finally, in Section V, I will discuss the possible role of excess liquidity, in particular, whether it could be considered an explanatory factor or simply another measure of the same phenomenon.

II. The Macro-Finance Literature and the RW Model

To speak about an interest rate conundrum implies having a specific model in mind that defines the benchmark for bond yields. Pure finance models are able to explain current low long-term bond yields with ease (see Kim and Wright [2005]). Moreover, there are even ample explanations as to why the yield curve could actually be negatively sloped in equilibrium due to the hedge long bonds provide against adverse consumption outcomes; see, for example, Campbell (1986) or Piazzesi and Schneider (2006). Empirically, there have been at least three other episodes where U.S. long-term bond yields declined after the first 100-basis-point hikes in short-term interest rates: in 1971, 1976–80, and 1983–84; see Credit Suisse (2006).

The use of the term conundrum thus only makes sense if attached to a specific model class. In the current paper, the authors chose the macro-finance class of models, and that is an excellent choice. The macro-finance literature is a most promising branch of the literature, and we have seen an important evolution in the last few years. Furthermore, the present authors have been at the frontier of this evolution, together with Ang, Piazzesi, Diebold, Li, and Hördahl, Tristani, and Vestin (HTV). The evolution we have seen has led to important improvements, because these models allow for time-varying risk premiums in a no-arbitrage consistent manner. The risk premiums depend on macroeconomic conditions, and mutual feedback between the macroeconomy and interest rates is allowed for.

The RW and HTV type models in the macro-finance literature usually describe the macroeconomy using a standard new Keynesian setup. They use a Phillips curve, an intertemporal Euler equation ("IS" curve), and a monetary policy rule (Taylor-type rule). The time-variable prices of risk are then determined as linear functions of the state of the economy. The prices of risk then translate one unit of factor volatility (which itself is constant over time) into a risk premium for each maturity, fully respecting the no-arbitrage conditions between different maturities.

In the RW model, the short rate is a function of two latent variables: the medium-term inflation goal and the Taylor rule. Four particular features of the RW model deserve to be mentioned. First, the policy instrument in the Taylor rule is defined in terms of the slope of the yield curve, not the short-term interest rate (one could of course ask whether the slope of the curve is really under the control of the central bank). Second, there is no forward-looking inflation term in the Phillips curve and, third, the real interest rate in the Euler equation is not derived with expected inflation but with the medium-term inflation goal. Fourth, the prices of risk are only functions of the two latent variables but not directly inflation and output. These choices might appear peculiar, but I have no evidence that they bias the results in any particular direction.

The second macro-finance model the authors use to assess the conundrum is the Bernanke-Reinhart-Sack (BRS [2004]) model, which describes the macroeconomy by means of a five-variable VAR instead of a structural model. This model uses only observable and no latent variables, so that the larger residuals compared to the RW model are no surprise. What is a bit surprising is the different size and even sign of residuals for some periods obtained after the authors reestimated the BRS model to include data up to December 2005. For example, the originally negative residuals in 2003 of about 150 basis points (BRS [2004, figure 9]) turned into positive residuals of about 50 basis points in the same period with the updated estimation. I will not comment further on the BRS model, as the RW model is more similar to the HTV model used later for comparison.

The macro-finance literature delivers some important results. Risk premiums are anti-cyclical, as higher risk premiums are requested in bad times. It follows that a positive output shock can lead to a fall in long-term government bond yields. This is the case when the negative effect on the risk premium is stronger than the positive effect on the risk-neutral rate. HTV (2006) have also shown that the HTV model has superior forecasting power for yields at all maturities. The reason is that macro-finance models can isolate the pure expectations component of future interest rates. They find for the euro area that the reduction in the average forecast error is 20 percent compared to a random walk, 40 percent compared to a pure finance model, and 100 percent with respect to a VAR.

The macro-finance literature is still evolving, as progress is slow. These models are computationally very demanding, and some shortcuts are needed simply to allow tractability. There is as yet no direct effect from long interest rates and risk premiums on the macroeconomy. Only the short rate affects the economy. Then there are some results that are indeed puzzling at first sight, for example, that an increase in the inflation target lowers the risk premiums for longer maturities while it increases them for shorter maturities. This is possibly because investors believe that the inflation target itself is mean reverting. Alternatively, lower long bond risk premiums could result (since the 1990s) from a positive correlation between inflation and consumption, so that a rise in inflation lowers risk premiums, because consumption is also expected to be higher in the future (see Piazzesi and Schneider [2006]). And finally, the micro-foundations of these models are not yet fully satisfactory. The pricing kernel, that is, the stochastic discount factor, should endogenously depend on consumption preferences, while in these models it is constant.

III. Conundrum or Conundra?

After these general remarks, we will investigate the performance of a macro-finance model in the euro area. Figure 1 shows evidence that using the macro-finance model by HTV (2006) estimated for German bond yields and euro area macro data between 1991 and 2006[26] reveals a similar conundrum for the euro area as for the

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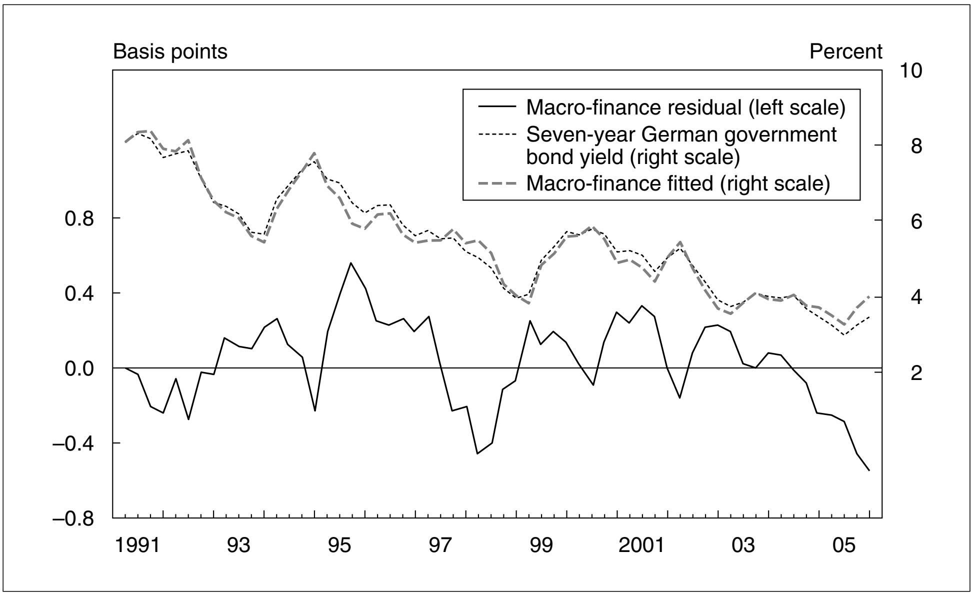
[26] The model allows for a structural break in the monetary policy rule in 1999/I.

Figure 1 HTV Macro-Finance Model for the Euro Area: A Euro-Conundrum?



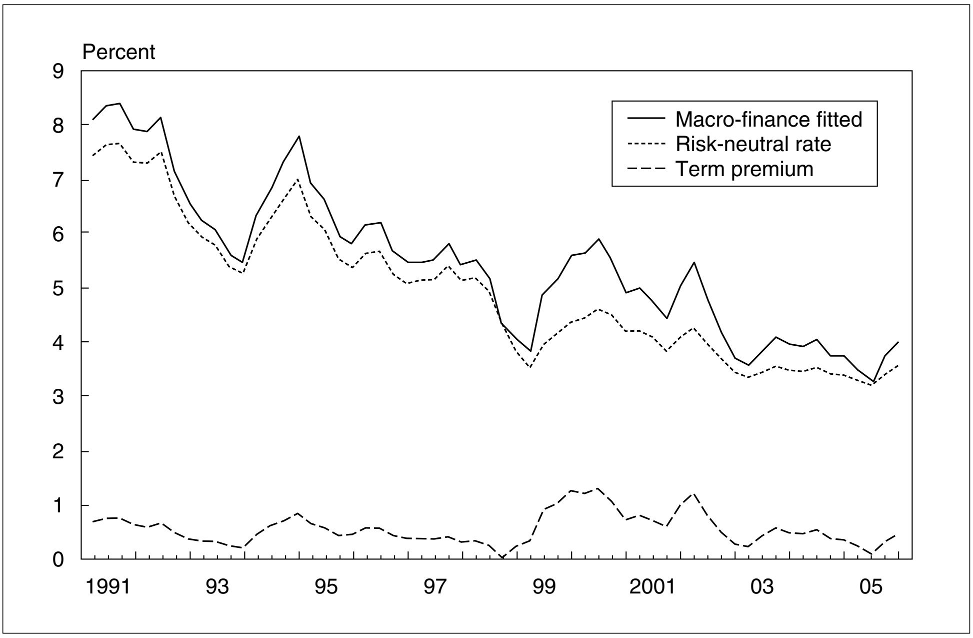
United States, although for the euro area there was one other period with similarly large residuals, in 1998. Given financial market integration, perhaps not surprisingly we can establish at least two conundra. The HTV model is very similar to the RW model, although it avoids those four rather special features mentioned in the previous section and is in this sense more standard in its macroeconomic setting. The results shown here for the period of European Monetary Union (EMU) are very preliminary, however, as one might guess from the fact that the residual series still seems to incorporate some autocorrelation. There is only one unobservable variable: the inflation target. HTV (2006) find that the risk premium on long-maturity bonds during the EMU period is mainly due to the inflation target risk and output risk, rather than the interest rate risk and inflation risk.

Interestingly, Figure 1 shows that the negative residual in German bond yields in the last two quarters where data were available (2005/IV and 2006/I) is due to the fact that the HTV model predicts a faster rise in yields than has actually happened. The changes in the macro-finance residual reported by Rudebusch, Swanson, and Wu (2006) for the U.S. 10-year government bond between June 2004 and June 2005 (the latest observation period in their sample) amount to –86.5 basis points in the BRS model and –32.2 basis points in the RW model. As depicted in Figure 1, the HTV residual reports a comparable residual change for the German seven-year bond of about –61 basis points between 2004/II and 2006/I.

Figure 2 portrays the same decomposition of the HTV fitted German yield into the risk-neutral rate and the term premium as the authors have for the BRS and RW models in their paper.

Similar to the RW model results, the term premium is relatively constant, while changes in the fitted rate are driven by the risk-neutral rate, which is a standard result for these models.

Figure 2 Decomposition of Fitted Rate in Risk-Neutral Rate and Term Premium



IV. Explaining the Euro-Conundrum

Rudebusch, Swanson, and Wu (2006) attempt to explain the residual using the "usual suspects" (derived from survey respondents' assessment of factors holding down long-term bond yields as of March 2005). The possible factors mentioned in the survey are demand by foreign central banks, increased demand by pension funds, reaching for yield type of behavior, a very low inflation risk, greater transparency of the Federal Reserve, excess global savings, and low volatility of economic growth. The authors quantify some of these factors and regress the model residuals on these variables (both univariate and multivariate regressions are used). They use implied volatility on longer-term Treasury securities, implied volatilities on six-month-ahead Eurodollar futures, implied volatility on the S&P 500, realized volatility of quarterly GDP growth, realized volatility of monthly core inflation, and foreign official purchases of U.S. Treasury securities as a percentage of U.S. debt in the hands of the public.

In one sense, the approach to regress the model residual on an easily available time series is inconsistent with this model class, which takes pride in allowing no arbitrage opportunities. On the other hand, the quantity of risk, as mentioned above, is at this stage constant in these models. So the idea to especially try to explain the residuals by variations in volatility of some of the factors is very reasonable (taking for granted that the first-best solution to include these exogenous variables directly in the model is intractable at this stage). Obviously, there is also a risk of reverse causality for some of those variables, for example, the low bond yield could attract foreign purchases due to a weak U.S. dollar, or small figures for bond yields could lead to low measured volatilities. But I will abstract from such issues here, as they are likely not to be crucial and simply perform the same exercises suggested by the authors for the German seven-year bond yield residual. The conclusions from Rudebusch, Swanson, and Wu (2006) have been that implied volatility of bond yields and to some smaller extent volatility of GDP growth can explain part of the conundrum. But the authors argue that the major part of the residuals still remains unexplained (48.6 basis points for the BRS model and 23.0 basis points for the RW model). They thus conclude that there really has been a conundrum, at the end of their sample period in mid-2005.

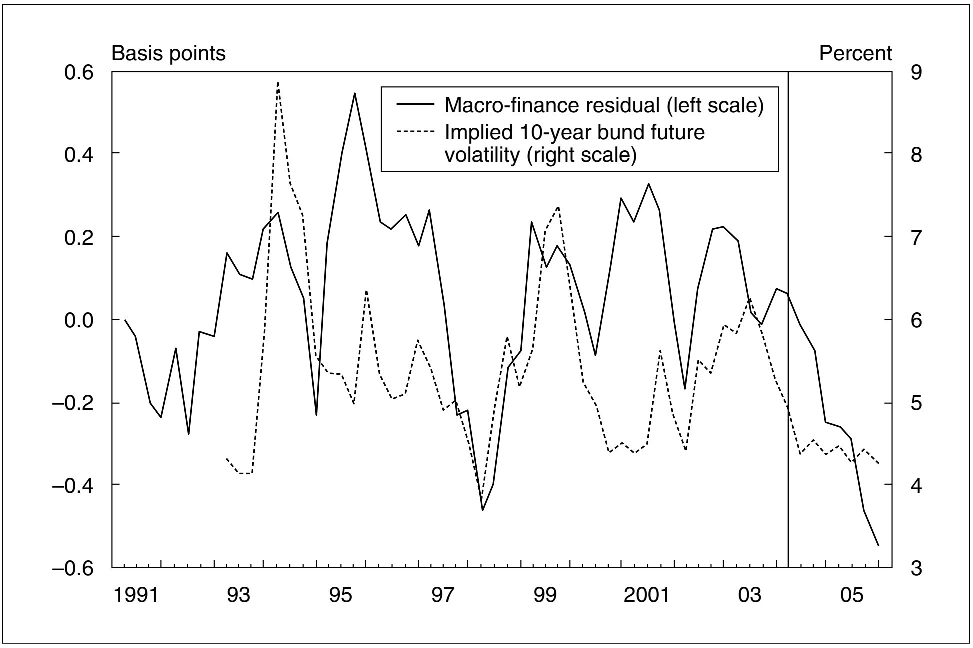
Let me include one remark on the authors' dismissal of the official foreign purchases hypothesis. The data the authors use to measure official foreign purchases are the custodian holdings by the New York Fed for all foreign official institutions (normalized by the U.S. stock of Treasury debt in private hands). This measure declined strongly from fall 2004, thus during the heydays of the conundrum, which is why this variable hardly contributes to explaining it. But there is anecdotal evidence that there has been a widening gap between foreign official purchases of U.S. Treasuries and total foreign purchases of U.S. Treasuries due to the reinvestment of OPEC oil revenues. OPEC investments are often not counted as official purchases by the Fed, because they are frequently invested in U.S. Treasuries via third parties in the City of London and Caribbean banking centers. It is thus possible that a temporary foreign demand factor contributed more to the conundrum than is acknowledged in the paper.

I then use similar measures for some of the "usual suspects" as Rudebusch, Swanson, and Wu (2006) to see how much these factors could explain the depicted euro-conundrum between 2004/II and 2006/I. The macro-finance residuals have been regressed on the Euro Interbank Offered Rate (Euribor) volatility, option-implied 10-year bund future (one-month) volatility, euro area real GDP growth volatility, euro area inflation volatility, and two measures of euro area excess liquidity. I tried both univariate as well as multivariate regressions, both in levels (allowing for autocorrelated residuals of order one) and in first differences, with and without a dummy for the EMU period since 1999 (before that, the euro area data are synthetically constructed from national data). The measures of excess liquidity will be defined below, while the other variables are defined as in Rudebusch, Swanson, and Wu (2006). In contrast to them, I use quarterly data as opposed to monthly data for this exercise.

Only implied bond yield volatility and the measures of excess liquidity had any statistically significant influence on the residual. I will concentrate on these two variables in the following.

Figure 3 plots the implied 10-year bund future volatility and the macro-finance residual. The vertical line indicates 2004/II, the time at which the authors date the start of the conundrum period. While in general Figure 3 visually confirms the regression results that bund future volatility is somehow positively correlated with the unexplained part of the macro-finance model, it is also evident that there has been no further decline in volatility in the period since 2004/II. It is thus not surprising that for the decomposition of the conundrum shown below, and in contrast to the results for the United States, by Rudebusch, Swanson, and Wu (2006), volatility contributed hardly anything to solving the puzzle of low bond yields.

Figure 3 Can Low Bond Volatility Explain the Euro-Conundrum?



The only other variable that turned out to be significant in the regression analysis was euro area excess liquidity. I used two measures of excess liquidity, which are regularly computed at the ECB, the monetary overhang and the real money gap, both corrected for portfolio shifts.

The monetary overhang and the real money gap are defined as follows:

$$\textit{overhang}_t = (m3_t - p_t) - (c + \beta y_t - \gamma(i_t - i_{M3,t})),$$

$$\textit{real money gap}_t = (m3_t - p_t) - (c + \beta y^* - \gamma(i^* - i^*_{M3})),$$

where $m3$ is the M3 monetary aggregate, p stands for the consumer price level, y for real GDP, and i is the nominal short-term interest rate, while i_{M3} is the own interest rate of assets included in M3. All variables are in logs, except the interest rates. c, β, and γ are estimated coefficients from a money demand equation. An asterisk highlights equilibrium values as opposed to time t-variables. While the overhang depicts the current real money balances in excess of money demand given current income and interest rates, the real money gap provides similar information, but money demand is measured by using estimated equilibrium income and equilibrium interest rates.[27]

Figure 4 shows that the typical correlation between the macro-finance residual and the monetary overhang is negative, except for the 1992–93 Exchange Rate Mechanism (ERM) crisis and the following year, as well as sometime in 2002–03. The positive correlation for the ERM crisis period is most likely due to money demand being

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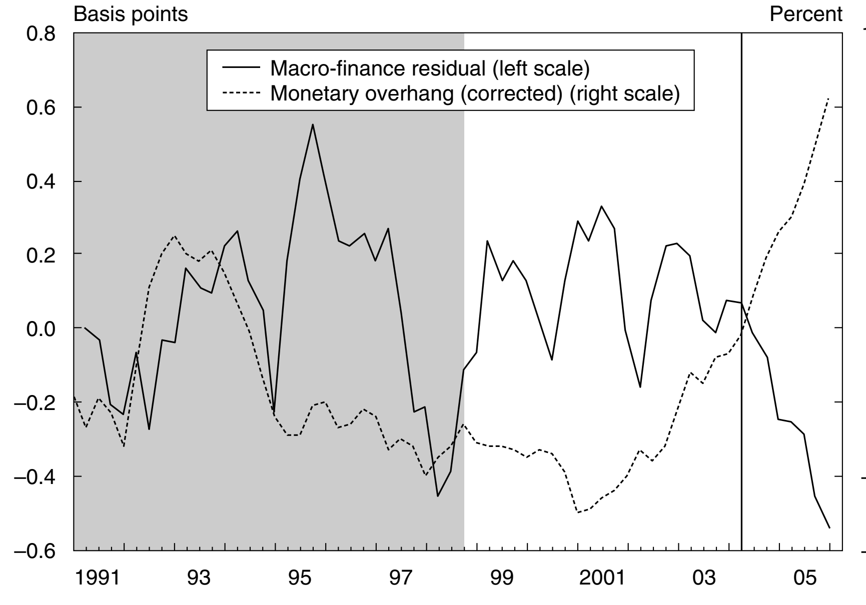
[27] See Masuch, Pill, and Willeke (2001).

Figure 4 "Excess Liquidity" and the Euro-Conundrum



Note: The shaded area depicts the pre-EMU period.

Table 1 Decomposition of the Euro-Conundrum

Contribution to the change in the macro-finance residual of seven-year German bond yield between 2004/II and 2006/I of –61 basis points

	Implied volatility	Monetary overhang	Real money gap
Full sample, levels	–4	–31	–34
Full sample, first difference	–4	–22	–27
EMU dummy, levels	–4	–41	–40
EMU dummy, first difference	–3	–59	–48

underpredicted during the crisis period. The reason was that the average euro area interest rate used in the computation of money demand had been increased by some countries defending the exchange rate peg toward Germany by substantially hiking short-term policy rates. The fitted low money demand probably does not reflect true euro area money demand at the time. Anyway, there is the issue of the appropriate estimation period for the excess liquidity measures. Below I will report results for the whole period and also include an EMU period dummy that is interacted with the excess liquidity measures, and which then focuses on the non-shaded sample period.

Table 1 reports the results of the decomposition of the three variables that turned out to be statistically significant. The estimated coefficients are multiplied with the change in the respective variable over the period from 2004/II to 2006/I to examine the contribution to the widening of the macro-finance residual by 61 basis points over this horizon. The monetary overhang and the real money gap are used alternatively but jointly with the implied volatility measure. Regressions were run in levels (with AR(1) terms if needed) and in first differences. The EMU dummy row shows results where the excess liquidity measures have been interacted with a dummy equaling one for the EMU period and zero otherwise.

The results in Table 1 come not surprisingly after having studied Figures 3 and 4. Implied volatility, despite being statistically significant over the whole period, contributes very little to explaining the conundrum, that is, it explains only 3–4 of the 61 basis points. Even if one starts the decomposition in 2003/III, just before the large drop in volatility (see Figure 3), the contribution would only be 10 basis points. The excess liquidity measures instead explain either about half, or 22–34 basis points, if the whole sample is used, or nearly all of the conundrum if only the EMU sample is used, that is, 40–59 basis points. Obviously the interpretation of this finding with respect to the excess liquidity measures deserves some comment. Depending on which of the differing views, discussed in the next section, is taken, the above results show that the euro-conundrum is either fully or not at all solved.

V. The Role of Excess Liquidity in Explaining the Conundrum

Interestingly, the notion of excess liquidity does not receive much attention in the academic literature, probably due to legitimate conceptual difficulties. However, beyond the academic world it is widely discussed, with a broad spectrum of theories covered. For example, browsing the Internet, one finds at one extreme articles on the dismal effects of monetary liquidity and excess money at the Ludwig von Mises Institute (a libertarian think tank) and at the other extreme a discussion of causes and effects of recent excess savings on the World Socialist Website, besides several investment bank publications on the topic.

The real issue is whether our measures of excess liquidity could provide at least to some degree an exogenous explanation of the bond yield conundrum or whether the measures of excess liquidity and the interest rate conundrum are simply different sides of the same coin. Exceptionally low yields show up in a negative macro-finance residual and simultaneously produce positive monetary overhangs and large real money gaps.

I will not be able to resolve this issue, but would like to provide some arguments why a quick and easy dismissal of the hypothesis that excess liquidity could play an important role in driving asset prices is premature. There are at least four arguments to consider.

First, the measures of euro area excess liquidity used above are corrected for estimated portfolio shifts in and out of M3, which are likely to be only reflections of temporary changes in liquidity preferences (see European Central Bank [2005]). This means that any aggregate increase in uncertainty—which would lead on the one hand to strong demand for safe government bonds and possibly to a negative macro-finance residual and on the other hand fuels excess liquidity via strong demand for assets included in M3—would most likely not be the reason for the negative correlation observed in the right half of Figure 4. Such a portfolio shift would be subtracted from the M3 aggregate used here. The ECB uses different indicators to form a judgment on the existence of portfolio shifts such as the net purchases of non-monetary assets by the money holding sector and the correlation of equity and long-term bond returns. The correction might not, of course, be perfect.

Second, let us assume that low short-term interest rates usually lead to strong excess liquidity. The correlation depicted in Figure 4 could then be the result of a mechanical relation between the macro-finance residual and the long bond yield (assuming the yield curve would move in parallel). Figure 5 shows that this is not the case. The correlation between the macro-finance residual and the level of the seven-year German bond yield, as revealed in Figure 5, has only been +0.18 over the whole sample.

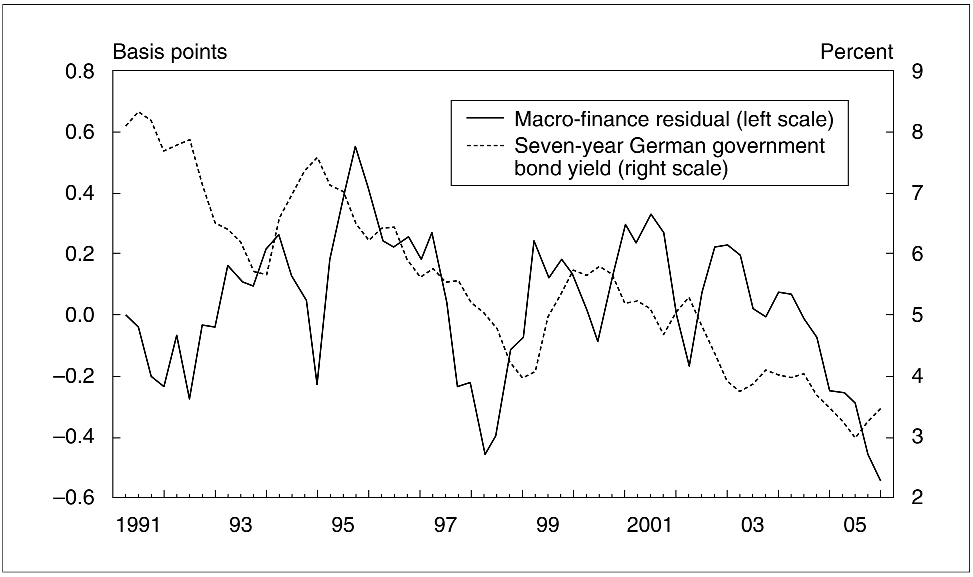
Third, the interest rates used in the computations of the overhang and real gap measures are short- to medium-term rates, while the macro-finance residuals refer to long-term bond yields. In practice and in theory, there is no mechanical link between short and long rates. The slope of the yield curve is endogenous in reality as well as in the macro-finance models. Thus, there is no reason to suspect that periods leading to excessive monetary growth would systematically and mechanically be associated with exceptionally low long-term bond yields.

Fourth, there is other empirical evidence that during asset price boom periods measures of excess liquidity (derived by extracting monetary shocks by means of VARs) contribute to explaining asset price dynamics (especially real residential real estate prices; see Adalid and Detken [2006]).

Still, even if one accepts that contemplating the possible role of excess liquidity is worth further effort, one of the open issues would be what actually determines the phenomenon of excess liquidity.

What is the final verdict on the euro-conundrum? Recently, we have observed a strong and negative correlation between euro area measures of excess liquidity and the

Figure 5 Correlation between Yield Level and Macro-Finance Residual



unexplained part of long-term German bond yields: the macro-finance residual. The degree to which one accepts excess liquidity to be an exogenous factor (or the degree to which it is itself explained by a factor that does not systematically and simultaneously also determine the long rate, such as, for example, the stance of monetary policy) can more or less account for the conundrum. To the degree that this is not the case, one would report the finding of a similar euro-conundrum in 2006/I that Rudebusch, Swanson, and Wu (2006) have reported for the United States in 2005/II. Casual observation of broad money growth in the United States suggests that there could be a similar issue with excess liquidity for the United States as for the euro area. Further research is certainly needed to explore these topics in more depth.

Finally it should be noted that at the time of the conference, the U.S. 10-year bond yield had increased by 100 basis points to about 5.00 percent from the levels reported by the authors for 2005/II. Similarly, the German seven-year bond yield had increased since 2006/I by 25 basis points to 3.75 percent. As of mid-July 2006, there has been another 15-basis-point increase in the respective German bond yield. Of course, the current fitted values of the macro-finance models are not available at the present stage. But given the small size of the conundra, that is, 61 basis points for the German yield and 86.5 basis points (BRS) and 32.2 basis points (RW) for the United States, of which only 48.6 basis points (BRS) and 23.0 basis points (RW) had remained unexplained, most likely the conundra had actually disappeared by the time of the conference.

References

Adalid, R., and C. Detken, "Asset Price Booms and 'Excess' Liquidity," paper presented at the SUERF Seminar "The Adoption of the Euro in the New Member States: Challenges and Vulnerabilities on the Last Stretch," May 4–5, 2006, Malta.

Bernanke, B., V. Reinhart, and B. Sack, "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment," *Brookings Papers on Economic Activity* 2, 2004, pp. 1–78.

Campbell, J., "Bond and Stock Returns in a Simple Exchange Model," *Quarterly Journal of Economics*, 101 (4), 1986, pp. 785–803.

Credit Suisse, "Yield Curve Conundrum," *Quantitative Research*, February 1, 2006.

European Central Bank, "Approaches to Identifying and Estimating Portfolio Shifts into and out of M3," *Monthly Bulletin*, January, 2005, pp. 13–16.

Hördahl, P., O. Tristani, and D. Vestin, "A Joint Econometric Model of Macroeconomic and Term-Structure Dynamics," *Journal of Econometrics,* 131 (1–2), 2006, pp. 405–444.

Kim, D., and J. Wright, "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates," Finance and Economics Discussion Paper No. 33, Board of Governors of the Federal Reserve System, 2005.

Masuch, K., H. Pill, and C. Willeke, "Framework and Tools of Monetary Analysis," in H. J. Klöckers and C. Willeke, eds. *Monetary Analysis: Tools and Applications*, European Central Bank, 2001.

Piazzesi, M., and M. Schneider, "Equilibrium Yield Curves," paper presented at the 2006 NBER Macroeconomic Annual Conference, Cambridge, Massachusetts, 2006.

Rudebusch, G., E. Swanson, and T. Wu, "The Bond Yield 'Conundrum' from a Macro-Finance Perspective," *Monetary and Economic Studies*, 24 (S-1), Institute for Monetary and Economic Studies, Bank of Japan, 2006, pp. 83–109 (this issue).

———, and T. Wu, "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy," working paper, Federal Reserve Bank of San Francisco, 2004.

Comment

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I. Introduction

The authors' paper starts with the observation that bond yields remained relatively low in 2004 and 2005, failing to rise appreciably even as the Federal Reserve increased its federal funds rate target from a low of 1 percent in mid-2004 to 4 percent by year-end 2005. Many hypotheses have been advanced to explain this phenomenon—which then-Fed Chairman Alan Greenspan famously labeled a "conundrum"—but this is the first paper to rigorously evaluate those hypotheses systematically. It does so using two state-of-the-art affine term structure models, both of which include macroeconomic factors, lending the analysis its "macro-finance" perspective. In the end, however, *none* of the candidate explanations provides an entirely satisfactory explanation for the phenomenon: Greenspan was right, it really *is* a conundrum.

The paper is commendable on several dimensions. First, leaving aside anything it might say about the conundrum specifically, it is a nice application of modern macro-finance methods, and very revealing about those methods. Its use of two alternative model specifications is also quite informative, and its analysis of the candidate explanations is impressive in its thoroughness. Finally, the authors are refreshingly modest in their claims. Negative results, like those in the paper, are useful—and all too rarely reported.

This comment has three parts. The first part summarizes some key features of the methods used in the analysis, posing some questions about affine term structure models in general. The second part contains some comments about the model specifications used for the exercise, while the third section sketches another, alternative "lens" for viewing the conundrum. None of this alters the paper's basic conclusion, however: the unusually low bond yields in 2004–05 really *do* represent a conundrum.

II. General Remarks on the Methods

The paper does an excellent job of summarizing the macro-finance methods developed by Ang and Piazzesi (2003), among others, for analyzing the term structure of interest rates. It is useful to note, however, that these methods entail two conceptually distinct elements. The first is a forecasting model for a vector of state variables (call it X_t). Layered on top of the forecasting model is a no-arbitrage asset pricing model, expressing the bond yields as a function of the underlying state variables.

What makes the whole thing tractable is the assumption that the price of risk, λ_t, is a linear function of the state variables: $\lambda_t = \lambda_0 + \lambda_1 X_t$. This assumption allows the bond yields to be written as linear functions of the state variables themselves, with coefficients determined by the λ_0 and λ_1 parameters, and by the coefficients in the forecasting model. In this context, it is useful to note that if the price of risk is zero (i.e., $\lambda_0 = \lambda_1 = 0$), then the framework collapses (approximately) to a risk-neutral expectations hypothesis model, with long-term yields determined (up to a Jensen's inequality term) by the sum of expected future short-term rates. This feature allows the authors to examine their models' risk-neutral yields, as well as the implied risk or term premiums.

To implement this approach, the authors choose two "off-the-shelf" specifications: one is the VAR used by Bernanke, Reinhart, and Sack (BRS [2004]), and the other is the small macro model developed by Rudebusch and Wu (RW [2004]). Having fit the models to the data, the paper confirms the conundrum's existence by comparing the fitted yields to the realizations. Then, in a separate analysis, the model residuals are regressed on proxies representing the candidate explanations. Nothing works: the proxies explain only a fraction of the residuals' variance.

Except for the puzzling 2004–05 episode, both models do an excellent job of fitting the observed yield curve data. And yet, two aspects of the method are vaguely unsatisfying—or even slightly troubling.

First, it is not at all clear (and the authors provide little guidance) as to what belongs in the state vector X_t. The BRS (2004) approach is to throw everything into X_t—conventional macroeconomic variables, plus survey measures of inflation expectations and Eurodollar futures rates. This "kitchen sink" approach makes it hard, if not impossible, to interpret the 30 estimated λ_0 and λ_1 parameters; nor is it clear how one would account for, in macroeconomic terms, fluctuations in bond risk premiums. Thus, while the no-arbitrage pricing model imposes some structure on the yield curve and its evolution over time, the link between the macroeconomy and bond term premiums is obscure, hidden in a reduced-form "black box."

With the price of risk depending on only two latent factors, S_t and L_t, determining the proximate cause of changes in the risk premiums is an easier task, compared with the BRS model. But these factors capture the level and slope of the term structure, rather than macroeconomic fundamentals. This use of yields to "explain" yields makes it difficult to discern the underlying macroeconomic factors affecting the term structure.

A second worrying aspect of the affine term structure methodology is the fact that the coefficients relating the bond yields to the state variables are highly nonlinear functions of the underlying parameters, making estimation "tricky in practice." The authors are commendably candid on this point, noting that the criterion function for the BRS model is characterized by many local minima—and that they had been able to improve significantly on the model's fit, relative to the results originally reported in BRS (2004). This is probably not a major concern if the sole objective is to maximize "fit" (assuming the global minimum in the 30-dimensional parameter space has been determined), but it would be more worrisome for questions involving interpretation or inference.

III. Specific Comments on the Results

Perhaps the most striking feature of the results is that, while both the RW and the BRS models track the term structure very closely, they have very different implications for the behavior of the "risk-free" yield (i.e., the forecast of future short-term rates) and the risk premium. The BRS model, with its 30 estimated coefficients in λ_0 and λ_1, fits the long rate quite closely by having a highly variable term premium. The more parsimonious RW (2004) specification, in contrast, produces a much less volatile term premium—a nearly constant 2 percent—while the highly volatile fitted "risk-free" yield gets most of the credit for fitting the observed bond yield.

For the purpose of this paper—which is simply to "fit" the term structure—exactly how the yield is decomposed into expected future rates and the term premium is of secondary importance. In many contexts this breakdown matters, however; policymakers, for one, would surely like to know whether an observed increase in yields reflects a change in expected future rates, as opposed to a movement in the risk premium. Both models fit the data fairly well, so one cannot choose between them on this basis, yet both cannot be correct. The paper's agnostic treatment of the models is one of its strengths, but it would nonetheless benefit from a discussion of the criteria one might use to distinguish between the two specifications.

One attractive feature of the RW (2004) specification is that it allows a "time-varying medium- to long-term inflation rate" to be extracted from the data as the latent L_t. This factor, which appears as expected inflation the IS and aggregate supply relations, and as the central bank's inflation objective in the monetary policy rule, is assumed to depend on its own lag, with an autoregressive coefficient ρ_L; and on current inflation π_t, and a random shock $\epsilon_{L,t}$.

A question that comes immediately to mind is whether this assumed process for L_t is a reasonable way to model the Fed's inflation objective. True, research by Gürkaynak, Sack, and Swanson (2005) suggests that bond yields behave as though the Fed's implicit inflation target was a distributed lag of past inflation, with weights summing to unity. The RW (2004) specification assumes inflation expectations are "unanchored" in the same way: a 1 percentage point positive inflation shock leads to a permanent increase in inflation—and inflation expectations—of roughly 0.33 percent. This might have been a reasonable description of Fed policy in the late 1980s and early 1990s—a period characterized by inflation in excess of 5 percent, followed by a period of "opportunistic" disinflation. One has to wonder, however, whether this characterization is still appropriate, now that inflation has descended into the range generally associated with price stability. Or, more to the point: an unmodeled shift toward a more "anchored" inflation expectations formation process could, perhaps, explain the apparent conundrum in 2004–05. Further investigation of this hypothesis is surely warranted.

In a similar vein, explanations involving shifts in the equilibrium real interest rate cannot easily be accommodated in either the RW model (which explicitly includes the assumption of a constant natural real interest rate) or the BRS model (in which all variables are assumed to be stationary). Recent research by Laubach and Williams (2003), on the other hand, clearly indicates that the equilibrium real rate has varied considerably over time. Extending the analysis in a way that allows for equilibrium real rate shifts seems worthwhile, especially since some of the candidate hypotheses (e.g., the "savings glut") imply just such a shift.

IV. An Alternative Conundrum Lens

The affine term structure techniques used in the paper are a powerful tool for understanding the bond yield conundrum of 2004–05. Their disadvantage, however, is that they are computation intensive—and as the results illustrate, highly sensitive to model specification. Examining one-year excess holding returns on the 10-year Treasury note provides an alternative lens for viewing the phenomenon: while it cannot explain the cross-section of yields, it provides a way to verify the existence of the conundrum that is independent of model specification. Conceptually, this approach is comparable to Hall's (1978) test of the permanent income hypothesis, whereas the paper's is analogous to Flavin's (1981) use of a VAR to estimate permanent income.

Expressed in terms of excess returns, the expectations hypothesis simply says that the expected one-year holding yield on an n-year bond should equal the (known) yield on a one-year bond—in other words, that the expected one-year excess return on the one-year bond should be zero (or a constant, departing from the "pure" expectations assumption). Thus, under the expectations hypothesis, a regression of this excess return on information known at time t should yield a zero R-squared; a forecastable component to the excess return implies a time-varying risk or term premium.

What is unusual about the 2004–05 episode is that bond yields remained relatively unchanged, despite the Fed's campaign to raise interest rates. This suggests that it would be useful to compare the excess returns over this period with those in other tightening periods. Since the object of analysis is the excess return over the following year, it makes sense to focus in on the excess returns on bonds held in the 12 months prior to the first increase in the federal funds rate. These "pre-tightening" periods are shown by the shaded areas in Figure 1, along with the federal funds rate itself.

Figure 2 plots the excess return, relative to a one-year security, on the 10-year note purchased at the date shown on the horizontal axis, and sold (as a nine-year note) one year later.[28] What is striking is that an investor holding a 10-year note during a "pre-tightening" period would have consistently experienced large, negative excess returns over the subsequent year. Consistently, that is, until 2004—when that investor would have earned about the same by holding a 10-year note for one year as he or she would have by holding a one-year security. Thus, it is the *absence* of a negative excess return in 2004 that is the conundrum.

The question, then, is *why* negative excess returns seemed to have been the predominant pattern until 2004. One hypothesis is that investors were always surprised by rate hikes, failing to incorporate forecasts of higher short-term interest rates into bond prices. An alternative hypothesis is that investors knew bond prices would fall—but willingly accepted below-average (or even negative) returns because of a decline in the risk premium. This simple approach of examining excess holding returns cannot distinguish between these two hypotheses, although the fully articulated RW and BRS models can potentially do so.

However, there is reason to suspect that investors' ability to forecast policy turning points has improved in recent years, as the Fed has become more "transparent" with

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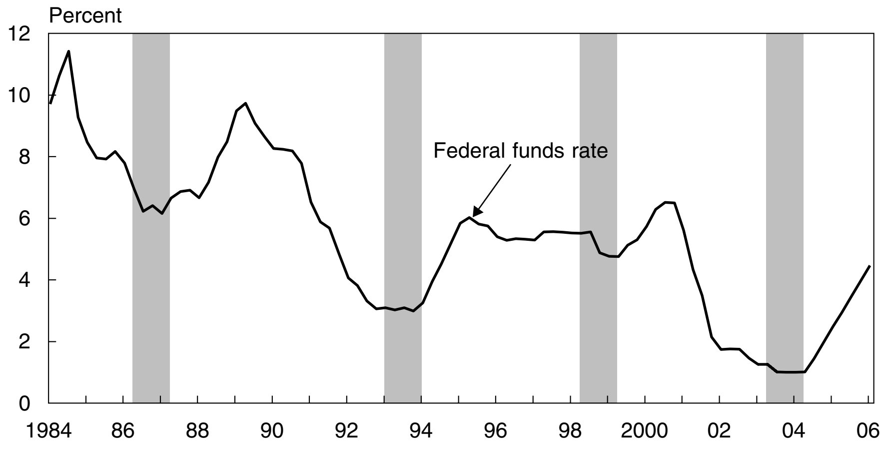
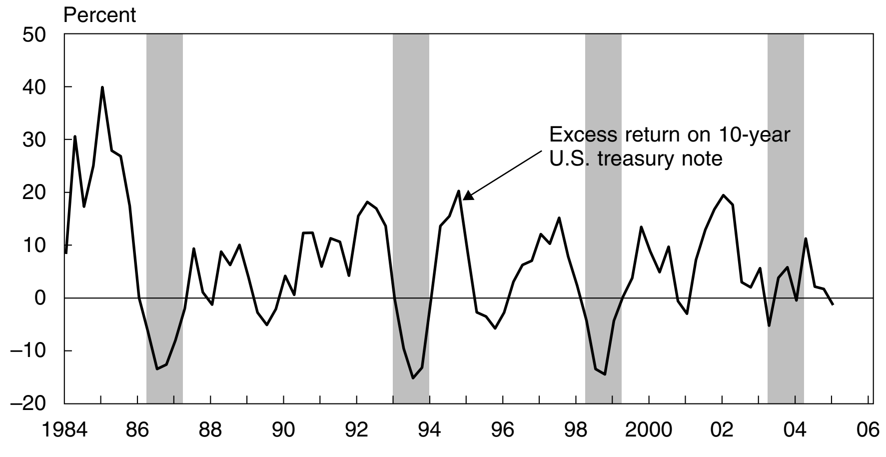
[28] The data are end-of-quarter hypothetical zero-coupon yields derived using the Nelson-Siegel-Svensson (Svensson [1995]) method, and are graciously provided by Eric T. Swanson.

Figure 1 The Federal Funds Rate and "Pre-Tightening" Periods, 1984–2006



Note: Shaded areas indicate "pre-tightening" periods.

Figure 2 The One-Year Excess Return on the 10-Year Bond

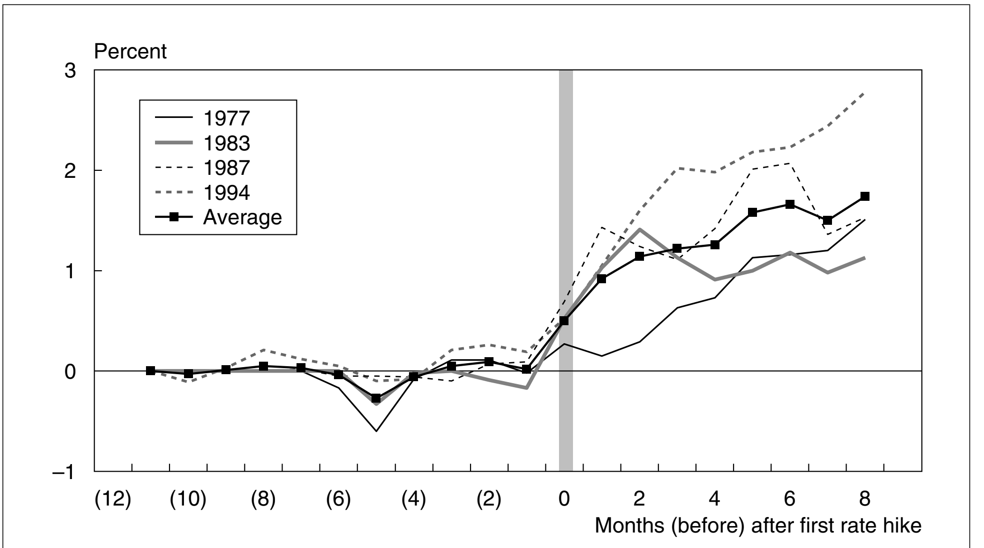


Note: Shaded areas indicate "pre-tightening" periods.

regard to its intentions. This is yet another candidate explanation for the 2004 conundrum, and by including the Eurodollar futures rate, the BRS (2004) specification may be able to capture this effect. As shown in Figure 3, in the 1977, 1983, 1987, and 1994 tightening cycles, there is virtually no tendency for the two-year yield to anticipate the increase in the federal funds rate; only once the tightening commences does the yield rise.

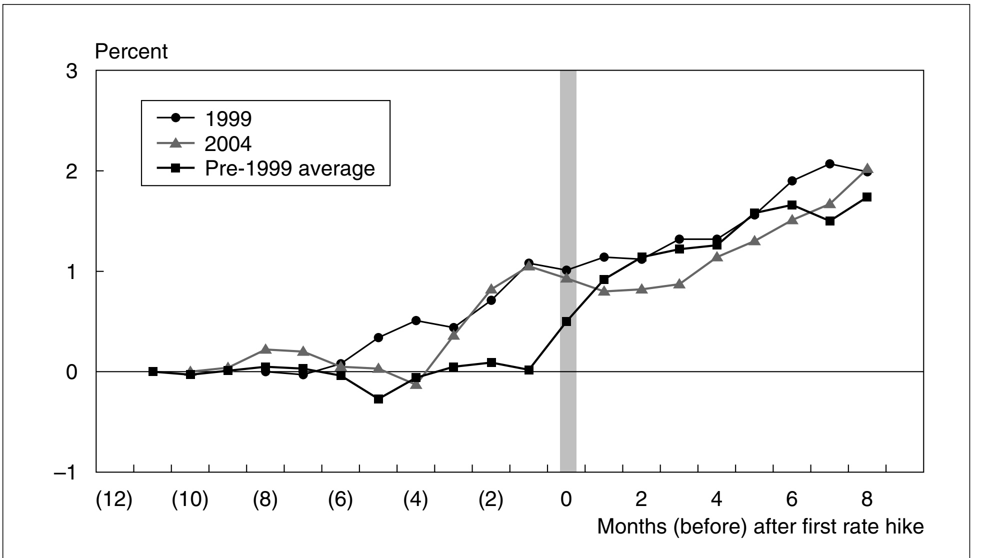
Figure 4 shows that the 2004–05 episode was, in fact, different: the two-year yield began rising three months before the first increase in the federal funds rate, and was

Figure 3 The Two-Year Yield during Tightening Periods, 1977–94



Note: Shaded area indicates the month of the first rate hike.

Figure 4 The Two-Year Yield during Tightening Periods, 1999 and 2004



Note: Shaded area indicates the month of the first rate hike.

a full percentage point higher when the tightening began. The problem, from the standpoint of explaining the conundrum, is that a similar pattern was observed before the 1999 tightening cycle, which was also apparently anticipated several months prior to the first rate hike. An increase in transparency therefore cannot explain both the near-zero excess returns in 2004 *and* the negative excess returns in 1999.

To see whether the negative pre-tightening returns might reflect a forecastable risk premium, the excess return was regressed on two of the most likely suspects: the forward rate factor identified by Cochrane and Piazzesi (2005), and the real activity factor constructed by Ludvigson and Ng (2005).[29] Both have been shown to have considerable forecasting power—the Cochrane-Piazzesi factor alone typically yields R-squareds of more than 0.40 in excess return regressions.

Neither can explain the prevalence of negative excess returns during the periods prior to Fed tightenings, however. While the Cochrane-Piazzesi factor performs well overall in forecasting the excess return on 10-year Treasuries, it does not generally predict negative returns in these pre-tightening episodes. And while the Ludvigson-Ng real activity does a good job of predicting excess returns at the short end of the maturity spectrum, it has relatively little forecasting power at the longer, 10-year maturity.

V. Conclusions

In the end, this cursory analysis, like that in the authors' paper, confirms the conundrum: long-term yields during the 2004–05 episode really did behave differently than they did in the past. The paper also provides an excellent example of how state-of-the-art macro-finance models might be used to address such questions, rigorously integrating finance theory with sensible macroeconomic models. That the analysis failed to uncover the proverbial "smoking gun" is in no way a shortcoming of the technique—although the analysis did reveal some questionable aspects of the affine term structure method.

Although the paper meticulously covered a number of interesting hypotheses, two did not receive the attention they deserved. One is that there has been a change in the expectations formation process, resulting in inflation expectations that are better "anchored" than in the past. The second is a reduction in the equilibrium real interest rate resulting from factors exogenous to the United States, such as a global savings glut. Perhaps future research will show that one of these—or some combination of the two—is capable of resolving Greenspan's conundrum.

References

Ang, Andrew, and Monika Piazzesi, "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables," *Journal of Monetary Economics*, 50 (4), 2003, pp. 745–787.

Bernanke, Ben S., Vincent R. Reinhart, and Brian P. Sack, "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment," *Brookings Papers on Economic Activity*, 2, 2004, pp. 1–78.

Cochrane, John, and Monika Piazzesi, "Bond Risk Premia," *American Economic Review*, 95 (1), 2005, pp. 138–160.

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[29] To forecast excess returns on two- through five-year securities, Cochrane and Piazzesi (2005) constructed a factor from the one-year rate and the two- through five-year-ahead forward rates. To forecast the excess return on a *10*-year security, a procedure analogous to theirs was used to construct a forward rate factor from the one-year rate, and the two- through nine-year-ahead forward rates.

Flavin, Marjorie A., "The Adjustment of Consumption to Changing Expectations about Future Income," *Journal of Political Economy,* 89 (5), 1981, pp. 974–1009.

Gürkaynak, Refet, Brian P. Sack, and Eric T. Swanson, "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," *American Economic Review*, 95 (1), 2005, pp. 425–436.

Hall, Robert E., "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86 (6), 1978, pp. 971–987.

Laubach, Thomas, and John Williams, "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, 85 (4), 2003, pp. 1063–1070.

Ludvigson, Sydney, and Serena Ng, "Macro Factors in Bond Risk Premia," NBER Working Paper No. 11703, National Bureau of Economic Research, 2005.

Rudebusch, Glenn D., Eric T. Swanson, and Tao Wu, "The Bond Yield 'Conundrum' from a Macro-Finance Perspective," *Monetary and Economic Studies*, 24 (S-1), Institute for Monetary and Economic Studies, Bank of Japan, 2006, pp. 83–109 (this issue).

———, and Tao Wu, "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy," working paper, Federal Reserve Bank of San Francisco, 2004.

Svensson, Lars E. O., "Estimating and Interpreting Forward Interest Rates: Sweden 1992–1994," *Sveriges Riksbank Quarterly Review*, 3, 1995, pp. 13–26.

General Discussion

As a rejoinder to the discussants, Glenn D. Rudebusch responded to the comments on specifications and results of the models. In response to Carsten Detken's comment on the model specifications and a latent slope factor in the Rudebusch-Wu model, he stated that the model was parsimonious and the latent slope factor worked remarkably well in terms of Taylor rule monetary policy interpretation. Regarding the inconsistency of the idea to explain the *ex post* residual on other variables, Rudebusch acknowledged that they could have been added to the models but did not come out successfully. In response to Kenneth N. Kuttner's question on how to choose the dependence of risk price variability on certain factors, Rudebusch explained that he chose it arbitrarily and there was a whole set of variables that could affect the risk price. Regarding the difficulties in estimation due to nonlinearity of coefficients relating yields to the state variables, Rudebusch acknowledged them as a drawback to these models.

Regarding the distinction between the models that yield different decomposition results into the risk-neutral yield and term premium, Rudebusch's co-author, Tao Wu (Federal Reserve Bank of Dallas), explained that the expectations of the future short-term rate would revert to some time-varying value in the Rudebusch-Wu model. He added that this was contrary to the Bernanke-Reinhart-Sack (BRS) model, where it would approach its sample mean very quickly. He stated that this was the reason why the estimates of the risk-neutral 10-year yield in the Rudebusch-Wu model were more volatile than in the BRS model. Rudebusch expressed his view that a drifting process better approximated inflation over the estimation period than a mean-reverting process.

In the general discussions, there were some comments on candidate explanations of the conundrum examined in the paper. Hung Tran (International Monetary Fund) claimed that the demand by pension funds for which the authors could not find a proxy was worth exploring because a variety of regulatory and accounting changes in the United States and Europe would keep the demand for long-term government bonds high. Rudebusch responded that if the conundrum had gone away in the last six months, pension fund reform would not explain the conundrum. Seng-Guan Toh (Bank Negara Malaysia) asked for a quantitative assessment of the contribution of a precommitment or the change in communication by the Federal Reserve to explaining the conundrum. Rudebusch responded that he tried to account for transparency at the Fed in terms of the implied volatility of the short-term Eurodollar options but it did not work. Thammarak Moenjak (Bank of Thailand) proposed to take a first difference of some variables used in regressions because they were not significant due to shifting means.

As alternative explanations for the conundrum, many participants stressed the importance of global factors. Masaaki Shirakawa (Bank of Japan) pointed out that if there was a conundrum in the U.S. bond yields, Japanese government bonds would be attractive and thus the conundrum would be exported from one country to another. Emphasizing the increased correlation between U.S. and Japanese bond yields, he proposed that the global factor be incorporated into the analysis. Kenneth J. Singleton (Stanford University) pointed out that a culprit for the conundrum might be the extensive degree to which carry trades are being taken on around the world. Having looked at the very similar conundrum in Europe that Detken showed, Fabrizio Perri (New York University) commented that a global explanation was probably in order. He also pointed out that high oil prices would help to explain the conundrum because one possible explanation of a world savings glut was high oil prices that increase the wealth of some oil-producing countries and these countries preferred to invest in euro and U.S. bonds. David Longworth (Bank of Canada) remarked that then-Chairman Greenspan's statement of the conundrum was not about the United States but about the world and therefore some of explanations that were put forward could not be assessed, unless we had a model in the context of the world.

Rudebusch agreed that the conundrum might be an international phenomenon. In response to Singleton, he stated that the carry trade was probably closely related to the conundrum but he had not thought about how to model it or structure a discussion. He agreed with Perri that high oil prices would be a manifestation of the global savings glut. He also pointed out that examining excess savings not just from oil-producing countries but also from Asia would be one way to think about the conundrum.

Some questions were raised on interpretations of the conundrum. Ulrich Kohli (Swiss National Bank) argued that if monetary policy was tightened more than expected, one would end up with lower inflation and lower nominal interest rates in the future and therefore it would not be surprising to see the long rates going down when interest rates were rising at the short end. Rudebusch responded that the rise in long rates with imperfect credibility was a little more typical than the decrease with perfect credibility. Shirakawa questioned the distinction between a conundrum and a bubble. Rudebusch responded that the term "bond market bubble" was not normally used, because a rational bubble with finite maturity was impossible whereas some irrational bubble is possible.