The capacitated lot sizing problem: a review of models and algorithms

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Abstract

Lot sizing is one of the most important and also one of the most difficult problems in production planning. This subject has been studied extensively in the literature. In this article, we consider single-level lot sizing problems, their variants and solution approaches. After introducing factors affecting formulation and the complexity of production planning problems, and introducing different variants of lot sizing and scheduling problems, we discuss single-level lot sizing problems, together with exact and heuristic approaches for their solution. We conclude with some suggestions for future research.
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1. Introduction

Production planning is an activity that considers the best use of production resources in order to satisfy production goals (satisfying production requirements and anticipating sales opportunities) over a certain period named the *planning horizon*.

Production planning typically encompasses three time ranges for decision making: *long-term*, *medium-term* and *short-term*. In long-term planning usually the focus is on anticipating aggregate needs and involves such strategic decisions as product, equipment and process choices, facility location and design, and resource planning. Medium-term planning often involves making decisions on material requirements planning (MRP) and establishing production quantities or lot sizing over the planning period, so as to optimise some performance criteria such as minimising overall costs, while meeting demand requirements and satisfying existing capacity restrictions. In short-term

planning, decisions usually involve day-to-day scheduling of operations such as job sequencing or control in a workshop.

In this review the focus is on medium-term production planning and especially on *single-level lot sizing* decisions. Lot sizing decisions give rise to the problem of identifying when and how much of a product to produce such that setup, production and holding costs are minimised. Making the right decisions in lot sizing will affect directly the system performance and its productivity, which are important for a manufacturing firm's ability to compete in the market. Therefore, developing and improving solution procedures for lot sizing problems is very important. The applicability of these problems arises commonly in operations such as forging and casting and in industries which consist of a single production process, or where all production process can be considered as a single operation, such as some medical or chemical industries.

After an introduction to lot sizing problems, this paper will focus on the capacitated lot sizing problem and will review the main contributions to this long standing but active research field focusing, particularly, on developments that have taken place since research was reviewed and compared in [1,2].

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2. Characteristics of lot sizing models

The complexity of lot sizing problems depends on the features taken into account by the model. The following characteristics affect classifying, modelling and the complexity of lot sizing decisions.

2.1. Planning horizon

The *planning horizon* is the time interval on which the master production schedule extends into the future. The planning horizon may be *finite* or *infinite*. A finite-planning horizon is usually accompanied by dynamic demand and an infinite planning horizon by stationary demand. In addition, the system can be observed continuously or at discrete time points, which then classifies it as a continuous- or discrete-type system. In terms of time period terminology, lot sizing problems fall into the categories of either *big bucket* or *small bucket* problems. Big bucket problems, are those where the time period is long enough to produce multiple items (in multi-item problem cases), while for small bucket problems the time period is so short that only one item can be produced in each time period. Another variant of the planning horizon is a *rolling horizon* usually considered when there is uncertainty in data. Under this assumption, optimal approaches for each horizon act as heuristics but cannot guarantee the optimal solution.

2.2. Number of levels

Production systems may be *single-level* or *multi-level*. In single-level systems, usually the final product is simple. Raw materials, after processing by a single operation such as forging or casting, are changed to final product. In other words, the end item is directly produced from raw materials or purchased materials with no intermediate subassemblies. Product demands are assessed directly from customer orders or market forecasts. This kind of demand, as will be further discussed later, is known as *independent demand*. In multi-level systems, there is a parent–component relationship among the items. Raw materials after processing by several operations change to end products. The output of an operation (level) is input for another operation. Therefore, the demand at one level depends on the demand for its parents' level. This kind of demand is named *dependent demand*. Multi-level problems are more difficult to solve than single-level problems.

Multi-level systems are further distinguished by the type of product structure, which includes *serial*, *assembly*, *disassembly and general* or *MRP* systems.

2.3. Number of products

The number of end items or final products in a production system is another important characteristic that affects the modelling and complexity of production planning prob-

lems. There are two principal types of production system in terms of number of products. In *single-item* production planning there is only one end item (final product) for which the planning activity has to be organised, while in *multi-item* production planning, there are several end items. The complexity of multi-item problems is much higher than that of single-item problems. van Hoesel and Wagelmans [3] provide theoretical results for the performance of algorithms for the single item capacitated lot sizing problem. (See also Section 4 of this paper.)

2.4. Capacity or resource constraints

Resources or capacities in a production system include manpower, equipment, machines, budget, etc. When there is no restriction on resources, the problem is said to be *uncapacitated*, and when capacity constraints are explicitly stated, the problem is named *capacitated*. Capacity restriction is important, and directly affects problem complexity. Problem solving will be more difficult when capacity constraints exist.

2.5. Deterioration of items

In the case that deterioration of items is possible, we encounter restrictions in the inventory holding time. This in turn is another characteristic which would affect problem complexity.

2.6. Demand

Demand type is considered as an input to the model of the problem. *Static demand* means that its value does not change over time, it is stationary or even constant, while *dynamic demand* means that its value changes over time. If the value of demand is known in advance (static or dynamic), it is termed *deterministic*, but if it is not known exactly and the demand values occurring are based on some probabilities, then it is termed *probabilistic*. In *independent* demand cases, an item's requirements do not depend on decisions regarding another item's lot size. This kind of demand can be seen in single-level production systems. In multi-level lot sizing, where there is a parent–component relationship among the items, because the demand at one level depends on the demand for their parents (pervious level), it is called *dependent*. Problems with dynamic and dependent demands are much more complex than problems with static and/or independent demands. Also, problems with probabilistic demand will be more complex than problems with deterministic demand.

2.7. Setup structure

Setup structure is another important characteristic that directly affects problem complexity. Setup costs and/or setup