The Capacity of the Ergodic MISO Channel with Per-antenna Power Constraint and an Application to the Fading Cognitive Interference Channel

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***Abstract*—To be considered for an IEEE Jack Keil Wolf ISIT Student Paper Award. This paper characterizes the ergodic capacity of the fading multiple-input single-output (MISO) channel with per-antenna power constraints (PerPC) with perfect Channel State Information (CSI) at all terminals. This turns out to be the sum-capacity achieving strategy for the ergodic fading Gaussian overlay cognitive interference channel (EGCIFC) in the strong interference regime. The EGCIFC is a two-user time-varying interference channel in which a primary / licensed transmitter and a secondary / cognitive transmitter share the same spectrum and where the cognitive transmitter has non-causal knowledge of the primary user's message. The MISO and the EGCIFC results are verified numerically for the case of independent Rayleigh fading gains. Different achievable strategies, corresponding to different amount of CSI, are compared to show the performance of the derived PerPC optimal power allocation.**

I. Introduction

Wireless communications are challenging because they must effectively deal with: *fading*, the time variation of the channel gains due to small scale effects of multi-path fading and large-scale effects such as path loss and shadowing, and *interference* between wireless users communicating over the same frequency band. In this work we study two wireless channel models that capture these effects.

We first characterize the ergodic capacity of the fading multi-input single-output (MISO) point-to-point Gaussian noise channel subject to long-term average per-antenna power constraints (PerPC) when the Channel State Information (CSI) is available to both the transmitter and the receiver.

We then consider an application to the fading two-user cognitive interference channel, where a secondary/cognitive user and a primary user share the same spectrum and where, according to the overlay paradigm [1], the secondary user has non-causal knowledge of the message the primary transmitter. We term this channel model the Ergodic Fading Gaussian Overlay Cognitive Interference Channel (EGCIFC). The EGCIFC is inspired by the idea of layered cognitive networks: the first layer consists of primary users and each additional layer consists of cognitive users that share the same spectrum. Each additional layer is given the codebook(s) of all previous layers. This hierarchical codebook knowledge enables them to causally learn the lower layers' messages and aid in their transmission. Thus studying the non-causal message knowledge setting [2] provides an upper bound to the more realistic case where the messages are causally learned by the cognitive transmitter(s). We characterize the ergodic sum-capacity of the EGCIFC when all nodes have perfect instantaneous CSI. In particular, we show that the optimal power allocation policy for the MISO channel with PerPC is sum-capacity optimal when the EGCIFC experiences *strong interference* – weak interference is treated in [3].

A. Prior work

In [4] the author derived the capacity for the MISO channel with PerPC under two different CSI models: (i) for a constant channel with CSI at both the transmitter and the receiver (numerically evaluated), and (ii) for a Rayleigh fading channel with CSI at the receiver only (analytical expressions). The MISO channel with PerPC is a more realistic model than the usual sum/trace power constraint (SumPC) across transmit antennas; for example, the downlink of cellular systems are power-limited per each transmit RF chain (per antenna). Through numerical examples, [4] compared the capacity with PerPC to the capacity with SumPC, which is achieved by the water-filling power allocation. In this work we *analytically* characterize the capacity of the fading MISO channel with CSI at *both* the transmitter and the receiver under PerPC.

In [5] the two-user Gaussian interference channel with ergodic fading was introduced and the optimal power allocation policy that maximizes the outer bound with CSI at all nodes was investigated. In [6] the authors also considered the two-user ergodic fading Gaussian interference channel with perfect CSI at all nodes and characterized the sum-capacity for certain channels; specifically, for the case of uniformly strong interference (where in every fading state each user experiences strong interference), and for the case of ergodic very strong interference (where on average each user experiences very strong interference but instantaneously the users can experience strong, mixed or weak interference). In these cases, [6] proved that encoding and decoding jointly across fading states achieves the sum-capacity and that the optimal power allocation is that of a compound Multiple Access Channel (MAC). In this work we consider a variation of the two-user interference channel where one of the users is *cognitive*, meaning that it non-causally knows the message of the other user [7]. The capacity of the static/non-fading cognitive interference channel is known exactly in some regimes and to within 1 bit otherwise [8]. In this work we remove the assumption of constant channel gains and consider instead the ergodic fading (time varying) case; for this fading channel we show that the sum-capacity in the strong interference regime is achieved by the strategy that attains the capacity of the fading MISO channel under PerPC. The derivation of the sum-capacity achieving power allocation policy for the general EGCIFC, i.e., not in strong interference, is not reported here for sake of space and can be found in [3].

B. Contributions and Paper Outline

Section II contains the main result: the capacity achieving strategy for the ergodic MISO channel with PerPC, full CSI at all terminals, and with an arbitrary number of transmit antennas. Section III introduces the single-antenna EGCIFC channel model, characterizes its sum-capacity, and shows that in the case of strong interference the sum-capacity is achieved by the strategy derived in Section II. Section IV provides numerical results for the case of independent Rayleigh fading, both for the MISO channel with PerPC and the single-antenna EGCIFC. Section V concludes the paper.

II. The ergodic MISO channel with PerPC

The MISO channel with n transmit antennas has output

$$Y = [H_1 \ H_2 \cdots H_n] \ \mathbf{X} + Z \in \mathbb{C}, \quad Z \sim \mathcal{N}(0,1),$$

where each entry of the input vector $\mathbf{X} := [X_1, \cdots, X_n]^T$ has a separate long-term average transmit power constraint $\mathbb{E}[|X_i|^2] \le \overline{P}_i$, for $i \in [1:n]$. The channel vector $[H_1 \ H_2 \cdots H_n]$ has complex-valued entries representing the channel gain coefficient from each transmit antenna to the receive antenna and is generated from an ergodic process whose instantaneous realization is assumed to be known to the transmitter and the receiver. We aim to characterize the ergodic capacity of this channel under PerPC, where achievable rate and capacity is defined as usual [9].

In the following, we indicate the instantaneous realization of the channel vector as $\mathbf{h} := [h_1 \ \cdots \ h_n] \in \mathbb{C}^n$ and the power allocated on antenna i in fading realization $\mathbf{h}$ as $P_i(\mathbf{h}), i \in [1:n]$. We let $\log^+(x) := [\log(x)]^+$ with $[x]^+ := \max\{0, x\}$.

Theorem 1. *The ergodic capacity of the Gaussian fading MISO channel with PerPC is the solution of*

$$\mathbb{E}\left[\max \ \log\left(1 + \Big(\sum_{i\in[1:n]} |h_i|\sqrt{P_i(\mathbf{h})}\Big)^2\right)\right] \quad (1)$$

where the maximization in (1) *is over all power allocation policies* $P_i(\mathbf{h}) \ge 0 : \mathbb{E}[P_i(\mathbf{h})] \le \overline{P}_i, \ i \in [1:n]$. *The optimal power allocation policy for antenna* $j \in [1:n]$ *is given by*

$$P_j^\star(\mathbf{h}) = \frac{\left[\sum_{i\in[1:n]} \frac{|h_i|^2}{\lambda_i} - 1\right]^+}{\left(\sum_{i\in[1:n]} \frac{|h_i|^2}{\lambda_i}\right)^2} \ \frac{|h_j|^2}{\lambda_j^2}, \quad (2)$$

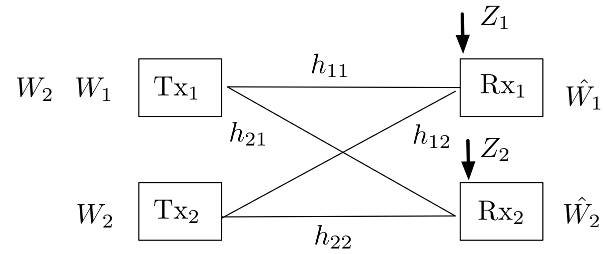


Fig. 1. The ergodic fading Gaussian cognitive interference channel.

where the Lagrange multipliers $\{\lambda_j, \ j \in [1:n]\}$ *solve the non-linear system of equations* $\mathbb{E}[P_j^\star(\mathbf{h})] = \overline{P}_j, \ j \in [1:n]$, *and attains*

$$C_{\mathrm{MISOPerPC}} = \mathbb{E}\left[\log^+\left(\sum_{i\in[1:n]} \frac{|h_i|^2}{\lambda_i}\right)\right] \quad (3)$$

Proof. The proof is based on solving the Lagrange dual problem to (1) and is provided in Appendix. The capacity in (3) can be obtained by *beamforming*: each antenna transmits

$$X_i = \exp\{-\mathrm{j}\angle h_i\}\sqrt{P_i^\star(\mathbf{h})} \ U, \ U \sim \mathcal{N}(0,1), \ i \in [1:n],$$

where $P_i^\star(\mathbf{h})$ is given by (2) and $\angle h_i$ is the phase of h_i . By taking the average over all fading states, the capacity can be expressed as in (3). □

Remark 1: If all Lagrange multipliers are equal to λ, then the optimal power allocation in (2) becomes

$$P_j^\star(\mathbf{h}) = \left[\frac{1}{\lambda} - \frac{1}{\|\mathbf{h}\|^2}\right]^+ \frac{|h_j|^2}{\|\mathbf{h}\|^2}, \ j \in [1:n], \quad (4)$$

with $\|\mathbf{h}\|^2 := \sum_{i\in[1:n]} |h_i|^2$. The expression in (4) corresponds to the water-filling power allocation optimal under SumPC, in which case the Lagrange multiplier would satisfy

$$\mathbb{E}\left[\sum_{i\in[1:n]} P_i^\star(\mathbf{h})\right] = \mathbb{E}\left[\left[\frac{1}{\lambda} - \frac{1}{\|\mathbf{h}\|^2}\right]^+\right] = \sum_{i\in[1:n]} \overline{P}_j.$$

This can happen if the power constraint on each antenna is the same and the distribution of the fading vector does not change by permuting its components, such as in i.i.d. fading.

Remark 2: As mentioned earlier, in [4] the analytical capacity of this channel model was derived under the assumption of CSI at the receiver only. Here we consider the case of CSI at both transmitter and receiver, and obtain the capacity with PerPC in closed-form, a problem left open in [4].

III. The ergodic fading Gaussian cognitive interference channel (EGCIFC)

The cognitive interference channel consists of two transmit-receive pairs Tx_1 to Rx_1 and Tx_2 to Rx_2 representing the cognitive and primary users, respectively, as shown in Fig. 1. The channel is cognitive in the sense that the secondary user Tx_1 has non-causal message knowledge of the primary user's message W_2. Each transmitter Tx_k wishes to convey to its destination Rx_k an independent message W_k, which is uniformly distributed over the set $[1 : 2^{NR_k}]$ for $k \in [1 : 2]$, where R_k is the rate in bits per channel use of message W_k, and N represents the codeword length. A rate vector (R_1, R_2) is said to be achievable if messages W_k can be simultaneously encoded at rates R_k, for $k \in [1 : 2]$, with the probability of decoding error made arbitrarily small. The sum-capacity is defined as the maximum achievable $R_1 + R_2$.

In Gaussian noise and with ergodic fading, the channel input-output relationship is given by

$$Y_1 = h_{11}X_1 + h_{12}X_2 + Z_1,\ Z_1 \sim \mathcal{N}(0, 1), \tag{5}$$

$$Y_2 = h_{22}X_2 + h_{21}X_1 + Z_2,\ Z_2 \sim \mathcal{N}(0, 1), \tag{6}$$

where $\mathbf{H} := \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ denotes the random channel gain matrix, with $[\mathbf{H}]_{i,j} = h_{ij} \in \mathbb{C},\ i, j \in [1 : 2]$ representing the fading channel gain between Tx_j and Rx_i. A realization of $\mathbf{H}$ is indicated as $\mathbf{h}$. Tx_j is subject to the long-term average power constraint $\mathbb{E}[|X_j|^2] \le \overline{P}_j,\ j \in [1 : 2]$. With CSI at all terminals, the transmitters can perform dynamic power allocation $P_j(\mathbf{h}) \ge 0,\ j \in [1 : 2]$, which is a function of the instantaneous channel gain matrix $\mathbf{h}$. We seek the optimal power allocation, which must satisfy $\mathbb{E}[P_j(\mathbf{h})] \le \overline{P}_j,\ j \in [1 : 2]$.

In the following, $A^\dagger$ represents the transpose and complex-conjugate of the matrix A, while $A^\star$ represents the optimal solution for a given optimization problem. Thanks to cognition, the channel inputs can be correlated; the correlation coefficient and the input covariance matrix in fading state $\mathbf{h}$ are

$$\rho(\mathbf{h}) := \frac{\mathbb{E}\left[X_{1i}X_{2i}^\dagger|\mathbf{H} = \mathbf{h}\right]}{\sqrt{P_1(\mathbf{h})\ P_2(\mathbf{h})}}, \tag{7}$$

$$\Sigma(\mathbf{h}) := \begin{bmatrix} P_1(\mathbf{h}) & \rho^\dagger(\mathbf{h})\sqrt{P_1(\mathbf{h})P_2(\mathbf{h})} \\ \rho(\mathbf{h})\sqrt{P_1(\mathbf{h})P_2(\mathbf{h})} & P_2(\mathbf{h}) \end{bmatrix}. \tag{8}$$

Our goal is to determine the ergodic sum-capacity of the EGCIFC. The sum-capacity, presented next, is the solution of a maximization problem in the variables $(P_1(\mathbf{h}), P_2(\mathbf{h}), \rho(\mathbf{h}))$, where the two powers are subject to a long-term average power constraint, while the correlation coefficient must satisfy $|\rho(\mathbf{h})| \le 1$ for each fading realization. We have:

Theorem 2. *The ergodic sum-capacity of the EGCIFC is the solution of the following optimization problem*

$$\max_{P_i(\mathbf{h}) \ge 0 : \mathbb{E}[P_i(\mathbf{h})] \le 1, i \in [1:2], |\rho(\mathbf{h})| \le 1} \mathbb{E}\Big[\log\left(1 + \mathbf{h}_2\Sigma(\mathbf{h})\mathbf{h}_2{}^\dagger\right) + \log\left(\frac{1 + (1 - |\rho(\mathbf{h})|^2)\max\{|h_{11}|^2, |h_{21}|^2\}P_1(\mathbf{h})}{1 + (1 - |\rho(\mathbf{h})|^2)|h_{21}|^2P_1(\mathbf{h})}\right)\Big] \tag{9}$$

with $\mathbf{h}_2 := [h_{21}\ h_{22}]$ (the index 2 refers to the second row of the instantaneous fading realization channel matrix $\mathbf{h}$).

Proof. The sum-capacity is upper-bounded, for any $\epsilon_N > 0$, by

$$\begin{aligned}
N(R_1 + R_2 - 2\epsilon_N) &\overset{(a)}{\le} I(W_1; Y_1^N|\mathbf{h}^N) + I(W_2; Y_2^N|\mathbf{h}^N) \\
&\overset{(b)}{\le} I(W_1; Y_1^N, Y_2^N|\mathbf{h}^N, W_2) + I(W_2; Y_2^N|\mathbf{h}^N) \\
&\overset{(c)}{=} I(W_1, Y_1^N|\mathbf{h}^N, W_2, Y_2^N) + I(W_1; Y_2^N|W_2, \mathbf{h}^N) \\
&\quad + I(W_2; Y_2^N|\mathbf{h}^N) \\
&\overset{(d)}{=} I(W_1; Y_1^N|\mathbf{h}^N, W_2, Y_2^N) + I(W_1, W_2; Y_2^N|\mathbf{h}^N) \\
&\overset{(e)}{=} I(X_1^N; Y_1^N|\mathbf{h}^N, X_2^N, Y_2^N) + I(X_1^N, X_2^N; Y_2^N|\mathbf{h}^N) \\
&\overset{(f)}{\le} \sum_{i=1}^N \big(I(X_{1i}; Y_{1i}|Y_{2i}, X_{2i}, h_i) + I(X_{1i}, X_{2i}; Y_{2i}^N|h_i)\big) \\
&\overset{(g)}{\le} \sum_{i=1}^N \big(I(X_{1Gi}; Y_{1i}|Y_{2i}, X_{2Gi}, h_i) + I(X_{1Gi}, X_{2Gi}; Y_{2i}^N|h_i)\big)
\end{aligned}$$

(a) Fano's Inequality, (b) Genie provides side information (Y_2^N, W_2) to Rx_1, (c) Chain Rule, (d) Recombining mutual information terms, (e) Data Processing Inequalities, (f) Removing conditioning increases entropy, and (g) Gaussian maximizes entropy, where X_{1Gi}, X_{2Gi} are jointly Gaussian with the same covariance matrix as X_{1i}, X_{2i}. We note that in (g) we can choose the correlation coefficient among Z_1 and Z_2 since the Rxs do not cooperate and hence the capacity region only depends on the noise marginal distributions (i.e., we can choose the worst noise correlation as long as the marginal distributions are preserved). From the results on the static channel [8] we know that the worst noise correlation is $\min\left\{\frac{|h_{11}|}{|h_{21}|}, \frac{|h_{21}|}{|h_{11}|}\right\}$. With this worst noise correlation and with the input covariance as in (8), the sum-capacity outer bound in (g) for the EGCIFC can be expressed as in (9).

The outer bound in (9) may be achieved by allowing the cognitive transmitter to assign part of its power to relay part of W_2 and use the remaining power to send its own message by "dirty paper coding" [10] against W_2, which it knows non-causally. This is similar to the scheme for the static channel [11] with the difference that at each channel use, the optimal parameters $(P_1^\star(\mathbf{h}), P_2^\star(\mathbf{h}), \rho^\star(\mathbf{h}))$ for that particular fading state that maximize (9) are used. We thus have

- Primary user sends $X_2 = \sqrt{P_2^\star(\mathbf{h})}U_2$, $U_2 \sim \mathcal{N}(0, 1)$;
- Cognitive user sends $X_1 = X_{2\text{R}} + X_{1\text{DPC}}$, where $X_{2\text{R}}$ relays U_2 as

$$X_{2\text{R}} = \sqrt{|\rho^\star(\mathbf{h})|^2P_1^\star(\mathbf{h})}e^{\text{j}(-\angle h_{21} + \angle h_{22})}U_2,$$

and generates, for U_1 independent of U_2, the signal

$$X_{1\text{DPC}} = \sqrt{(1 - |\rho^\star(\mathbf{h})|^2)P_1^\star(\mathbf{h})}U_1,\ U_1 \sim \mathcal{N}(0, 1),$$

treating the following as non-causally known interference

$$S = \left(h_{12}\sqrt{P_2^\star(\mathbf{h})} + h_{11}\sqrt{|\rho^\star(\mathbf{h})|^2P_1^\star(\mathbf{h})}e^{\text{j}(-\angle h_{21} + \angle h_{22})}\right)U_2;$$

- The received signals are

$$\begin{aligned}
Y_1 =& h_{11}\sqrt{(1 - |\rho^\star(\mathbf{h})|^2)P_1^\star(\mathbf{h})}U_1 + S + Z_1, \\
Y_2 =& e^{\text{j}\angle h_{22}}\left(|h_{22}|\sqrt{P_2^\star(\mathbf{h})} + |h_{21}|\sqrt{|\rho^\star(\mathbf{h})|^2P_1^\star(\mathbf{h})}\right)U_2 \\
&+ h_{21}\sqrt{(1 - |\rho^\star(\mathbf{h})|^2)P_1^\star(\mathbf{h})}U_1 + Z_2;
\end{aligned}$$

- Since Tx_1 used dirty paper coding on U_1 to "pre-cancel" the interference due to S, we have

$$R_1 = \log\Big(1 + |h_{11}|^2(1-|\rho^\star(\mathbf{h})|^2)P_1^\star(\mathbf{h})\Big);$$

Rx_2 decodes U_2 by treating U_1 as noise, yielding

$$R_2 = \log\left(1 + \frac{(|h_{21}||\rho^\star(\mathbf{h})|\sqrt{P_1^\star(\mathbf{h})} + |h_{22}|\sqrt{P_2^\star(\mathbf{h})})^2}{1+|h_{21}|^2(1-|\rho^\star(\mathbf{h})|^2)P_1^\star(\mathbf{h})}\right).$$

Re-arranging and taking expectation yields (9). □

The problem in (9) can be solved by using Lagrange duality. The general solution is not reported here for sake of space, and can be found in [3]. Notice that when $\max\{|h_{11}|^2, |h_{21}|^2\} = |h_{21}|^2$ the second logarithm in (9) is zero, in which case the expression to be optimized is that of the MISO with PerPC in (1). Hence it follows that:

Lemma 3. *When* $\mathbb{P}[|h_{11}|^2 \le |h_{21}|^2] = 1$*, referred to as the* strong interference regime*, the sum-capacity achieving power allocation policy for the EGCIFC is given by* (2) *for* $j \in [1:2]$*; the sum-capacity is given by* (3) *with* $[h_1\ h_2] = [h_{21}\ h_{22}]$.

Proof. The proof follows from the discussion immediately preceding the statement of the Lemma. □

IV. Numerical Results

In this section we numerically evaluate Theorems 1 and 2 under the assumptions that the channel gains are independent Rayleigh fading random variables, not necessarily with the same mean parameter.

A. The Rayleigh fading MISO with PerPC

We first consider a 2×1 MISO channel where the channel gains are independent and exponentially distributed with means $\mathbb{E}[|h_1|^2] = \gamma_1 = 5$ and $\mathbb{E}[|h_2|^2] = \gamma_2 = 2$, respectively. The transmit antennas are subject to the average power constraints $\overline{P}_1$ and $\overline{P}_2$. In Fig. 2(a) four surfaces representing the sum-capacity under different power allocations for the MISO channel are plotted vs. $\overline{P}_1$ and $\overline{P}_2$. The sum-capacities correspond to: (a) the MISO channel with PerPC $C_{\text{MISOPerPC}}$ given in (3) with $P_1^\star(\mathbf{h})$ and $P_2^\star(\mathbf{h})$ being the optimal power allocation described by (2) for $i \in [1:2]$ (b) the MISO channel with constant power allocation and beam forming (where the instantaneous phases are assumed to be known at the transmitter to allow for coherent beamforming)

$$C_{\text{dep,cte}} = \mathbb{E}\left[\log\left(1 + \left(\sqrt{|h_1|^2\overline{P}_1} + \sqrt{|h_2|^2\overline{P}_2)}\right)^2\right)\right];$$

(c) the MISO channel with constant power allocation, independent signaling (here the instantaneous phases are not known at the transmitter preventing from coherent beamforming; since the phases are assumed to be uniformly distributed in $[0, 2\pi]$ as in [4] independent inputs are optimal)

$$C_{\text{indep,cte}} = \mathbb{E}\left[\log\left(1 + |h_1|^2\overline{P}_1 + |h_2|^2\overline{P}_2\right)\right];$$

and (d) the MISO channel with a SumPC $C_{\text{MISOSumPC}}$ formally given by (3) with $P_1^\star(\mathbf{h})$ and $P_2^\star(\mathbf{h})$ being the optimal power allocation described by (4) for $i \in [1:2]$. Fig. 2(a) shows, as expected, that

$$C_{\text{indep,cte}} \le C_{\text{dep,cte}} \le C_{\text{MISOPerPC}} \le C_{\text{MISOSumPC}},$$

i.e., the MISO sum-capacity with PerPC ($C_{\text{MISOPerPC}}$) is upper and lower bounded by that of the MISO with SumPC ($C_{\text{MISOSumPC}}$) and that of the MISO with constant power allocation ($C_{\text{dep,cte}}$), respectively. Also, dependent constant inputs ($C_{\text{dep,cte}}$) outperform independent constant inputs ($C_{\text{indep,cte}}$).

Remark 3: In [4] the MISO channel with PerPC with Rayleigh fading with no CSI at the transmitters was compared to that with SumPC and with independent signaling in which case the author noted that $C_{\text{MISOPerPC}} = C_{\text{indep,cte}}$; since we assume the transmitter has CSI the conclusion made in [4] is not relevant and we have $C_{\text{MISOPerPC}} \ge C_{\text{indep,cte}}$.

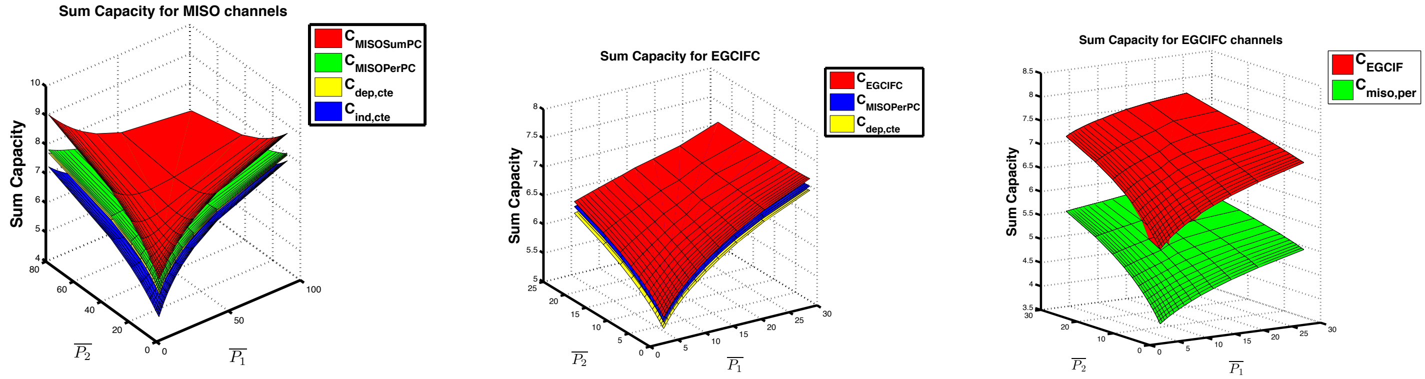
Remark 4: In the case of dependent inputs and beam forming, the channel is assumed to have CSI at the transmitter to account for the channel gains' phases and the ability to coherently beam form. This explains why $C_{\text{MISOPerPC}}$ is almost the same as $C_{\text{dep,cte}}$, which may have practical implications on what CSI is more valuable at the transmitter.

B. The Rayleigh fading EGCIFC

We now consider two different cases of the EGCIFC that differ by the channel gain mean parameters: 1) the means are chosen such that the channel experiences strong interference with high probability, and 2) the means are chosen to experience weak interference states with high probability. The goal is to numerically evaluate the performance attainable by the MISO with PerPC strategy in Theorem 1 compared to the optimal power allocation in Theorem 2 (see [3]) for which the MISO with PerPC strategy is optimal only in some fading states as per Lemma 3.

Fig. 2(b) shows the sum-capacity of the EGCIFC C_{EGCIFC} having independent and exponentially distributed channel gains with mean $\mathbb{E}[|h_{11}|^2] = \gamma_0 = 1$, $\mathbb{E}[|h_{21}|^2] = \gamma_1 = 5$ and $\mathbb{E}[|h_{22}|^2] = \gamma_2 = 2$. This choice of mean values 'skews' the EGCIFC to be in strong interference regime (defined as $|h_{21}|^2 \ge |h_{11}|^2$) with high probability since $\mathbb{P}\left[|h_{21}|^2 \ge |h_{11}|^2\right] = \frac{\gamma_1}{\gamma_1+\gamma_0} = \frac{5}{6}$. It also shows the sum-capacity if the MISO with PerPC transmit strategy is used $C_{\text{MISOPerPC}}$ along with the MISO with constant power allocation and dependent inputs $C_{\text{dep,cte}}$. Similar observations as in Remark 4 can be made here. Moreover, we notice that the surface representing the sum-capacity $C_{\text{MISOPerPC}}$ approaches that of C_{EGCIFC}, due to the fact that with high probability the channel experiences strong interference.

Fig. 2(c) shows the sum-capacity of the EGCIFC having mean parameters $\mathbb{E}[|h_{11}|^2] = \gamma_0 = 5$, $\mathbb{E}[|h_{21}|^2] = \gamma_1 = 1$ and $\mathbb{E}[|h_{22}|^2] = \gamma_2 = 2$, which 'skew' the EGCIFC to be in strong interference regime (defined as $|h_{21}|^2 \ge |h_{11}|^2$) with low probability since $\mathbb{P}\left[|h_{21}|^2 \ge |h_{11}|^2\right] = \frac{\gamma_1}{\gamma_1+\gamma_0} = \frac{1}{6}$. It also shows the sum-capacity achieved with MISO with PerPC $C_{\text{MISOPerPC}}$. As expected, the MISO scheme with PerPC does not perform as well in the regime where channel is skewed to weak interference.



(a) MISO with PerPC upper and lower bounded by that of MISO with SumPC and that of constant power allocation (dependent and independent inputs).

(b) EGCIFC skewed to be in strong interference, the MISO with PerPC is capacity achieving for the EGCIFC.

(c) EGCIFC skewed to be in weak interference, the MISO with PerPC is not optimal.

V. Conclusion

In this work we analytically characterized the ergodic capacity of the MISO channel with PerPC and perfect CSI at all nodes. This result is sum-capacity achieving for the EGCIFC when the channel is in strong interference.

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Appendix

The ergodic capacity of the MISO channel with PerPC is the solution of (1). By Lagrange duality, let $\lambda_i \geq 0$ be the Lagrange multiplier associated with the power constraint on antenna $i \in [1:n]$. The Lagrangian of (1) is then

$$\mathcal{L} = \log\left(1 + \Big(\sum_{i\in[1:n]} |h_i|\sqrt{P_i}\Big)^2\right) - \left(\sum_{i\in[1:n]} \lambda_i P_i\right).$$

By differentiating $\mathcal{L}$ with respect to P_i we get

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\theta}{1+\theta^2}\frac{|h_i|}{\sqrt{P_i}} - \lambda_i = 0, \quad \forall i \in [1:n], \tag{10}$$

with $\theta := \sum_{i\in[1:n]} |h_i|\sqrt{P_i(\mathbf{h})}$. By solving (10) we get $\sqrt{P_i(\mathbf{h})} = \frac{\theta}{1+\theta^2}\frac{|h_i|}{\lambda_i}, \quad \forall i$ which implies

$$\theta = \sum_{i\in[1:n]} |h_i|\sqrt{P_i(\mathbf{h})} = \frac{\theta}{1+\theta^2}\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}$$

$$\iff 1+\theta^2 = \sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i} \geq 1$$

$$\iff \sqrt{P_j(\mathbf{h})} = \frac{\sqrt{\left(\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i} - 1\right)^+}}{\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}}\frac{|h_j|}{\lambda_j}$$

$$\iff \lambda_j P_j(\mathbf{h}) = \left(1 - \frac{1}{\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}}\right)^+ \frac{\frac{|h_j|^2}{\lambda_j}}{\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}},$$

thus proving (2). With these powers, the rate is given by (3) and the Lagrange multipliers solve

$$\lambda_j\, \overline{P}_j = \mathbb{E}\left[\left[1 - \frac{1}{\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}}\right]^+ \frac{1}{\sum_{i\in[1:n]}\frac{|h_i|^2}{\lambda_i}}\,\frac{|h_j|^2}{\lambda_j}\right].$$

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