The circular hall plate : approximation of the geometrical correction factor for small contacts

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by

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E I N D H O V E N   U N I V E R S I T Y   O F   T E C H N O L O G Y

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Abstract - The circular Hall plate is considered with four equal finite line contacts symmetrical with respect to two orthogonal axes. A proof is given of an approximation of the geometrical correction factor $C(\theta,m)$, if θ tends to zero. The angle θ corresponds to the length of each contact. The parameter m depends on the magnetic field, which need not be small.

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1. Introduction

Consider a circular Hall plate with four finite contacts of equal length (Fig. 1). It is convenient to take the radius equal to one unit of length. We shall deal with an n-type semiconductor. The current I enters the sample

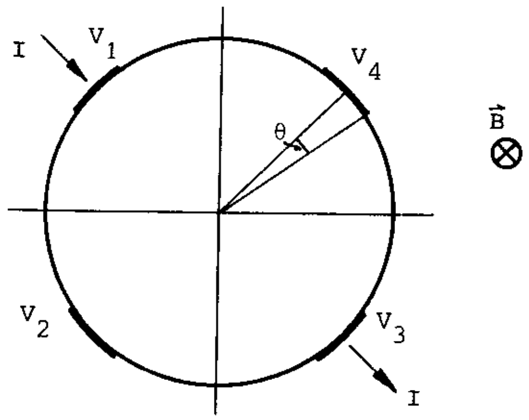

Fig. 1 *Circular Hall plate with four contacts which are equal in length Radius = 1 unit of length*

at contact 1 and leaves it at contact 3. The Hall electrodes draw no current. The constant magnetic induction B is directed backwards and perpendicular to the plane surfaces of the sample. It is assumed that the sample is homogeneous and has a specific resistivity ρ and a uniform thickness d. The last-named is very short with respect to the radius of the sample. As is well known [1] the method of conformal transformations can then be applied in order to obtain the potential distribution.

We have analysed the circular structure in [2]. Let V_j denote the potential of contact j (see Fig. 1). The geometrical correction factor $C(\theta,m)$ is defined by the relation

$$\frac{d}{\rho}\frac{V_H}{I} = C(\theta,m)\tan\beta \tag{1}$$

where the Hall voltage $V_H = V_2 - V_4$, β is the Hall angle, $m = 2\beta/\pi$ and the angle θ corresponds to half a contact. We have proved that [2]

$$C(\theta,m) = \frac{1}{\sin\beta}\,\frac{J_4(\theta,m) - J_4(\theta,-m)}{J_3(\theta,m)} \tag{2}$$

The function $J_3(\theta,m)$ is given by

$$J_3(\theta,m) = \operatorname{sgn} p \int_{t_1}^{t_2} dy\ (y-t_1)^{-b}(t_2-y)^{b-1} f_3(y,m) \tag{3}$$

with

$$f_3(y,m) = \left(y-p\right)\left(y+\frac{1}{p}\right)\left\{\left(y+t_1\right)\left(y+\frac{1}{t_2}\right)\left(\frac{1}{t_1}-y\right)\right\}^{b-1} \left\{\left(y+t_2\right)\left(y+\frac{1}{t_1}\right)\left(\frac{1}{t_2}-y\right)\right\}^{-b}$$

Furthermore, the function $J_4(\theta,m)$ is defined by

$$J_4(\theta,m) = \operatorname{sgn} p \int_p^{t_1} dy \; (t_1-y)^{-b} \; f_4(y,m) \tag{4}$$

with

$$f_4(y,m) = \left(y-p\right)\left(y+\frac{1}{p}\right)\left\{\left(t_2-y\right)\left(y+t_1\right)\left(y+\frac{1}{t_2}\right)\left(\frac{1}{t_1}-y\right)\right\}^{b-1} \left\{\left(y+t_2\right)\left(y+\frac{1}{t_1}\right)\left(\frac{1}{t_2}-y\right)\right\}^{-b}$$

In these integrals the following parameters t_1, t_2, b and p occur:

$$t_1 = \tan\left(\frac{\pi}{8}-\frac{\theta}{2}\right), \; t_2 = \tan\left(\frac{\pi}{8}+\frac{\theta}{2}\right), \; b = \frac{1+m}{2} \tag{5}$$

and

$$p = \frac{1}{2}\left\{Q \pm \left(4+Q^2\right)^{\frac{1}{2}}\right\} \tag{6}$$

where

$$Q = \frac{J_1(\theta,m)+J_1(\theta,-m)}{J_2(\theta,-m)-J_2(\theta,m)} \tag{7}$$

The integrals $J_1(\theta,m)$ and $J_2(\theta,m)$ are defined by

$$J_1(\theta,m) = \int_{t_1}^{t_2} dy \; (1-y^2) \; f(y,m) \tag{8}$$

and

$$J_2(\theta,m) = \int_{t_1}^{t_2} dy \; y \; f(y,m) \tag{9}$$

respectively ; where

$$f(y,m) = \left\{\left(y-t_1\right)\left(y+t_2\right)\left(\frac{1}{t_2}-y\right)\left(y+\frac{1}{t_1}\right)\right\}^{-b} \left\{\left(y+t_1\right)\left(t_2-y\right)\left(y+\frac{1}{t_2}\right)\left(\frac{1}{t_1}-y\right)\right\}^{b-1}$$

Note that t_1 and t_2 are positive but less than one. The sign in formula (6) has to be chosen such that $|p| < 1$.

In this paper we shall derive an approximative expression for the geometrical correction factor $C(\theta,m)$ if θ tends to zero. It has already been given without proof in [2].

In the next section an approximation will be obtained for t_1 and t_2, if θ approaches zero. In Section 3 the integral $J_1(\theta,m)$ will be approximated. Section 4 contains an analysis which provides approximations of the integral $J_2(\theta,m)$ and of p. We are then in a position to ascertain the behaviour of $J_3(\theta,m)$ and of the difference $J_4(\theta,m) - J_4(\theta,-m)$ if θ tends to zero. This is done in Sections 5 and 6. Finally, we shall derive an approximative formula for the geometrical correction factor $C(\theta,m)$ in Section 7.

2. Approximation of t_1 and t_2.

Let $\delta = \theta$ where δ is a small positive number. Define $\varepsilon = t_2 - t_1$. If δ tends to zero, then ε also approaches zero. It is convenient to approximate $C(\delta,m)$ by a simple function of ε. Later on, the relationship between δ and ε will be used in order to obtain an approximation in δ. It is easily seen from (5) that

$$\varepsilon = \delta(4-2\sqrt{2}) + O(\delta^3) \qquad (\delta \to 0). \tag{10}$$

Then we find

$$t_1 = \tan\left(\frac{\pi}{8} - \frac{\delta}{2}\right)$$

$$= -1 + \sqrt{2} - \frac{1}{2}\varepsilon + \frac{\sqrt{2}}{16}\varepsilon^2 + O(\varepsilon^3) \qquad (\varepsilon \to 0) \tag{11}$$

Furthermore,

$$t_2 = \tan\left(\frac{\pi}{8} + \frac{\delta}{2}\right)$$

$$= -1 + \sqrt{2} + \frac{1}{2}\varepsilon + \frac{\sqrt{2}}{16}\varepsilon^2 + O(\varepsilon^3) \qquad (\varepsilon \to 0) \tag{12}$$

The integral $J_1(\theta,m)$ will be considered in the next section.

3. Approximation of integral $J_1(\theta,m)$

Let

$$L(\delta,m,k) = \int_{t_1}^{t_2} dy\,(y-t_1)^{k-b}(t_2-y)^{b-1} \tag{13}$$

where k is zero or a natural number. Introducing a new variable $v = (y-t_1)/\varepsilon$, we obtain

$$L(\delta,m,k) = \varepsilon^k\, B(k+1-b,b) \tag{14}$$

where $B(x,y)$ is the beta function

$$B(x,y) = \int_0^1 t^{x-1}\,(1-t)^{y-1}\,dt \qquad (R\{x\}>0,\ R\{y\}>0)$$

The integral $J_1(\delta,m)$ can be written in the form

$$J_1(\delta,m) = \int_{t_1}^{t_2} dy\,(y-t_1)^{-b}\,(t_2-y)^{b-1}\,g(y,m) \tag{15}$$

where

$$g(y,m) = \left(1-y^2\right)\left\{\left(y+t_2\right)\left(\frac{1}{t_2}-y\right)\left(y+\frac{1}{t_1}\right)\right\}^{-b}$$
$$\left\{\left(y+t_1\right)\left(y+\frac{1}{t_2}\right)\left(\frac{1}{t_1}-y\right)\right\}^{b-1}$$

We now develop the function $g(y,m)$ in a series in $y-t_1$

$$g(y,m) = A(m)\left\{B_o(m) + B_1(m)\,(y-t_1) + \ldots\right\} \tag{16}$$

Using (11) and (12), an elementary calculation yields

$$A(m) = \frac{2+\sqrt{2}}{16}\left[1+\varepsilon\left(\frac{\sqrt{2}}{2}-b\,\frac{\sqrt{2}}{4}\right)+\frac{\varepsilon^2}{16}\left\{5+\sqrt{2}-b\,(19+10\sqrt{2})+b^2\right\}\right]+$$
$$+\,O(\varepsilon^3)$$

$$B_0(m) = -2+2\sqrt{2}+\varepsilon(\sqrt{2}-1)+O(\varepsilon^2)$$

$$B_1(m) = -1-\frac{1}{2}\sqrt{2}-\varepsilon(1+\frac{1}{2}\sqrt{2})+\varepsilon b\,\frac{5}{4}(1+\sqrt{2})+O(\varepsilon^2)$$

From (14), (15) and (16) one finds

$$J_1(\delta,m) = A(m)\left\{B_0(m)\ L(\delta,m,0) + B_1(m)\ L(\delta,m,1)\right\} + O(\varepsilon^2)$$

$$= \frac{1}{8}\sqrt{2}\ B(1-b,b) + \frac{\varepsilon}{16}\left\{(2 + \sqrt{2} - b)\ B(1-b,b) - (3+2\sqrt{2})\ B(2-b,b)\right\} \quad (17)$$

if ε tends to zero.

4. Approximation of integral $J_2(\theta,m)$ and of p

The integral $J_2(\delta,m)$ can be rewritten as

$$J_2(\delta,m) = \int_{t_1}^{t_2} dy\ (y - t_1)^{-b}\ (t_2-y)^{b-1}\ h(y,m) \quad (18)$$

where

$$h(y,m) = y\left\{\left(y+t_2\right)\left(\frac{1}{t_2} - y\right)\left(y + \frac{1}{t_1}\right)\right\}^{-b} \left\{\left(y+t_1\right)\left(y + \frac{1}{t_2}\right)\left(\frac{1}{t_1} - y\right)\right\}^{b-1} \quad (19)$$

The function h(y,m) is developed in a series in $y - t_1$:

$$h(y,m) = A(m)\cdot\left\{C_0(m) + C_1(m)\ (y - t_1) + C_2(m)\ (y - t_1)^2 + \dots\right\}$$

Using (11) and (12), after some straightforward calculations we obtain

$$C_0(m) = -1 + \sqrt{2} - \frac{1}{2}\varepsilon + \varepsilon^2\frac{\sqrt{2}}{16} + O(\varepsilon^3)$$

$$C_1(m) = -\frac{1}{2} + \frac{3}{4}\sqrt{2} + \frac{5}{8}(1 + \sqrt{2})b\varepsilon - \frac{1}{4}(1 + \sqrt{2}) + O(\varepsilon^2)$$

$$C_2(m) = \frac{1}{8}(-5 + \sqrt{2}) + O(\varepsilon)$$

The value of A(m) was already obtained in the preceding section. From (14), (18) and (19) it is easily seen that

$$J_2(\delta,m) = A(m)\left\{C_0(m)\ B(1-b,b) + \varepsilon\ C_1(m)\ B(2-b,b) + \varepsilon^2\ C_2(m)\ B(3-b,b)\right\} \quad (20)$$

if ε tends to zero.

From (6), (7), (17) and (20) we can derive an expression for p (note that $|p| < 1$). Extensive but simple calculations yield

$$p = -\frac{2+\sqrt{2}}{8}(1-2b)\,\varepsilon + O(\varepsilon^3) \qquad (\varepsilon\to 0) \qquad (21)$$

5. Approximation of function $J_3(\theta,m)$

From (3), (8) and (9) we have

$$J_3(\delta,m) = \operatorname{sgn} p \left\{- J_1(\delta,m) + J_2(\delta,m)\left(\frac{1}{p} - p\right)\right\} \qquad (22)$$

From (17), (20), (21) and (22) we find :

$$J_3(\delta,m) = \frac{\operatorname{sgn} p}{\varepsilon(1-2b)}\,\frac{1-\sqrt{2}}{2}\,B(1-b,b) + O(\varepsilon) \qquad (\varepsilon\to 0) \qquad (23)$$

It should be noted that $J_3(\theta,m)$ is invariant for reversal of the magnetic field: $J_3(\theta,m) = J_3(\theta,-m)$.

6. Approximation of difference $J_4(\theta,m) - J_4(\theta,-m)$

Eqn.(4) can be rewritten as

$$J_4(\theta,m) = \operatorname{sgn} p \int_p^{t_1} dy \left\{y^2-1 + \left(\frac{1}{p} - p\right) y\right\}(t_1-y)^{-b} \,*$$

$$*\,(t_2 - y)^{b-1}\,h_4(y,m) \qquad (24)$$

where

$$h_4(y,m) = \left\{\left(y+t_1\right)\left(y+\frac{1}{t_2}\right)\left(\frac{1}{t_1} - y\right)\right\}^{b-1}\left\{\left(y+t_2\right)\left(y+\frac{1}{t_1}\right)\left(\frac{1}{t_2} - y\right)\right\}^{-b}$$

Because of the singularity of the integrand in (24) it is found to be necessary to split the interval of integration into four parts. Introducing a positive number a less than one, we have

$$J_4(\delta,m) = \operatorname{sgn} p \left\{J_{41}(\delta,m,a) + \left(\frac{1}{p} - p\right) J_{42}(\delta,m,a) + \right.$$

$$\left. - J_{43}(\delta,m) + J_{44}(\delta,m,a)\right\} \qquad (25)$$

with

$$J_{41}(\delta,m,a) = \int_0^{at_1} dy\,(y^2-1)\,(t_1-y)^{-b}\,(t_2-y)^{b-1}\,h_4(y,m) \qquad (26)$$

$$J_{42}(\delta,m,a) = \int_0^{at_1} dy\, y(t_1-y)^{-b}\ (t_2-y)^{b-1} h_4(y,m) \tag{27}$$

$$J_{43}(\delta,m) = \int_0^{p} dy \left\{y^2-1 + (\frac{1}{p} - p)\ y\right\}\ (t_1-y)^{-b}(t_2-y)^{b-1} h_4(y,m)$$

and

$$J_{44}(\delta,m,a) = \int_{at_1}^{t_1} dy \left\{y^2-1 + (\frac{1}{p} - p)\ y\right\} (t_1-y)^{-b}(t_2-y)^{b-1} h_4(y,m) \tag{28}$$

respectively. Since $J_{43}(\delta,m) = O(\varepsilon)$, if ε tends to zero, we find

$$\begin{aligned} J_4(\delta,m) - J_4(\delta,-m) = {} & \operatorname{sgn} p \left\{J_{41}(\delta,m,a) + J_{41}(\delta,-m,a)\right\} + \\ & + \operatorname{sgn} p\ (\frac{1}{p} - p) \left\{ J_{42}(\delta,m,a) - J_{42}(\delta,-m,a)\right\} + \\ & + \operatorname{sgn} p \left\{J_{44}(\delta,m,a) + J_{44}(\delta,-m,a)\right\} + \\ & + O(\varepsilon) \qquad (\varepsilon \to 0) \end{aligned} \tag{29}$$

Successively, the termson the right-hand side of (29) will be determined. The value which the number a should have in order to obtain an accurate approximation will be clear later on.

6.1 Approximation of $J_{41}(\delta,m,a) + J_{41}(\delta,-m,a)$

From (26) it follows that the constant term in the approximation of $J_{41}(\delta,m,a) + J_{41}(\delta,-m,a)$ is

$$J_{41}(0,m,a) + J_{41}(0,-m,a) = 2 \int_0^{at_1} dy \frac{y^2-1}{\left(t_1-y\right) \left(y+t_1\right) \left(y+\frac{1}{t_1}\right) \left(\frac{1}{t_1} - y\right)}$$

in which $t_1 = -1 + \sqrt{2}$. By elementary integration we obtain the result (independent of m)

$$J_{41}(0,m,a) + J_{41}(0,-m,a) = \frac{\sqrt{2}}{4} \ln \frac{(1-a)\ \{1-a(3-2\sqrt{2})\}}{(1+a)\ \{1+a(3-2\sqrt{2})\}} \tag{30}$$

The linear term in the Taylor series is evidently of the order ε.

6.2 Approximation of $J_{42}(\delta,m,a) - J_{42}(\delta,-m,a)$

From its definition (27) it is easy to see that $J_{42}(0,m,a)$ is independent of the magnetic field. Hence, the constant term in the Taylor series of $J_{42}(\delta,m,a) - J_{42}(\delta,-m,a)$ is zero. In order to obtain its linear term we need the derivation of $J_{42}(\delta,m,a)$:

$$\frac{dJ_{42}(\delta,m,a)}{d\varepsilon} = \frac{\partial J_{42}(\delta,m,a)}{\partial t_1}\frac{dt_1}{d\varepsilon} + \frac{\partial J_{42}(\delta,m,a)}{\partial t_2}\frac{dt_2}{d\varepsilon}$$

By differentiating (27), we find, for example

$$\left[\frac{\partial J_{42}(\delta,m,a)}{\partial t_1}\right]_{\varepsilon=0} = \frac{a^2 t_1}{(1-a^2)\ (1-a^2 t_1{}^4)} +$$

$$+\left(1+\frac{1}{t_1{}^2}\right)\int_0^{at_1} dt\ t^2 \left\{\left(t_1{}^2 - t^2\right)\left(\frac{1}{t_1{}^2} - t^2\right)\right\}^{-1} *$$

$$*\left[-b\left\{\left(t_1 - t\right)\left(t+\frac{1}{t_1}\right)\right\}^{-1} + (1-b)\left\{\left(t+t_1\right)\left(\frac{1}{t_1} - t\right)\right\}^{-1}\right]$$

in which $t_1 = -1 + \sqrt{2}$. The variables t_1 and t_2 are represented as functions of ε in (11) and (12) respectively. Carrying out laborious but straightforward integrations, we obtain the first-order approximation

$$J_{42}(\delta,m,a) - J_{42}(\delta,-m,a) = \varepsilon(2b-1)\left(1+\frac{1}{t_1{}^2}\right) *$$

$$*\left[\frac{at_1{}^2}{16(a^2t_1{}^4-1)} + \frac{a}{16(1-a^2)} + \frac{\sqrt{2}}{64}\ln\frac{(1-a)(1-at_1{}^2)}{(1+a)(1+at_1{}^2)}\right] \qquad (31)$$

where $t_1 = -1 + \sqrt{2}$.

6.3 Approximation of $J_{44}(\delta,m,a)$

The problem of deriving an approximation of the function $J_{44}(\delta,m,a)$ for small values of δ is very complicated.

First, we develop the function $\left\{y^2-1 + (\frac{1}{p} - p)\ y\right\} h_4(y,m)$ in a Taylor series in the variable $y - t_1$:

$$\left\{y^2-1 + \left(\frac{1}{p} - p\right) y\right\} h_4(y,m) = D(m) \left\{E_0(m) + E_1(m) (y-t_1) + + E_2(m) (y-t_1)^2+\dots\right\} \quad (32)$$

Using (11), (12) and (21), after some straightforward calculations we find that

$$D(m) = A(-m)$$

$$E_0(m) = 2-2\sqrt{2} + \left\{4 - 3\sqrt{2} + \frac{\varepsilon}{2} (2 - \sqrt{2})\right\} \frac{4}{\varepsilon(2b-1)}$$

$$E_1(m) = \frac{2 + \sqrt{2}}{2} + \left\{5 - 2\sqrt{2} + \varepsilon \frac{1}{4} \sqrt{2} - \frac{5}{8} \varepsilon b\sqrt{2}\right\} \frac{4}{\varepsilon(2b-1)}$$

$$E_2(m) = \frac{12-7\sqrt{2}}{2\varepsilon(2b-1)} - \frac{3+6\sqrt{2}}{4} + \frac{3\sqrt{2}}{2(2b-1)}$$

We refer to Section 3 for an expression for $A(-m)$.
Second, let

$$K(\delta,m,a,k) = \int_{at_1}^{t_1} dy\ (t_1-y)^{k-b}(t_2-y)^{b-1} \quad (33)$$

where k is zero or a natural number. By introducing a new variable $v = (y-at_1)/\{(1-a)t_1\}$, we obtain

$$K(\delta,m,a,k) = (1-a)^k t_1{}^k x^{1-b} \int_0^1 dv\ (1-v)^{k-b}(1-xv)^{b-1}$$

in which

$$\frac{1}{x} = 1 + \frac{\varepsilon}{t_1(1-a)}$$

It is easily seen that

$$K(\delta,m,a,k) = (1-a)^k\ t_1{}^k x^{1-b} *$$
$$* \sum_{n=0}^{k} (-1)^n \binom{k}{n} B(n+1,\ 1-b)\ F(1-b,\ n+1;\ 2-b+n;\ x) \quad (34)$$

where $B(x,y)$ is the beta function and $F(a,b;c;x)$ is the hypergeometric function

$$F(a,b;c;x) = \frac{\Gamma(c)}{\Gamma(b)\ \Gamma(c-b)} \int_0^1 du\ u^{b-1}(1-u)^{c-b-1}(1-xu)^{-a}$$

if $b > 0$, $c-b > 0$ and $|x| < 1$. It is known that [3]

$$F(p,q;p+q;x) = \frac{1}{B(p,q)} \sum_{i=0}^{\infty} \frac{(p)_i (q)_i}{(i!)^2} (1-x)^i f_i(p,q,x) \qquad (35)$$

$$(|1-x| < 1)$$

with

$$f_i(p,q,x) = 2\,\psi(i+1) - \psi(p+i) - \psi(q+i) - \ln(1-x)$$

$$\psi(z) = \Gamma'(z)/\Gamma(x)$$

$$(p)_i = \Gamma(p+i)/\Gamma(p) \text{ for } i=1,2,3\ldots;\ (p)_0=1.$$

Third, using the above results, we shall now determine an approximation for $J_{44}(\delta,m,a)$. From (28), (32) and (33) we obtain

$$J_{44}(\delta,m,a) = D(m)\left\{E_0(m)\ K(\delta,m,a,0) - E_1(m)\ K(\delta,m,a,1) + \right.$$
$$\left. + E_2(m)\ K(\delta,m,a,2)\right\} \qquad (36)$$

It is assumed here that $(1-a)$ is a small positive number. Applying (35) in (34), we find after some straightforward calculations that

$$K(\delta,m,a,0) = \psi(1) - \psi(1-b) - \ln\frac{1+\sqrt{2}}{1-a} - \ln\varepsilon +$$
$$+\varepsilon\left(\frac{1-b}{1-a} - \frac{1}{2}\right)(1+\sqrt{2}) + O(\varepsilon^2) \qquad (\varepsilon\to 0)$$

Note that we used the relation

$$\psi(z+1) - \psi(z) = \frac{1}{z}$$

and that the term with index i in (35) is of the order of $O(\varepsilon^i)$. In the same way the functions $K(\delta,m,a,1)$ and $K(\delta,m,a,2)$ have been approximated, viz.

$$K(\delta,m,a,1) = (1-a)\ (-1+\sqrt{2})\left\{1 + \frac{1-b}{1-a}(1+\sqrt{2})\varepsilon\ln\varepsilon + \right.$$
$$\left. +\varepsilon\left[-\frac{1+\sqrt{2}}{2} + \frac{1-b}{1-a}(1+\sqrt{2})\left\{-\psi(2) + \psi(2-b) + \ln\frac{1+\sqrt{2}}{1-a}\right\}\right]\right\} +$$
$$+ O(\varepsilon^2 \ln\varepsilon)$$

and

$$K(\delta,m,a,2) = \frac{(1-a)^2(3-2\sqrt{2})}{2}\left[1 - \varepsilon\ (1+\sqrt{2})\left\{\frac{2(1-b)}{1-a} + 1\right\}\right]$$

Let a = 0.9. Then, from (36) we obtain after straightforward calculations

$$\begin{aligned} J_{44}(\delta,m,0.9) &= \frac{0.20711}{2b-1}\ \frac{\ln\varepsilon}{\varepsilon} + \\ &+ \left\{0.79077 + 0.20711\ \psi(1-b)\right\}\frac{1}{\varepsilon(2b-1)} + \\ &+ 2.64276 - \frac{0.14017}{1-b} + \\ &+ \frac{1}{2b-1}\left\{- 2.39956 + \frac{0.14017}{1-b}\right\} + \\ &+ O(\varepsilon\ \ln\ \varepsilon) \qquad (\varepsilon\to 0) \end{aligned} \tag{37}$$

Note that for a <0.9 the approximation of $J_{44}(\delta,m,a)$ by three terms (see (36)) may not be accurate enough (see (34)).

7. Approximation of the geometrical correction factor

In this section an approximation for the geometrical correction factor $C(\theta,m)$ will be derived. From (37) it is easily seen that

$$\begin{aligned} J_{44}(\delta,m,0.9) + J_{44}(\delta,-m,0.9) &= \frac{0.20711}{\varepsilon(2b-1)}\left\{\psi(1-b) - \psi(b)\right\} + \\ &+ 5.28552 \qquad (\varepsilon\to 0) \end{aligned} \tag{38}$$

From (30) we obtain

$$J_{41}(\delta,m,0.9) + J_{41}(\delta,-m,0.9) = - 1.15108 + O(\varepsilon)\quad (\varepsilon\to 0) \tag{39}$$

From (21) and (31) it is shown that

$$\left(\frac{1}{p} - p\right)\left\{J_{42}(\delta,m,0.9) - J_{42}(\delta,-m,0.9)\right\} = - 3.42757 \tag{40}$$

Furthermore, from (29), (38), (39) and (40) it is found that

$$J_4(\delta,m) - J_4(\delta,-m) = \text{sgn}\ p\left[\frac{0.20711}{\varepsilon(1-2b)}\left\{\psi(b) - \psi(1-b)\right\} + 0.70687\right] \tag{41}$$

Next, we derive from (2), (23) and (41) that

$$C(\delta,m) = 1 + \frac{3.41304\ (1-2b)\varepsilon}{\psi(b) - \psi(1-b)} \qquad (\varepsilon \to 0)$$

Since $\psi(b) - \psi(1-b) = \pi \tan(\beta)$ and $\varepsilon = \delta(4-2\sqrt{2}) + O(\delta^3)$, if δ tends to zero, we finally obtain

$$C(\delta,m) = 1 - 0.8103\ \beta \cot\text{an}\ (\beta)\ \delta \qquad (\delta \to 0) \qquad (42)$$

Herewith we have found an important formula for the geometrical correction factor $C(\theta,m)$ if θ tends to zero. Data on the accuracy of formula (42) have been given in Section 5 of [2].

8. Conclusion

It has been shown that the geometrical correction factor $C(\theta,m)$ can be approximated by a simple analytical expression if the angle θ tends to zero. For the accuracy of the approximation reference is made to [2].

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