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THE CLOUD–INTERCLOUD PHASE-CHANGE IN THE INTERSTELLAR MEDIUM

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SUMMARY

Pressure equilibrium is not the only condition necessary for the equilibrium of an interstellar cloud—the interface between cloud and intercloud medium as a whole must be in thermal balance as heat diffuses from regions of the interface that are heated excessively to those that would otherwise cool. On the other hand this equilibrium is only attained on a very long time-scale so that earlier treatments of clouds as discrete particles are quite justified.

Use of an improved equation of state shows that interstellar clouds are always gravitationally unstable if their mass exceeds $\sim 2000\, M_\odot$. Such a figure would indicate star formation to be a more efficient process than hitherto thought but inclusion of effects such as magnetism and rotation would increase this mass and modify this conclusion.

1. INTRODUCTION

For many years now the concept of interstellar clouds has been generally accepted by astronomers. The observations of the interstellar lines (Adams 1949) show that the interstellar medium is very clumpy and 21-cm observations (Heiles 1967) have resolved features that suggest they are individual clouds. Theoreticians have used the existence of the clouds to develop a model of star formation (Oort 1954) in which cloud collisions lead to more massive clouds which are gravitationally unstable and collapse to form star clusters. On the other hand our understanding of the clouds themselves, their equilibrium and stability, has until recently been marginal. It is with this understanding that the present paper is concerned.

In a recent important paper, Field, Goldsmith & Habing (1969—hereinafter referred to as FGH) presented a model of the interstellar clouds which explained many of the observed properties of the interstellar medium. Under the influence of cosmic ray heating and cooling by collisional excitation and subsequent emission of a variety of spectral lines from different atoms and ions, the interstellar gas comes to an equilibrium temperature, which is a function of the gas density and increases with decrease in density. If the pressure in these equilibrium states is plotted against density, it is found that there is a range of pressures for which three density states exist corresponding to a single pressure. Of these states one is thermally unstable and FGH identify the other two with the interstellar clouds and the intercloud medium which can thus exist together in pressure equilibrium.

Now Evans & Penston (1969) have pointed out the similarity of the equation of state found by FGH for the interstellar medium with an isotherm of a Van der Waal's gas. In the latter case the denser state is identified as liquid and the rarer state as vapour. However for a Van der Waal's gas the true equation of state jumps from one state to the other at the saturated vapour pressure of the liquid, so that there is only one pressure for a given temperature at which gas and liquid can co-exist in true equilibrium. It is true that drops of liquid may briefly exist in the vapour at pressures other than the saturation pressure but then evaporation or condensation brings the situation into balance. We conclude that pressure equilibrium is not the only criterion for true equilibrium and shall investigate the relevance of this idea to the equilibrium of the interstellar clouds.

In addition to the above topic, this paper also investigates the gravitational instability of gas clouds. This topic was studied extensively a number of years ago and is particularly associated with the names of Ebert (1955), Bonnor (1956) and McCrea (1957). However, all these authors assumed that the gas in interstellar clouds was essentially isothermal and, in view of our improved knowledge of the equation of state of the interstellar gas, the time seems ripe for a rediscussion of this problem.

2. THEORY OF THE PHASE-CHANGE

In this section we shall examine the equilibrium of the interstellar clouds with respect to evaporation or condensation. We shall not concern ourselves with the special case when interstellar clouds are evaporated by an external radiation field such as that from young stars. This problem has already been treated by Dyson (1968). It is rather our intention to discuss these processes in the interstellar medium itself, taking as our guide the sort of evaporation or condensation that also applies to a liquid at pressures other than its saturated vapour pressure.

In such cases exemplified by that of the Van der Waal's gas, one may use thermodynamic arguments since the medium is all at constant temperature. Then the saturation pressure is found to be that pressure at which the work done changing from one state to the other along a line of constant pressure is equal to that done when making the same change along the isotherm. However, the state of the interstellar medium is not defined by an isotherm but rather by the condition that the heat losses balance the heat gains and indeed the states identified by FGH as cloud and intercloud are at quite different temperatures. Thus the interstellar medium is not in thermodynamic equilibrium and a thermodynamic description of the cloud–intercloud phase change would require the methods of non-equilibrium thermodynamics. We shall not adopt such methods here, choosing rather to examine the detailed structure of the interface between cloud and intercloud.

We shall find it useful to define the generalized loss rate of the interstellar gas $L(p, T)$ as the excess of the cooling rate over the heating rate per unit volume at pressure p and temperature T. Now of course the equation

$$L(p, T) = 0 \tag{1}$$

together with the perfect gas law defines the sequence of states discussed by FGH. The medium may exist in more than one state at pressures at which the solution of equation (1) gives more than one root for the temperature. FGH showed that for a range of pressures relevant to the interstellar medium this equation has three roots but that one of them is unimportant since the corresponding state is thermally unstable (Field 1965). The other two states are identified as the interstellar clouds and the intercloud medium with which we are concerned.

Now since as already remarked the clouds and intercloud medium are at different temperatures, they will thus be separated by a region in which heat diffusion is important. Since this region is surrounded by material in detailed heat balance we should expect the equilibrium condition to be that the front as a whole is in heat balance as excess losses in one region are balanced by gains in another. The process of diffusion is most likely thermal conduction, so that we see that clouds are surrounded by conduction boundary layers in which the temperature changes from its cloud to intercloud value. We shall examine the structure of such layers surrounding condensing, evaporating clouds or clouds in equilibrium on the assumption that they may be regarded as plane-parallel. If clouds are regular structures this will be a good approximation since the distance over which conduction operates (the mean free path) is much smaller than the typical diameter of an interstellar cloud.

We now note that for the equilibrium of the cloud–intercloud interface the pressure must be uniform throughout the layer since the model of FGH proposes pressure equilibrium between cloud and intercloud. In fact any pressure irregularities are removed rapidly on the sound-travel time scale. Even when the evaporation or condensation of the cloud causes motions, variations in the pressure will not be important unless the velocity of the gas approaches the sound speed. The first law of thermodynamics is

$$dQ = dU + p\,dV$$

where Q is the heat, U the internal energy per unit mass and V the specific volume equal to $1/\rho$ if ρ is the density.

We write

$$dQ = d(U + pV) - V\,dp$$

and since the enthalpy per unit mass

$$H(p, T) = U + pV$$

at the constant pressure applying in the interface region, we have

$$dQ = dH(p, T).$$

Then we may write down an equation describing the rate of change of heat per unit volume in the interface

$$\rho \frac{DH}{Dt}(p, T) = \frac{\partial}{\partial z} K(T) \frac{\partial T}{\partial z} - L(p, T)$$

where z is the distance variable measured normal to the surface of the cloud, t the time variable and $K(T)$ the coefficient of thermal conductivity. If the structure of the front remains the same as the cloud condenses or evaporates and z remains fixed with respect to the front,

$$\frac{\partial H}{\partial t}(p, T) = 0$$

so that

$$\rho v \frac{\partial H}{\partial z}(p, T) = \frac{\partial}{\partial z} K(T) \frac{\partial T}{\partial z} - L(p, T)$$

where v is the velocity of the interstellar gas evaporating away from the cloud. We see that $\rho v = m$ is the flux of mass through the front per unit area and is independent of z for a steady front. Finally then we have:

$$\frac{\partial}{\partial z}\left\{m\mathrm{H}(p, T)-K(T)\frac{\partial T}{\partial z}\right\} = -L(p, T). \tag{2}$$

This equation may be interpreted as follows—both terms on the left-hand side of equation (2) represent fluxes of heat and these fluxes make up the excess loss rate at values of T that do not correspond to FGH's equilibrium states. In fact $mH(p, T)$ is the flux of heat carried by the motions through the front ($\mathrm{H}(p, T)$ is the ' latent heat ' in the phase change) and $-K(T)(\partial T/\partial z)$ is the flux due to conduction.

Now at both sides of this interface are the equilibrium states of cloud and intercloud, therefore in these regions no conduction occurs and we may write down the boundary conditions of equation (2) as the fact there is no temperature gradient in these regions—

$$\frac{\partial T}{\partial z}\rightarrow 0 \quad \text{as} \quad z\rightarrow \pm\infty.$$

The physical significance of equation (2) is more clearly seen if we take its formal first integral:

$$\left[m\mathrm{H}(p, T)-K(T)\frac{\partial T}{\partial z}\right]_{-\infty}^{+\infty} = \int_{-\infty}^{+\infty} L(p, T)\,dz$$

or taking into account the boundary conditions:

$$m[\mathrm{H}(p, T)]_{\mathrm{cloud}}{}^{\mathrm{intercloud}} = -\int_{\mathrm{front}} L(P, T)\,dz. \tag{3}$$

Thus the total loss of heat (or gain) in the front due to the heating and cooling mechanisms drives the condensation or evaporation so that this loss is balanced by the latent heat of the phase change from the gas flowing through the front.

The condition that no condensation or evaporation takes place is $m = 0$. In this case equation (3) gives:

$$\int_{\mathrm{front}} L(p, T)\,dz = 0$$

so that the front as a whole is in thermal balance as expected from the eralier discussion. This is clearly a consequence of the equilibrium of the regions on both sides, neither of which can supply heat to the front. However, this condition involves the structure of the interface so that in order to find the pressure at which the equilibrium occurs we must solve equation (2) with $m = 0$,

$$\text{i.e.}\ \frac{\partial}{\partial z}K(T)\frac{\partial T}{\partial z} = L(p, T) \tag{4}$$

with its associated boundary conditions. The problem thus is an eigen-value problem for the pressure p. By analogy with the case of an imperfect gas (Evans & Penston 1970) we shall call this eigenvalue the ' saturated vapour pressure ' of the interstellar medium. In the next section we shall give a numerical value for this saturation pressure for realistic forms of the loss rate and thermal conductivity in the interstellar medium.

It is only at the saturated vapour pressure then that the clouds and intercloud medium are in true equilibrium. At other pressures, just as in the case of a Van der

Waals gas, this lack of equilibrium is relieved by evaporation at pressures below the saturation pressure and condensation above this pressure. In order to find this evaporation rate at any pressure we must solve equation (2) at the chosen pressure for the eigenvalue m. We also give some typical numerical values for evaporation and condensation rates of clouds in the interstellar medium in the next section.

So far our discussion has only mentioned the equilibrium of the interface between cloud and intercloud but there are two questions concerning the stability of this interface that deserve attention. First, is the front thermally stable? Although this question merits further investigation, we are encouraged to believe that it is thermally stable since Field (1965) has shown that thermal instability is suppressed at scales over which conduction is important. Second, is the front stable under deformations? In other words if the geometry of the front is not plane-parallel, may (for example) the evaporation rate be enhanced in regions of the front which lie closer to the centre of the cloud? We note that for a van der Waal's gas the vapour pressure over a surface is indeed affected by the geometry of the surface (Evans & Penston 1969). However, we shall leave both these questions of stability for future discussion.

3. NUMERICAL ESTIMATES

It is now our intention to determine some numerical values for the saturation pressure, p_{sat}, of the interstellar medium and for the evaporation rate, m, as a function of pressure. Of these the most interesting are the evaporation rates since they enable us to compute the lifetime of a cloud. In our present state of knowledge it does not seem worthwhile to aim at extreme accuracy but we shall hope for a final value of the evaporation rate accurate to a factor two or three. An order of magnitude estimate however would not satisfy us since the values of temperature, density, loss rate and so on all vary by at least one hundred across the front.

It therefore behoves us to solve the eigenvalue problems posed by equations (2) and (4) in the last section and we first pick numerical forms of the loss rate $L(p, T)$, the conductivity $K(T)$ and the enthalpy $\mathrm{H}(p, T)$ relevant to the interstellar medium.

The loss rate

In view of our remarks above concerning the accuracy for which we are aiming and for the sake of simplicity we consider only heating due to cosmic rays and only two cooling mechanisms. We shall see that these give sufficiently good results. At high temperatures the major source of cooling is collisional excitation of atomic hydrogen by electrons and subsequent emission of a photon. This mechanism is cut off sharply at temperatures $\sim 10^4\,°\mathrm{K}$ when colliding electrons cease to have sufficient energy to excite the hydrogen. Below this temperature the excitation of C^+ ions becomes important. Collisions of C^+ with atomic hydrogen are most important and since the relevant excited state of C^+ is only $\sim 0{\cdot}008$ eV above the ground level this cooling mechanism continues to very low temperatures ($\sim 10\,°\mathrm{K}$).

From the discussion of Appendix I, our formula for the loss rate takes the form:

$$L = \left[3{\cdot}1\,.\,10^5\, n_{\mathrm{H}}^{1/2} T^{1/4} (1 + 1{\cdot}27\,.\,10^{-5}\, T) \exp\left(\frac{-1{\cdot}18\,.\,10^5}{T}\right) + 6{\cdot}83\, n_{\mathrm{H}} \exp\left(\frac{-92}{T}\right) - 2{\cdot}17\right] n_{\mathrm{H}} \times 10^{-27}\ \mathrm{erg\ cm^{-3}\ s^{-1}} \quad (5)$$

where n_H and T are the density in H atoms cm^{-3} and degrees Kelvin respectively.

When we define the pressure $p = n_H T$ we have

$$L(p, T) = \left[3 \cdot 1 . 10^5 \, p^{1/2} T^{-1/4} (1 + 1 \cdot 27 . 10^{-5} \, T) \exp\left(\frac{-1 \cdot 18 . 10^5}{T}\right) + 6 \cdot 83 \, pT^{-1} \exp\left(\frac{-92}{T}\right) - 2 \cdot 17\right] pT^{-1} \times 10^{-27} \text{ erg cm}^{-3} \text{ s}^{-1}. \quad (6)$$

Now p is not the total gas pressure in the interstellar medium but is truly the partial pressure of hydrogen. We have noted that $n_e \ll n_H$ so that the partial pressure of the electrons may be ignored (except possibly at the lowest densities). The only other important partial pressure is that of helium, which if present in its normal abundance, will raise the total pressure by a factor of 1·1.

We have used equation (6) to solve

$$L(p, T) = 0$$

and plot in Figs 1 and 2 the resulting loci in the pressure–density and temperature–density diagrams. The general similarity of these curves to those of FGH show that our adoption of only two coolants for our loss rate (6) gives a representation of the full loss-rate adequate for our present purpose.

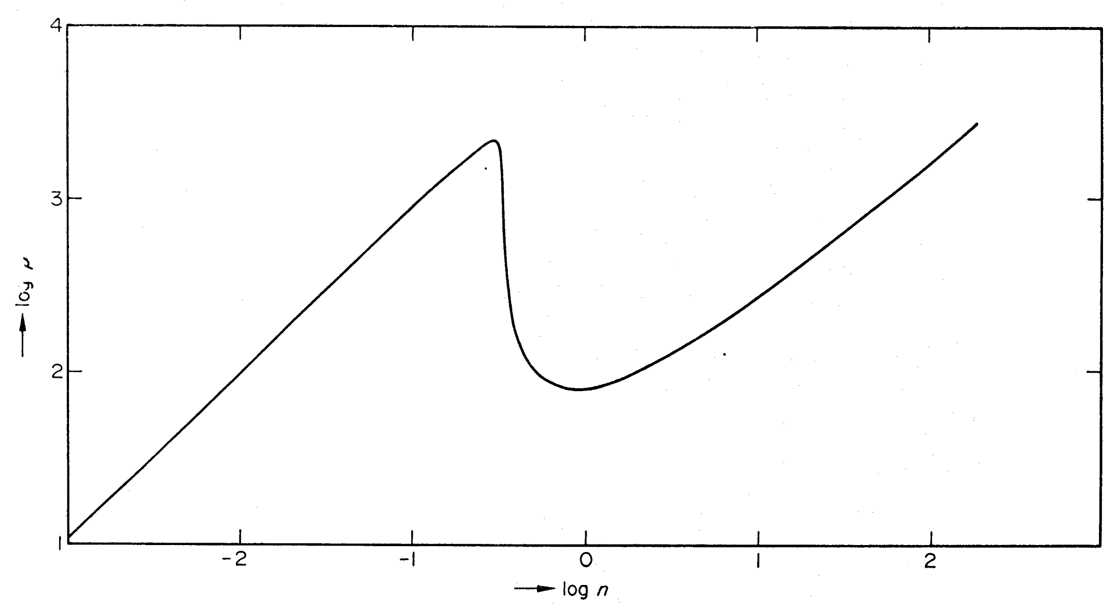

FIG. 1. *Locus of zero loss rate in the pressure–density diagram.*

It is interesting to note that if we have chosen an incorrect value for the ionization rate ζ, the form of the pressure–density and temperature–density relationships is not changed and only in scale do differences occur. If we transform

$$n_H \to n_H \left(\frac{\zeta}{4 . 10^{-16}}\right), \quad p \to p \left(\frac{\zeta}{4 . 10^{-16}}\right) \quad \text{and} \quad T \to T,$$

we find the same equilibrium loci as Figs 1 and 2, since then

$$L \to L \left(\frac{\zeta}{4 . 10^{-16}}\right)^2.$$

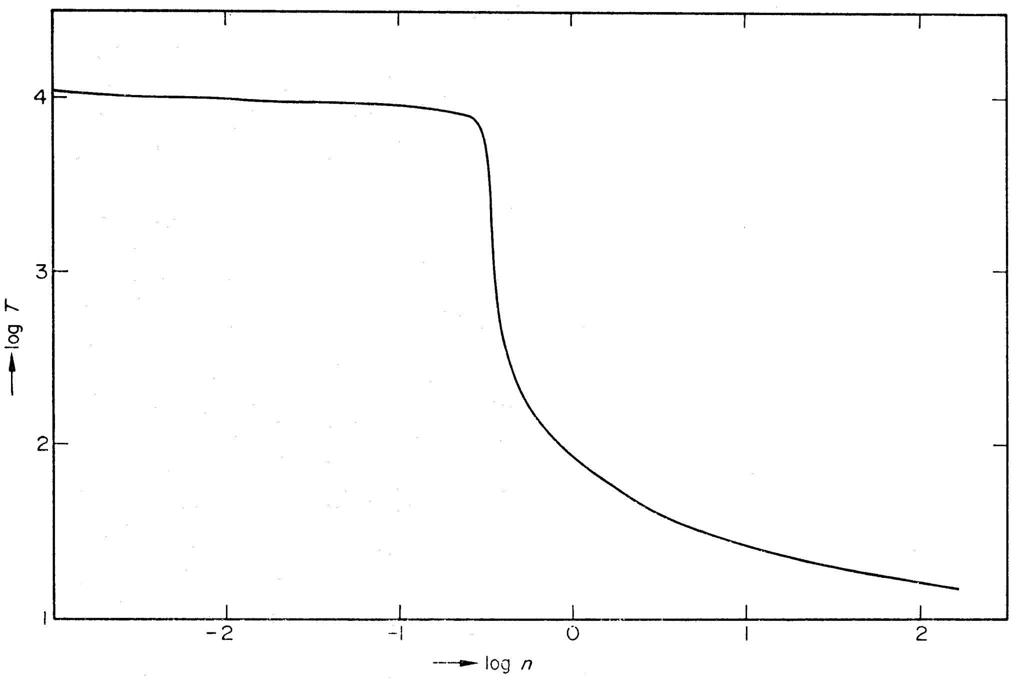

FIG. 2. *Locus of zero loss rate in the temperature–density diagram.*

Thermal conductivity

We shall consider two types of heat transport—firstly ordinary conduction of heat by particles and secondly heat diffusion by the motion of turbulent elements in the cloud–intercloud front. In either case the general formula for the conductivity is given in works on kinetic theory (for example Jeans 1940). We find

$$K = \tfrac{1}{3} n \bar{c} \lambda \frac{d\bar{E}}{dT} \tag{7}$$

where n is the number density of the conducting particles, $\bar{c}$ their mean velocity, λ their mean free path and $\bar{E}$ their mean energy. Jeans also gives formulae for $\bar{c}$, $\bar{E}$ and $n\lambda$ when the particles are atoms:

$$\bar{c} = \sqrt{\left(\frac{8}{\pi}\frac{\mathcal{R}T}{\mu}\right)}, \quad \bar{E} = \frac{1}{\gamma - 1}\frac{\mathcal{R}T}{\mu} m_{\mathrm{H}} \quad \text{and} \quad n\lambda = \frac{1}{\sqrt{2}\,\sigma}$$

where $\mathcal{R}$ is the gas constant, m_{H} the mass of the hydrogen atom and μ and σ the mean molecular weight and collision cross-section of the conducting atoms. Thus for conduction by atomic hydrogen setting $\sigma = 4\pi a_0^2$ (Massey & Burhop 1956) and inserting numerical values we find

$$K(T) = 2{\cdot}0 . 10^3\, T^{1/2}. \tag{8}$$

We note that not only is the conductivity of ionized hydrogen less at the temperatures we shall be considering but also such transport will be greatly inhibited if any magnetic field is present.

Turning to the case where heat is diffused by turbulence we note that the velocity of turbulent elements is limited by the sound speed. Accordingly we set

$$\bar{c} = \alpha \sqrt{\frac{\mathcal{R}T}{\mu}}$$

where α is a constant of order unity. If the radius of the turbulent elements is r then we find

$$\bar{E} = \left(\frac{1}{\gamma-1}+\frac{\alpha^2}{2}\right)\frac{\mathcal{R}T}{\mu}.\frac{4\pi}{3}\rho r^3 \quad \text{and} \quad n\lambda = \frac{1}{\sqrt{2}\pi r^2}$$

where ρ is the ambient density. Thus neglecting factors of order unity, equation (7) gives

$$K \sim \left(\frac{\mathcal{R}}{\mu}\right)^{3/2} T^{1/2}\rho r$$

so that the largest elements are most important for our purpose. However, the turbulent elements must not be so large that the heat cannot diffuse out of them after they come to rest—otherwise the heat they transport is not passed on to the medium as a whole.

The time for the heat to diffuse from the turbulent elements by normal atomic conduction is of order

$$\frac{\mathcal{R}}{\mu}\frac{\rho r^2}{K_{\text{atomic}}}$$

where K_{atomic} is the conductivity of equation (8). Now this time cannot be much greater than the collision time of the turbulent elements λ/c. Since the concept of turbulence supposes $\lambda \sim r$ we find that the maximum value of ρr for which the heat can diffuse from the elements is of order

$$K_{\text{atomic}}\left(\frac{\mathcal{R}}{\mu}\right)^{3/2} T^{1/2}.$$

Thus we conclude that

$$K \sim K_{\text{atomic}}$$

so that including turbulence increases the conductivity by only a factor of ~ 2 over that given in equation (8).

Enthalpy

We have already defined the enthalpy

$$\text{H} = U + pV.$$

If we include in the internal energy of the interstellar gas (assumed monatomic) the ionization energy we find

$$\text{H} = \frac{5}{2}\frac{\mathcal{R}T}{\mu} + \chi\frac{n_e}{n_{\text{H}} m_{\text{H}}}$$

where $\chi = 13 \cdot 6$ eV the ionization energy of hydrogen.

Once again we shall neglect the presence of any helium since this only alters H by a small factor. Then we find

$$\frac{1}{\mu} = 1 + \frac{n_e}{n_{\text{H}}}$$

and

$$\mathrm{H} = \frac{5\mathcal{R}}{2}T\left(1+\frac{n_e}{n_{\mathrm{H}}}\right)+\chi\,\frac{n_e}{n_{\mathrm{H}}m_{\mathrm{H}}}.$$

We turn now to the solution of the first eigenvalue problem—that posed by equation (4) for the saturated vapour pressure, p_{sat}. We have inserted our formulae for the loss rate and conductivity from equations (6) and (8), into equation (4) which we have solved numerically by the Runge-Kutta–Gill method. Although the boundary conditions truly apply as $z \to \pm\infty$ we find no difficulty in starting our integration near one of the equilibrium states (at finite z) since any small initial change in temperature from its equilibrium value or a small temperature gradient lead to solutions that *initially* converge strongly to the true solution. However, as the other equilibrium state is approached, the solutions diverge equally strongly. It seems that the best method for solution is that used in stellar structure problems where the same difficulties arise—we have integrated in from both sides of the front and matched solutions in the middle of the range.

Integrations following the above scheme have shown that the saturation pressure lies in a range given by $2{\cdot}53 < \log p_{\mathrm{sat}} < 2{\cdot}54$. In other words

$$p_{\mathrm{sat}} = 343 \pm 4\,{}^{\circ}\mathrm{K\ cm^{-3}}.$$

We have plotted the resulting profiles of temperatures, conductive flux and loss rate in the equilibrium front in Fig. 3. Most of the structure of the front lies near the low temperature end of the front and accordingly we have plotted these profiles against $\int n_{\mathrm{H}}\,dz$ rather than z. In addition we have plotted the loss rate as L/n_{H} the loss rate per hydrogen atom since then the algebraic area under the profile in such a plot is zero.

We have also found solutions to the eigenvalue problem for m the evaporation rate by solving equation (2) for a variety of pressures $p \neq p_{\mathrm{sat}}$. In Table I are the results of these integrations. For each pressure we have considered we present the evaporation rate with an error $\pm 0{\cdot}025 . 10^{-21}$ together with the velocity of the front into the cloud region and the time, t, required to evaporate a spherical cloud of one solar mass. These quantities are given positive if evaporation occurs and negative when the cloud is condensing. Note that the time-scales are much longer than the collision time between clouds, so that for the most part such evaporation or condensation may be ignored. We shall discuss further the significance of these results in Section 5.

Table I

log p	log (°K cm^{-3})	3·2	3·0	2·8	2·6	2·4	2·2	2·0
m	10^{-21} g cm^{-2} s^{-1}	−1·275	−0·575	−0·225	−0·025	0·075	0·125	0·175
v	cm s^{-1}	−8·0	−6·5	−4·25	−0·9	4·8	18·0	55·0
t	10^9 yr	−5·8	−8·8	−15·6	−92·0	20·0	7·8	3·2

Just as the equilibrium states could be scaled on change of the ionization rate, so the values of the saturated vapour pressure and the evaporation rate can be scaled on changes in either the ionization rate or the constant in equation (8) for the conductivity. If equation (8) is generalized to

$$K(T) = K_0 T^{1/2}$$

as it might be if turbulence was active in carrying heat. Then our solutions apply if

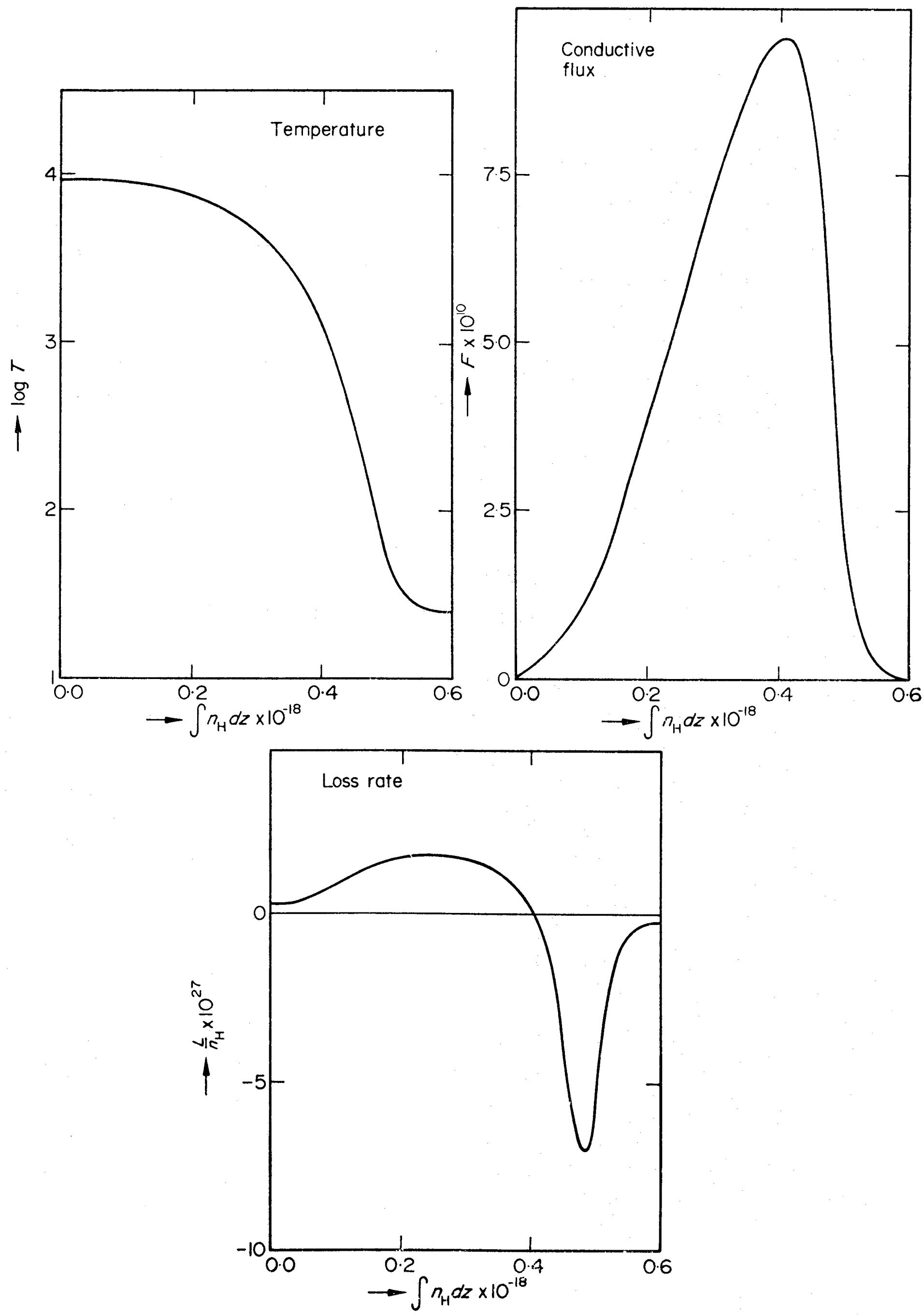

FIG. 3. *Profiles of temperature conductive flux and loss rate through the equilibrium cloud–intercloud interface. The ordinate is* $\int n_{\mathrm{H}}\,dz$ *and the loss rate is plotted as* L/n_{H}*—the loss rate per hydrogen atom.*

we rescale as before and add the transformations

$$z \rightarrow z\left(\frac{\zeta}{4.10^{-16}}\right)^{-1}\left(\frac{K_o}{2{\cdot}0.10^3}\right)^{1/2}$$

$$p_{\mathrm{sat}} \rightarrow p_{\mathrm{sat}}\left(\frac{\zeta}{4.10^{-16}}\right)$$

$$\mathrm{H} \rightarrow \mathrm{H}\left(\text{in as much as } \frac{n_e}{n_{\mathrm{H}}} \ll 1\right)$$

and

$$m \to m \left(\frac{\zeta}{4 . 10^{-16}}\right)\left(\frac{K_0}{2{\cdot}0 . 10^3}\right)^{1/2}.$$

We note that p_{sat} does not change as K_0 changes—so that the saturation pressure is not affected if turbulence is included. On the other hand the evaporation rate is increased by turbulence but only by a very modest factor and of course if the dependence of K on either pressure or temperature were changed, both p_{sat} and m would have to be recalculated afresh.

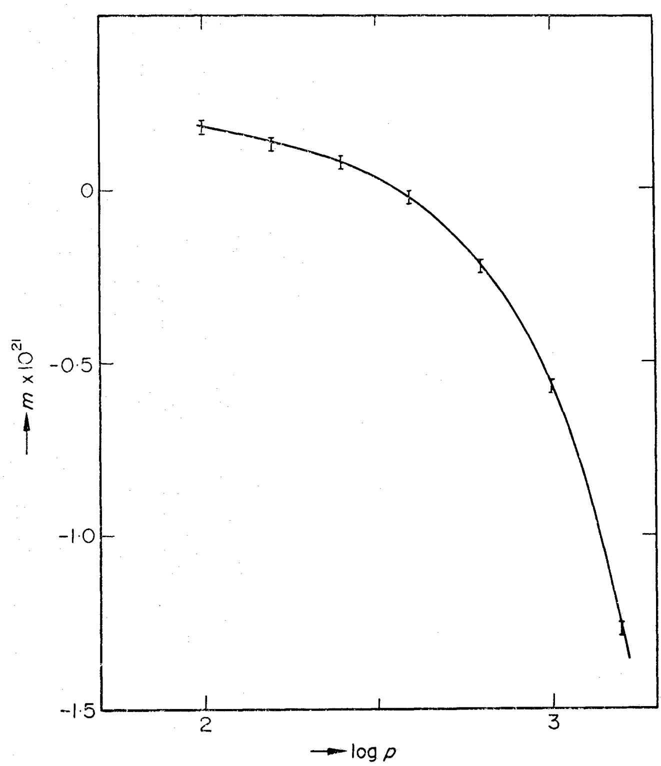

FIG. 4. *The relationship of the evaporation rate m to the pressure in non-equilibrium fronts. Error bars denote maximum errors.*

4. GRAVITATIONAL INSTABILITY

We now consider a different problem concerning the equilibrium and stability of interstellar clouds—that of their gravitational instability. It is this instability that is believed to fix the upper limit of cloud masses. If clusters form from the collapse of interstellar clouds, their mass too is related to this upper limit.

Several years ago the problem of the gravitational instability of the isothermal gas sphere under external pressure was explored by Ebert (1955), Bonnor (1956) and McCrea (1957). Ebert's approach to the problem was to consider first-order perturbations to the gas sphere while Bonnor studied the external pressure required to contain a cloud of given mass as a function of the central condensation, finding a maximum pressure which indicates the onset of instability. McCrea used the virial theorem to predict the instability in a more general way. All three approaches show that the maximum mass M_{max} of a stable isothermal sphere contained by an external pressure $p_{\rm ext}$ is given by a relation of the form:

$$M_{\rm max} = 1{\cdot}2 \left(\frac{\mathscr{R}T}{\mu}\right)^2 G^{-3/2} p_{\rm ext}^{-1/2}. \qquad (9)$$

It is our intention to rediscuss the problem of gravitational instability of interstellar clouds relaxing the assumption of isothermality in favour of adopting the equation of state given by setting the loss rate equal to zero (i.e. that displayed in Fig. 1). In this equation of state the exponent γ in the relationship $p \propto \rho^\gamma$ varies from $\sim 0{\cdot}9$ at a temperature of 10 °K to zero at 91·7 °K corresponding to the minimum pressure of the cloud phase (the minimum of the locus in Fig. 1). This minimum pressure $p_{\rm min} = 79{\cdot}2$ °K cm^{-3} will play an additional role in our problem. It is the lowest external pressure that can be exerted on the surface of an interstellar cloud, since material in the cloud phase would move dynamically from the surface of a cloud if it were immersed in a medium at a lower pressure. For a recent complete discussion of the gravitational instability of an isothermal sphere we refer the reader to Lynden-Bell & Wood (1968).

Elsewhere (Penston 1969) one of us has discussed the gravitational instability of non-isothermal spheres by use of the virial theorem following McCrea's (1957) method. Here we shall use the approach taken by Bonnor (1956). We consider a spherical gas cloud and pick a central density, then we integrate up the hydrostatic equation from the centre using our adopted equation of state to give runs of mass, pressure, density and temperature against radius. We choose an external pressure, stop the integrations when this pressure is reached and read off the mass of the resulting cloud. By varying the central density we soon find there is a maximum mass for given external pressure. There is then no equilibrium configuration for interstellar clouds more massive than this limit under the chosen external pressure. If this chosen external pressure is $p_{\rm min}$ this limit is an absolute maximum mass for spherical equilibrium configurations of material in the cloud phase, where the only support is from thermal pressure.

The equations governing the hydrostatic equilibrium of our gas spheres are:

$$k\frac{dp}{dr} = -\frac{GM_r}{r^2} m_{\rm H} n_{\rm H}$$

$$\frac{dM_r}{dr} = 4\pi m_{\rm H} n_{\rm H} r^2$$

where M_r is the mass within radius r, k is Boltzman's constant and G the gravitational constant and we have assumed the sphere is pure hydrogen—the presence of helium does not affect the maximum by a large factor and we shall return to this point later. To the hydrostatic equations we add:

$$L(p, T) = 0$$

and

$$p = n_{\rm H} T.$$

For a set of central values of $n_{\rm H}$ running from 1·35 to 800 cm^{-3} we have integrated these equations by the Runge-Kutta-Gill method from $r = 0$ until the pressure drops to the value $p = p_{\rm min}$, at which point no further decrease in pressure is possible in the cloud phase.

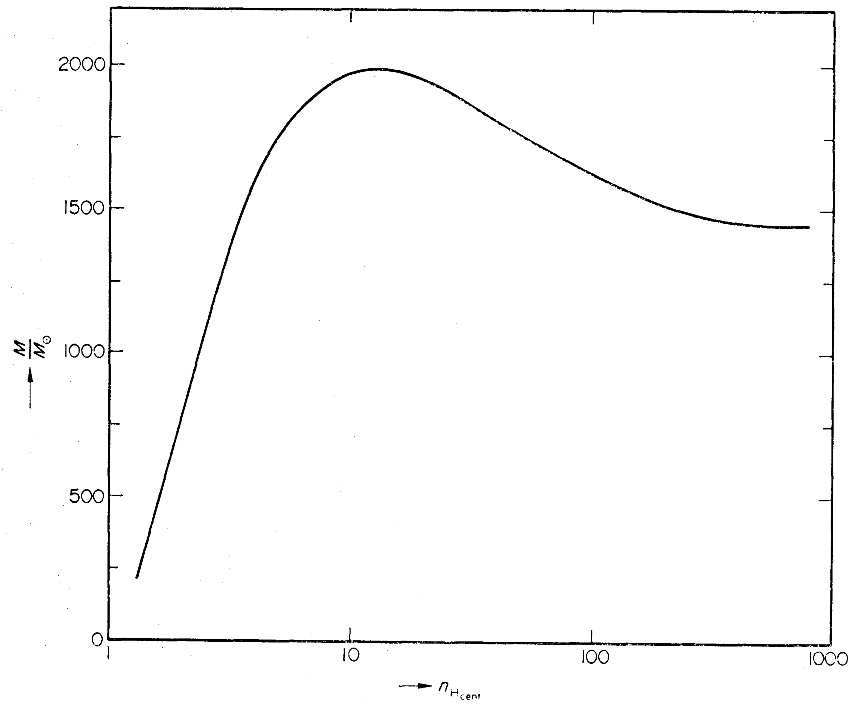

FIG. 5. *The mass in a spherical cloud in hydrostatic equilibrium plotted against the central density when the external pressure is equal to* p_{min}.

If the external pressure $p_{ext} = p_{min}$ then we find the relationship between the mass of the cloud and the central density displayed in Fig. 5 and deduce the maximum mass $M_{max} \sim 2000\, M_{\odot}$. This as mentioned above is the absolute maximum mass that can exist as a spherical cloud in hydrostatic equilibrium and other choices of p_{ext} give lower mass. For example if we choose $p_{ext} = p_{sat}$ (although the long time-scales for evaporation in Table I give us no reason for thinking this more likely than any other pressure) we find a maximum mass of only $\sim 320\, M_{\odot}$.

These masses are much less than previous values deduced for the maximum mass gravitationally stable interstellar clouds which have been thought to be $\sim 40\,000\, M_{\odot}$ (Field & Saslaw 1965). This difference is not primarily caused by relaxing the assumption of isothermality but rather by using the lower temperatures given by $L(p, T) = 0$ in preference to the traditional value of 100 °K. It can be seen from equation (9) the maximum mass is sensitive to the value of the temperature assumed. Indeed by using for T in equation (9) the central value (which is also the lowest value) of the temperature in our most massive sphere (~ 30 °K) we find the maximum mass of an isothermal sphere of this temperature under an external pressure $p_{ext} = p_{min}$ is also $\sim 2000\, M_{\odot}$. It seems that the destabilizing effects of the lower values of the exponent γ in the outer parts of the gas sphere are counteracted by the increased temperature in these regions.

We shall discuss the astrophysical consequences of this result for the maximum mass in the next section. Here we shall make further comments on the changes in scale which result either from changes in ζ, the ionization rate, or including the effects of helium. If we assume our gas sphere has a helium to hydrogen ratio by mass of Y, then we make the transformations

$$p \to p(1 + Y/4)\left(\frac{\zeta}{4.10^{-16}}\right) \quad \text{and} \quad \rho \to \rho(1 + Y)\left(\frac{\zeta}{4.10^{-16}}\right).$$

When we add

$$r \to r(1+Y/4)^{1/2}(1+Y)^{-1}\left(\frac{\zeta}{4.10^{-16}}\right)^{-1/2}$$

and

$$M \to M(1+Y/4)^{3/2}(1+Y)^{-2}\left(\frac{\zeta}{4.10^{-16}}\right)^{-1/2}$$

we find the same functional forms apply for the solutions for equilibrium gas spheres.

5. RELEVANCE TO STAR FORMATION

It remains to discuss the relevance of our conclusions in earlier sections for current ideas on star formation and the interstellar medium.

In Section 3 we showed that clouds evaporate or condense only on a very long time-scale. Thus the true equilibrium of clouds and intercloud medium at the saturation pressure of the interstellar medium is never attained, and for practical purposes pressure equilibrium is all we need to consider. Although this is a negative conclusion, an understanding of the physics of the phase-change mechanism is important as it removes doubts as to the nature, cause and time scale of this effect. In particular it is reassuring to find that treatments of the interstellar clouds as discrete objects (Field & Saslaw 1965; Field & Hutchins 1968; Penston *et al.* 1969), are entirely justified.

On the other hand, the problem we have studied of an isolated cloud at rest in an otherwise infinite uniform medium is somewhat artificial. In particular some clouds may be moving supersonically through the intercloud medium and such motions would pose severe problems in obtaining a hydrodynamic version of our theory. We have already mentioned in Section 2 two other problems relating to the stability of the interface region that need to be solved: the thermal stability and the geometrical stability of the interface. These remain as work for the future.

Turning to the results of Section 4, we must compare our most massive interstellar cloud with the masses of clusters. According to the Oort (1954) model of star formation such massive clouds represent the basic units from which star formation occurs and we should not expect cluster masses much in excess of this value. Now certainly the masses of globular clusters greatly exceed our value of $\sim 2000\ M_\odot$ but we may surmise that the globulars formed by a different process at a time when the galaxy was collapsing. Masses for the galactic clusters NGC 188 and M 67 are given by van den Bergh & Sher (1960) as being $\sim 1000\ M_\odot$, but their data also shows that NGC 7789 is a cluster with a mass a factor of 3 to 4 greater. While it does not seem possible to be sure these figures are absolutely in conflict, it does seem clear that a maximum cloud mass of $\sim 2000\ M_\odot$, indicates that star formation has an efficiency of 25–100 per cent as against the value of 3 per cent given by Field & Saslaw (1965). On the other hand, it seems likely that the maximum unstable mass might be considerably increased if a magnetic pressure, turbulence or rotation were present. For example the exponent γ relevant to the magnetic case is 4/3 which is considerably greater than that applying to the gas and it has been shown elsewhere (Penston 1969) that truncated gas spheres are always gravitationally stable if $\gamma \geqslant 4/3$. Thus when modified by other effects the true maximum mass may well be in excess of that we have derived but this can only be so if interstellar clouds are not primarily supported by thermal pressure.

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Since this paper was first submitted for publication, a very similar piece of work by Zeldovich & Pikelner (1969) has appeared in English Translation. Although the present work was done quite independently, it must be acknowledged that the Russian authors were the first to derive the results presented here in Section 2. On the other hand, our discussion of Section 3, although reaching the same conclusions as Zeldovich and Pikelner, is an improvement over their treatment as fewer approximations were made here.

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APPENDIX I

DETAILS OF THE HEAT-LOSS FUNCTION

The heating rate from cosmic rays is given by

$$\Gamma = n_0 \zeta \langle E \rangle$$

where n_0 is the number density of *neutral* hydrogen, ζ is the cosmic ray ionization rate and $\langle E \rangle$ the average kinetic of the liberated electrons. Following FGH we adopt $\zeta = 4 . 10^{-16}\ \mathrm{s}^{-1}$, and take $\langle E \rangle = 3{\cdot}4$ eV from Spitzer & Tomasko (1968).

The cooling rate for collisional excitation of neutral hydrogen and emission of Lyα is given by Burbidge, Gould & Pottasch (1963) as

$$\Lambda_1 = n_e n_0 E_2 \langle \sigma v \rangle$$

where n_e is the electron density, $E_2 = 10{\cdot}2$ eV and

$$\langle \sigma v \rangle = 0{\cdot}9\ \pi a_0^2 \left(\frac{E_2}{2m_e}\right)^{1/2} \left(1 + \frac{3kT}{2E_2}\right) \exp\left(-\frac{E_2}{kT}\right)$$

where a_0 is the Bohr radius, m_e the electronic mass and k Boltzmann's constant.

For cooling by carbon ions, Spitzer & Tomasko (1968) give the rate

$$\Lambda_2 = 1{\cdot}72 . 10^{-23}\ n_0 n_{C^+} \exp\left(-\frac{92}{T}\right)\ \mathrm{erg\ cm^{-3}\ s^{-1}}$$

and we adopt a carbon abundance

$$\frac{n_{C^+}}{n_H} = 4 . 10^{-4}.$$

To find the electron density, we equate cosmic ray ionization and recombinations

$$n_0 \zeta = \alpha n_e n_p$$

where the recombination coefficient

$$\alpha = \frac{6{\cdot}21 . 10^{-11}}{T^{1/2}}\ \mathrm{cm^{-6}\ s^{-1}}.$$

Using charge neutrality we find

$$2n_e = n_i - \frac{\zeta}{\alpha} + \left(n_i^2 + 2\frac{\zeta}{\alpha} n_i + \frac{\zeta^2}{\alpha^2} + 4n_H\right)^{1/2}$$

where n_i is the total number density of electron donors such as C^+, Mg^+ and Fe^+ which are almost completely ionized by starlight and n_H is the *total* density of protons and neutral hydrogen atoms.

This formula corrects that of Spitzer and Tomasko which assumed $n_H \gg n_e$. We are only interested in the case

$$n_H \gg n_e \gg n_i$$

when

$$n_e = \left(\frac{\zeta}{\alpha} n_H\right)^{1/2}.$$

Finally we adopt then the heat-loss function

$$L = \Lambda_1 + \Lambda_2 - \Gamma.$$