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THE COHN-JORDAN EXTENSION AND SKEW MONOID RINGS OVER A QUASI-BAER RING

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Abstract. A ring R is called *(left principally) quasi-Baer* if the left annihilator of every (principal) left ideal of R is generated by an idempotent. Let R be a ring, G be an ordered monoid acting on R by β and R be G-compatible. It is shown that R is (left principally) quasi-Baer if and only if skew monoid ring $R_\beta[G]$ is (left principally) quasi-Baer. If G is an abelian monoid, then R is (left principally) quasi-Baer if and only if the Cohn-Jordan extension $A(R,\beta)$ is (left principally) quasi-Baer if and only if left Ore quotient ring $G^{-1}R_\beta[G]$ is (left principally) quasi-Baer.

Introduction

Throughout this paper R denotes an associative ring with unity, $Inj(R)$ and $Aut(R)$ the set of all injective endomorphisms and the set of all automorphisms of R respectively. Recall from [1] that R is a *Baer* ring if the right annihilator of every nonempty subset of R is generated by an idempotent. In [19], Kaplansky introduced Baer rings to abstract various properties of von Neumann algebras and complete $*$-regular rings. The class of Baer rings includes the von Neumann algebras. In [7], Clark defines a ring to be *quasi-Baer* if the left annihilator of every left ideal is generated, as a left ideal, by an idempotent. Moreover, he shows the left-right symmetry of this condition by proving that R is quasi-Baer if and only if the right annihilator of every right ideal is generated, as a right ideal, by an idempotent. Further work on quasi-Baer rings appears in [3]-[6], [15]-[16] and [23]. Every prime ring is a quasi-Baer ring. In [4], Birkenmeier et al. defines a ring to be called *left* (resp. *right*)

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principally quasi-Baer if the left (resp. right) annihilator of a principal left (resp. right) ideal of R is generated by an idempotent. Observe that every biregular ring and every quasi-Baer ring is left (right) principally quasi-Baer.

A natural question for a given class of rings is: How dose the given class behave with respect to polynomial extensions? In 1974, Armendariz considered the behavior of a polynomial ring over a Baer ring by obtaining the following result: Let R be a *reduced* ring (i.e. R has no nonzero nilpotent elements). Then $R[x]$ is Baer if and only if R is Baer [1, Theorem B]. Armendariz provided an example to show that the reduced condition is not superfluous. Note that in a reduced ring R, R is Baer if and only if R is quasi-Baer. In [3], Birkenmeier et al. showed that a ring R is right principally quasi-Baer if and only if $R[x]$ is right principally quasi-Baer. In [15], Hirano considered relations between the set of annihilators in R and the set of annihilators in $R[x]$. In [5], Birkenmeier et al. showed that the quasi-Baer condition is preserved by many polynomial extensions including $R[[x;\alpha]]$ and $R[[x,x^{-1};\alpha]]$, where α is an automorphism of R. In [17], Hong et al. showed that, an *α-rigid* ring R (where α is an endomorphism of R such that for each $a \in R$, $a\alpha(a) = 0$ implies that $a = 0$) is quasi-Baer if and only if $R[x;\alpha,\delta]$ is quasi-Baer, where δ is an α-derivation on R.

A monoid G is *ordered* if G has a total ordering $\leq$ such that whenever $s < s'$ we have $s_1 s < s_1 s'$ and $s s_1 < s' s_1$ for all $s_1 \in G$. Let R be a ring, let G be an ordered monoid and $\beta : G \to Inj(R)$ a homomorphism of monoids which map identity of G to identity of $Inj(R)$ (or simply, G acts on R by β). We denote by β_g the image of $g \in G$ under β. The skew monoid ring $R_\beta[G]$ is a ring which as a left R-module is free with basis G and multiplication defined by the rule $gr = \beta_g(r)g$. Let α be an endomorphism of R. We say that R is *α-compatible* if for each $a, b \in R$, $ab = 0 \Leftrightarrow a\alpha(b) = 0$. In this case, clearly the endomorphism α is injective. Note that this definition is a generalization of rigid concept.

In this paper we show that for an G-compatible ring R, the skew monoid ring $R_\beta[G]$ is (left principally) quasi-Baer if and only if R is (left principally) quasi-Baer. When G is an abelian monoid, we show that R is (left principally) quasi-Baer if and only if the Cohn-Jordan extension $A(R,\beta)$ is (left principally) quasi-Baer if and only if left Ore quotient ring $G^{-1}R_\beta[G]$ is (left principally) quasi-Baer.

As a consequence, for an α-compatible ring R, the Ore extension $R[x;\alpha]$ is (left principally) quasi-Baer if and only if skew Laurent extension $R[x,x^{-1};\alpha]$ is (left principally) quasi-Baer if and only if R is (left principally) quasi-Baer. Also if G is an ordered monoid, then R is (left principally) quasi-Baer if and only if monoid ring $R[G]$ is (left principally) quasi-Baer.

For a nonempty subset X of R, $r_R(X)$ and $\ell_R(X)$ denote the right and left annihilators of X in R respectively.

We start by providing example of α-compatible ring which is not α-rigid:

EXAMPLE 1. ([11, Example 1.2]) Let R be an α-rigid ring. Let

$$R_3 = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \middle| \; a, b, c, d \in R \right\}$$

be a subring of the 3×3 upper triangular matrix ring over R. The endomorphism α of R is extended to the endomorphism $\overline{\alpha} : R_3 \to R_3$ defined by $\overline{\alpha}((a_{ij})) = (\alpha(a_{ij}))$. Then

(i) R_3 is an $\overline{\alpha}$-compatible ring;

(ii) R_3 is not $\overline{\alpha}$-rigid.

The following example shows that there exists an α-compatible (left principally) quasi-Baer ring R such that R is not α-rigid:

EXAMPLE 2. Let R_1 be a (left principally) quasi-Baer ring, D be a domain and $R = T_n(R_1) \oplus D[y]$, where $T_n(R_1)$ is the $n \times n$ upper triangular matrix ring over R_1. Let $\alpha : D[y] \to D[y]$ be an injective endomorphism which is not surjective. Then we have the following:

(1) R is (left principally) quasi-Baer;

(2) Let $\overline{\alpha} : R \to R$ be an endomorphism defined by $\overline{\alpha}(A \oplus f(y)) = A \oplus \alpha(f(y))$ for each $A \in T_n(R_1)$ and $f(y) \in D[y]$. Then $\overline{\alpha}$ is injective but not surjective and R is $\overline{\alpha}$-compatible which is not $\overline{\alpha}$-rigid.

LEMMA 3. ([11, Lemma 2.1]) *Let R be an α-compatible ring. Then we have the following:*

(i) *If $ab = 0$, then $a\alpha(b) = \alpha(a)b = 0$.*

(ii) *If $\alpha(a)b = 0$ for some $a, b \in R$, then $ab = 0$.*

LEMMA 4. ([11, Lemma 2.2]) *Let α be an endomorphism of a ring R. Then R is α-compatible reduced if and only if R is α-rigid.*

DEFINITION 5. A ring R is said to be G-compatible if for each $g \in G$, β_g is a compatible endomorphism of R.

Hiranio [15], defines a ring R to be *quasi-Armendariz* if whenever tow polynomials $f(x) = \sum_{i=0}^{m} a_i x^i, g(x) = \sum_{j=0}^{n} b_j x^i \in R[x]$ satisfy

$f(x)R[x]g(x) = 0$, we have $a_iRb_j = 0$ for each i and j. We extend this definition as follows:

DEFINITION 6. Let R be a ring and G be an ordered monoid acting on R by β. We say that R is skew quasi-Armendariz if whenever $fR_\beta[G]g = 0$ for $f = \sum_{i=1}^{n} r_i f_i$, $g = \sum_{j=1}^{m} s_j g_j \in R_\beta[G]$ with $r_i, s_j \in R$, $f_i, g_j \in G$ satisfying $f_i < f_j$ and $g_i < g_j$ if $i < j$, then $r_iRs_j = 0$ for each i and j.

For a ring R put $rAnn_R(id(R))=\{r_R(U) \mid U \text{ is an ideal of } R\}$. By a similar way as in the proof of [11, Proposition 2.5] we can prove the following result.

PROPOSITION 7. *Let R be an G-compatible ring and S be the skew monoid ring $R_\beta[G]$. Then the following statements are equivalent:*
(1) *R is skew quasi-Armendariz;*
(2) *$\psi : rAnn_R(id(R)) \to rAnn_S(id(S)); A \to AS$ is bijective.*

DEFINITION 8. (Tominaga [30]) An ideal I of R is said to be *left s-unital* if, for each $a \in I$ there is an $x \in I$ such that $xa = a$. If an ideal I of R is left s-unital, then for any finite subset F of I, there exists an element $e \in I$ such that $ex = x$ for all $x \in F$.

By a similar way as in the proof of [11, Theorem 2.7] we can prove the following result.

PROPOSITION 9. *Let R be an G-compatible ring and S be the skew monoid ring $R_\beta[G]$. Then the following are equivalent:*
(1) *$r_R(aR)$ is left s-unital in R for any element $a \in R$;*
(2) *$r_S(fS)$ is left s-unital in S for any element $f \in S$. In this case R is skew quasi-Armendariz.*

Since (left principally) quasi-Baer rings satisfy hypothesis of Proposition 9, hence we have:

THEOREM 10. *Let R be an G-compatible ring. Then R is (left principally) quasi-Baer if and only if $R_\beta[G]$ is (left principally) quasi-Baer; In this case R is skew quasi-Armendariz.*

By [27, Lemma 13.1.6 and Corollary 13.2.8], torsion-free nilpotent groups and free groups are ordered groups. Hence any submonoid of a torsion-free nilpotent group or a free group is an ordered monoid. So we have the following corollary:

COROLLARY 11. *Let G be a submonoid of a free group or of a torsion-free nilpotent group. If R is G-compatible, then the skew monoid ring $R_\beta[G]$ is a (left principally) quasi-Baer ring if and only if R is a (left principally) quasi-Baer ring.*

A ring R is called a *right* (resp. *left*) *p.p.-ring* if the right (resp. left) annihilator of an element of R is generated, as a right (resp. left), ideal by an idempotent of R. R is called a *p.p.*-ring if it is both right and left *p.p.* If R is a reduced ring, then R is *p.p.*-ring if and only if R is a left principally quasi-Baer ring. Hence we have the following corollary, which is a generalization of [1, Theorem A].

COROLLARY 12. *Let R be a reduced ring and G be an ordered monoid acting on R by β. If R is an G-compatible ring, then the skew monoid ring $R_\beta[G]$ is a p.p.-ring (resp. Baer ring) if and only if R is a left principally quasi-Baer (resp. quasi-Baer) ring.*

Example 2 shows that there is an α-compatible quasi-Baer ring which is not reduced. Therefore our Corollary 13 is not implied from Hong et al.' result [17, Theorem 11].

COROLLARY 13. *Let R be an α-compatible ring. Then R is (left principally) quasi-Baer if and only if $R[x;\alpha]$ is (left principally) quasi-Baer.*

COROLLARY 14. (Hirano [15]) *Let R be a ring and G be an ordered monoid. Then R is (left principally) quasi-Baer if and only if ordered monoid ring $R[G]$ is (left principally) quasi-Baer.*

The following example shows that α-compatible condition on R in Corollary 13 is not superfluous:

EXAMPLE 15. (Han et al. [10, Example 2.8]) There is an example of a quasi-Baer ring R and an endomorphism α of R such that $R[x;\alpha]$ is not quasi-Baer. In fact, let $R = F[t]$ be the polynomial ring over a field F and α be the endomorphism given by $\alpha(f(t)) = f(0)$. Then the ring $R[x;\alpha]$ is not quasi-Baer.

In the sequel, G is an abelian orderd monoid acting on R by β.

DEFINITION 16. Let A be a ring and $\tau : G \longrightarrow Aut(A)$ be a monoid homomorphism. We say that the pair (A, τ) is the Cohn-Jordan extension of the pair (R, β), if A and τ satisfy the following conditions:

(1) R is a subring of A and τ_u is an extension of β_u for each $u \in G$, and

(2) $A = \bigcup_{g\in G} \tau_g^{-1}(R)$.

The ring A which is constructed by D. A. Jordan [18], is called the Cohn-Jordan extension of R by means of β and is denoted by $A(R, \beta)$.

PROPOSITION 17. *Let R be a ring and G be an abelian ordered monoid acting on R by β. Then we have the following:*

(1) *The set G is a left denominator set in $R_\beta[G]$.*

(2) *The set $A = \{u^{-1}ru \mid u \in G,\ r \in R\}$ is a subring of the left Ore quotient ring $G^{-1}R_\beta[G]$.*

(3) *Let us consider the monoid homomorphism $\tau : G \longrightarrow Aut(A)$ given by the equality $\tau_u(b) = ubu^{-1}$ for all $u \in G$ and $b \in A$. Then the pair (A, τ) is the Cohn-Jordan extension of (R, β) and $G^{-1}R_\beta[G] \cong A_\tau[G, G^{-1}]$ where $A_\tau[G, G^{-1}]$ is a skew group ring.*

Proof. The equality $u^{-1}ru + v^{-1}sv = (uv)^{-1}(\beta_v(r) + \beta_u(s))(uv)$ for all $u, v \in G$ and $r, s \in R$ shows that A is closed under addition. It is easy to see that A is closed under multiplication. Therefore A is a subring of $G^{-1}R_\beta[G]$. If $r \in R$ and $u \in G$, then $\beta_u(r) = uru^{-1} = \tau_u(r)$ and so τ_u is an extension of β_u. Thus condition (1) of Definition 16 holds. Condition (2) of Definition 16 follows from definition τ_g. The ring homomorphism $\lambda : G^{-1}R_\beta[G] \longrightarrow A_\tau[G, G^{-1}]$ given by the equality $\lambda(u^{-1}rv) = \tau_u^{-1}(r)u^{-1}v$ for all $u, v \in G$, $r \in R$ is an automorphism. □

The method of construction of the Cohn-Jordan extension described above due to Mushrub [26], is a generalization of the method presented by D. A. Jordan in [18]. The Cohn-Jordan extension (A, τ) of (R, β) has the following universal property. If B is an overring of R and $\varphi : G \longrightarrow Aut(B)$ such that $\varphi_g(r) = \beta_g(r)$ for all $g \in G$ and $r \in R$, then A embeds in B by means of the mapping $\tau_g^{-1}(r) \longrightarrow \varphi_g^{-1}(r)$. The Cohn-Jordan extension of R by means of β solves the universal problem and is therefore unique up to isomorphism. It is clear that $A(R, \beta) = R$ if and only if $Im(\beta) \subseteq Aut(R)$.

Proposition 18. *A ring $A = \{g^{-1}rg \mid g \in G,\ r \in R\}$ is G-compatible if and only if R is G-compatible.*

Proof. Clearly subring of G-compatible ring is G-compatible. Let R be an G-compatible ring and $g^{-1}rg, v^{-1}sv \in A$ with $g, v \in G$ and $r, s \in R$, such that $(g^{-1}rg)\tau_h(v^{-1}sv) = 0$ for some $h \in G$. Then $\beta_v(r)\beta_{gh}(s) = 0$. Since R is G-compatible, $\beta_v(r)\beta_g(s) = 0$. Thus $(g^{-1}rg)(v^{-1}sv) = 0$. By a similar argument one can show that if $(g^{-1}rg)(v^{-1}sv) = 0$ then $(g^{-1}rg)\tau_h(v^{-1}sv) = 0$ for each $h \in G$. Therefore A is G-compatible. □

Proposition 19. *Let R be a ring and G be an abelian ordered monoid acting on R by β. If R is an G-compatible ring, then R is (left principally) quasi-Baer if and only if $A(R, \beta)$ is (left principally) quasi-Baer.*

Proof. Suppose that A is quasi-Baer and I is an ideal of R. Then $I_\beta = \sum_{g \in G} R\beta_g(I)R$ is an ideal of R and $\beta_h(I_\beta) \subseteq I_\beta$ for each $h \in G$.

Let $\Delta(I_\beta) = \{u^{-1}ru \mid r \in I_\beta, u \in G\}$. Then $\Delta(I_\beta)$ is an ideal of A. Since A is quasi-Baer, there is an idempotent $e \in R$ and $u \in G$ such that $r_A(\Delta(I_\beta)) = (u^{-1}eu)A$. We show that $r_R(I) = eR$. Since $I(u^{-1}eu) = 0$ and R is G-compatible, we have $Ie = 0$. Thus $eR \subseteq r_R(I)$. Now let $t \in r_R(I)$. By G-compatibility of R, we have $\Delta(I_\beta)t = 0$. Thus $t = (u^{-1}eu)t$. Hence $\beta_u(t) = e\beta_u(t)$ and by G-compatibility of R, $t = et$. Thus $r_R(I) = eR$. Therefore R is quasi-Baer.

Suppose that R is quasi-Baer and J is an ideal of A. Then $J_\tau = \sum_{g\in G} \tau_g(J)$ is an ideal of A and $\tau_h(J_\tau) \subseteq J_\tau$ for each $h \in G$. Since by Proposition 18, A is G-compatible so $r_A(J) = r_A(J_\tau)$. For each $g \in G$, let $J^g_\tau = \{r \in R \,|\, g^{-1}rg \in J_\tau\}$. Then J^g_τ is an ideal of R and $J^g_\tau \subseteq J^1_\tau$, where 1 is an identity element of G. Since R is quasi-Baer, $r_R(J^1_\tau) = eR$ for some idempotent $e \in R$. We claim that $r_A(J) = eA$. Since $J^1_\tau e = 0$ and $J^g_\tau \subseteq J^1_\tau$ and A is G-compatible, we have $Je = 0$. Now let $a \in r_A(J)$. Then $a = u^{-1}ru$ for some $u \in G$ and $r \in R$. Since $r_A(J) = r_A(J_\tau)$ and A is G-compatible, we have $r \in r_R(J^1_\tau)$. Hence $r = er$ and by Lemma 3, $r = \beta_u(e)r$. Thus $a = u^{-1}ru = (u^{-1}\beta_u(e)u)(u^{-1}ru) = ea$. Therefore $r_A(J) = eA$. □

THEOREM 20. *Let R be a ring and G be an abelian ordered monoid acting on R by β. Let R be an G-compatible ring. Then the following conditions are equivalent:*

(1) *R is (left principally) quasi-Baer;*
(2) *A is (left principally) quasi-Baer;*
(3) *$G^{-1}R_\beta[G]$ is (left principally) quasi-Baer.*

PROOF. The equivalence of (1) and (2) follows from Proposition 19. By Proposition 18, R is G-compatible if and only if A is G-compatible and by Proposition 17, $G^{-1}R_\beta[G] \cong A_\tau[G, G^{-1}]$. So (2) $\Leftrightarrow$ (3) follows from Theorem 10. □

COROLLARY 21. *Let G be an abelian submonoid of a free group or of a torsion-free nilpotent group. If R is G-compatible, then R is a (left principally) quasi-Baer ring if and only if left Ore quotient ring $G^{-1}R_\beta[G]$ is a (left principally) quasi-Baer ring.*

COROLLARY 22. *Let R be an α-compatible ring. Then R is (left princi-pally) quasi-Baer if and only if $R[x, x^{-1}; \alpha]$ is (left principally) quasi-Baer.*

The following example [11, Example 3.6] or [17, Example 9] shows that there exists a ring R which $R[x, x^{-1}; \alpha]$ and $R[x; \alpha]$ are (left principally) quasi-Baer, but R is not (left principally) quasi-Baer, hence

α-compatibility condition on R in Corollaries 13 and 22 is not superfluous.

EXAMPLE 23. Let $\mathbb{Z}$ be the ring of integers and consider the ring $\mathbb{Z} \oplus \mathbb{Z}$ with the usual addition and multiplication. Let $R = \{(a, b) \in \mathbb{Z} \oplus \mathbb{Z} \mid a \equiv b \pmod 2\}$ the subring of $\mathbb{Z} \oplus \mathbb{Z}$. Then:

(i) R is not quasi-Baer.

(ii) Now let $\alpha : R \to R$ be defined by $\alpha((a, b)) = (b, a)$. Then α is an automorphism of R which is not α-compatible.

(iii) $R[x, x^{-1}; \alpha]$ and $R[x; \alpha]$ are quasi-Baer.

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