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THE COLLAPSE PHASE OF EARLY SOLAR EVOLUTION

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SUMMARY

That phase of the contraction of a star in which H_2 molecules are being dissociated and H and He atoms are being ionized is examined, and it is found that a stellar configuration must be unstable against dynamical collapse during this phase. Quantitative calculations have been made with polytropes of indices 1.5 and 3. The gravitational instability sets in for stars of one solar mass at a radius of about 100 A.U. and ceases at about 1/3 A.U. If a star is rotating, having conserved angular momentum during its collapse, then it should flatten into a nebular disk without forming a central body in hydrostatic equilibrium. Some physical parameters of possible disks are given.

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INTRODUCTION

The riddle of solar system formation is one of the oldest unsolved problems of scientific philosophy. Serious speculation on this topic dates back to Descartes. More than 300 years ago, he advanced some ideas concerning the condensation of the sun and planets from a chaotic mass of dust and gas; these speculations bear a remarkable resemblance to some of the ideas now accepted about the formation of the solar system. In the intervening three centuries, a great many schemes for the formation of the solar system have been put forward, some of them in remarkable mathematical detail; however, they are only as plausible as their assumed set of initial conditions, and the initial conditions vary enormously.

The basic difficulty with the development of such a theory is that we have very little concrete information upon which to base it. Until recent years, this information consisted only of a number of remarkable regularities observed within the solar system itself. There are basically two regularities involved. One is the tendency for alignment of the angular momentum vectors in the solar system. This means that: (1) the rotations of the sun, planets, and satellites tend to have the same equatorial planes, and (2) the motions of the satellites about the planets, and of the planets about the sun, tend also to lie in the same plane. The other regularity is that there seems to be a smoothly increasing spacing between the orbits of the planets. When we examine the nature of the regularities in orbital spacings, the latter, known as Bode's law, is often more impressive in its departures from strict regularity than in its adherence to it.

In more recent years we have obtained much additional information about the early history of the solar system from the study of meteorites. These objects are usually believed to be fragments of asteroidal bodies that have been destroyed by collisions. Thus, a study of the chemical and physical properties of meteorites can yield much information about conditions in the interiors of asteroidal bodies. This subject is extremely complicated, and the resulting interpretations are still very controversial.

One of the more exciting discoveries of recent years concerning the meteorites is the presence in some of them of anomalous isotopic compositions of certain elements. These anomalies take the form of excess abundances of isotopes which are decay daughters of certain radioactivities with half-lives of a few million years. It appears that these radioactivities, though now extinct, must have been present in the meteorite parent bodies when they were formed. From the abundances of these radioactivities (deduced from the anomalous abundance of the daughter isotopes in the daughter elements), much can be inferred as to the time at which chemical isolation of the meteorite parent bodies occurred.

We may expect to obtain a great deal of additional information about the early history of the solar system when we begin systematic explorations of the moon. Because of: (1) the absence of erosion on its surface, and (2) its apparently minimal amount of volcanic activity, the moon has preserved a record of the solar system's early history that has long since been eradicated from the earth. Thus, within the next few years, it should become possible to extrapolate the history of the solar system backward in time with a great deal more assurance than can be done at present. It is doubtful, however, that we shall ever properly understand the processes involved in the formation of the solar system solely from studies of this kind. We must combine such findings with studies of star formation. Most investigators have now come to view the planets as having been formed by some sort of condensation process of a nebular disk of gas and dust that once encircled the sun. There is considerable controversy as to how this disk was formed, but many feel that it must have been a natural consequence of the sun's formation. Thus, it is desirable to study all phases of the star formation process.

STAR FORMATION

The study of star formation and its relation to the formation of the solar system has begun much more recently than has speculation about the formation of the solar system itself. Such studies were not very meaningful until recent years, when it became apparent that considerable gas and dust lying between the stars is available for condensation into new stellar systems; and that some stars have been formed in our galaxy within the last few million years. This has lead to much speculation about the conditions necessary for the formation of stars.

Other studies in recent years have shown that the galaxy is much older than had previously been thought. In particular, detailed studies of stellar evolution conducted with large electronic computers have given results that can be compared with the Hertzprung-Russell diagrams of old galactic and globular star clusters, and from which the ages of the clusters can be deduced. There is still much uncertainty about the resulting ages, but it appears that the galaxy is at least 1×10^{10}, and possibly as great as 2.5×10^{10} years old. For comparison, studies of the relative isotopic composition of lead in the earth and in the meteorites have shown that the solar system is some 4.5×10^{9} years old. Thus, the galaxy is very much older than the solar system. Since the galaxy was presumably composed entirely of gas in the beginning, and since that gas now constitutes only about 2 per cent of the galactic mass, it seems evident that star formation must have gone on at a much greater rate in the early history of the galaxy than it does at present. It seems a reasonable conclusion from this that at the time the solar system was formed, the galaxy was not very different in appearance from that of today. Hence, we have some justification for believing that observations of star formation today are relevant to the conditions under which the sun was formed.

Nearly all studies of the star formation process have concluded that stars are probably not formed singly, but in associations or clusters resulting from the contraction of an interstellar cloud. The interstellar gas seems to be subject to great fluctuations in both its density and velocity distributions, the regions of greater density being called clouds. There appears to be a general concentration of the dense clouds toward the spiral arms of the galaxy (the positions where most of the newer stars are formed); there also appears to be a direct connection between the presence of dense clouds of gas and the formation of stars in the arms.

We can readily understand the tendency of new stars to form from dense interstellar clouds. Such clouds are rarely in equilibrium; they usually have a tendency to expand or contract. They will expand if their internal energy, in the form of heat and turbulent motion, is comparable to or greater than their self-gravitational potential energy. In such cases, the expansion will usually be limited by an external pressure, perhaps by the interstellar magnetic field or by surrounding gas at higher temperature. However, if the internal energy becomes small enough as compared to the gravitational potential energy, the cloud tends to contract. Once such contraction is well under way, it is difficult to visualize its stopping short of star formation. As the density of the cloud increases, the efficiencies of its cooling processes also increase, and hence, its internal energy becomes very small as compared with its gravitational potential energy. This greatly reinforces the tendency toward contraction.

As the contraction of a cloud proceeds, the gravitational potential energy rapidly increases. Consequently, small parts of the cloud are able to contract individually, since the internal energy of the gas in such small regions becomes small compared with the gravitational potential energy of the region itself. Thus, the cloud can fragment into many different stars, ceasing when the ultimate fragment becomes opaque to its own radiation, so that the internal energy of its constituent gas can no longer decrease relative to its gravitational potential energy. Bodies have then formed with masses approximately equal to that of the sun. It is with the subsequent evolution of such bodies that we shall be mainly concerned in this discussion.

ANGULAR MOMENTUM

One of the principal questions with which we shall be concerned regarding the early evolution of a star, is the amount of angular momentum it contains. We have some idea as to the amount of angular momentum of the interstellar cloud before the contraction starts. The motions of the interstellar clouds seem to be constrained, at least to some extent, by the presence of an interstellar magnetic field. The ions of the interstellar gas, both those produced by the photoionization processes of starlight and those produced by the ionizing effects of cosmic ray particles, are bound in close spiral motions about the magnetic lines of force. The collisions of the neutral atoms in the interstellar gas with these ions are sufficient to cause all of the gas to stay closely associated with the magnetic lines of force. Thus motion along these lines is easily possible, while motion at right angles to them is very difficult. Consequently, the interstellar magnetic field must be trapped in the gas that condenses to form the stars. Before the condensation starts, the magnetic field may well constrain the motion of the interstellar cloud so that it rotates once per revolution about the center of the galaxy. This corresponds to an angular velocity of 10^{-15} rad/sec at the sun's distance from the center of the galaxy. Random motions of the interstellar clouds may well induce larger angular velocities, but this value is the minimum angular velocity that we might expect a cloud to possess.

When a cloud contracts and fragments to form a cluster or association of stars, a general conservation of angular momentum must prevail, and some of the cloud's initial angular momentum will go into the orbital motion of the star fragments about the center of mass. The fragments themselves will also rotate more and more rapidly as their contraction proceeds. The major question seems to be: What role, if any, is played by the trapped interstellar magnetic field in transmitting angular momentum from the contracting cloud to the stable surrounding interstellar medium?

A recent study by the present author indicates that the contracting cloud does not twist appreciably during its contraction and fragmentation stage, so that presumably very little angular momentum is transmitted to the interstellar medium at such a time. A more difficult question, which cannot be answered at the present time, concerns the extent to which the individual fragments may lose angular momentum through torque transmitted from that part of the interstellar magnetic field trapped inside the fragments when they are formed. No studies have yet been made of stellar models in this earliest stage of contraction; we do not know how fast they contract, nor whether they contain large internal convection zones, which would tend to scramble the internal magnetic field and isolate it from that of the surrounding medium. Perhaps it is best to proceed with two alternate assumptions: that during early contraction phases the protostars (1) lose *most* of their angular momentum, or (2) lose *none* of their angular momentum.

HYDROSTATIC EQUILIBRIUM

Once a protostar is formed, an approximate condition of hydrostatic equilibrium will be set up throughout. This means that there will be a central concentration of mass in the protostellar body, with the central density becoming considerably greater than that in the outer layers. Again applying the principle of conservation of angular momentum, we would conclude that the central region of such protostars would rotate at a faster rate than the outer regions. However, such a differential rotation throughout the protostar cannot long be maintained owing to the action of the magnetic field trapped inside the protostar. The lines of force of the magnetic field stay well glued to the interior gas of the protostar, regardless of low temperature. This condition is caused by the persistent state of ionization being maintained by radioactive substances contained in the gas (particularly the radioactivity of the isotope K^{40}). The differential rotation in the protostar draws out this magnetic field into a spiral pattern, thus crowding the magnetic lines of force closer together, and increasing the energy in the magnetic field. This additional magnetic energy can be obtained only at the expense of the differential rotational energy, and hence differential rotation ceases. We may therefore regard the protostellar bodies as being rigid rotators during the course of their contraction.

The author has attempted to determine, in a rough way, the behavior of a contracting protostar by considering how polytropic spheres contract. A polytropic sphere is a spherical mass of gas in which the pressure is related throughout to the density by some given power law. Such polytropes have been studied in considerable detail since the latter part of the 19th century, and have provided useful approximations in some cases for the conditions to be expected in a stellar internal. If, throughout the polytrope, we have a relation between the pressure P and the density ρ given by

$$P = K\rho^{(n+1)/n} ,$$

where K and n are constants, then n is called the index of the polytrope. Two values of this index have been used in this study to represent probable extreme conditions between which the internal density distribution of the protostars may fall. In one extreme we let $n = 1.5$, corresponding to a sphere in which there is convection throughout. In such a sphere, the central density is approximately 6 times the average density. In the other extreme, the index $n = 3$ corresponds to a stellar model in which energy transport is principally by radiation. In such a sphere, the central density is about 54 times the average density.

As the protostar contracts, half of the gravitational potential energy released will be stored as internal heat, and the remaining half will be radiated away from the surface. Indeed, the rate at which this excess energy can be radiated from the surface governs the rate at which the protostar can contract. Until more realistic models of the protostar are constructed, we cannot say how rapid this contraction would be.

As the contraction proceeds, the gravitational potential energy being stored as internal heat results in an increase in the central temperature. This temperature is probably a few hundred degrees Kelvin when the protostar is first formed as a fragment, and increases in proportion to the decreasing radius of the protostar.

When the central temperature has risen to the vicinity of 1800°K, some rather interesting phenomena take place. By far the most abundant constituent of the gas is hydrogen, which will be in the molecular form at such low temperatures. When the temperature rises toward 1800°K, however, not all of the remaining half of the gravitational potential energy is radiated away from the surface of the protostar; some of it goes into the excitation of higher rotational and vibrational bands of the hydrogen molecules. Furthermore, at 1800°K, the dissociation of molecular into atomic hydrogen begins; and this too occurs at the expense of the remaining half of the gravitational potential energy. Indeed, the demand for energy to dissociate the hydrogen soon becomes so great that the temperature at the center of the protostar stops rising as rapidly as one might expect. Far more energy would be required to dissociate all of the hydrogen throughout the protostar than is possessed by its gravitational potential energy when the central temperature has reached 1800°K. Hence, the central temperature cannot increase appreciably beyond 1800°K until a very large shrinkage of the protostar has occurred and the excess gravitational potential energy has gone into the dissociation of the hydrogen molecules. Under these circumstances, the protostar can no longer remain approximately in hydrostatic equilibrium, but must undergo a *collapse* in which the gravitational potential energy necessary to dissociate molecular hydrogen is rapidly released.

For a sphere of polytropic index $n = 1.5$, the relation between the mass, radius, and temperature is

$$\frac{M}{R} = 1.73 \times 10^{-5} \frac{T}{\mu} ,$$

where M is the mass in solar units, R is the radius in astronomical units, T is the temperature in degrees Kelvin, and μ is the mean molecular weight of the material. For solar material containing hydrogen molecules, $\mu = 2.5$. With $T = 1800°K$ and $M = 1$, we have $R = 80$ astronomical units. For a polytropic sphere of index $n = 3$, the similar relation is

$$\frac{M}{R} = 1.093 \times 10^{-5} \frac{T}{\mu} .$$

With $\mu = 2.5$, $T = 1800°K$, and $M = 1$, we have $R = 127$ astronomical units. Thus, a protostar of 1 solar mass unit, in which hydrogen molecules are dissociated, would become unstable against collapse when its radius shrinks to about 100 astronomical units.

Let us consider a polytrope of index $n = 1.5$ having no rotation. We wish to see how far the collapse of such a polytropic sphere would proceed. Let us assume its configuration to be at the stage when most of the hydrogen molecules have been dissociated throughout. It requires 4.48 electron volts to dissociate each hydrogen molecule, and hence the total dissociation energy of the hydrogen molecules throughout the protostar is $2.69 \times 10^{45} M$ ergs, where M is the mass in solar units.

We first wish to find out if the collapse will stop when this total dissociation energy has become equal to $7.62 \times 10^{45}\ M^2/R$ ergs, half the magnitude of the potential energy. If the collapse ceased at that point, then, following the collapse, $M = 0.353\ R$. We would then have $\mu = 1.4$, and the central temperature would become $T = 8.1 \times 10^4\ M/R$, or 2.86×10^4 °K. However, at this central temperature, the hydrogen atoms are almost fully ionized. Considerably more energy is required to ionize a hydrogen atom than to dissociate a hydrogen molecule; thus, it is evident that the collapse cannot cease after the dissociation of the molecular hydrogen, but must continue through the ionization of the resulting atomic hydrogen. If we were to make a similar calculation to find the polytrope configuration at the end of the hydrogen ionization stage, we would find that helium (the second most abundant element in the protostar) had become singly ionized throughout much of the protostar's central region. Helium ionization requires the absorption of yet more gravitational potential energy, and therefore the collapse will continue. In fact, the collapse must continue until not only the hydrogen, but also the helium, has become fully ionized, with each helium atom losing two electrons. This condition of instability was first pointed out by L. Biermann and T. G. Cowling*, but it has since received very little discussion.

We may now ask what final configuration the polytrope assumes, once all the ionization has taken place. The sum of the hydrogen molecule dissociation energy and all the ionization energies is $3.33 \times 10^{46}\ M$. Following the collapse, we will have $M = 4.37\ R$; and, since $\mu = 0.665$, the central temperature will be $T = 3.84 \times 10^4\ M/R$, or 1.68×10^5 °K. Thus, one solar mass will have collapsed to a final radius of approximately 0.228 astronomical unit. In the case of the sun, this is well inside the present orbit of Mercury.

A similar set of conclusions follows for a polytrope of index $n = 3$; and we would find that the final collapsed configuration was reached with $M = 2.50\ R$ and a central temperature T of 1.52×10^5 °K.

*Biermann, L., and Cowling, T.G., "Chemisch Zusammensetzung und dynamische Stabilität der Sterne. II," *Zeit. für Astrophysik* 19: 1-10, 1939.

The time required for a collapse of this sort would be only a few hundred years, starting from the initial configuration.

The protostar has now reached the configuration at which F. Hoyle*, in a recent theory concerning the origin of the solar system, assumed that the sun became rotationally unstable at the equator owing to the conservation of angular momentum. The subsequent shrinkage of the protosun must have been accompanied by the loss of mass in the equatorial plane, in order to conserve angular momentum. Hoyle assumes that this gas was accelerated outward to larger radii by means of a magnetic interaction between the gas in the nebular disk and the sun. This interaction both slowed down the rotation of the sun and drew the released gas out into that region now occupied by the planets. The planets supposedly condensed out of this gas.

Let us now consider the alternative possibility in which the contracting protostars lose *none* of their angular momentum to the surrounding interstellar medium. In this case, they become unstable against the loss of mass at the equator when their radii have shrunk to a few hundred astronomical units. At the time that the central instability sets in (resulting in their rapid collapse), we would expect such protostars to be already losing mass in order to conserve angular momentum. Such protostars should remain rigid rotators until their central temperatures reach 1800°K. Once the collapse gets underway, it progresses too rapidly for the magnetic field to maintain rigid rotation throughout the collapsing polytrope; consequently, there will be a local conservation of angular momentum. Thus a given element of mass will be shed when its angular velocity becomes sufficient to permit a Kepler rotation of the element in the gravitational field of the remaining mass at smaller distances from the center of gravity.

A schematic representation of the process is shown in Figure 1, where we see a protostar with considerable distortion in the equatorial plane, owing to its rotation. When the collapse starts at the center of the protostar, mass is continually shed in the equatorial plane, and a nebular disk is formed. Some gas may remain to form a central star in hydrostatic equilibrium, about which the nebular disk will rotate.

PARAMETERS OF THE NEBULAR DISK

The author has made some numerical calculations concerning the formation of such a nebular disk. For this purpose, the mass distribution was computed in cylindrical shells about an axis of rotation through the center of the polytrope. It was assumed that the mass *of* such a cylindrical shell would be shed when the polytrope's radius became small enough — that is, when its angular velocity reached the Kepler angular velocity of a particle orbiting in the gravitational field due to the polytrope's fractional mass *enclosed by* that cylinder. The latter mass was assumed to be concentrated at the system's center of gravity. The resulting gravitational potential in which the nebular gas is presumed to move is an approximation, but it should be accurate enough for this preliminary investigation.

*Hoyle, F., "On the Origin of the Solar Nebula," *Quart. J. Roy. Astron. Soc.* 1(1): 28-55, September 1960.

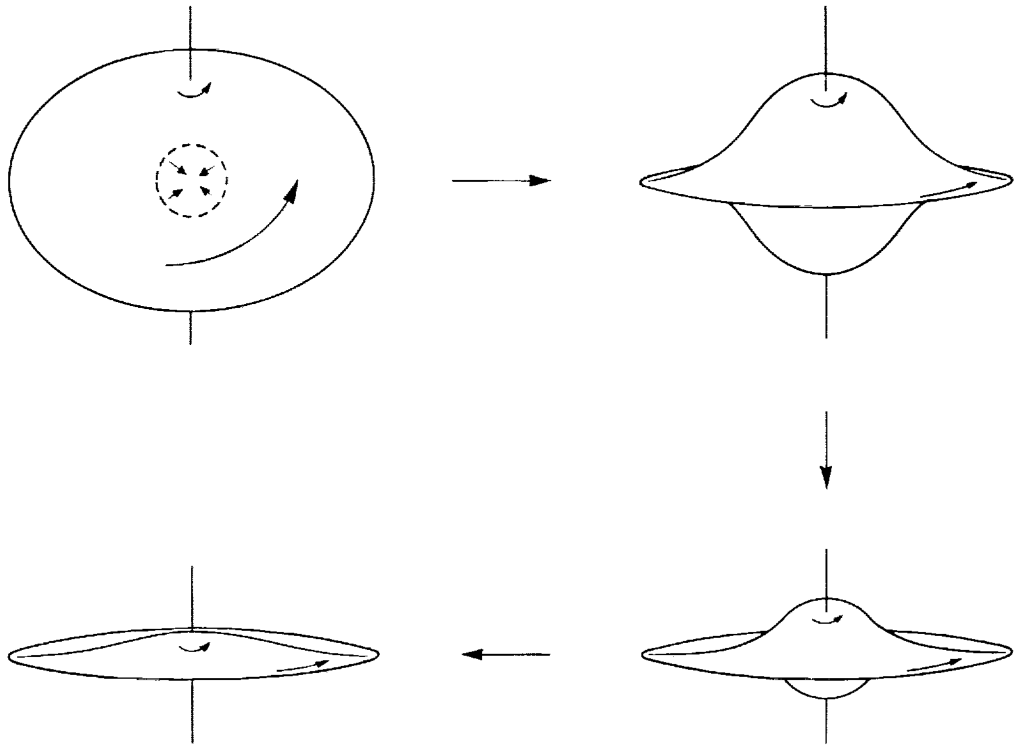

Figure 1—Schematic representation of the formation of a nebular disk from a collapsing polytrope.

The first significant result to emerge from this analysis was the fact that there is no central body formed in hydrostatic equilibrium. For both classes of polytropes, the collapse leads solely to the formation of a nebular disk, in which all of the helium has been ionized. Although the density of the nebula near the center of gravity is considerably greater than that near the edge, there is no residual body at the center.

Since the collapse of the polytropes is initiated by the dissociation of hydrogen molecules, and continues owing to the ionization of hydrogen and helium (involving temperatures in a range well above 10^3 °K), it is evident that the nebular disk will be initially formed at a high temperature. The height of the nebular disk above the central plane depends upon its temperature, but the pressure at the center of the plane is independent of the temperature to a first approximation. Owing to the high temperature in the disk at its formation, the metals throughout the disk, as well as many of the other lighter constituents, will be ionized. The magnetic field, trapped in the body since its formation from the interstellar medium, will therefore remain trapped in the nebular disk. The nebular disk will be subject to differential rotation in its own gravitational field, the inner portion rotating more rapidly than the outer. This motion will tend to draw out the magnetic lines of force, thus crowding them closer together and increasing the magnetic field strength at the expense of the energy in the differential rotation. This is a process that tends to disrupt the nebular disk and, during the process, the nebular disk must conserve its own angular momentum. Conservation will occur if most of the mass in the disk flows inward toward the center of gravity, while the remaining mass flows outward and carries with it much of the angular momentum of the disk. From this point of view, the sun would be formed from the mass flowing toward the center of the nebular disk, as a result of magnetic friction. The precise amount of mass flowing inward to form the protosun may thus depend upon accidental configurations of the magnetic field; the effects would differ considerably from one case to another. It may be that in some cases the action of the magnetic field would result in the formation of two or more centers of condensation, causing the formation of a binary or multiple star system.

The mass deposited inside an arbitrary orbital radius is shown in Figure 2. Four different initial cases are presented: polytropes of indices 1.5 and 3, and masses of 2 and 4 solar masses M_0. From this figure it may be seen that, if the initial configuration is a polytrope of index 1.5, then 1 solar mass must be collected from within roughly the orbit of Saturn. On the other hand, if the initial configuration is a polytrope of index $n = 3$, then 1 solar mass would be contained inside the earth's orbit.

Polytropes of considerably greater mass than that of the sun were initially assumed because, after the sun has formed from the nebula, there may be significant loss of mass. Herbig has observed very rapid rates of mass loss from stars in the later stages of their contraction toward the main sequence. Such stars are called T Tauri stars.

The surface densities in the nebular disk corresponding to these different assumptions are shown in Figure 3. The surface density is the total amount of nebular disk mass (gm) per cm^2 of area in the central plane. The corresponding pressures in the disk's central plane are shown in Figure 4.

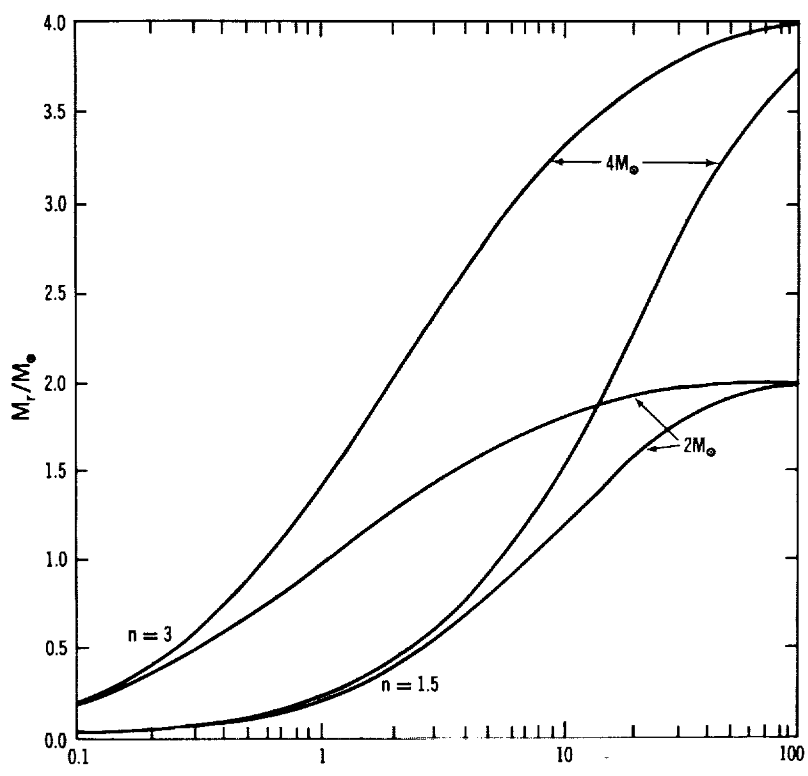

Figure 2—Mass contained in nebular disks inside the indicated orbital radius.

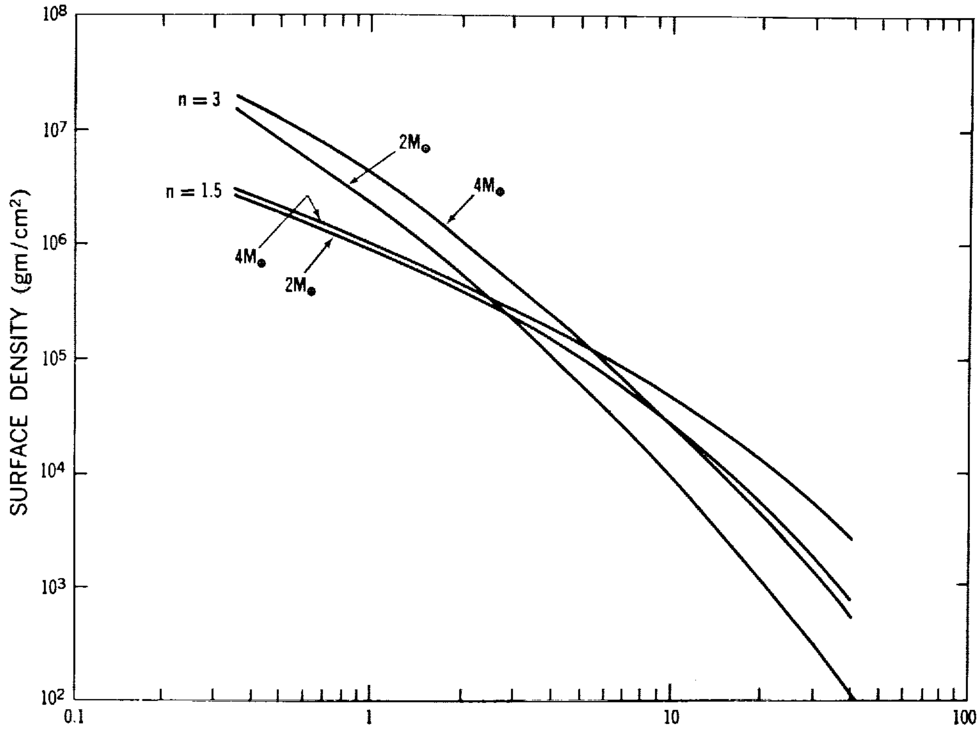

Figure 3—Surface density of gas in the nebular disks.

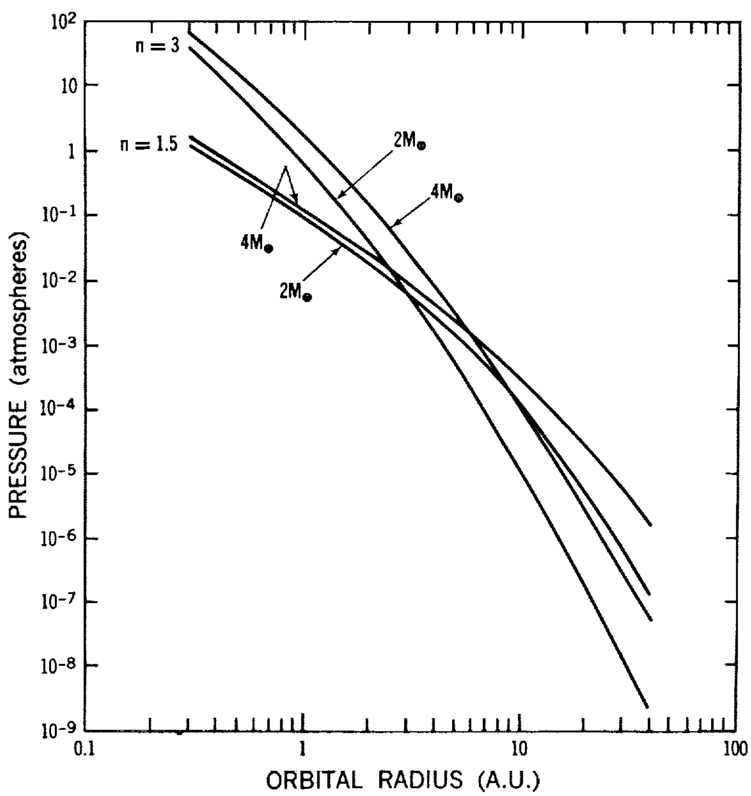

Figure 4—Gas pressure in the nebular disks' central plane.

Some interesting chemical and physical properties are associated with high gas temperatures and the range of pressure in the nebular gas indicated by Figure 4. The gas will form at a temperature of several thousand degrees, and will cool at essentially constant pressure in the central portion of the disk. According to studies of J. A. Wood, as long as the pressure in a gas of solar composition remains above 10^{-2} atmosphere, the cooling curve will pass through a region of temperature and pressure where liquid silicates and iron will condense out. Wood has concluded that chondrules, a very common feature of meteorites, have condensed in this fashion. From Figure 4, we see that a pressure of 10^{-2} atmosphere is likely to be exceeded in the nebula out to about the orbital radii of the asteroids. This is encouraging in view of the fact that meteorites are probably asteroidal debris.

The present combined mass of the planets represents a very inefficient collection of condensable material from this nebular disk. If we consider the inner planets to be composed principally of metallic oxides, and the outer ones of metallic oxides, water, ammonia, and methane, then it appears that the present planets represent only about 1 percent of the mass that was potentially available for collection. Presumably only those bodies that grew to a sufficiently large size in a relatively short time were able to survive the dissipation of the nebula; most of the smaller bodies were swept along with the flow of gases required to form the sun. We may therefore expect that the outer portion of the solar system contains a large number of rather small solid bodies that have not collected to form planets. Such bodies may include the comets.

CONCLUDING REMARKS

It appears that the collapse phase of early solar evolution is a very interesting phenomenon requiring much additional study. Depending on the amount of angular momentum possessed by the earliest phase of the contracting protostar, entirely different configurations may result at the end of the collapse period; and entirely different views arise as to how the solar nebula, from which the planets condensed, was formed. Thus, studies of the problem of star formation are becoming intimately associated with the investigations of the early history of the solar system.

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