The collective model of household consumption: a nonparametric characterization*

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Abstract

We provide a nonparametric characterization of a general collective model for multi-person household consumption, which includes externalities and public consumption. Next, we institute necessary and sufficient conditions for data consistency with collective rationality that only include observed price and quantity information, and that are formally similar to the *GARP* condition for the unitary model. In addition, we derive the minimum number of commodities and observations that enable the falsification of the general model.
Key words: collective household models, intrahousehold allocation, revealed preferences, nonparametric analysis.

1. Introduction

Traditionally, household consumption behaviour is crammed into the so-called unitary approach, which assumes that a household acts as if it were a single decision maker; it maximizes a well-behaved (single) utility function subject to a household budget constraint. The collective model, which was first presented by Chiappori (1988, 1992),

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differs from the unitary model in that it explicitly recognizes that the individual household members have own, possibly diverging, rational preferences. These individuals are assumed to engage into a bargaining process that results in a Pareto efficient intrahousehold allocation.

Browning and Chiappori (1998) have provided a general characterization of the collective model. They start from the 'minimalistic' assumptions that the empirical analyst cannot determine which commodities are privately and/or publicly consumed within the household, and that the quantities that are privately consumed by the different household members cannot be observed. In addition, they consider general individual preferences that allow for altruism and other externalities. Their core result for two-person households is that under collectively rational household behaviour the pseudo-Slutsky matrix can be written as the sum of a symmetric negative semi-definite matrix and a rank one matrix.

Browning and Chiappori focus on a so-called *parametric* setting, which requires some (non-verifiable) functional structure that is imposed on the household decision process (i.e., the household member preferences and the intrahousehold bargaining process). In this paper, we follow a *non*parametric approach, which analyzes household behaviour without imposing any parametric structure on, e.g., preferences; see, among others, Afriat (1967) and Varian (1982). This nonparametric approach was first adapted to the collective model by Chiappori (1988), who restricted attention to a labour supply setting that involves a number of convenient simplifications for the empirical analyst (e.g., observability of individuals' leisure/labour supply and no public consumption).

We aim at generalizing Chiappori's work by providing a nonparametric characterization of the collective consumption model *à la* Browning and Chiappori, which includes both public consumption and (*in casu* positive) externalities. In Section 2, we institute necessary and sufficient nonparametric conditions for data consistency with this general model. As we will discuss, these conditions imply *unobservable* (member-specific) quantity and price information. In Sections 3 and 4, we subsequently establish necessary and sufficient conditions that only require *observed* prices and aggregate household quantities. Interestingly, this implies nonparametric tests for collective rationality that are finite in nature and do not require finding a solution to a system of (possibly nonlinear) inequalities.[1] As a by-product, we derive the minimum number of commodities and observations that enable falsification of collective rationality. Section 5 contains some concluding remarks. The Appendix contains the proofs of our results.

[1] We see at least two important differences between our approach and that of Snyder (2000), who addresses a similar research question for Chiappori's (1988) original labour supply model. First, Snyder focuses on a more restricted model that includes egoistic agents and observable leisure. Second, we do not make use of semi-algebraic theory for quantifier elimination. A well-known limitation of these latter techniques is that they become computationally cumbersome for large data sets. For example, Snyder restricts to settings of only two observations, while we consider the general case of T observations.

2. A general characterization of collective rationality for two-person households

We consider a two-person (1 and 2) household.[2] Each household purchases the (non-zero) n-vector of commodities $\mathbf{q} \in \Re^n_+$ with corresponding prices $\mathbf{p} \in \Re^n_{++}$. These commodities can be consumed privately, publicly or both. Generally, we have $\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H$ for $\mathbf{q}$ the (observed) aggregate consumption, $\mathbf{q}^1$ and $\mathbf{q}^2$ the (unobserved) private consumption bundles of each household member, and $\mathbf{q}^H$ the (unobserved) public consumption bundle.

Following Browning and Chiappori (1998), we consider general preferences for the household members that may depend not only on own consumption and public consumption, but also (positively) on the other individual's consumption bundle; this allows for altruism and/or externalities.[3] Formally, this means that the preferences of household member m ($m = 1, 2$) can be represented by a utility function of the form $U^m\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right)$ that is monotonously increasing in its arguments $\mathbf{q}^1$, $\mathbf{q}^2$ and $\mathbf{q}^H$.

Suppose that we have T household observations. For each observation $j \in \{1, ..., T\}$ we use $\mathbf{q}_j$ and $\mathbf{p}_j$ to denote the observed quantity and price vector, respectively; and $S = \{(\mathbf{q}_j; \mathbf{p}_j), j = 1, ..., T\}$ represents the set of all observations. Given this, we can generally define collective rationalization as (with $\mathbf{0}^n$ the n-vector of zeroes):

Definition 1. *A pair of utility functions U^1 and U^2 provides a* collective rationalization (CR-2) *of the observed set S, if there exist T combinations of two vectors $\mathbf{q}^1_j$ and $\mathbf{q}^2_j$, both $\in \Re^n_+$, and a scalar $\mu_j \in \Re_{++}$ such that ($j \in \{1, ..., T\}$):*

(i) $\mathbf{0}^n \leq \mathbf{q}^m_j \leq \mathbf{q}_j, m = 1, 2$ *and* $\mathbf{0}^n \leq \mathbf{q}_j - \mathbf{q}^1_j - \mathbf{q}^2_j$;

(ii) $U^1\left(\mathbf{q}^1_j, \mathbf{q}^2_j, \mathbf{q}_j - \mathbf{q}^1_j - \mathbf{q}^2_j\right) + \mu_j U^2\left(\mathbf{q}^1_j, \mathbf{q}^2_j, \mathbf{q}_j - \mathbf{q}^1_j - \mathbf{q}^2_j\right) \geq U^1\left(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^H\right) + \mu_j U^2\left(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^H\right)$ *for all* $\left(\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^H\right) \in \left(\Re^n_+\right)^3$ *with* $\mathbf{p}'_j\left(\mathbf{z}^1 + \mathbf{z}^2 + \mathbf{z}^H\right) \leq \mathbf{p}'_j \mathbf{q}_j$.

In this definition, the μ_j's represent the 'bargaining power' of the different household members; see Browning and Chiappori (1998) for a detailed discussion. In this collective set-up, optimal consumption bundles maximize the weighted household utility function

[2] Generalizations for M-person households are found in Cherchye *et alii* (2004).

[3] This setting generalizes Chiappori's (1988) altruistic model in two ways: it does not assume the observability of any commodity; and it allows for public consumption. Admittedly, the assumption of positive externalities, which is not needed in a parametric setting (see Browning and Chiappori, 1998), may be restrictive in some instances. However, its restrictive nature should not be overestimated. Even though a negative externality may be associated with e.g. tobacco consumption, the non-smoker's positive valuation of the smoker's utility generated by smoking might well outweigh that negative externality. In addition, within-household mechanisms may be instituted that decrease or even eliminate the negative externalities; see, e.g., the widespread practice of smoking outside in households consisting of smokers as well as non-smokers.

given in part (ii) of the definition. This weighted function reflects the Pareto efficiency assumption regarding observed household consumption.

Before providing a nonparametric characterization of collective rationality, we recapture the *Generalized Axiom of Revealed Preference* (*GARP*).[4]

Definition 2. *If* $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$ *then* $\mathbf{q}_i \in DRP_j$*, where* DRP_j *represents the* directly revealed preferred set *associated with the bundle* $\mathbf{q}_j$*. Next, if* $\mathbf{q}_i \in DRP_k$, $\mathbf{q}_k \in DRP_l$, ..., $\mathbf{q}_z \in DRP_j$ *for some sequence of observations* $(k, l, ..., z)$ *then* $\mathbf{q}_i \in RP_j$*, where* RP_j *represents the* revealed preferred set *associated with the bundle* $\mathbf{q}_j$*.*

Definition 3. *A set of observations* S *satisfies the* Generalized Axiom of Revealed Preference (GARP) *if for all* $j \in \{1, ..., T\} : \mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_r \in RP_j} \mathbf{p}_j'\mathbf{q}_r$.

The implicit idea is that observation $j \in \{1, ..., T\}$ is (theoretically) utility maximizing under its budget constraint if and only if it is expenditure minimizing over its 'better than' set; in the (empirical) *GARP* condition this last set is approximated by the 'revealed preferred' set RP_j. Varian (1982; p.948) demonstrated that (price and quantity) data consistency with the *GARP* at the level of the household as a whole is necessary and sufficient for observed household behaviour to be consistent with the unitary model (i.e., for the existence of a (single) utility function that rationalizes the consumption observations). We will repeatedly refer to this result in our following discussion.

We can now establish the nonparametric conditions for a *CR-2* of a set S.

Proposition 1. *There exists a pair of concave, monotonously increasing, continuous utility functions* U^1 *and* U^2 *that provide a* CR-2 *of the observed set* S *if and only if* $\forall j \in \{1, ..., T\}$ *there exist vectors* $\mathbf{q}_j^1$, $\mathbf{q}_j^2$, $\boldsymbol{\pi}_j^1$, $\boldsymbol{\pi}_j^2$, $\boldsymbol{\pi}_j^H \in \Re_+^n$ *with* $\mathbf{0}^n \leq \mathbf{q}_j^m \leq \mathbf{q}_j$ $(m = 1, 2)$, $\mathbf{0}^n \leq \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2$, $\mathbf{0}^n \leq \boldsymbol{\pi}_j^k \leq \mathbf{p}_j$ $(k = 1, 2, H)$*, such that one of the following equivalent conditions is met:*

(i) the data $(\widehat{\mathbf{q}}_j; \boldsymbol{\pi}_j)$ *on the one hand and* $(\widehat{\mathbf{q}}_j; \widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)$ *on the other both satisfy the* GARP *conditions for*

$$\boldsymbol{\pi}_j = \begin{pmatrix} \boldsymbol{\pi}_j^1 \\ \boldsymbol{\pi}_j^2 \\ \boldsymbol{\pi}_j^H \end{pmatrix}, \ \widehat{\mathbf{p}}_j = \begin{pmatrix} \mathbf{p}_j \\ \mathbf{p}_j \\ \mathbf{p}_j \end{pmatrix} \text{ and } \widehat{\mathbf{q}}_j = \begin{pmatrix} \mathbf{q}_j^1 \\ \mathbf{q}_j^2 \\ \mathbf{q}_j - \mathbf{q}_j^1 - \mathbf{q}_j^2 \end{pmatrix};$$

(ii) there exist numbers U_j^m *and* $\lambda_j^m > 0$ $(m = 1, 2)$ *such that* $\forall\, i, j \in \{1, ..., T\}$:

$$U_i^1 - U_j^1 \leq \lambda_j^1 (\boldsymbol{\pi}_j)' (\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j) \text{ and } U_i^2 - U_j^2 \leq \lambda_j^2 (\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)' (\widehat{\mathbf{q}}_i - \widehat{\mathbf{q}}_j).$$

[4] In Definition 2, the sets RP_j incorporate the sets DRP_j and additionally include transitivity of the preferences. For brevity, we will indicate such a transitivity property as $\mathbf{q}_i \in DRP_j \Rightarrow (\mathbf{q}_i \in RP_j \wedge RP_i \subseteq RP_j)$ when we deal with individual household members' preferences.

Following Chiappori (1988), the different commodities may be interpreted as 'public goods', given that they all enter both individuals' utility functions. Accordingly, the personalized prices $\boldsymbol{\pi}_j^k$ and $\left(\mathbf{p}_j - \boldsymbol{\pi}_j^k\right)$ $(k = 1, 2, H)$ may be understood as 'Lindahl prices': they must add-up (over the household members) to the observed market prices in order to be consistent with Pareto efficiency. Thus, no qualitative distinction should be made between publicly and privately consumed commodities (where private consumption may be associated with externalities). Yet, there is a clear quantitative difference: household members may accord another marginal valuation to private consumption than to public consumption.

To conclude, it is interesting to compare the conditions in Proposition 1 to the standard rationality conditions in a unitary setting (see Definition 3). Just like in the latter setting, the nonparametric characterization requires certain aspects of household behaviour to obey the *GARP* conditions. Importantly, however, in the collective setting these *GARP* conditions apply to the price-quantity bundles of the *individual household members*. Contrary to the unitary case, these member-specific prices and quantities are usually unobserved. Therefore, it is only imposed that there should exist *at least one* intrahousehold allocation that satisfies the above conditions.

3. Observable necessity restrictions

Proposition 1 institutes nonlinear (necessary and sufficient) conditions for a *CR-2* of the data, which makes their implementation in general computationally infeasible. This section derives a finitely computable necessary condition for collective rationality, which no longer requires the construction of unobservable personalized prices and quantities. The next section presents the complementary sufficiency condition.

We start with defining member-specific revealed preferred sets in terms of the (unobservable) personalized prices and quantities.

Definition 4. *Consider a specification of the personalized prices and quantities* $(\widehat{\mathbf{q}}_j; \boldsymbol{\pi}_j)$ *for all* $j \in \{1, ..., T\}$. *If* $\boldsymbol{\pi}_i' \widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i' \widehat{\mathbf{q}}_j$ *then* $\widehat{\mathbf{q}}_i \in DRP_j^1$, *and if* $(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)' \widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)' \widehat{\mathbf{q}}_j$ *then* $\widehat{\mathbf{q}}_i \in DRP_j^2$, *where* DRP_j^m $(m = 1, 2)$ *represents the* m-th member's directly revealed preferred set *associated with the (decomposed) bundle* $\widehat{\mathbf{q}}_j$. *Next, if* $\widehat{\mathbf{q}}_i \in DRP_j^m$ *then* $\widehat{\mathbf{q}}_i \in RP_j^m$ *and* $RP_i^m \subseteq RP_j^m$, *where* RP_j^m *represents the corresponding* m-th member's revealed preferred set.

For a given observation j, the sets RP_j^m are the (collective) member-specific analogues of the (unitary) revealed preferred set RP_j in Definition 2. Of course, the specification of the sets DRP_j^m and RP_j^m will vary with the (unobserved) personalized price-quantity constellation. To conceive operational necessary and sufficient conditions, we construct inner bound approximations for the sets DRP_j^m (and, consequently, RP_j^m), hereby exploiting the limited available price-quantity information. We first define the general concept of *observable* directly revealed preferred sets.[5]

Definition 5. *The sets* $\widehat{DRP}_j \subseteq \{\mathbf{q}_1, ..., \mathbf{q}_T\}$, $j \in \{1, ..., T\}$, *represent a collection of* observable directly revealed preferred sets *if, for all feasible specifications of the personalized quantities and prices* $(\widehat{\mathbf{q}}_j; \boldsymbol{\pi}_j)$ *with corresponding* DRP_j^m $(m = 1, 2)$, *it is possible to construct* $\widehat{DRP}_j^m$ *such that* $\bigcup_{m=1,2} \widehat{DRP}_j^m = \widehat{DRP}_j$ *and* $\mathbf{q}_i \in \widehat{DRP}_j^m$ *implies* $\widehat{\mathbf{q}}_i \in DRP_j^m$ $(i \in \{1, ..., T\})$.

Thus, for any possible specification of the personalized prices and quantities, the (observable) sets $\widehat{DRP}_j$ should be decomposable into member-specific sets $\widehat{DRP}_j^m$ that provide inner bounds for the true directly revealed preferred sets DRP_j^m. In other words, these (empirical) sets approximate the (theoretical but unobservable) member-specific directly revealed preferred sets by accounting for *all* conceivable price-quantity intra-household scenarios (which effectively avoids constructing the $\widehat{\mathbf{q}}_i$ and $\boldsymbol{\pi}_i$, $i \in \{1, ..., T\}$).

From an empirical point of view, a crucial question is whether we can provide an operational characterization of the sets $\widehat{DRP}_j$. Interestingly, we find that the 'maximal' (collective) observable set of directly revealed preferred bundles is the (unitary) set DRP_j (introduced in Definition 2).

Lemma 1. *The collection of the sets* DRP_j, $j \in \{1, ..., T\}$ *constitutes a collection of observable directly revealed preferred sets. Moreover, we have* $\widehat{DRP}_j \subseteq DRP_j$ *for any collection of observable directly revealed preferred sets* $\widehat{DRP}_j$.

In the collective setting, the set DRP_j has a subtly different interpretation than in the unitary setting. This difference essentially pertains to the explicit recognition of the household's two-person nature in the collective approach. Specifically, it follows from our above discussion that $\mathbf{q}_i \in DRP_j$ (i.e., $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$) may imply $\widehat{\mathbf{q}}_i \in DRP_j^1$ as well as $\widehat{\mathbf{q}}_i \in DRP_j^2$. Intuitively, if $\mathbf{q}_i$ has been chosen when $\mathbf{q}_j$ was equally feasible, then at least one household member should prefer the (decomposed) former bundle above the (decomposed) latter bundle; this reflects the Pareto efficient nature of household behaviour in the collective model.

Because of Lemma 1, we can use DRP_j as the starting point in our empirical conditions. Specifically, we characterize member-specific observable revealed preferred sets.

[5] The 'feasible' personalized prices and quantities in Definition 5 are non-negative and add up to observed prices and quantities (see Proposition 1). The additional *CR-2* restrictions on these prices and quantities (see conditions (i) and (ii) in Proposition 1) are contained in Propositions 2 and 4.

Definition 6. *The sets* $\widehat{RP}_j^m \subseteq \{\mathbf{q}_1, ..., \mathbf{q}_T\}$, $j \in \{1, ..., T\}$ *and* $m \in \{1, 2\}$, *represent a collection of* observable member-specific revealed preferred sets *if*

(i) $\mathbf{q}_i \in DRP_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m$ $(m = 1$ *or* $2)$,
(ii) $\mathbf{q}_i \in \widehat{DRP}_j^m \Rightarrow (\mathbf{q}_i \in \widehat{RP}_j^m \wedge \widehat{RP}_i^m \subseteq \widehat{RP}_j^m)$,
(iii) $(\mathbf{q}_i \in \widehat{DRP}_j^m \wedge \mathbf{q}_j \in \widehat{RP}_i^m) \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^l$ $(m, l \in \{1, 2\}; m \neq l)$, *and*
(iv) $(\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i' (\mathbf{q}_{j_1} + \mathbf{q}_{j_2}) \wedge \mathbf{q}_{j_1} \in \widehat{RP}_i^m) \Rightarrow \mathbf{q}_i \in \widehat{DRP}_{j_2}^l$ $(m, l \in \{1, 2\}; m \neq l)$.

Proposition 2 will state the necessary nature of properties (i)-(iv) for any collection of observable member-specific revealed preferred sets. The intuition of property (i) has been discussed above (following Lemma 1): we construct revealed preferred sets that (only) include observed directly revealed preferred bundles, i.e. $DRP_j = \bigcup_{m=1,2} \widehat{DRP}_j^m$. This construction should additionally respect the properties (ii)-(iv). Property (ii) reveals the transitivity idea that also underlies Definition 2 of the revealed preferred sets in the unitary model. Basically, conditions (i) and (ii) reflect the empirical implications of rational household behaviour *for one and the same household member*; they are formally similar to the unitary conditions. The following conditions then pertain to rationality *across household members*; this distinguishes the collective setting from the unitary setting. First, property (iii) expresses that, if the household member m is indifferent between $\mathbf{q}_i$ and $\mathbf{q}_j$, then the choice of $\mathbf{q}_i$ (when $\mathbf{q}_j$ was equally obtainable) can be rationalized only if the other member l prefers $\mathbf{q}_i$ over $\mathbf{q}_j$. Next, the meaning of property (iv) is that, if $\mathbf{q}_i$ can be 'exchanged' for the sum of $\mathbf{q}_{j_1}$ and $\mathbf{q}_{j_2}$ while the household member m has revealed its preference for $\mathbf{q}_{j_1}$ over $\mathbf{q}_i$, then the only possibility for rationalizing the choice of $\mathbf{q}_i$ is that the other member l prefers $\mathbf{q}_i$ to $\mathbf{q}_{j_2}$.

Using Definition 6, we have the following necessary condition:

Proposition 2. *A* necessary condition *for the existence of utility functions* U^1 *and* U^2 *that provide a* CR-2 *of the observed set* S *is that there exists a collection of observable revealed preferred sets* $\widehat{RP}_j^m$ $(m = 1, 2)$, $j \in \{1, ..., T\}$ *such that* $\mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_{r_m} \in \widehat{RP}_j^m, m=1,2} \mathbf{p}_j'(\mathbf{q}_{r_1} + \mathbf{q}_{r_2})$ *and* $\mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_r \in \widehat{RP}_j^1 \cap \widehat{RP}_j^2} \mathbf{p}_j'\mathbf{q}_r$.

For given (observable) member-specific revealed preferred sets, this condition checks all combinations of consumption bundles $\mathbf{q}_{r_1} \in \widehat{RP}_j^1$ and $\mathbf{q}_{r_2} \in \widehat{RP}_j^2$. The interpretation of the necessary condition is then complementary to that of property (iv) in Definition 6: if household members 1 and 2 reveal that they prefer respectively $\mathbf{q}_{r_1}$ and $\mathbf{q}_{r_2}$ over $\mathbf{q}_j$, then the choice of $\mathbf{q}_j$ can be rationalized only if it cannot be exchanged for the sum of $\mathbf{q}_{r_1}$ and $\mathbf{q}_{r_2}$ (or, *stricto sensu*, under the prices $\mathbf{p}_j$ the bundle $\mathbf{q}_j$ should not be associated with a strictly higher expenditure level than the sum $\mathbf{q}_{r_1}+ \mathbf{q}_{r_2}$). In the special case where both members have revealed their preference for the same bundle $\mathbf{q}_r$

over $\mathbf{q}_j$ (i.e., $\mathbf{q}_r \in \widehat{RP}_j^1 \cap \widehat{RP}_j^2$), the condition states that $\mathbf{q}_j$ should not be exchangeable for that (single) bundle $\mathbf{q}_r$.

As an illustration, we next provide a numerical price-quantity data structure for which a *CR-2* cannot be obtained.

Example 1. *It follows from Proposition 2 that a* CR-2 *is impossible for a combination of three observations with:*

$$\mathbf{p}_1'\mathbf{q}_1 > \mathbf{p}_1'(\mathbf{q}_2 + \mathbf{q}_3), \ \mathbf{p}_2'\mathbf{q}_2 > \mathbf{p}_2'(\mathbf{q}_1 + \mathbf{q}_3) \ \text{and}\ \mathbf{p}_3'\mathbf{q}_3 > \mathbf{p}_3'(\mathbf{q}_1 + \mathbf{q}_2).$$

Using the first two inequalities, Definition 6 implies that $\mathbf{q}_{r_1} \in \widehat{RP}_3^1$ *and* $\mathbf{q}_{r_2} \in \widehat{RP}_3^2$ *($r_1, r_2 \in \{1, 2\}$); and the data do not meet the associated condition* $\mathbf{p}_3'\mathbf{q}_3 \leq \mathbf{p}_3'(\mathbf{q}_1 + \mathbf{q}_2)$ *(see Proposition 2). This specific data structure applies to:*

$$\begin{aligned} \mathbf{q}_1 &= \begin{pmatrix} 8 & 2 & 1 \end{pmatrix}', \mathbf{q}_2 = \begin{pmatrix} 2 & 1 & 8 \end{pmatrix}', \mathbf{q}_3 = \begin{pmatrix} 1 & 8 & 2 \end{pmatrix}'; \\ \mathbf{p}_1 &= \begin{pmatrix} 5 & 2 & 1 \end{pmatrix}', \mathbf{p}_2 = \begin{pmatrix} 2 & 1 & 5 \end{pmatrix}', \mathbf{p}_3 = \begin{pmatrix} 1 & 5 & 2 \end{pmatrix}'. \end{aligned}$$

This example implies that it is sufficient to have three commodities and three observations for rejecting collective rationality. The following proposition institutes that this is also necessary.

Proposition 3. *There do not always exist utility functions U^1 and U^2 that provide a* CR-2 *of the observed set S if and only if (i) the number of commodities $n \geq 3$ and (ii) the number of observations $T \geq 3$.*

We only sketch the basic idea for the necessity result; a detailed proof is found in Cherchye *et alii* (2004). First, consistency with the *CR-2* conditions for two commodities can always be achieved for an intrahousehold allocation with each m-th ($m = 1, 2$) household member consuming exclusively the m-th commodity (and no public consumption). Next, consistency with the *CR-2* conditions for two observations can always be achieved for an intrahousehold allocation with each m-th household member consuming everything in the m-th household observation.

We conclude that the collective model can be falsified (or empirical testing is meaningful) as soon as there are at least three commodities and three observations. Interestingly, the lower bound of three commodities is actually below the lower bound derived by Browning and Chiappori (1998) in their parametric setting: empirical falsification of their (parametric) collective model necessitates at least five commodities. This is due to the fact that in their differentiable framework, one needs at least five commodities to come to testable implications of pseudo-Slutsky symmetry, while only three commodities are needed to test pseudo-Slutsky negativity.[6]

[6] We are grateful to an anonymous referee for pointing this out.

Essentially, testing data consistency with the necessity requirement is a finite process because the number of subsets $\widehat{DRP}_j^m$ consistent with property (i) in Definition 6 is finite in nature.[7] For each specification of the $\widehat{DRP}_j^m$, one may use the Warshall algorithm proposed by Varian (1982, p. 949) for reconstructing the sets $\widehat{RP}_j^m$. The necessity test consequently checks the associated closing conditions in Proposition 2.

4. Observable sufficiency restrictions

While the conditions in Proposition 2 are necessary for a *CR-2* of the data, they are in general not sufficient.[8] This follows from Example 2, which contains data that satisfy the conditions but cannot be collectively rationalized in the sense of Proposition 1.

Example 2. *We proof in the appendix that a* CR-2 *cannot be obtained for a combination of seven observations with:*

$$\begin{aligned}
\forall i &\in \{1,...,7\} : \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j \ \forall j \in \{1,...,7\} \setminus \{i\}, \\
\forall i &\in \{1,7\} : \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'(\mathbf{q}_j + \mathbf{q}_k) \ \forall j,k \in \{1,...,7\} \setminus \{i\} \ \text{with } j \neq k, \text{ and} \\
\forall i &\in \{2,...,6\} : \mathbf{p}_i'\mathbf{q}_i = \mathbf{p}_i'(\mathbf{q}_j + \mathbf{q}_k) - \varepsilon \ \forall j,k \in \{1,...,7\} \setminus \{i\} \ \text{with } j \neq k,
\end{aligned}$$

where $\frac{\min_{i,\nu}\langle\mathbf{p}_i\rangle_\nu \min_{i,\nu}\langle\mathbf{q}_i\rangle_\nu}{6} > \varepsilon > 0$ *(for* $\langle\mathbf{x}_i\rangle_\nu$ *the* ν*-th entry of the vector* $\mathbf{x}_i$*;* $i \in \{1,...,7\}$ *and* $\nu \in \{1,...,n\}$*). For example, that structure applies to*

$$\begin{aligned}
\forall i &\in \{1,...,7\} : \langle\mathbf{q}_i\rangle_i = 3 \text{ and } \langle\mathbf{q}_i\rangle_\nu = 1 \text{ if } \nu \neq i, \\
\forall i &\in \{1,7\} : \langle\mathbf{p}_i\rangle_i = 11 \text{ and } \langle\mathbf{p}_i\rangle_\nu = 1 \text{ if } \nu \neq i, \text{ and} \\
\forall i &\in \{2,...,6\} : \langle\mathbf{p}_i\rangle_i = 10 - \varepsilon \text{ and } \langle\mathbf{p}_i\rangle_\nu = 1 \text{ if } \nu \neq i,
\end{aligned}$$

where $(1/6) > \varepsilon > 0$.

We next provide a sufficient condition for collective rationality that solely uses observable (aggregate) price and quantity information. Like before, this condition implies (*in casu* sufficiency) tests for collective rationality that involve a finite number of steps.

[7] It can be verified that the maximum number of configurations of member-specific directly revealed preferred sets that are consistent with Definition 6 is in theory of order 3^{T^2}. While this may seem computationally cumbersome in the case of large datasets, it is worth stressing that strategies exist for considerably enhancing the computational efficiency; see Cherchye *et alii* (2005) for an illustrative application. A similar qualification applies to the sufficiency condition in Proposition 4; in that case, the maximum number of conceivable scenarios is of order 2^T.

[8] In fact, it can be verified that the necessary condition in Proposition 2 is also sufficient for $T \leq 4$ (for compactness, we abstract from a formal statement). While Example 2 uses $T = 7$ for mathematical elegance of the proof, it is worth stressing that similar (but less elegant) arguments can be established for $4 < T < 7$.

Proposition 4. *A* sufficient condition *for the existence of utility functions U^1 and U^2 that provide a* CR-2 *of the observed set S is that there exist a collection of observable revealed preferred sets $\widehat{RP}_j^m$ $(m = 1, 2)$, $j \in \{1, ..., T\}$ that enables to construct N_m such that $\bigcup_{m=1,2} N_m = \{1, ..., T\}$ and $N_m = \{j \in \{1, ..., T\} \mid \mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_r \in \widehat{RP}_j^m} \mathbf{p}_j'\mathbf{q}_r\}$ with $\forall i, j \in N_m : \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m$.*

To interpret the condition, we introduce the concept of 'situation-dependent totalitarianism': when labelling the unitary model as 'totalitarian' (i.e., one and the same household member always has the full decision power), 'situation-dependent' totalitarianism indicates that the identity of the household member with full decision power may vary according to the specific situation.[9] In that interpretation, all observations in the set N_m have the household member m as the totalitarian decision maker; and the closing sufficiency condition then states that each situation-dependent (totalitarian) decision maker should act rationally, i.e., cost minimizing over the corresponding revealed preferred set. The additional restriction $\forall i, j \in N_m : \mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_j^m$ indicates that, if household member m is the decision maker in situations i and j, then the choice of $\mathbf{q}_i$ when $\mathbf{q}_j$ was equally obtainable under the prices $\mathbf{p}_i$ can be rationalized only if $\mathbf{q}_i \in \widehat{DRP}_j^m$.

In summary, violation of the necessary condition in Proposition 2 means that a *CR-2* of the data is impossible, while consistency with the sufficient condition in Proposition 4 entails the opposite conclusion. As for data that meet the necessity but not the sufficiency condition, we cannot directly tell from the observable (aggregate) price and quantity information whether a *CR-2* of the data is effectively possible. For instance, the proof of the inconsistency result in Example 2 starts from the necessity condition (which, like the unitary *GARP* condition, focuses on the full consumption bundles), to subsequently consider the construction of the personalized prices and quantities for individual commodities. In general, such practice boils down to checking the inequalities in Proposition 1 that are nonlinear in unobservables (which is avoided here only because of our specific condition for ε).

Still, even though the necessary condition should not generally coincide with the sufficient condition, we may expect the two conditions to become equally powerful (or 'converge') when the sample size increases.[10] Specifically, for $j \in \{1, ..., T\}$ we have that $\min_{\mathbf{q}_r}\{\mathbf{p}_j'\mathbf{q}_r \mid \mathbf{q}_r \in \widehat{RP}_j^1 \wedge \mathbf{q}_r \notin \widehat{RP}_j^2\}$ or $\min_{\mathbf{q}_r}\{\mathbf{p}_j'\mathbf{q}_r \mid \mathbf{q}_r \in \widehat{RP}_j^2 \wedge \mathbf{q}_r \notin \widehat{RP}_j^1\}$ will generally get closer to zero for larger T. Hence, the empirical requirement $\mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_{r_m} \in \widehat{RP}_j^m, m=1,2} \mathbf{p}_j'(\mathbf{q}_{r_1} + \mathbf{q}_{r_2})$ in Proposition 2 will approach $\mathbf{p}_j'\mathbf{q}_j \leq \min_{\mathbf{q}_r \in \widehat{RP}_j^m} \mathbf{p}_j'\mathbf{q}_r$

[9] We stress that, for data that are consistent with the sufficiency condition, this may not be the only data rationalizing interpretation; the sole implication of the sufficiency result is that situation-dependent totalitarianism *always* constitutes a possible interpretation.

[10] See, e.g., Bronars (1987) for power notions in the context of nonparametric rationality tests.

for $m = 1$ or 2 in Proposition 4.

The associated 'convergence rate' will then of course depend (positively) upon the price-quantity variation in the data and, hence, we may expect it to increase with the number of consumption commodities. For a given number of commodities, the speed of convergence will vary with the specific data generating process that underlies the aggregate household consumption data, which in turn depends on the member-specific utilities and on the characteristics of the within-household bargaining process. But, in general, we can safely argue that the empirical implications of the fairly rudimentary 'situation-dependent totalitarian' solution (see the sufficient condition) will get closer to those of any more refined intrahousehold decision process (see the necessary condition) when the sample size increases.

5. Concluding remarks

To conclude, we recall that the model under study considers general (altruistic) member-specific preferences, and only assumes that the empirical analyst observes the aggregate household consumption quantities and prices. Attractively, the model encompasses a large variety of alternative behavioural models as special cases, which include additional prior information regarding the personalized prices and quantities. For example, such additional structure may pertain to observability of quantities that are privately and/or publicly consumed or to the nature of the individual member preferences (namely, egoistic rather than altruistic); notable examples are the traditional unitary model and the collective model *à la* Chiappori (1988). For each of these special cases, we may generally expect more stringent (observable) necessary and sufficient conditions for data consistency with the model implications. (These conditions may be obtained along similar lines as in the proofs of Propositions 2 and 4.)

As a final note, we indicate that the observable collective rationality restrictions have a formally analogous structure as the (unitary) $GARP$ restrictions, which allows for easy adaptations of the existing power and goodness-of-fit measures for nonparametric consumption models (see respectively Bronars, 1987, and Varian, 1990). Specifically, the necessary and sufficient conditions may generate upper and lower bounds for each of these measures. (If these upper and lower bounds are situated close to each other, one possible interpretation is that the empirical content of the necessary and sufficient conditions is practically the same for the data under study.)

Appendix

A. Proof of Proposition 1

Varian (1982) proves the equivalence between conditions (i) and (ii) of the proposition. We may therefore restrict our following proof to conditions (ii).[11]

(i; necessity) Each bundle $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right)$ $(j = 1, ..., T)$ solves the problem

$$\max_{\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H} U^1\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) + \mu_j U^2\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) \text{ s.t. } \mathbf{p}_j'\left(\mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H\right) \leq \mathbf{p}_j'\mathbf{q}_j.$$

Given concavity, both individual utility functions are subdifferentiable, which carries over to their weighted sum $U^1 + \mu_j U^2$. An optimal solution to the above maximization problem should therefore satisfy (for η_j the Lagrange multiplier associated with the budget constraint)

$$U_{\mathbf{q}^k}^1 + \mu_j U_{\mathbf{q}^k}^2 \leq \eta_j \mathbf{p}_j,$$

where $U_{\mathbf{q}^k}^m$ is a subgradient of the utility function U^m $(m = 1, 2)$ defined for the vector $\mathbf{q}^k$ $(k = 1, 2, H)$ and evaluated at $\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right)$. Letting $\boldsymbol{\pi}_j^k = \frac{U_{\mathbf{q}^k}^1}{\eta_j}$, $\lambda_j^1 = \eta_j$ and $\lambda_j^2 = \frac{\eta_j}{\mu_j}$ thus gives

$$U_{\mathbf{q}^k}^1 = \lambda_j^1 \boldsymbol{\pi}_j^k \text{ and } U_{\mathbf{q}^k}^2 \leq \lambda_j^2\left(\mathbf{p}_j - \boldsymbol{\pi}_j^k\right). \tag{A.1}$$

Next, concavity of the functions U^1 and U^2 implies $(m = 1, 2)$

$$U^m\left(\mathbf{q}_i^1, \mathbf{q}_i^2, \mathbf{q}_i^H\right) - U^m\left(\mathbf{q}_j^1, \mathbf{q}_j^2, \mathbf{q}_j^H\right) \leq \sum_{l=1,2,H} U_{\mathbf{q}^l}^m\left(\mathbf{q}_i^l - \mathbf{q}_j^l\right). \tag{A.2}$$

Substituting (A.1) in (A.2) and setting $U_k^m = U^m\left(\mathbf{q}_k^1, \mathbf{q}_k^2, \mathbf{q}_k^H\right)$ $(m = 1, 2;\ k = i, j)$ obtains the conditions (ii) of the proposition.

(ii; sufficiency) As a first step, we define

$$U^1\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) = \min_{j \in \{1,...,T\}}\left[U_j^1 + \lambda_j^1 \sum_{l=1,2,H}\left(\boldsymbol{\pi}_j^l\right)'\left(\mathbf{q}^l - \mathbf{q}_j^l\right)\right] \text{ and} \tag{A.3}$$

$$U^2\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) = \min_{j \in \{1,...,T\}}\left[U_j^2 + \lambda_j^2 \sum_{l=1,2,H}\left(\mathbf{p}_j - \boldsymbol{\pi}_j^l\right)'\left(\mathbf{q}^l - \mathbf{q}_j^l\right)\right]. \tag{A.4}$$

[11] This proof generalizes that of Chiappori (1988), who focuses on the specific case of household labour supply. Another difference is that Chiappori focuses on (a strong version of) the *SARP* conditions while our proof uses the (less stringent) *GARP* conditions. It is worth pointing out that all our results for the *GARP* can be adapted to apply for the (strong) *SARP*.

Varian (1982) proves that $U^1\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right)=U_j^1$ and $U^2\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right)=U_j^2$. Next, given $\mu_j\in\Re_{++}$, we have for all $\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)$ such that $\mathbf{p}_j'\left(\mathbf{q}^1+\mathbf{q}^2+\mathbf{q}^H\right)\leq\ \mathbf{p}_j'\left(\mathbf{q}_j^1+\mathbf{q}_j^2+\mathbf{q}_j^H\right)$

$$\begin{aligned}&U^1\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)+\mu_jU^2\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)\\ \leq\ & U_j^1+\lambda_j^1\sum_{l=1,2,H}\left(\boldsymbol{\pi}_j^l\right)'\left(\mathbf{q}^l-\mathbf{q}_j^l\right)+\mu_j\left[U_j^2+\lambda_j^2\sum_{l=1,2,H}\left(\mathbf{p}_j-\boldsymbol{\pi}_j^l\right)'\left(\mathbf{q}^l-\mathbf{q}_j^l\right)\right].\end{aligned}$$

Without losing generality, we concentrate on $\mu_j=\left(\lambda_j^1/\lambda_j^2\right)$, which obtains

$$U^1\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)+\mu_jU^2\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)\leq U_j^1+\mu_jU_j^2+\lambda_j^1\left(\mathbf{p}_j'\sum_{l=1,2,H}\left(\mathbf{q}^l-\mathbf{q}_j^l\right)\right).$$

Since $\mathbf{p}_j'\left(\mathbf{q}^1+\mathbf{q}^2+\mathbf{q}^H\right)\leq\mathbf{p}_j'\left(\mathbf{q}_j^1+\mathbf{q}_j^2+\mathbf{q}_j^H\right)$, we thus have

$$U^1\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)+\mu_jU^2\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)\leq U_j^1+\mu_jU_j^2=U^1\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right)+\mu_jU^2\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right),$$

which proves that $\left(\mathbf{q}_j^1,\mathbf{q}_j^2,\mathbf{q}_j^H\right)$ maximizes $U^1\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)+\mu_jU^2\left(\mathbf{q}^1,\mathbf{q}^2,\mathbf{q}^H\right)$ subject to $\mathbf{p}_j'\left(\mathbf{q}^1+\mathbf{q}^2+\mathbf{q}^H\right)\leq\mathbf{p}_j'\mathbf{q}_j$. We conclude that the functions U^1 and U^2 in (A.3) and (A.4) provide a *CR-2* of the data. These functions are concave, monotonously increasing and continuous (see again Varian, 1982). ■

B. Proof of Lemma 1

(i) As a preliminary step, we note that $\mathbf{q}_i\in\widehat{DRP}_j$ is equivalent to: for all $\boldsymbol{\pi}_i$, $\widehat{\mathbf{q}}_k$ $(k=i,j)$ that satisfy the restrictions in Proposition 1, we have (see Definition 5)

$$\widehat{\mathbf{q}}_i\in DRP_j^1\Leftrightarrow\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i\geq\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j\text{ or }\widehat{\mathbf{q}}_i\in DRP_j^2\Leftrightarrow\left(\widehat{\mathbf{p}}_i-\boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_i\geq\left(\widehat{\mathbf{p}}_i-\boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_j.$$

(ii) We first derive that the collection of the sets DRP_j, $j\in\{1,...,T\}$ is a collection of observable directly revealed preferred sets. The result follows from the fact that $\mathbf{p}_i'\mathbf{q}_i\geq\mathbf{p}_i'\mathbf{q}_j$ is incompatible with the existence of some $\boldsymbol{\pi}_i,\widehat{\mathbf{q}}_k$ $(k=i,j)$ such that $(\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i<\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j\wedge\left(\widehat{\mathbf{p}}_i-\boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_i<\left(\widehat{\mathbf{p}}_i-\boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_j)$. Indeed, summing these last inequalities immediately yields $\mathbf{p}_i'\mathbf{q}_i<\mathbf{p}_i'\mathbf{q}_j$; whence we may conclude $\mathbf{q}_i\in DRP_j\Rightarrow\mathbf{p}_i'\mathbf{q}_i\geq\mathbf{p}_i'\mathbf{q}_j\Rightarrow\forall\boldsymbol{\pi}_i,\widehat{\mathbf{q}}_k$ $(k=i,j):(\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i\geq\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j\vee\left(\widehat{\mathbf{p}}_i-\boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_i\geq\left(\widehat{\mathbf{p}}_i-\boldsymbol{\pi}_i\right)'\widehat{\mathbf{q}}_j)$.

(iii) We next establish that $\widehat{DRP}_j\subseteq DRP_j$ for any collection of observable directly revealed preferred sets $\widehat{DRP}_j$, $j\in\{1,...,T\}$. The result is obtained by noting that

$\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j \Rightarrow \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i + (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i < \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j + (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j$ for all possible $\boldsymbol{\pi}_i$, $\widehat{\mathbf{q}}_k$ $(k = i, j)$. It is then easy to see that $\mathbf{p}_i'\mathbf{q}_i < \mathbf{p}_i'\mathbf{q}_j \Rightarrow \exists \boldsymbol{\pi}_i, \widehat{\mathbf{q}}_k$ $(k = i, j) : (\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i < \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j \wedge (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j)$; e.g., one may use $\boldsymbol{\pi}_k^1 = (1/2)\,\mathbf{p}_k$ and $\mathbf{q}_k^1 = \mathbf{q}_k$ $(k = i, j)$. Hence, we have $\forall \boldsymbol{\pi}_i, \widehat{\mathbf{q}}_k$ $(k = i, j) : (\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_j \vee (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j)$ only if $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j$, i.e., $\mathbf{q}_i \in DRP_j$. ■

C. Proof of Proposition 2

Consider an arbitrary specification of the personalized prices and quantities $\boldsymbol{\pi}_k$ and $\widehat{\mathbf{q}}_k$ $(k \in \{1, ..., T\})$, which entails the (member-specific) directly revealed preferred sets DRP_k^m (for $i \in \{1, ..., T\}$) :

$$\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_k \Rightarrow \widehat{\mathbf{q}}_i \in DRP_k^1 \text{ and } (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i \geq (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_k \Rightarrow \widehat{\mathbf{q}}_i \in DRP_k^2,$$

This in turn implies the sets RP_j^m; see Definition 4. In the following, we show that these sets are consistent with the (direct analogues of the) properties (i)-(iv) in Definition 6 and the necessary condition in Proposition 2 when the data meet the *CR-2* conditions in Proposition 1. These properties carry over to the ('inner bound') observable sets $\widehat{DRP}_k^m$ and $\widehat{RP}_k^m$ under *CR-2* consistency of the data.

Property (i) follows directly from Lemma 1. Next, Property (ii) easily follows from the transitivity relationships implied by the *GARP* requirements (at the level of the individual household members) in the conditions (i) of Proposition 1.

As for property (iii), we first recall that property (i) implies that $\mathbf{q}_j \in \widehat{DRP}_i^m$ $(m \in \{1, 2\})$ only if $\mathbf{q}_j \in DRP_i$ or $\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i$. Using this, we should establish that $(\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i \wedge \widehat{\mathbf{q}}_i \in RP_j^1) \Rightarrow \widehat{\mathbf{q}}_j \in DRP_i^2$ (the argument for $(\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i \wedge \widehat{\mathbf{q}}_i \in RP_j^2) \Rightarrow \widehat{\mathbf{q}}_j \in DRP_i^1$ is directly analogous). Under consistency with the *CR-2* conditions, $\widehat{\mathbf{q}}_i \in RP_j^1$ requires $\boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_i$. Given $\mathbf{p}_j'\mathbf{q}_j \geq \mathbf{p}_j'\mathbf{q}_i$, this last inequality implies $(\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_j \geq (\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_i$ or $\widehat{\mathbf{q}}_j \in DRP_i^2$, which gives the result.

To derive property (iv), suppose that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\,(\mathbf{q}_{j_1} + \mathbf{q}_{j2})$ in combination with $\widehat{\mathbf{q}}_{j_1} \in RP_i^1$ and $\widehat{\mathbf{q}}_i \notin DRP_{j2}^2$. On the one hand, $\widehat{\mathbf{q}}_i \notin DRP_{j2}^2$ means that $(\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_i < (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_{j_2}$. On the other hand, $\widehat{\mathbf{q}}_{j_1} \in RP_i^1$ requires that $\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \leq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_{j_1}$ for the data to be consistent with the *CR-2* conditions. Combining these two inequalities would imply $\mathbf{p}_i'\mathbf{q}_i < \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_{j_1} + (\widehat{\mathbf{p}}_i - \boldsymbol{\pi}_i)'\,\widehat{\mathbf{q}}_j \leq \mathbf{p}_i'\,(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$, which contradicts $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\,(\mathbf{q}_{j_1} + \mathbf{q}_{j_2})$. Thus, we conclude that $(\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\,(\mathbf{q}_{j_1} + \mathbf{q}_{j2}) \wedge \widehat{\mathbf{q}}_{j_1} \in RP_i^1) \Rightarrow \widehat{\mathbf{q}}_i \in DRP_{j2}^2$. A directly analogous argument yields $(\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\,(\mathbf{q}_{j_1} + \mathbf{q}_{j2}) \wedge \widehat{\mathbf{q}}_{j_1} \in RP_i^2) \Rightarrow \widehat{\mathbf{q}}_i \in DRP_{j2}^1$.

Finally, under $\widehat{\mathbf{q}}_{r_1} \in RP_j^1$ and $\widehat{\mathbf{q}}_{r_2} \in RP_j^2$ consistency with the *CR-2* requirements is obtained only if $(\boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_{r_1}) \wedge ((\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_j \leq (\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_{r_2})$. This last result immediately yields $\mathbf{p}_j'\mathbf{q}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_{r_1} + (\widehat{\mathbf{p}}_j - \boldsymbol{\pi}_j)'\,\widehat{\mathbf{q}}_{r_2} \leq \mathbf{p}_j'\,(\mathbf{q}_{r_1} + \mathbf{q}_{r_2})$ if $\mathbf{q}_{r_1} \neq \mathbf{q}_{r_2}$ and, similarly, $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_r$ if $\mathbf{q}_{r_1} = \mathbf{q}_{r_2} = \mathbf{q}_r$. The observation that such an inequality should hold for any combination $\widehat{\mathbf{q}}_{r_1} \in RP_j^1$ and $\widehat{\mathbf{q}}_{r_2} \in RP_j^2$ then immediately entails the stated necessary condition for collective rationality. ■

D. Proof of the result in Example 2

For the specific data structure, consistency with the necessity condition implies $\forall i, j \in \{1, ..., 7\}, \ i \neq j : (\mathbf{q}_i \in \widehat{RP}_j^m \wedge \mathbf{q}_i \notin \widehat{RP}_j^l)$ for $m, l \in \{1, 2\}, \ m \neq l$; and we cannot have $(\mathbf{q}_i \in \widehat{RP}_k^1 \wedge \mathbf{q}_j \in \widehat{RP}_k^2)$ for $k \in \{1, 7\}$ and $\forall i, j \in \{1, ..., 7\} \setminus \{k\}$. Given this, one possible member-specific revealed preference structure is[12]

$$\forall i, j \in \{1, ..., 7\} : (i > j \Rightarrow \mathbf{q}_j \in \widehat{RP}_i^1) \text{ and } (i < j \Rightarrow \mathbf{q}_j \in \widehat{RP}_i^2).$$

Combining the associated conditions for *CR-2* consistency obtains $\forall i \in \{2, ..., 6\}$

$$\forall j \in \{1, ..., 7\} : (i > j \Rightarrow \mathbf{p}_i' \mathbf{q}_j - \varepsilon \leq \boldsymbol{\pi}_i' \widehat{\mathbf{q}}_j \leq \mathbf{p}_i' \mathbf{q}_j) \text{ and } (i < j \Rightarrow 0 \leq \boldsymbol{\pi}_i' \widehat{\mathbf{q}}_j \leq \varepsilon). \quad \text{(D.1)}$$

Next, because $\langle \mathbf{q}_j \rangle_\nu = \left\langle \mathbf{q}_j^1 \right\rangle_\nu + \left\langle \mathbf{q}_j^2 \right\rangle_\nu + \left\langle \mathbf{q}_j^H \right\rangle_\nu$, we obtain that $\mathbf{p}_i' \mathbf{q}_j - \varepsilon \leq \boldsymbol{\pi}_i' \widehat{\mathbf{q}}_j \leq \mathbf{p}_i' \mathbf{q}_j$ implies $\forall \nu \in \{1, ..., n\}$

$$\langle \mathbf{p}_i \rangle_\nu \langle \mathbf{q}_j \rangle_\nu - \varepsilon \leq \sum_{m \in \{1,2,H\}} \langle \boldsymbol{\pi}_i^m \rangle_\nu \left\langle \mathbf{q}_j^m \right\rangle_\nu \leq \langle \mathbf{p}_i \rangle_\nu \langle \mathbf{q}_j \rangle_\nu ,$$

which in turn entails $\forall m \in \{1, 2, H\}$

$$\langle \mathbf{p}_i \rangle_\nu - \frac{\varepsilon}{\left\langle \mathbf{q}_j^m \right\rangle_\nu} \leq \langle \boldsymbol{\pi}_i^m \rangle_\nu \leq \langle \mathbf{p}_i \rangle_\nu .$$

Similarly, the restriction $0 \leq \boldsymbol{\pi}_i' \widehat{\mathbf{q}}_j \leq \varepsilon$ requires

$$\left[0 \leq \sum_{m \in \{1,2,H\}} \langle \boldsymbol{\pi}_i^m \rangle_\nu \left\langle \mathbf{q}_j^m \right\rangle_\nu \leq \varepsilon \right] \Rightarrow \left[\forall m \in \{1, 2, H\} : 0 \leq \langle \boldsymbol{\pi}_i^m \rangle_\nu \leq \frac{\varepsilon}{\left\langle \mathbf{q}_j^m \right\rangle_\nu} \right].$$

Let us concentrate on $\nu = 1$ and consider a σ such that $0 < \sigma < \min_{j \in \{1,...,7\}, \nu \in \{1,...,n\}} \langle \mathbf{q}_j \rangle_\nu$. The Pigeon Hole Principle implies $\forall j \in \{1, ..., 7\} : \exists m_j \in \{1, 2, H\} : \left\langle \mathbf{q}_j^{m_j} \right\rangle_1 > (\sigma/3)$, so that we get

$$\left[\mathbf{p}_i' \mathbf{q}_j - \varepsilon \leq \boldsymbol{\pi}_i' \widehat{\mathbf{q}}_j \leq \mathbf{p}_i' \mathbf{q}_j \right] \Rightarrow \left[\exists m_j \in \{1, 2, H\} : \langle \mathbf{p}_i \rangle_1 - \frac{3\varepsilon}{\sigma} \leq \left\langle \boldsymbol{\pi}_i^{m_j} \right\rangle_1 \leq \langle \mathbf{p}_i \rangle_1 \right] \text{ and}$$

$$\left[0 \leq \boldsymbol{\pi}_i' \widehat{\mathbf{q}}_j \leq \varepsilon \right] \Rightarrow \left[\exists m_j \in \{1, 2, H\} : 0 \leq \left\langle \boldsymbol{\pi}_i^{m_j} \right\rangle_1 \leq \frac{3\varepsilon}{\sigma} \right].$$

[12] The following argument can be repeated for any alternative preference structure that meets the necessity conditions.

Remark that $\frac{\min_{j,\nu}\langle\mathbf{p}_j\rangle_\nu \min_{j,\nu}\langle\mathbf{q}_j\rangle_\nu}{6} > \varepsilon$ implies $\langle\mathbf{p}_i\rangle_1 - \frac{3\varepsilon}{\sigma} > \frac{3\varepsilon}{\sigma}$. Using this, the preference structure in (D.1) obtains $\forall i \in \{2, ..., 6\}$

$$\forall j_1, j_2 \in \{1, ..., 7\} : (i > j_1 \wedge i < j_2 \Rightarrow m_{j_1} \neq m_{j_2}); \qquad \text{(D.2)}$$

the reasoning is that $(i > j_1 \Rightarrow \langle\mathbf{p}_i\rangle_1 - \frac{3\varepsilon}{\sigma} \leq \left\langle \boldsymbol{\pi}_i^{m_{j_1}} \right\rangle_1 \leq \langle\mathbf{p}_i\rangle_1)$ and $(i < j_2 \Rightarrow 0 \leq \left\langle \boldsymbol{\pi}_i^{m_{j_2}} \right\rangle_1 \leq \frac{3\varepsilon}{\sigma})$, which excludes $m_{j_1} = m_{j_2}$. Impossibility of a *CR-2* follows as (D.2) implies $m_{j_1} \neq m_{j_2}$ for all $j_1, j_2 \in \{1, 3, 5, 7\}, j_1 \neq j_2$; and this contradicts $m_j \in \{1, 2, H\}$ $\forall j \in \{1, .., 7\}$. ■

E. Proof of Proposition 4

Suppose that there exists a collection of observable revealed preferred sets $\widehat{RP}_j^m$ $(m = 1, 2)$, $j \in \{1, ..., T\}$ that satisfies the sufficiency condition in Proposition 4. Given this, we can construct a partitioning $\widehat{N}_1, \widehat{N}_2$ $(\widehat{N}_1 \cup \widehat{N}_2 = \{1, ..., T\}; \widehat{N}_1 \cap \widehat{N}_2 = \emptyset)$ with associated personalized prices and quantities that meet the conditions for a *CR-2* of the data. Specifically, we define $j \in N_1 \Rightarrow j \in \widehat{N}_1$ and $j \in \widehat{N}_2$ in the other case (which implies $j \in N_2$); and we use the personalized price-quantity specifications

$$\begin{aligned} \widehat{\mathbf{q}}_i &= \begin{pmatrix} (\mathbf{q}_i)' & (\mathbf{0}^n)' & (\mathbf{0}^n)' \end{pmatrix}' \text{ for } i \in \widehat{N}_1,\ \widehat{\mathbf{q}}_i = \begin{pmatrix} (\mathbf{0}^n)' & (\mathbf{q}_i)' & (\mathbf{0}^n)' \end{pmatrix}' \text{ for } i \in \widehat{N}_2, \\ \boldsymbol{\pi}_i &= \begin{pmatrix} (\mathbf{p}_i)' & (\mathbf{0}^n)' & (\mathbf{0}^n)' \end{pmatrix}' \text{ for } i \in \{1, .., T\}. \end{aligned}$$

We next establish that this price-quantity allocation satisfies conditions (i) in Proposition 1.[13]

For the sake of brevity, we restrict attention to the first household member; but a directly analogous reasoning applies to the second household member. The *GARP* requirement states that $\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_k$, ..., $\boldsymbol{\pi}_y'\widehat{\mathbf{q}}_y \geq \boldsymbol{\pi}_y'\widehat{\mathbf{q}}_j$ (for some sequence of household observations $(k, ..., y)$) implies $\boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_i$. As a preliminary step, we note that under the above specification $\forall \boldsymbol{\pi} \in \Re_+^{3n}$: $\boldsymbol{\pi}'\widehat{\mathbf{q}}_z = 0$ if $z \in \widehat{N}_2$. This makes that the only interesting case is i, j, k, ..., $y \in \widehat{N}_1$. Hence, obtaining $\boldsymbol{\pi}_i'\widehat{\mathbf{q}}_i \geq \boldsymbol{\pi}_i'\widehat{\mathbf{q}}_k$, ..., $\boldsymbol{\pi}_y'\widehat{\mathbf{q}}_y \geq \boldsymbol{\pi}_y'\widehat{\mathbf{q}}_j \Rightarrow \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_j \leq \boldsymbol{\pi}_j'\widehat{\mathbf{q}}_i$ boils down to verifying $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k$, ..., $\mathbf{p}_y'\mathbf{q}_y \geq \mathbf{p}_y'\mathbf{q}_j \Rightarrow \mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$ for any possible sequence of observations $(i, k, ..., y, j)$ with $i, j, k, ..., y \in \widehat{N}_1$.

Using that $\forall\ k_1, k_2 \in \widehat{N}_1 : \mathbf{p}_{k_1}'\mathbf{q}_{k_1} \geq \mathbf{p}_{k_1}'\mathbf{q}_{k_2} \Rightarrow \mathbf{q}_{k_1} \in \widehat{DRP}_{k_2}^1$ (for $k_1, k_2 \in \widehat{N}_1$ follows from $k_1, k_2 \in N_1$), we have $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_k$, ..., $\mathbf{p}_y'\mathbf{q}_y \geq \mathbf{p}_y'\mathbf{q}_j \Rightarrow \mathbf{q}_i \in \widehat{DRP}_k^1$, ..., $\mathbf{q}_y \in \widehat{DRP}_j^1$, which in turn implies $\mathbf{q}_i \in \widehat{RP}_j^1$; see the transitivity property (ii) in Definition 6. The sufficiency condition consequently guarantees $\mathbf{p}_j'\mathbf{q}_j \leq \mathbf{p}_j'\mathbf{q}_i$, i.e. the *GARP* requirement for the first household member is met. ■

[13] The following proof does not explicitly use the properties (iii) and (iv) in Definition 6. These properties are implied by the closing sufficient condition in Proposition 4.

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