

# The collision rate of monodispersed particles in turbulent flows with gravity

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## Abstract

It is of great importance to investigate the collisions of monodispersed particles in turbulent flows both in the mechanical engineering and in the cloud microphysics. A number of previous studies investigated the collision phenomena but they neglected the effect of gravity acting on particles under the assumption of the isotropy of particle motions. However, it is not concluded whether the gravity effect can be neglected or not. In this study, therefore, we investigated the effect of gravity on the collision frequency by using a direct numerical simulation. The results show that the gravity acting on particles decreases the collision frequency, compared to the zero-gravity case. When the terminal velocity of a particle is comparable to or greater than the root mean square value of fluid velocity, the effect of gravity is significant.

*Keywords: collision frequency, monodispersed particles, turbulent flow, multi-phase flow.*

## 1 Introduction

Collisions of monodispersed particles in turbulent flows have much effect on the development of convective clouds [1] as well as on turbulence modulation in multi-phase flows [2]. Therefore, it is of great importance to investigate the particle collision mechanism in both fields; cloud microphysics and mechanical engineering. In the previous studies, the particles are uniformly distributed, that is, the Stokes number of particles was assumed to be zero or enough large [3, 4]. On the other hand, Sundaram & Collins [5] derived the formula of the collision frequency which can be applied even for the Stokes number around unity, by considering the preferential concentration. A number of previous studies have used the formula by Sundaram & Collins. However, Sundaram & Collins



neglected the gravity acting on particles in order to assume the isotropy of particle motion. The assumption of zero-gravity may involve the risk of low estimation of particle collision frequency [1], but no study has clarified how the gravity affects the collision frequency. Therefore, we aimed to investigate the effect of the gravity on the collision frequency among monodispersed particles in turbulent flows, by means of a direct numerical simulation (DNS).

## 2 Overviews of direct numerical simulation

### 2.1 Direct numerical simulation of fluid motion

Gas phase turbulent flows were computed by using the DNS based on the spectrum method. Periodic boundary conditions were applied to a cubic domain of length  $2\pi$ , which corresponds to  $3.14 \times 10^{-3}$  m in this study. Time advancement was done using an efficient forth-order Runge-Kutta scheme. The steady flow field was achieved by maintaining the strength of the vortex whose wave number is unity. Table 1 shows the typical turbulence statistics in gas phase turbulent flows.  $U_{rms}$  in the table is the RMS value of fluid velocity fluctuation and the Reynolds number,  $Re_\lambda$ , was calculated by

$$Re_\lambda = \frac{l_\lambda U_{rms}}{\nu}, \quad (1)$$

where  $l_\lambda$  is the Taylor's micro scale and  $\nu$  is the kinematic viscosity of fluid.

Table 1: Typical turbulence statistics in turbulent flows.

	$U_{rms}$ [m/s]	Kolmogorov scale $l_\eta$ [m]	Taylor's micro scale $l_\lambda$ [m]	$Re_\lambda$
RUN-1	0.130	$5.83 \times 10^{-4}$	$8.92 \times 10^{-3}$	77.3
RUN-2	0.173	$4.60 \times 10^{-4}$	$7.73 \times 10^{-3}$	89.2
RUN-3	0.242	$3.54 \times 10^{-4}$	$6.56 \times 10^{-3}$	106

### 2.2 Direct numerical simulation of particle motion

Minute water droplets were used here as dispersed particles, since cloud droplets were targeted in this study. The typical radius of a cloud droplet being about  $10 \mu\text{m}$ , we used particles with  $r < 40 \mu\text{m}$ , that is,  $Re_p < 1$ , which are regarded as the Stokes particles. In the computational domain,  $4096 (=16 \times 16 \times 16)$  particles were initially put. Its number density  $n$  was  $3.2 \times 10^8 \text{m}^{-3}$  and was comparable to that in a typical cloud. The present mass and volume fraction of particles were so small that we neglected the turbulence modulation by particles. The relations between the particle radius, the relaxation time  $\tau_p$  and the particle Reynolds number  $Re_p$  are shown in table 2. We considered only binary collisions because particle concentration was estimated to be sufficiently low that binary collisions were overwhelmingly dominant. We assumed that one of the two particles



vanish after the binary collision, in order to avoid an additional collision induced by the momentum exchange via hard-sphere collisions [6].

Table 2: Parameters of particles.

Radius $r$ [ $\mu\text{m}$ ]	Relaxation time $\tau_p$ [s]	$Re_p$ ( $=2rV_\infty/\nu$ )
15	$2.81 \times 10^{-3}$	0.0550
20	$5.00 \times 10^{-3}$	0.131
25	$7.81 \times 10^{-3}$	0.255
30	$1.12 \times 10^{-2}$	0.440
35	$1.53 \times 10^{-2}$	0.700
40	$2.00 \times 10^{-2}$	1.05

### 3 Results and discussion

#### 3.1 Collision kernel

In order to investigate the effect of gravity acting on particles on the collision frequency, we computed the collision kernel  $K_c$  which is defined by  $K_c = N_c/n^2$ . Here,  $N_c$  is the collision frequency and  $n$  is the particle number density. The collision kernels with and without gravity are shown in figure 1. The vertical axis is normalized by using the local velocity gradient  $\lambda = (\varepsilon/\nu)^{1/2}$  and particle radius  $r$  [3]. From this figure, it is found that neglecting the gravity causes the overestimation of the collision kernel and the tendency becomes stronger as the Stokes number increases even in the same turbulent flow field or as the turbulence becomes weak even for the same Stokes number.

When we consider that the collision kernel depends on the state of particle distribution and the duration time in which a particle is kept in and influenced by a vortex, we can guess two possible reasons why the collision kernel is overestimated under the zero-gravity condition. The first one is that the exact particle distribution cannot be computed and the other is that the duration time cannot be correctly predicted if the gravity acting on particles is neglected.

#### 3.2 Particle distribution

In order to examine the effect of the gravity on the particle distribution, we compared the cluster scales. Fig. 2 shows the effect of the gravity on the cluster scales. Here, we employed the Fessler *et al.*'s definition [7] of the cluster scale. The vertical axis is normalized by Kolmogorov length scale  $l_\eta$  in the figure. It is found that the cluster scales are not strongly influenced by the gravity. Therefore, the particle distribution has no influence on the overestimation of the collision kernel under the zero-gravity condition.



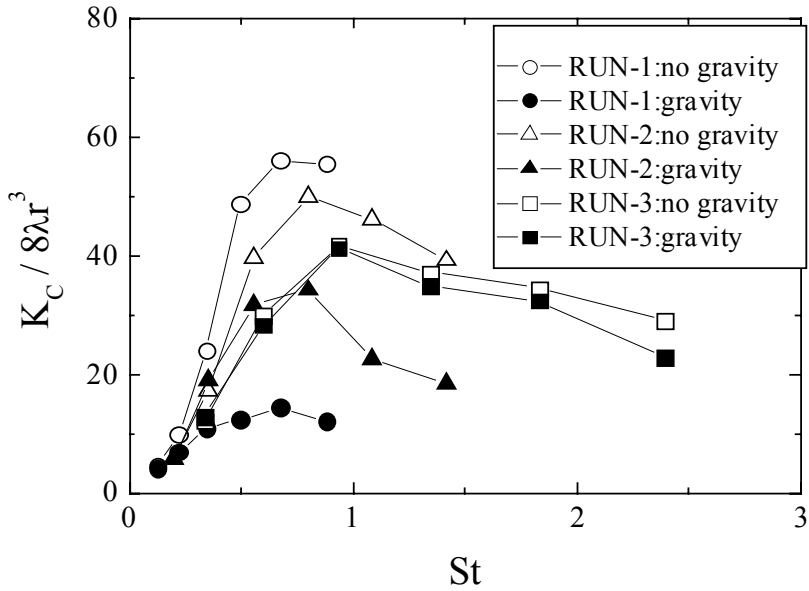


Figure 1: Averaged collision kernel normalized by  $8 \lambda r^3$  as a function of the Stokes number. Open symbols denote the results without gravity and closed symbols denote the results with gravity.

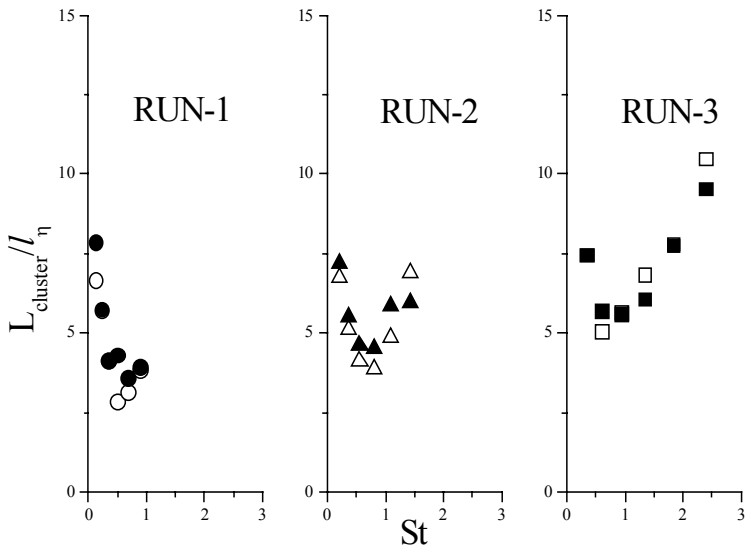


Figure 2: Cluster scales in the cases of (a) RUN-1, (b) RUN-2 and (c) RUN-3. Symbols as in Fig. 1.



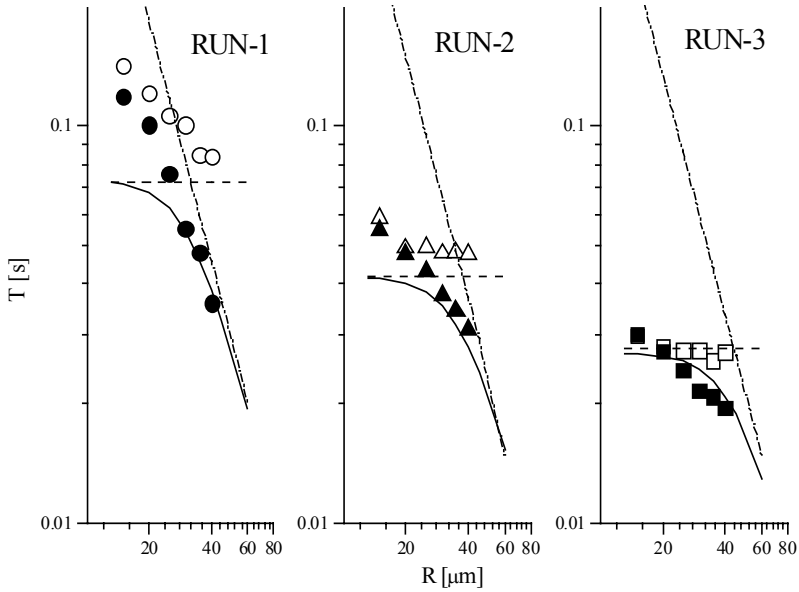


Figure 3: Averaged duration time of a particle in a vortex in the cases of (a) RUN-1, (b) RUN-2 and (c) RUN-3. Symbols as in Fig. 1 and lines as follows:  $\cdots$ ,  $T_1$ ;  $-\cdot-$ ,  $T_2$ ;  $-$ ,  $T_3$ .

### 3.3 Particle duration time

In order to define the duration time in which a particle is kept in a vortex, we used a spanwise vorticity of the fluid at the center of a particle. When the vorticity becomes zero at a time  $T_a$  and, after some time, it becomes zero again at a time  $T_b$ , we defined the  $T_b - T_a$  as the duration time. By this definition, we can obtain the duration time in which a particle is kept in a vortex whose size is comparable to the Taylor's micro scale  $l_\lambda$ . The duration times under gravity and zero-gravity conditions are shown in figure 3. The dashed lines indicate the duration time,  $T_1 = l_\lambda / U_{rms}$ , derived under the assumption that a particle moves with the ambient fluid velocity. The chain lines indicate the duration time,  $T_2 = l_\lambda / V_\infty$ , derived under the assumption that a particle moves with its terminal velocity  $V_\infty$ . The solid lines indicate the duration time,  $T_3 = l_\lambda / V_\infty \sqrt{\alpha^2 + 1}$ , derived under the assumption that a particle moves with the sum of the ambient fluid velocity and its terminal velocity. Here  $\alpha$  is given by  $U_{rms} / V_\infty$ . In figure 3, the duration time approaches  $T_1$  as the turbulence becomes stronger under the zero-gravity condition. As the turbulence becomes stronger under the gravity condition, the duration time is between  $T_2$  and  $T_3$  and it approaches  $T_3$ . Further, it is found that the duration time is overestimated when the gravity is neglected. The duration time is more overestimated as the Stokes number increases or as the turbulence becomes weaker. This tendency agrees



with that of the collision kernel. Therefore, it is concluded that the overestimation of the collision kernel is caused by the overestimation of the duration time. Especially, a big overestimation of the duration time can be seen at the case of  $T_1 > T_2$ , that is,  $V_\infty > U_{rms}$ . This shows that, at the time of  $V_\infty > U_{rms}$ , the assumption of the zero-gravity causes the serious error on the estimation of the collision kernel.

## 4 Conclusions

The effect of the gravity on the collision of monodispersed particles in turbulent flows was investigated by means of a direct numerical simulation. The results from this study can be summarized as follows.

- (1) When the gravity acting on particles is neglected, the collision frequency between the monodispersed particles in turbulent flows is overestimated.
- (2) The collision frequency under the zero-gravity condition is also overestimated, since the duration time of a particle in a vortex is overestimated.
- (3) When the terminal velocities of particles are larger than the r.m.s value of fluid velocity fluctuation, the zero-gravity condition causes serious errors on the estimation of the particle collision kernel.

We have applied the above conclusions to the study of Sundaram & Collins [5], in which the gravity was neglected. Translating their computational conditions to those in a representative convective cloud whose dissipation rate is  $0.01 \text{ m}^2/\text{s}^3$ ,  $\alpha$  reaches 2.33 when the Stokes number is assumed to be unity. This value of  $\alpha$  bigger than unity suggests that the collision frequencies computed by a model of Sundaram & Collins are useless in predicting the cloud drop growth by collision-coalescence in convective clouds.

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