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# The collisions of two ion acoustic solitary waves in a magnetized nonextensive plasma

**Research Article**

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**Abstract:** Using the extended Poincaré–Lighthill–Kuo (EPLK) method, the interaction between two ion acoustic solitary waves (IASWs) in a multicomponent magnetized plasma (including Tsallis nonextensive electrons) has been theoretically investigated. The analytical phase shifts of the two solitary waves after interaction are estimated. The proposed model leads to rarefactive solitons only. The effects of colliding angle, ratio of number densities of (positive/negative) ions species to the density of nonextensive electrons, ion-to-electron temperature ratio, mass ratio of the negative-to-positive ions and the electron nonextensive parameter on the phase shifts are investigated numerically. The present results show that these parameters have strong effects on the phase shifts and trajectories of the two IASWs after collision. Evidently, this model is helpful for interpreting the propagation and the oblique collision of IASWs in magnetized multicomponent plasma experiments and space observations.

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**Keywords:** negative-ion plasma, extended Poincaré–Lighthill–Kuo method, solitons, phase shift, nonextensive electrons



## 1. Introduction

Ion acoustic solitary waves (IASWs) have been studied for several decades both theoretically and experimentally. They have been first considered in [1] where their fully nonlinear features were studied using a mechanical analogy. Later on, these nonlinear waves have received considerable attention both theoretically and experimentally [2]. It has been reported that only compressive IASWs involving density humps exist in unmagnetized two-component plasmas. Plasmas containing an appreciable amount of negative ions (the so- called multicomponent plasmas) have been the subject of intense investigations [3–13]. This interest is mainly due to their wide technological applications [14–17] and role in astrophysical

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plasmas. Negative ions have been detected in the Earth's ionosphere [18], cometary comae [19], and the upper regions of Titan [20]. Moreover, multicomponent plasmas, such as Ar/$SF_6$ and K/$SF_6$ plasmas, are generally used to perform basic research on IASWs in dc discharge devices and Q-machines [21–28]. Since the early space observations [29], it has been admitted that the Maxwellian distribution is not always a realistic distribution [30–32]. Due to a variety of different process, a plasma may deviate from its thermodynamic equilibrium state to evolve into a nonequilibrium stationary state. For instance, a background turbulence may contribute to the appearance of new distribution functions that deviate noticeably from Maxwellian [33]. Most of the natural space plasma distribution functions exhibit non-Maxwellian high-energy tails or flat tops with pronounced shoulders. Recently, a great deal of attention has been devoted to an appropriate generalization of the Boltzmann–Gibbs–Shannon (BGS) entropy [34]. The later is considered valid universally for macroscopic ergodic equilibrium systems. It also seems to be inadequate to describe systems with long-range oblique collisions, such as plasma and gravitational systems . A nonextensive generalization of the BGS entropy for statistical mechanics was first proposed by Rényi [35] and, later on, constructed by Tsallis [36]. This extends the standard additivity of the entropies to the nonlinear, nonextensive case where one particular parameter, the entropic index $q$, characterizes the strength of nonextensivity. This new nonadditive entropy has been successfully applied to a wide range of phenomena (self-gravitating systems, some kinds of plasma turbulence etc.) [37, 38]. Recent evidences suggest that $q$-entropy would be used to establish a suitable frame for investigating many astrophysical phenomena; in stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy clusters. In addition, the experimental results of electrostatic plane-wave propagation in a collisionless thermal plasma yield to a class of Tsallis's velocity distribution described by a nonextensive $q$ parameter less than unity [37–48].

Additionally, the excitation, propagation, stability and oblique collision of solitary waves in plasmas still deserve to be carefully perused and examined. The interesting features of the collision between solitary waves are now well known: when two solitary waves approach closely, they interact, exchange their energies and positions with each other and then scatter, regaining their original wave forms [50, 51]. During the oblique collision process, the solitary waves are remarkably stable entities, preserving their identities; the collision changes the phase shift only. We will focus our attention studying the oblique collision of two IASWs based on evaluating their phase shifts and trajectories after an oblique collision occurs. It may be worth mentioning that in one-dimensional systems there are two distinct types of the solitary wave collisions; overtaking and head-on [49–51]. The overtaking collision of solitary waves (where the angle between the two solitary waves propagation directions $\delta$, vanishes) can be studied by the inverse scattering transformation method [52]. In the head-on collision, this angle $\delta$ is $\pi$. The latter type have been investigated using the extended Poincaré- Lighthill–Kuo (EPLK) method [50, 51, 53–56]. Indeed, what cannot be ignored is, the one-dimensional geometry may not be the realistic situation in laboratory devices and/or in space. However, the oblique collision (i.e., $0 < \delta < \pi$) of solitary waves in a three-dimensional geometry is more realistic in magnetized multicomponent plasma. Therefore, the main purpose of this manuscript is to investigate the oblique collision of two IASWs in a three-dimensional magnetized multicomponent plasma using the EPLK method. Recently, some authors [57–61] have focused on the interaction of two solitary waves taking into account arbitrary collision angles in different plasma models. Xue [57] discussed how the magnetic field significantly modifies the solitons collision properties. The influence of the colliding angle for two dust acoustic waves oblique collision has been investigated in a three-dimensional magnetized plasma model, Liang *et al.* [58]. Accordingly, we are interested in investigating the effects of the external magnetic field, colliding angle, number densities of (positive/negative) ions species, temperature ratio of the plasma species and the electron nonextensive parameter on the main characteristics of the two IASWs oblique collision in a multicomponent magnetized plasma including Tsallis nonextensive distributed electrons.

## 2. Governing equations and oblique collision of two IASWs

We consider a multicomponent magnetized plasma model whose constituents are singly charged cold positive ions (subscript $i$), singly charged hot negative ions (subscript $n$), and nonextensive electrons (subscript $e$). The dynamics of nonlinear IASWs in this proposed plasma system is governed by the following normalized equations [11, 12, 48],

$$\frac{\partial n_n}{\partial t} + \vec{\nabla} \cdot (n_n \vec{u}) = 0, \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right) \vec{u} + \mu \left(- \vec{\nabla}\phi + \frac{5}{3}\frac{\theta_n}{n_n^{1/3}} \vec{\nabla} n_n\right) + \Omega\, \vec{u} \times \hat{z} = 0, \tag{2}$$

the nonextensive electron number density is

$$n_e = \left[1 + (q-1)\,\phi\right]^{(q+1)/[2(q-1)]}, \tag{3}$$

the ions are assumed to be Boltzmann species

$$n_i = \beta \exp\left(-\theta_i \ \phi\right), \tag{4}$$

the system is closed by Poisson equation which is,

$$\nabla^2 \phi = n_e - n_i + n_n. \tag{5}$$

In Eqs. (1)-(5), $n_j$ is the $j-$ species number density ($j = n$ for negative ion, $i$ for positive ion, and $e$ for electrons), $\vec{u}$ is the negative ion fluid velocity with the components $(v, w, \psi)$, and $\phi$ is the electrostatic wave potential. The system is exposed to an external magnetic field, $\vec{B} = B_o \hat{z}$. The variables appearing in Eqs. (1)-(4) have been scaled by appropriate quantities. Thus, $n_j$ is normalized by the unperturbed electron number density $n_{eo}$, $\vec{u}$ is scaled by the ion sound speed $C_s = (k_B T_e/m_i)^{1/2}$ and the potential $\phi$ by $(k_B T_e/e)$. The time by the ion plasma period $\omega_{pi}^{-1} = (4\pi e^2 n_{eo}/m_i)^{-1/2}$, and the space variables $(x, y, z)$ are in units of the Debye radius $\lambda_D = (k_B T_e/4\pi e^2 n_{eo})^{1/2}$, where $T_e$ is the electron temperature and $k_B$ is the Boltzmann constant. We have defined $\beta = n_{io}/n_{eo}$, $\theta_i = T_e/T_i$, $\mu = m_i/m_n$ and $\theta_n = T_n/T_e$ where $m_j\left(T_j\right)$ is the $j$-species mass (temperature) and $\Omega = \left(eB_o/m_n c\right)/\omega_{pi}$. Note, we restrict the study for $\mu < 1$, i.e. heavy negative ion fluid. Therefore, it is reasonable to consider Boltzmann positive ion species due to (positive/negative) mass order. The neutrality condition implies $\beta = 1+\alpha$, where $\alpha = n_{no}/n_{eo}$.

In Eq. (3), the parameter $q$ is the strength of nonextensivity. For $q < -1$, the nonextensive electron distribution (not given here) is unnormalizable. In the extensive limiting case $q \to 1$, the electron density, (3), reduces to the well-known Maxwell–Boltzmann counterpart.

To investigate the oblique collision of two IASWs, we follow the procedures presented in Refs. [50, 51]. Let us study the effects of quasielastic oblique collision of two solitons $S_1$ and $S_2$ in the present multicomponent magnetized plasma. We also assume that they are, asymptotically, far apart in the initial state and travel toward each other. After some time they interact, and the amplitude of the overlapping waves is greater than the algebraic sum of the individual solitons before collision. Furthermore, the amplitude slightly dips immediately after the collision and returns to its value before next collision occurring at a later time. In order to analyze the effects of this collision, we employ an EPLK method. According to this method, the dependent variables are expanded in power of $\epsilon$ as,

$$\left.\begin{aligned} n_n &= \alpha + \epsilon^2 n_{n1} + \epsilon^3 n_{n2} + \epsilon^4 n_{n3} + \ldots, \\ v &= \epsilon^3 v_1 + \epsilon^4 v_2 + \epsilon^5 v_3 + \ldots, \\ w &= \epsilon^3 w_1 + \epsilon^4 w_2 + \epsilon^5 w_3 + \ldots, \\ \psi &= \epsilon^2 \psi_1 + \epsilon^3 \psi_2 + \epsilon^4 \psi_3 + \ldots, \\ \phi &= \epsilon^2 \phi_1 + \epsilon^3 \phi_2 + \epsilon^4 \phi_3 + \ldots. \end{aligned}\right\} \tag{6}$$

However, the independent variables are presented as [55, 59],

$$\left.\begin{aligned} \xi &= \epsilon(\kappa_1 x + \ell_1 y + \chi_1 z - c_1 t) + \epsilon^2 P_o\left(\eta, \tau\right) + \epsilon^3 P_1\left(\xi, \eta, \tau\right) + \ldots, \\ \eta &= \epsilon(\kappa_2 x + \ell_2 y + \chi_2 z + c_2 t) + \epsilon^2 Q_o\left(\xi, \tau\right) + \epsilon^3 Q_1\left(\xi, \eta, \tau\right) + \ldots, \\ \tau &= \epsilon^3 t. \end{aligned}\right\} \tag{7}$$

where $\xi$ $[\eta]$ denotes the trajectory of two solitary waves propagating respectively in different directions; $\overrightarrow{R}_1 = (\kappa_1 + \ell_1 + \chi_1)$ $[\overrightarrow{R}_2 = (\kappa_2 + \ell_2 + \chi_2)]$ at $P_o(\eta, \tau) = Q_o(\xi, \tau) = 0$. Furthermore, if two waves interact, their trajectories will change and accordingly $P_o(\eta, \tau) \neq 0$ and $Q_o(\xi, \tau) \neq 0$. Here, $c_1$ and $c_2$ are the unknown phase velocities of two IASWs (to be determined later). Before going into details, let us determine the angle $\delta$ between the two waves propagation directions, which can be calculated from $\cos\delta = (\kappa_1\kappa_2 + \ell_1\ell_2 + \chi_1\chi_2) / \left[(\kappa_1^2 + \ell_1^2 + \chi_1^2)(\kappa_2^2 + \ell_2^2 + \chi_2^2)\right]^{1/2}$, where $\kappa_1, \ell_1, \chi_1$ $(\kappa_2, \ell_2, \chi_2)$ are the directional cosines of the first (second) wave vector along the $x-$, $y-$, and $z-$axes, respectively.

Substituting Eqs. (6) and (7) into the basic equations; Eqs. (1)-(5) and collecting terms of the same powers of $\epsilon$. For the first-order perturbed quantities, we have

$$\left.\begin{aligned}
&\left(-c_1\tfrac{\partial}{\partial\xi}+c_2\tfrac{\partial}{\partial\eta}\right)n_{n1}+\alpha\left(-\chi_1\tfrac{\partial}{\partial\xi}+\chi_2\tfrac{\partial}{\partial\eta}\right)\psi_1=0,\\
&-\mu\left(\kappa_1\tfrac{\partial}{\partial\xi}+\kappa_2\tfrac{\partial}{\partial\eta}\right)\phi_1+\gamma\left(\kappa_1\tfrac{\partial}{\partial\xi}+\kappa_2\tfrac{\partial}{\partial\eta}\right)n_{n1}+\Omega w_1=0,\\
&-\mu\left(\ell_1\tfrac{\partial}{\partial\xi}+\ell_2\tfrac{\partial}{\partial\eta}\right)\phi_1+\gamma\left(\ell_1\tfrac{\partial}{\partial\xi}+\ell_2\tfrac{\partial}{\partial\eta}\right)n_{n1}-\Omega v_1=0,\\
&\left(-c_1\tfrac{\partial}{\partial\xi}+c_2\tfrac{\partial}{\partial\eta}\right)\psi_1-\mu\left(\chi_1\tfrac{\partial}{\partial\xi}+\chi_2\tfrac{\partial}{\partial\eta}\right)\phi_1+\gamma\left(\chi_1\tfrac{\partial}{\partial\xi}+\chi_2\tfrac{\partial}{\partial\eta}\right)n_{n1}=0,\\
&\left[\tfrac{1}{2}(q_1+1)+\beta\ \theta_i\right]\phi_1+n_{n1}=0,
\end{aligned}\right\} \tag{8}$$

where $\gamma=\frac{5}{3}\alpha^{-1/3}\mu\ \theta_n$. Solving Eq. (8), we obtain explicit expressions for these first-order perturbed quantities as,

$$\left.\begin{aligned}
\phi_1(\xi,\eta,\tau)&=\phi_{11}(\xi,\tau)+\phi_{12}(\eta,\tau),\\
n_{n1}(\xi,\eta,\tau)&=n_{n11}(\xi,\tau)+n_{n12}(\eta,\tau)\\
&=\alpha\mu\left(\tfrac{\chi_1^2}{T_1}\phi_{11}(\xi,\tau)+\tfrac{\chi_2^2}{T_2}\phi_{12}(\eta,\tau)\right),\\
v_1(\xi,\eta,\tau)&=v_{11}(\xi,\tau)+v_{12}(\eta,\tau)\\
&=\tfrac{\mu}{\Omega}\left(\tfrac{\ell_1c_1^2}{T_1}\tfrac{\partial\phi_{11}(\xi,\tau)}{\partial\xi}+\tfrac{\ell_2c_2^2}{T_2}\tfrac{\partial\phi_{12}(\eta,\tau)}{\partial\eta}\right),\\
w_1(\xi,\eta,\tau)&=w_{11}(\xi,\tau)+w_{12}(\eta,\tau)\\
&=-\tfrac{\mu}{\Omega}\left(\tfrac{\kappa_1c_1^2}{T_1}\tfrac{\partial\phi_{11}(\xi,\tau)}{\partial\xi}+\tfrac{\kappa_2c_2^2}{T_2}\tfrac{\partial\phi_{12}(\eta,\tau)}{\partial\eta}\right),\\
\psi_1(\xi,\eta,\tau)&=\psi_{11}(\xi,\tau)-\psi_{12}(\eta,\tau)\\
&=\mu\left(\tfrac{c_1\chi_1}{T_1}\phi_{11}(\xi,\tau)-\tfrac{c_2\chi_2}{T_2}\phi_{12}(\eta,\tau)\right),
\end{aligned}\right\} \tag{9}$$

with $T_1=\alpha\gamma\chi_1^2-c_1^2$ and $T_2=\alpha\gamma\chi_2^2-c_2^2$. Moreover, the phase velocities; $c_1$ and $c_2$ are determined to have the forms,

$$\left.\begin{aligned}
c_1&=\chi_1\sqrt{\alpha\left(\gamma+\tfrac{\mu}{\frac{1}{2}(q+1)+\beta\theta_i}\right)},\\
c_2&=\chi_2\sqrt{\alpha\left(\gamma+\tfrac{\mu}{\frac{1}{2}(q+1)+\beta\theta_i}\right)},
\end{aligned}\right\} \tag{10}$$

It is remarked here that the unknown functions $\phi_{11}(\xi,\tau)$ and $\phi_{12}(\eta,\tau)$ will be determined at higher orders of $\epsilon$. Therefore, at the next-order of $\epsilon$, we have a system of equations whose solutions are

$$\left.\begin{aligned}
\phi_2(\xi,\eta,\tau)&=\phi_{21}(\xi,\tau)+\phi_{22}(\eta,\tau),\\
n_{n2}(\xi,\eta,\tau)&=n_{n21}(\xi,\tau)+n_{n22}(\eta,\tau)=\alpha\mu\left(\frac{\chi_1^2}{T_1}\phi_{21}(\xi,\tau)+\frac{\chi_2^2}{T_2}\phi_{22}(\eta,\tau)\right),\\
v_2(\xi,\eta,\tau)&=v_{21}(\xi,\tau)+v_{22}(\eta,\tau)\\
&=\frac{\mu}{\Omega}\left[\frac{1}{\Omega}\left(\frac{\kappa_1c_1^3}{T_1}\frac{\partial^2\phi_{11}(\xi,\tau)}{\partial\xi^2}-\frac{\kappa_2c_2^3}{T_2}\frac{\partial^2\phi_{12}(\eta,\tau)}{\partial\eta^2}\right)+\frac{\ell_1c_1^2}{T_1}\frac{\partial\phi_{21}(\xi,\tau)}{\partial\xi}+\frac{\ell_2c_2^2}{T_2}\frac{\partial\phi_{22}(\eta,\tau)}{\partial\eta}\right],\\
w_2(\xi,\eta,\tau)&=w_{11}(\xi,\tau)+w_{12}(\eta,\tau)\\
&=\frac{\mu}{\Omega}\left[\frac{1}{\Omega}\left(\frac{\ell_1c_1^3}{T_1}\frac{\partial^2\phi_{11}(\xi,\tau)}{\partial\xi^2}-\frac{\ell_2c_2^3}{T_2}\frac{\partial^2\phi_{12}(\eta,\tau)}{\partial\eta^2}\right)-\left(\frac{\kappa_1c_1^2}{T_1}\frac{\partial\phi_{21}(\xi,\tau)}{\partial\xi}+\frac{\kappa_2c_2^2}{T_2}\frac{\partial\phi_{22}(\eta,\tau)}{\partial\eta}\right)\right],\\
\psi_2(\xi,\eta,\tau)&=\psi_{21}(\xi,\tau)+\psi_{22}(\eta,\tau)=\mu\left(\frac{c_1\chi_1}{T_1}\phi_{21}(\xi,\tau)+\frac{c_2\chi_2}{T_2}\phi_{22}(\eta,\tau)\right),
\end{aligned}\right\} \tag{11}$$

Going further to the next higher-order in perturbation theory, we obtain

$$\begin{aligned}
-2(c_1\chi_2+c_2\chi_1)\psi_3=&\frac{2c_1\mu\chi_1^2}{T_1}\int\left[\frac{\partial\phi_{11}}{\partial\tau}+A_1\phi_{11}\frac{\partial\phi_{11}}{\partial\xi}+B_1\frac{\partial^3\phi_1}{\partial\xi^3}\right]d\eta+\frac{2c_2\mu\chi_2^2}{T_2}\int\left[\frac{\partial\phi_{12}}{\partial\tau}-A_2\phi_{12}\frac{\partial\phi_{12}}{\partial\eta}-B_2\frac{\partial^3\phi_{12}}{\partial\eta^3}\right]d\xi\\
&+\int\int\left[\left(C_1\frac{\partial P_o}{\partial\eta}+D_1\phi_{12}\right)\frac{\partial^2\phi_{11}}{\partial\xi^2}+\left(C_2\frac{\partial Q_o}{\partial\xi}+D_2\phi_{11}\right)\frac{\partial^2\phi_{12}}{\partial\eta^2}\right]d\eta d\xi,
\end{aligned} \tag{12}$$

where

$$\left.\begin{array}{l}
A_1=\frac{T_1}{2c_1\mu}\left[2a+\left(\frac{\mu\chi_1}{T_1}\right)^2\left(3c_1^2-\frac{1}{3}\alpha\gamma\chi_1^2\right)\right],\\
A_2=\frac{T_2}{2c_2\mu}\left[2a+\left(\frac{\mu\chi_2}{T_2}\right)^2\left(3c_2^2-\frac{1}{3}\alpha\gamma\chi_2^2\right)\right],\\
B_1=\frac{1}{2c_1}\left[-\frac{T_1}{\frac{1}{2}(q+1)+\beta\theta_i}+\left(\frac{c_1^2}{\chi_1\Omega}\right)^2\left(\ell_1^2+\kappa_1^2\right)\right],\\
B_2=\frac{1}{2c_2}\left[-\frac{T_2}{\frac{1}{2}(q+1)+\beta\theta_i}+\left(\frac{c_2^2}{\chi_2\Omega}\right)^2\left(\ell_2^2+\kappa_2^2\right)\right],\\
C_1=\frac{4\mu\chi_1\chi_2c_1^2}{T_1},\\
C_2=-\frac{4\mu\chi_1\chi_2c_2^2}{T_2},\\
D_1=\chi_1^2\left[2a+\frac{\mu^2\chi_2}{T_1T_2}\left(\chi_2c_1^2-\chi_1F\right)\right],\\
D_2=-\chi_2^2\left[2a+\frac{\mu^2\chi_1}{T_1T_2}\left(\chi_1c_2^2-\chi_2F\right)\right],\\
a=\frac{\mu}{4(q+1+2\beta\theta_i)}\left[(q+1)(3-q)-4\beta\,\theta_i^2\right],\\
F=2c_1c_2+\frac{1}{3}\alpha\gamma\chi_1\chi_2.
\end{array}\right\}\qquad(13)$$

The first- (second-) term on the right hand side of Eq. (12) is proportional to $\eta$ ($\xi$) because the integrated function is independent of $\eta$ ($\xi$) . Thus, the first two terms of Eq.(12) are secular terms, which must be eliminated to avoid spurious resonances [50, 51]. Hence we have

$$\left.\begin{array}{l}
\frac{\partial\phi_{11}}{\partial\tau}+A_1\phi_{11}\frac{\partial\phi_{11}}{\partial\xi}+B_1\frac{\partial^3\phi_{11}}{\partial\xi^3}=0\\
\frac{\partial\phi_{12}}{\partial\tau}-A_2\phi_{12}\frac{\partial\phi_{12}}{\partial\xi}-B_2\frac{\partial^3\phi_{12}}{\partial\xi^3}=0
\end{array}\right\}\qquad(14)$$

Equation (14) represents two–side travelling Korteweg de Vries (KdV) wave equations in the reference frames of $\xi$ and $\eta$, respectively, where their solutions are

$$\left.\begin{array}{l}
\phi_{11}(\xi,\tau)=\phi_A\,\mathrm{sech}^2\left[\sqrt{\frac{A_1\phi_A}{12B_1}}\left(\xi-\frac{1}{3}A_1\phi_A\tau\right)\right],\\
\phi_{12}(\eta,\tau)=\phi_B\,\mathrm{sech}^2\left[\sqrt{\frac{A_2\phi_B}{12B_2}}\left(\eta+\frac{1}{3}A_2\phi_B\tau\right)\right],
\end{array}\right\}\qquad(15)$$

$\phi_A=3U_A/A_1$ and $\phi_B=3U_B/A_2$ are the amplitudes of the two solitons $S_1$ and $S_2$ in their initial positions. $U_A\,(U_B)$ is the initial velocity of soliton $S_1\,(S_2)$.

It is clear that the third-and fourth- terms in Eq. (12) are not secular terms in this order, but they would generate secular behaviours in the next orders [50, 51]. Therefore, they must vanish to control the equations of the phase shifts

$$\left.\begin{array}{l}
\frac{\partial P_o}{\partial\eta}=-\frac{D_1}{C_1}\phi_{12},\\
\frac{\partial Q_o}{\partial\xi}=-\frac{D_2}{C_2}\phi_{11}.
\end{array}\right\}\qquad(16)$$

Hence, up to $O\left(\epsilon^2\right)$, the trajectories of the two IASWs, for weak oblique collision, are,

$$\left.\begin{array}{l}
\xi=\epsilon\left(\kappa_1x+\ell_1y+\chi_1z-c_1t\right)-\epsilon^2\frac{D_1}{C_1}\sqrt{\frac{12B_2\phi_B}{A_2}}\left\{\tanh\left[\sqrt{\frac{A_2\phi_B}{12B_2}}\left(\eta+\frac{1}{3}A_2\phi_B\tau\right)\right]+1\right\}+...,\\
\eta=\epsilon(\kappa_2x+\ell_2y+\chi_2z-c_2t)-\epsilon^2\frac{D_2}{C_2}\sqrt{\frac{12B_1\phi_A}{A_1}}\left\{\tanh\left[\sqrt{\frac{A_1\phi_A}{12B_1}}\left(\xi-\frac{1}{3}A_1\phi_A\tau\right)\right]-1\right\}+...
\end{array}\right\}\qquad(17)$$

Now, to obtain the actual phase shifts after the oblique collision of the two solitons, we suppose that the two solitons $S_1$ and $S_2$ are far from each other at an initial time ($\tau=-\infty$), i.e., soliton $S_1$ is at $\xi=0$, $\eta=-\infty$ and the other soliton, $S_2$ is at $\eta=0$, $\xi=+\infty$, respectively. After a collision ($\tau=+\infty$), the soliton $S_1$ is propagating to the right of the second soliton $S_2$ , i.e., soliton $S_1$ becomes at $\xi=0$, $\eta=+\infty$ and $S_2$ is at $\eta=0$, $\xi=-\infty$. Using Eqs. (15)-(17), we can calculate the phase shift changes; $\Delta P_o$ and $\Delta Q_o$, [50, 51] as,

$$\left.\begin{array}{l}
\Delta P_o=-\frac{2\epsilon^2D_1}{C_1}\sqrt{\frac{12\phi_BB_2}{A_2}},\\
\Delta Q_o=\frac{2\epsilon^2D_2}{C_2}\sqrt{\frac{12\phi_AB_1}{A_1}}.
\end{array}\right\}\qquad(18)$$

## 3. Discussions and conclusion

In this section, we present a number of numerical illustrations to show the dependence of the calculated collision phase shift on the plasma parameter variations. The selected parameters values are inspired by the recorded recent experimental data of multicomponent plasma experiments [21–24, 28], though we focus on the case of heavier negative ion magnetized multicomponent plasma $Ar^+$-$SF_6^-$ and $Xe^+$-$SF_6^-$ ($\mu<1$). The selected physical parameters are taken as $\epsilon=0.1$, $\mu=0.1$, $\theta_n=0.02$, $\theta_i=10$, $\beta=1.5$, $\Omega=0.4$ and $\delta=\pi/2.55$ (with $\kappa_1=\ell_1=\chi_1=-\kappa_2=\ell_2=\chi_2=1/\sqrt{3}$). Any changes in these parameters will be stated in the figure caption. First, let us examine the polarity of the IASWs. Since

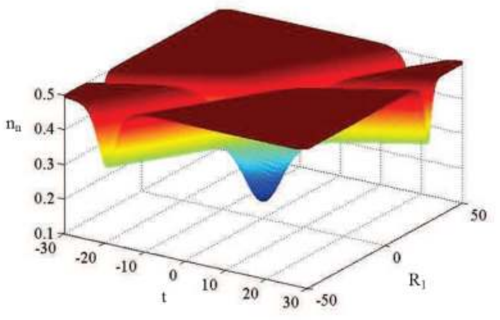


**Figure 1.** The colliding process of two IASWs. The solitons have negative polarity and are rarefactive in negative ion number density, $n_n$.

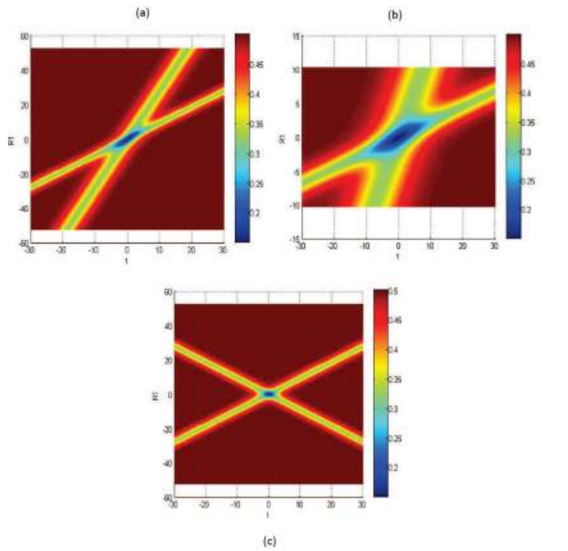


**Figure 2.** Space-time plots of two colliding IASWs for (a) $\delta = \pi/2.55$ (with $\kappa_1 = \ell_1 = \chi_1 = -\kappa_2 = \ell_2 = \chi_2 = 1/\sqrt{3}$) (b) $\delta = \pi/2$ (with $\kappa_1 = -\kappa_2 = 1$, $\ell_1 = \chi_1 = \ell_2 = \chi_2 = 1/\sqrt{2}$), and (d) $\delta = \pi$ (with $\kappa_1 = \ell_1 = \chi_1 = -\kappa_2 = -\ell_2 = -\chi_2 = 1/\sqrt{3}$).

$B_1$and $B_2$ are always positive, the IASWs are compressive if $A_1$ and $A_2 > 0$ and rarefactive if $A_1$ and $A_2 < 0$. For the case at hand, it is found that both $A_1$ and $A_2$ are always negative. Therefore, in the system under consideration, there are only rarefactive IASWs. Figure 1 shows the oblique collision of two nonlinear IASWs which results as rarefaction of negative ion number density. It is shown that when two IASWs obliquely collide, a new nonlinear wave is formed during their collision (i.e., blue region) which moves ahead of the colliding IASWs; both its amplitude and width are larger than those of colliding IASWs, as depicted in Fig. 1. Owing to the formation of this new nonlinear wave, the IASWs after the oblique collision are delayed. Thus, the phase shift depends directly on a new formed nonlinear wave structure. It is remarked that the phase shifts within the range $0 < \delta < \pi/2$ are larger and noticeable than that of $\pi/2 < \delta < \pi$. Space-time contour plots of two colliding IASWs are presented in Fig 2 for different angles $\delta$. They show that increasing $\delta$ results in increasing the width of the produced IASW at the point of collision for $0 < \delta < \pi/2$. However, the opposite response is occurred against increasing $\delta$ for $\pi/2 < \delta < \pi$. These features can be recognized by comparing the blue region in the center of each panel (the region where collision occurs; a new wave is created) with other panels. We note that, during the oblique collision an essentially motionless composite structure is created for some time. Figure 3 show contour plots of $\Delta Q_o$ variations in $\mu - \beta$ plan in panel (a), in $q - \theta_n$ plan in panel (b) and in panel (c) in $\Omega - \alpha$ plan. It reveals that $\Delta Q_o$ increases as either $\mu$, $\beta$ or $q$ increases, though it decreases as $\theta_n$, $\alpha$ or $\Omega$ increases. In other words, introducing either heavier or hotter negative ions results in a smaller phase shift. Moreover, increasing the number density of negative ion species results in a decline of phase shifts. The influence of stronger magnetic field is to reduce the phase shift. On contrary, including more nonextensive electrons increases the collision phase shifts.

To conclude, we have presented a study of the oblique collision of two nonlinear IASWs in a hot magnetized multicomponent plasma consisting of heavy negative ions fluid, positive ions and nonextensive electrons. The present model supports rarefactive solitons only. The phase shifts and the trajectories describing the collision of two IASWs are calculated using the EPLK method. The analytical findings are numerically investigated revealing that the magnitude of the phase shift depends directly on the electron nonextensive parameter. However, it is revisal proportional to their number density and mass and the strength of the applied magnetic field.

Finally, it may be pointed out that the present results are very useful in explaining the oblique collision of IASWs waves in multicomponent plasma experiments with nonextensive electrons. The proposed theoretical model would be applied to other astrophysical situations where nonextensive electrons are present by appropriate choices of the physical parameter numerical values.

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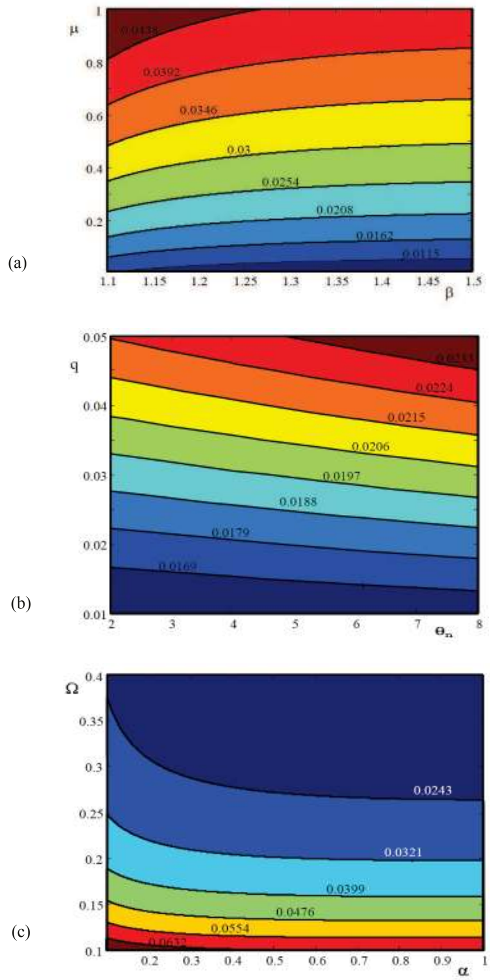


**Figure 3.** The variation of $\Delta Q_o$ are plotted for $\delta = \pi/2.55$ (with $\kappa_1 = \ell_1 = \chi_1 = -\kappa_2 = \ell_2 = \chi_2 = 1/\sqrt{3}$) and for different plans; in (a) for $\mu - \beta$ plan, with $q = 2$, in (b) in the $q - \theta_n$ plan, and in (c) in the $\Omega - \alpha$ plan. The number appeared besides each contour indicates the value of the corresponding phase shift $\Delta Q$.

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