The Company that You Keep:

When to Buy a Competitor's Keyword

by

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Dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
in the Department of Business Administration
in the Graduate School of Duke University

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<u>ABSTRACT</u>

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Abstract

Search advertising refers to the practice where advertisers place their text-based advertisement on the search engine's result page along with the organic search results. With its growing importance, search advertising has seen a recent surge in academic interest. However, the literature has been ignoring some practical yet important problems of advertisers, including the keyword selection problem. In my dissertation, I focus on the keyword selection problem, more specifically, the choice of branded keywords in search advertising.

My dissertation begins with an observation on different patterns of branded keyword purchase behavior by the brand owner and its competitor. Under some branded keywords, we observe in the sponsored link, only the brand owner or only the competitor. However, under some other branded keywords, we observe both firms, or neither of them. I aim to understand what drives this puzzling pattern in a competitive environment. To this purpose, I develop a duopoly model where two firms compete in the product market with both horizontally and vertically differentiated products. Their products are evaluated by consumers whose perception is affected by what they see in search advertising. With this setup, Then I derive a subgame perfect equilibrium of the two stage game.

In a pricing equilibrium, I find that any benefit a firm gets from search advertising either from an exposure benefit or from a contrast or assimilation effect, helps this firm charge a higher price while forcing the other firm charge a lower price. This result affects the incentive for each firm to buy the branded keyword in the advertising stage. First, firms have an incentive to buy the keyword when the cost of advertising is less than the exposure benefit accrued the advertisement. Moreover, if the quality difference between the brand owner (i.e., the firm associated with the keyword) and the competitor is large and thus there exists a contrast between the two firms, the competitor with low quality product refrains from buying the keyword, because the contrast effect hurts the competitor. On the other hand, if the quality difference is small and thus two brands are assimilated, the brand owner with high quality product refuses to buy the keyword, because it is hurt by the assimilation effect. If the quality difference is in the intermediate range so that neither context effect is harmful to either firm, both firms buy the keyword at the same time. On probing further the underlying incentives, I find that in some cases, the brand owner may buy its own keyword only to defend itself from the competitor's threat. In contrast, I also identify the case where the brand owner chooses to buy its own keyword and precludes the competitor from buying it. My result also suggests that both firms may be worse off by engaging in advertising, as in the prisoner's dilemma case.

In an extension, I provide an analysis on the impact of the insufficient advertising budget. If the budget is limited, both firms may have an incentive to hurt the other firm taking the higher slot, by increasing the bid amount and thus quickly exhausting the competitor's budget. The budget constraint can also deprive the advertisers of the incentive to buy the keyword and thus, the budget-constrained advertisers may refuse to match the competitor's purchase of the keyword.

Finally, the experimental investigation based on survey data shows the existence of the exposure effect and the context effects. Another study based on secondary data that compares the use (nonuse) of keyword advertising in a number of hybrid car manufacturers also supports the model prediction.

Dedication

To my dearest wife Sangmi, to whom I owe everything.

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1. Introduction

1.1 An Overview of Search Advertising Market

Search advertising refers to the practice where advertisers place their text-based advertisement on the search engine's result page along with the organic search results. Starting in the late 1990s, search advertising speeded up its growth by means of numerous developments in its mechanism. Now, it has grown to be one of the most crowded advertising media. According to *IAB Internet Advertising Revenue Report*, search advertising revenues totaled $3.8 billion in 2004, $5.1 billion in 2005, $6.8 billion in 2006, $8.8 billion in 2007, and $10.5 billion in 2008. Even in 2009, when advertisers cut their advertising budgets due to economic downturn, search advertising revenues in the first half were reported to be $5.1 billion, while the overall online advertising spending has decreased by 5.3 percent (IAB 2010). The same report has shown that search advertising accounted for 47 percent of overall online advertising spending. This means that search advertising has become a major means of advertising and that many advertisers are now switching from other advertising media to search advertising.

Then how does search advertising work? For a certain keyword, there can be multiple advertising slots on the right-hand side of the search engine result page. The search engine's problem is to allocate these slots to advertisers who wish to buy the slot. What the search engines use in reality is a position auction where it ranks the advertisers, by some function taking into account their bid amount as well as their

relevance to the keyword. The relevance here is known to be a function of a click-through rate, a landing page quality, and some other factors, although no search engine has ever revealed the exact relevance formula. In this position auction, each advertiser in each slot is charged whenever they get a click from a search engine user. This is often called the cost-per-click scheme. Here, the per-click payment is determined by the result of an auction. In particular, in Google and many other search engines, a winner of a slot gets charged the minimum amount required to get placed at the current rank, which in fact is based on the next slot winner's bid amount and thus is called a generalized second-price auction rule. In addition to these rules, there are other mechanisms that makes search advertising more attractive to advertisers, including the local search option (where advertisers can choose to specify the geographical region to show their advertisement), daily budget stipulation (where advertisers can specify the maximum advertising spending per day), and cost-per-impression bidding (where brand advertisers can choose to bid on basis of a thousand impression, instead of a click).

Because search advertising involves a complicated position auction, there have been a significant amount of explorations on the development of a mechanism. For example, topics including the first-price auction vs. the second-price auction rule (Edelman and Ostrovsky 2007), the ranking mechanism (Balachander and Kanna 2007), and the payment scheme (Dellarocas and Viswanathan 2008), have been investigated. In addition, due to its significance to advertising and to marketing, a growing number of

researchers have focused on the advertiser side of the phenomenon. This includes the performance measurement of a campagin (Rutz and Bucklin 2007b) and the optimal bidding strategy (Ghose and Yang 2009). However, there remain lots of advertiser-relevant problems to be investigated in search advertising. One of the important problems is the keyword selection. Because the choice of the keywords determines the audience of the advertising campaign, the keyword selection is essential problem in terms of targeting. Also, if a firm chooses a set of keywords that it is less relevant to, it will suffer from high cost of advertising and thus, the keyword choice directly affects the profitability of the campaign. Hence this paper deals with the keyword selection problem of advertisers. More specifically, it investigates the choice of a branded keyword, which will become clear with the following examples.

1.2 Introduction

Type Gucci in Google's search box. You will see Gucci taking the first slot of both sponsored links and organic links. Now, type Dior. You will find only the organic link but not the sponsored link of Dior. What about Camry? Toyota comes out in the sponsored section as well as in the organic section. However, you will also observe some of its competitors listed in the sponsored section: Chevy Malibu and Nissan Altima. Finally, try Harvard MBA. Do you see Harvard Business School in the sponsored link? No. Only its competitors show up there. (See Figure 1 for screen captures. We have observed each example consistently showing the same pattern for more than a year.)
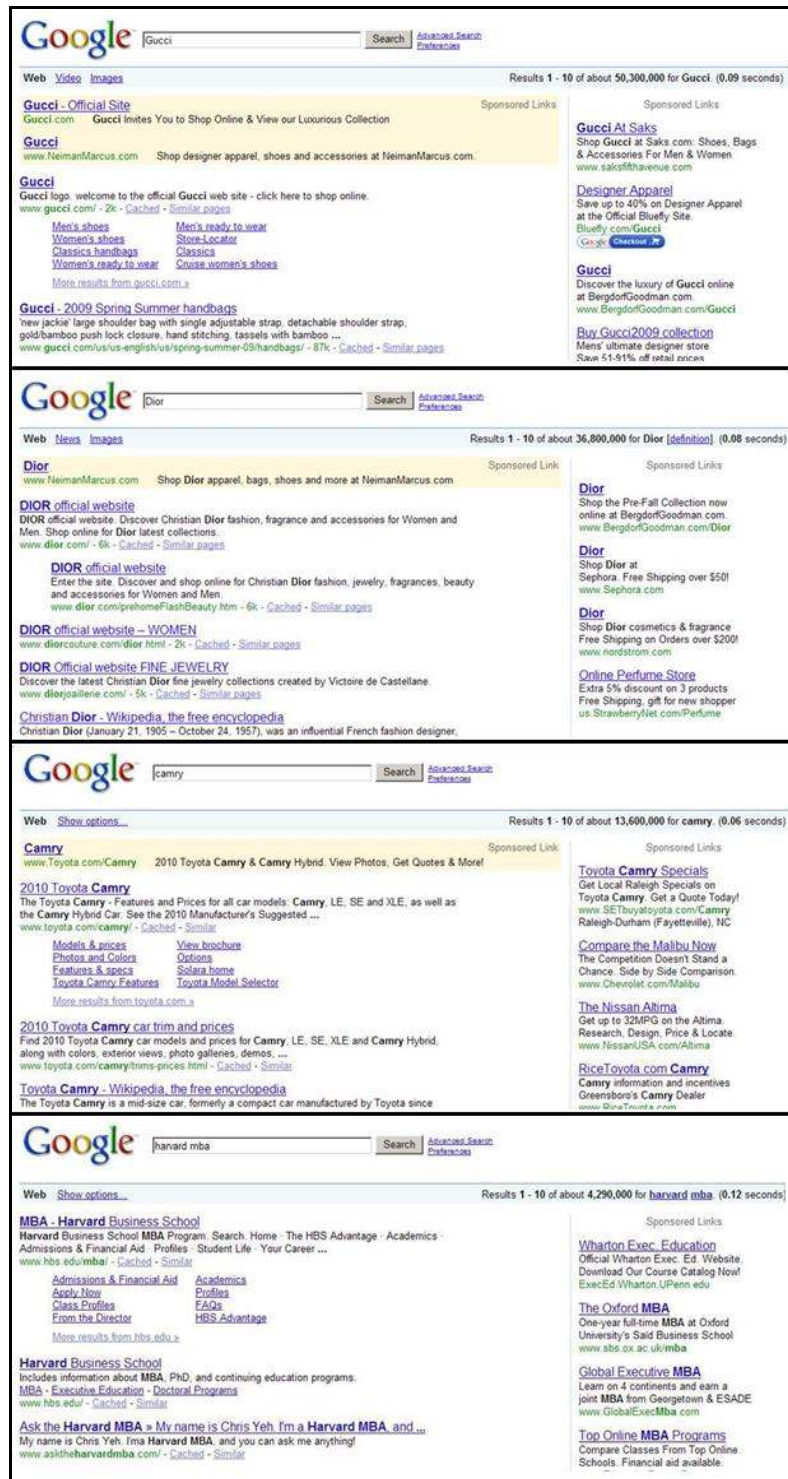
Figure 1: Examples of Branded Keywords

Why does Gucci buy its own name, while Dior does not? Why do Toyota's competitors advertise under Toyota's brand Camry, while Gucci's or Dior's competitors do not advertise under Gucci or Dior? Why does Harvard let its competitors make use of its own brand name? In this paper, we attempt to understand why we observe different patterns of branded keyword purchase behavior. In particular, we investigate the purchase decision of the brand owner (i.e., the firm associated with the keyword) and the competitors as well as their underlying incentives for the purchase or no purchase decision. Further, we aim to provide a normative guideline in the selection of the branded keyword.

Branded keywords have been proved to be effective in increasing the click-through rate (Rutz and Bucklin 2007), and many advertisers use the branded keyword to collect more relevant clicks to their sites. However, increasing the number of clicks is not the only objective for the use of a branded keyword. An advertiser may also be concerned with the brand awareness or the brand's perceived quality. Thus, in this paper, we focus on the understudied yet important aspect of the keyword search advertising: brand advertising, and investigate how it affects the strategic decision of advertisers.

In this regard, we ask the following three questions: (1) When do we observe one purchase pattern over another? (2) Why do brand owners buy their own branded

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keyword? (3) Why do some firms forgo the opportunity to advertise their less known brand names under the competitor's well-known brand name?

To answer these questions, we develop a model of duopoly where two firms compete with each other in a product market as well as in the search advertising market. Both offer a vertically and horizontally differentiated product, and the brand owner provides the higher quality product.[1] The two firms also engage in search advertising to promote their brands. Specifically, we describe the benefit of advertising using two well-documented phenomena: the exposure effect and the context effect. The exposure effect captures an increase in the brand awareness or the brand's perceived quality through exposure of the brand's advertisement. The context effect refers to a positive or negative bias associated with consumers seeing the two brand names simultaneously. Specifically, consumers perceive brands that differ substantially in quality to be more different than they actually are and similar brands to be more similar than they actually are.

Based on these two effects, our model answers the first question by providing an integrated framework to define the condition for each pattern of purchase decision: (1) both firms buying (YY), (2) only the brand owner buying (YN), (3) only the competitor buying (NY), and (4) neither buying (NN). First, if the exposure effect is small, neither

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[1] Our model generalizes to the situation where the brand owner has the lower quality product. We discuss this issue later.

firm will buy, because they cannot justify the cost of advertising. If the exposure effect is above a certain threshold, at least one of them will buy. What matters now is the context effect because firms consider how they are viewed by the consumer when they are compared with their competitor. If there is a large contrast effect, only the brand owner buys the keyword because the competitor with an inferior product would be hurt by the contrast effect. If there is a large assimilation effect, only the competitor is observed in equilibrium because now the brand owner would be hurt by the assimilation effect. If neither effect is large, both firms choose to advertise together because neither firm finds the effect of comparison too detrimental.

We also examine the strategic concerns of both firms in order to provide a deeper understanding of their incentives. First, we find that in some cases, the brand owner may purchase its own branded keyword solely to defend itself from the competitor's encroachment. This happens when the exposure effect is somewhat weak and neither the contrast nor assimilation effect is large. Because of the weak exposure, the brand owner would not buy the keyword if its competitor does not buy. However, if the competitor buys the keyword, the brand owner has an incentive to buy its own keyword because the brand owner is negatively affected in the product market by letting the competitor advertise alone. This negative impact can be reduced if the brand owner also purchases the keyword. Note however, the brand owner does not succeed in driving out its competitor.

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Our result also identifies the case where by purchasing its own keyword, the brand owner succeeds in effectively precluding the competitor from buying the owner's branded keyword. We observe this pattern when both the exposure effect and the contrast effect are large. In this case, though the competitor would prefer to advertise because of the large exposure effect, the brand owner discourages this attempt by purchasing its own keyword. This exclusion is possible because the large contrast effect makes the brand owner's threat credible and at the same time, it deprives of the competitor the incentive to match the brand owner's purchase decision. When the assimilation effect is large, we obtain the opposite result. The competitor may discourage the brand owner's attempt to utilize its own brand name. Even though the brand owner would like to advertise without the competitor's advertisement, it would withdraw in the presence of the competitor, in fear of the loss due to assimilation.

Two firms, when purchasing the branded keyword, may expect to see an increase in their profits. However, they are not always better off. Instead, for some intermediate values of the context effect (i.e., low contrast or low assimilation), we observe a prisoner's dilemma case where both firms are worse off by engaging in search advertising. This is because when the comparative benefit (of a contrast or an assimilation) is not large, the direct benefit dictates the profitability, but if both of them expect to get a direct benefit by advertising together, these direct benefits are canceled out by each other.

Finally, we investigate how the advertising incentives change when there is an advertising budget constraint. For this, we first characterize the strategic interaction in the bidding game. We start with the observation that in the context of the generalized second-price auction format, the second slot winner can hurt the first slot winner by increasing the advertising cost of the first slot winner. This increased cost coupled with the first firm's budget constraint can result in the first firm not always appearing in the sponsored link. This allows the second firm to appear in the first slot at times. This in turn benefits the second firm in the product market competition. This interesting interaction between the two competitors also affects the participation decisions of both firms. In particular, the firm which could take the first slot in the bidding game, may refuse to buy the keyword at all under some conditions.

Our theoretical model is based on two well-established effects. In addition, these effects were subjected to empirical investigation in the context of search advertising. We conducted two experiments where we exposed participants to a search engine result page with different sets of brands (brand owner only, competitor only, both, or neither) in the sponsored link section of the page, and collected the consumers' quality perceptions of each brand. We also varied the quality difference between the brand owner and the competitor by using two different competitor brand names in one study. We confirmed an existence of the exposure effect and the context effects - the contrast effect and the assimilation effect - and showed that our model assumption is well-

grounded. In addition, we found that our theoretical prediction on the keyword

purchase pattern is consistent with the empirical observations based on the experimental

result.

Finally, we tested the qualitative prediction of the model. In particular, we see

whether the quality difference between the brand owner and the competitor can

moderate the participation decision of both firms. By investigating the link between the

predicted purchase pattern together with the empirical purchase pattern in a product

category (hybrid sedans), we find that as predicted in our theory, as the quality

difference gets larger, the purchase pattern generally changes from NY to YY, and to

YN, if the brand owner has higher quality. While our data does not show a perfect

match between the theory and the empirical observation, we can at least conclude that

the general pattern follows our model prediction.

The rest of the dissertation is organized as follows. In the next section we review

some related literature. Then we develop our theoretical model and analyze it, assuming

firms have unlimited advertising budget. In an extension, we show how the bidding

incentive and the advertising incentive change when the advertising budget is

constrained. Finally, we provide an experimental investigation of our assumption on the

exposure effect and the context effect, and show how our model prediction matches the

empirical observation. Then we discusses our findings and opportunities for future

research.

2. Related Literature

Search advertising is a form of advertising in a new medium. Thus we start this section with a brief review of the advertising literature. Then we review the research on this new advertising medium, i.e., search advertising, by defining three sub-areas: position auction mechanism, advertiser strategy, and consumer search with search advertising. As we will discuss, our work falls on the second area, because it deals with advertiser's strategy of keyword choice, especially on branded keywords. After showing how our work fits the big picture, we will briefly discuss other related literature: (1) the contrast and assimilation effect literature, as we use these effects as one of main drivers of our results, and (2) the raising rivals' cost literature, as our model extension is an application of this idea.

2.1 Advertising Research

While advertising literature is huge, here we focus on three sub-areas that are most relevant to our research: effect of advertising exposure, competitive advertising, and comparative advertising.

2.1.1 Effect of Advertising Exposure

The literature on the effect of advertising expousre traces back to Zajonc (1968)'s research on mere exposure effect. In this seminal monograph, he showed that mere exposure to an unfamiliar stimulus such as nonsense words or symbols can lead to positive attitude change. This implies that repeated exposure to an advertisement will

increase the liking of the brand. Thus the mere exposure has been applied to the study of the effect of advertising exposure. For example, Winter (1973) showed that advertising exposure has a favorable effect on individual brand attitude change and identified the brand familiarity and the prior attitude towards the brand as the moderating factors in this change. In particular, he showed people with low brand familiarity or unfavorable prior attitude experience large attitude change by advertising exposure. Both Obermiller (1985) and Janiszewski (1993) attempted to uncover the underlying mechanism of the mere exposure effect. Obermiller (1985) found that neither the noncognitive mediation nor the uncertainty reduction can fully explain why repeated exposure increases the liking. However, he showed that different processing styles (e.g., an advertisement processed for product information, processed for the aesthestic characteristics of picture, or not processed consciously at all) influence evaluation of stimuli differently, which calls for advertiser's attention to the situation where consumers receive advertising message. Janiszewski (1993) suggested a preattentive processing where consumers use the hemispheric ressources in storing and accessing an incidentally viewed brand name. This research implies that even incidental mere exposures can have an impact on consumer's attitude change and that attention is not necessary in this process.

In addition to the mere exposure effect, others have investigated how consumers react to the advertiisng exposure by processing information from the advertisement. While the mere exposure effect literature focuses on the attitudinal change by

advertising exposure, this literature pushes the impact of advertising exposure further to the brand choice. Among others, Nedungadi et al. (1993) offered a framework to understand the link between the advertising exposure and the brand choice, by analyzing the consumer's choice process. In this process, they emphasized the role of memory. Baker (1993) proposed the relevance-accessibility model (RAM) to study the effect of advertising on brand choice. The RAM basically says that an advetising message is effective when it is both relevant and accessible. By offering three choice processes (optimizing, satisficing, and indifference) and three levels of information (relative performance information, quality cues, and affect) that consumers use in their brand choice, the model predicts that an advertising message is most influential when the choice objective is the most relevant and when the message matches the level of information the conusumer seek for at the time of brand choice. This prediction has also been empirically tested (Baker and Lutz 2000).

In our paper, we model the exposure effect of advertising. In doing so, we do not distinguish the mere exposure effect from the exposure effect due to information proccessing. In fact, our exposure effect includes both types: the search engine users may revise their quality perception whenever they see the sponsored link of a brand (mere exposure effect), but also when they read the advertising description in the sponsored link (exposure effect due to information processing).

2.1.2 Competitive Advertising Strategy

There also has been theoretical literature that deals with the strategic advertising decision of firms in a competitive environment. Here the effectiveness of advertising is usually modeled by some parameters which increase the market share or consumer's evaluation, or decrease the price elasticity, and its impact is investigated in a competitive setting. For example, Iyer et al. (2005) studied the impact of targeted advertising. In particular, they found that if firms choose to target their advertising more to loyals but less to comparison shoppers, they can increase differentiation between them and thus make more profits. In our setting, the keyword choice itself can be seen as the targeting of advertising, because the keywords detemine the audience of a search advertising campaign. Thus the problem of whether to buy own vs. competitor's keywords is equivalent to whether to target own vs. competitor's loyals in advertising. Although our model just focuses on these two loyal segments, the comparison shoppers can also be considered by the choice of buying generic keywords.

Chen at al. (2009) investiaged the consequence of combative advertising where firms attempt to shift consumer preference towards themselves. As a result of this type of advertising, firm profits will increase. However, they also found that firm profits may decrease, because advertising competition leads to price competition. This latter result is similar to what we obtain in our model, where we show that advertising firm can

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increase its own prices and profits unless the comparison of two brands have negative impact on the quality assessment on the advertising firm.

As in these two papers, our work also investigates advertising competition by modeling the effectiveness of advertising in the competitive environment. However, unlike these papers, we base our discussion of the effectiveness of advertising on our own experimental results where we actually observed the changes in the consumer's quality perception due to advertising. In this sense, we are integrating the approaches of the two streams of research (one investigating consumer's reaction to advertising, and the other focusing on firm strategies): we start from studying the consumer behavior by experiments and then investigate its implications on the firm strategies.

2.1.3 Comparative Advertising

The last stream of advertising research that is relevant to our work is the research on comparative advertising. While search advertising itself is not comparative advertising, it shares some similarities with comparative advertising: for example, two firms are shown together and thus compared with each other. However, there are some differences too. As Gorn and Weinberg (1984) have shown, comparative advertising increases brand similarity between the leader and the challenger. Thus, an inferior challenger can use this type of advertising in order to catch up with the leader. In contrast, in the search advertising setting, advertisers can aim to induce contrast as well

as assimilation with the competitor. Thus, both the leader and the challenger make their own decision on whether or not to run a search advertising campaign.

2.2 Search Advertising Research

In search advertising, we have seen three big areas of research: position auction mechanism design, advertiser strategy, and consumer search with search advertising.

2.2.1 Position Auction Mechanism

The first area starts with an equilibrium characterization: Edelman et al. (2007) and Varian (2007) characterized and offered solutions to the keyword search auction problem. In particular, Edelman et al. (2007) described the continuously dynamic bidding game as the one-shot static game of advertisers, with an assumption that the bidding process will converge to a resting point in the long run. In this setting, because advertisers gradually learn the valuation of other advertisers, one can derive a solution under complete information. They characterized a solution to this generalized second-price auction where every slot winner pays the bid of the next slot winner, and named it as the locally envy-free equilibrium. This solution implies that in equilibrium, no advertiser has an incentive to move up to the higher slot, because then, the upper slot winner retaliates by slightly underbidding and thus switching the order. Varian (2007) also investigated the same problem and developed a similar solution concept, called Symmetric Nash Equilibrium. In this equilibrium, no advertiser has an incentive to switch its position with any other advertiser. Both locally envy-free equilibria and

Symmetric Nash Equilibria share the same lower bound but only Symmetric

NashEequilibria have an upper bound. Finally, Lahaie (2006) also considered various

ways to model the bidding game among advertisers (complete vs. incomplete

information, first vs. second price auction) and offer solutions whenever they exist.

Based on the equilibrium derivation, there has been a series of papers

investigating the search engine's mechanism design. First, Balachander and Kannan

(2008), Desai and Shin (2006), and Lahaie (2006) dealt with the advertiser ranking

determination problem. There, the issue is whether to rank them by bid only or by the

product of bid and relevance. Balachander and Kannan (2008) and Desai and Shin (2006)

showed that contrary to our expectation, ranking by bid only may be more profitableto

the search engine in some cases. This happens when the relevance and valuation are

positively correlated. This is because under that condition, both scheme choose the

identical set of winners but in the second scheme (ranking by bid × relevance), the

winners pay less per click because of their high relevance. Second, there has been

theoretical research on the optimal minimum bids on the position auction (Desai and

Shin 2009, Liu et al. 2008). This is of particular interest, because the search engines'

automated system allows the search engine set up different minimum bids for different

advertisers. More specifically, both Liu et al. (2008), by using type-specific minimum

bids, and Desai and Shin (2009), by using advertisers-specific minimum bids, showed

that the search engine can increase its profits by using different minimum bids for

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different advertisers. Third, Dellarocas and Viswanathan (2008) compared different payment schemes in search advertising, including cost-per-impression and cost-per-action. They found that under some conditions, cost-per-action may lead to higher per-click price, lower adverage quality of advertisers, and lower profits for every stakeholder. In this regard, Feng and Xie (2009) also investigated the performance-based pricing scheme and its consequence on the consumer's inference on quality. In particular, they found that switching to performance-based scheme makes it harder for consumers to infer quality of advertisers. They also identified a case where both the search engine and advertiser can make more profits by use of performance-based pricing scheme. Finally, Yao and Mela (2009) investigated the profitability of different auction formats in a counterfactual experiment using their structural econometric model of the Search engine, advertisers, and search users.

2.2.2 Advertiser Strategy

The second area deals with advertiser strategy in search advertising. The most obvious question in this area is how to derive the optimal bidding strategy. While every model deriving an equilibrium solution involves the derivation of the optimal bid, Katona and Savary (2010) enriched the discussion on the optimal bid by addressing detailed consideration of advertisers in competition. Their model provides a potential explanation on the seemingly unpredictable patterns in search engine results: Advertisers with a concave valuation of the number of clicks do not always want to be

listed higher if they can collect enough clicks from the organic search result. The optimal bidding strategy has also been empirically derived in Ghose and Yang (2009), in their estimation and counterfactual.

Another essential problem of advertisers is to measure the performance of search advertising campaigns. Rutz and Bucklin (2007a) specified the problem in measuring performance: because the click-through or conversion data are sparse in their nature, it is hard to get a reliable estimates of the click-through rate and the conversion rate. They provided a method to overcome this problem. In addition, other issues that may be of interest of advertisers have been investigated: for example, Agarwal et al. (2008) investigated the performance of advertising campaign by the position and found that the click-through rate decreases with position but the conversion rate initially increases and then decreases. This implies that the most profit position in the search engine result page is not necessarily the top mostslot but somewhere in the middle. Yang and Ghose (2010) examined the relationship between the organic and the sponsored links and found that both types of links have positive interdependence with each other, in terms of the click-through rate. However, this interdependence is asymmetric in that organic links have stronger impact on the sponsored link than vice versa.

Finally, one important problem that has not been paid much attention to is the keyword selection problem. Regarding this problem, Rutz and Bucklin (2007a) investigated the spill over effect from generic to branded keyword and thus, suggests

that the effectiveness of generic keywords has been underestimated. This generates

some implications on the choice of branded vs. generic keyword. However, there is a

lack of research on which set of keywords to use in the search advertising campaign.

Our work fills this gap by providing a normative guideline on when to buy own brand

name as keyword and when to buy the competitor's brand name as keyword.

2.2.3 Consumer Search and Search Advertising

There also have been attempts to link the search advertising model to the

consumer search model (Athey and Ellison 2008, Chen and He 2008, Jerath et al. 2009).

By this type of model, they explain many different aspects of the keyword auction. For

example, Jerath et al. (2009) explains why firms with high quality product is listed below

the firms with low quality product, by investigating the firm incentives based on the

consumer search process. With consumers search from top to bottom, they found that

the high-qulaity firm can be at a lower position but still get some traffic, because

consumers may continue their search even after clicking on the low-quality firm at the

top slot. However, the low-quality firm does not have this chance and thus becomes

more desparate to get the top position. In contrast, Chen and He (2008) derived an

equilibrium where the listing order in the sponsored link contains some information

about the relevance (matching probability in their term) of a product and consumers use

this information in their search for the product. Lastly, although it does not fit in any of

above categories, Wilbur and Zhu (2009) investigated an important problem of the click-

fraud in the search advertising setting. They found that the saerch engine may be better off by allowing click-fraud if the search advertising market is not competitive and thus advertisers may not prefer to cut their budget for the risk of click-fraud.

2.2.4 Contribution

We contribute to the search advertising literature in three regards. First, we go beyond academia's focus on the click-generating role of search advertising, and examine an increasing interest of the industry, the brand-lifting role (Google and Media Screen 2008, Enquiro Research and Google 2007). We model the brand advertising aspect of search advertising, to our knowledge, for the first time. Second, in doing so, we investigate the link between the search advertising market and the product market. Although one can easily think that the advertising decision affects the product market outcome, the search advertising literature has generally ignored the link between the search advertising market and the product market competition.[1] We specifically consider the impact of advertising decision on the product market outcome such as the equilibrium price and sales, by jointly modeling the product market competition and the competition in search advertising. Finally, we focus on branded keywords. We believe

[1]The exception is Xu et al. (2009), which explicitly linked the pricing decision in the product market to the bidding decision in search advertising. Chen and He (2006) also consider product market competition by endogenizing prices but showed that firms set monopoly prices in equilibrium, following the logic of Diamond (1971). Thus, they failed to show the impact of the search advertising decision on the product market outcome. Similarly, other models of consumer search and search advertising (Athey and Ellison 2008, and Jerath et al. 2009) consider the product market but do not explicitly model the firm's reaction to the advertising market outcome in the product market by assuming exogenous profit of firms.

this is important because branded keywords will be used more in consumer search as more firms try to link search advertising with other media advertising (Online Publishers Association 2009). In addition, recent lawsuits between Google and several advertisers on the use of trademark reflects an increasing interest in the issue (Helft 2009, Orey 2009). By analyzing the effect of using one's own as well as the competitor's branded keywords, we offer insight to a practical problem in the field.

2.3 The Contrast and Assimilation Effect

We also base our research on the contrast and assimilation effect literature. Hovland et al. (1957) showed a contrast effect and an assimilation effect in a social communication context. They defined a latitude of acceptance, wihtin which individuals accept the communication and outside of which they reject that communication. This implies that assimilation occurs with less discrepancy from one's position but contrast occurs with greater discrepancy. In fact, they also stated that "whether assimilation or contrast effect occurs would be a function of the relative distance between [subject]'s own stand and the position of communication". Appyling this idea to the context of search advertising, when two firms are shown in the search engine reuslts page, they will be contrasted with each other if the relative distance (in terms of quality) between the two firms is large, while they will be assimilated if the relative distance is small. We use this idea in developing our model in the next section.

In a different context, Sherif et al. (1958) showed that anchoring stimuli affects judgments in the similar pattern as above. In their weight-lifting study, by varying anchors, they showed how the subjects' judgments were displaced in relation to these anchors. In particular, when the anchor was close, their judgments were displaced towards the anchor but when it was far, their judgments were displaced away from the anchor. This is in line with the aforementioned contrast and assimilation effect.

In marketing, the idea of the contrast and assimilation effect has been applied in the reference price research (Lichtenstein and Bearden 1989, Grewal et al. 1998) and in the context of product evaluation (Olshavsky and Miller 1972, Anderson 1973). For example, Anderson (1973) showed in his experiment where he manipulated the expectation of subjects on the product quality, that product ratings were assimilated toward expectations unless expectations were too large but if expectations were large, contrated away from expectations.

Recently, Stewart and Malaga (2009) showed an assimilation and a contrast of internet links of different levels of familiarity to consumers. In particular, they showed that consumers' trust on the unfamiliar web site may be higher if the link to this site is surrounded by links to the familiar web sites (assimilation effect), while consumer's trust on the familiar web site may be higher if the link to this site is surrounded by links to the unfamiliar web sites (contrast effect). Likewise, we also examine the contrast effect

and the assimilation effect on the internet links, especially in the context of search advertising.

2.4 Raising Rivals' Cost

The last stream of research that is relevant to our paper is the Raising Rivals' Cost (RRC) research. Salop and Scheffman (1983) first develop this concept and characterize the conditions for a profitable RRC strategy. The idea of the RRC strategy is to disadvantage the competitor by raising competitor's cost and to increase its own profit. Salop and Scheffman (1987) show various applications of cost-raising strategies, including overbuying supply materials and vertical integration. We apply this strategy to the search advertising context and show that competing firms may have an incentive to increase competitor's cost of advertising first by bidding on the competitor's keyword and by increasing its own bid amount in the context of the generalized second-price auction. The net result is that the focal firm is better off even though it had to spend more money, because the competitor is worse off and thus becomes a weaker competitor.

3. Model

In this section, we develop a model to investigate how firms make advertising decisions regarding branded keywords in search advertising, while at the same time competing in the product market. We start by describing the search advertising market. We then lay out the rules of the product market.

3.1 Search Advertising Market

Consider two firms competing in the product market by offering both vertically and horizontally differentiated products. These two firms are asymmetric in terms of their quality: One firm has a higher quality product than the other firm. We call the former Firm H and the latter Firm L. In order to make their products known to consumers and induce consumers to buy their own products, they choose to advertise in a search engine. Thus the benefit they seek from the search advertising is not just click-throughs but also the exposure itself. To highlight the impact of search advertising on the product market competition, we focus on the brand enhancing role of search advertising rather than the generation of clicks. This brand advertising aspect of search advertising is particularly suitable in the context of product market competitors who attempt to increase the value of their brands and build long-term reputations in the market (Lee 2009).

While firms can choose a variety of keywords, we focus on their purchase of a branded keyword. In particular, we investigate both firms' purchase decision of Firm

H's branded keyword. While Firm L's keyword can also be bought, our analysis can be

easily generalized to this case. Also, Firm H's branded keyword is useful in investigating

how a lower quality firm strategically appropriates the high-quality competitor's

established brand name, and how the high-quality competitor responds to this

competitive threat. This is somewhat analogous to the comparative advertising in the

traditional media, where the weaker brand (with low market share) attacks the stronger

brand (with high market share) (Batra et al. 1996). In most mature industries, it is

usually the case that the firm with high market share is likely to be one with the high

quality product, because this firm has been successful in the market by continuously

improving its quality, although there also exist examples of unknown or small firms

which have very high technologies. Thus in this paper, we will investigate the nature of

competition around the keyword of Firm H. Accordingly, ``the keyword'' always refers

to the branded keyword of Firm H from this point on. Further, we consider only one

keyword and thus abstract away from the issue of keyword portfolio decision.

The two firms in the search advertising market make a participation decision and

if participating, make a bid. Depending on their participation decision, there can arise

four different scenarios: (1) Only Firm H advertises, (2) Only Firm L advertises, (3) Both

Firm H and Firm L advertise, or (4) Neither firm advertises. We denote participation by

Y and no participation by N so that each scenario can be denoted by (1) YN, (2) NY, (3)

YY, or (4) NN. If only one of them participates, the participating firm takes the top slot

among all available advertising slots in the search engine result page. If both advertise,

they compete for the top slot in a position auction held by the search engine. Depending

on the result of the bidding game, either Firm H or Firm L can take the first slot. In

addition to the sponsored links, there are organic links in the search engine result page.

Under Firm H's branded keyword, Firm H always appears on the organic links at the

top slot but Firm L is never listed in that section.[1] Thus, Firm H is exposed to consumers

regardless of its purchase decision, while Firm L is shown only when it purchases Firm

H's branded keyword.

3.1.1 Effect of Brand Advertising

As a result of advertisement, each firm gets exposure to consumers, which in

turn increases consumers' quality perceptions of the advertised products. In particular,

whenever a firm appears in the sponsored link, it achieves a nonnegative increase in its

quality perception. This may be due to either mere exposure (Zajonc 1968) or the

information gain from the advertising text. We call this change in the perceived quality

the exposure effect and denote it by E_1 which can take any nonnegative value. One may

think of the possibility of the exposure effect being negative. However, search

advertising is not as intrusive as other means of advertising and thus we do not allow

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[1]We conducted searches on Google with more than 100 branded keywords and found no case of
Firm H's not appearing or Firm L's appearing in the organic links.

for this possibility in our model development, although our model can easily be

generalized to account for negative exposure effect.

In addition, when both firms are shown in the search engine result page, the two

brands are contrasted or assimilated so that their product qualities are perceived

differently than if they were seen separately. Specifically, when the quality difference

between the two products is greater than a threshold q^0, a contrast occurs and Firm H's

product is perceived to be of higher quality than its actual quality, while that of Firm L is

seen to be of lower quality. On the other hand, when the quality difference is less than a

threshold q^0, an assimilation occurs and Firm H is disadvantaged in its quality

perception while Firm L benefits. This threshold model is also consistent with the

pattern observed in the literature on the contrast and the assimilation effect, and has

been explained by a latitude of acceptance (Hovland et al. 1957). We label this change

due to coexistence of two brands' links in the search engine result page the context effect

and denote it by E_2. This parameter takes a positive value when there is a contrast and a

negative value when there is an assimilation. Although we discuss our model and

results in terms of the contrast or the assimilation effect, we put more structure on this

effect and define it as a function of the quality difference $\Delta q (\equiv q_H - q_L$, with q_i being

Firm i's qulaity): $E_2 \equiv \phi(\Delta q - q^0)$, where q^0 is exogenously given, depending on the

brands and the industry they are in. For simplicity, we normalize the scale parameter ϕ

to be one. As discussed in the related literature section, a similar pattern of assimilation

and contrast has been observed in various contexts in the psychology and marketing

literatures (For example, see Hovland et al. 1957, Sherif et al. 1958, Anderson 1973,

Lichtenstein and Bearden 1989, and Ledgerwood and Chaiken 2007). Later in the paper,

we provide empirical evidence of the existence of both exposure effect and context effect

in an internet search situation, using data collected from a survey where consumers

viewed different types of web page showing different conditions being investigated.

The exposure effect may be different across firms. To capture this difference, we

introduce a differentiation factor $\gamma \in (-1,1)$ and use $(1+\gamma)E_1$ to represent the exposure

effect of Firm H, while keeping E_1 as the exposure effect of Firm L. Thus, if $-1 < \gamma < 0$,

Firm L has a larger exposure effect than Firm H, potentially because Firm H is already

known to consumers who search with Firm H's branded keyword, and thus does not

gain much additional benefit. On the other hand, if $0 < \gamma < 1$, Firm H has a larger

exposure effect than Firm L. This can be observed when Firm H's organic link has a

synergistic effect with its sponsored link. In fact, Yang and Ghose (2010) reported a

positive interdependence of these two types of links in terms of click-throughs: the

existence of organic link lifted the click-through of the sponsored links (and vice versa).

Though we focus on the brand advertising aspect rather than the click-through, their

research may suggest a similar interdependence of both links in terms of the quality

perception of Firm H. Finally, if $\gamma = 0$, both firms show an identical level of exposure

effect. While we choose not to focus on what drives γ or how it is determined, we construct a model as general as possible that includes all the possibilities.

The exposure effect can also be contingent on the rank in the sponsored link. In particular, the exposure is greater in the first slot than in the second slot, whether or not these sponsored links are located on the top of the page or on the right-hand side of it. According to a few eye tracking study results, search engine users start scanning the search engine result page from top to bottom and from left to right, thus forming so-called `golden triangle' in the upper left part of the search engine result page (Hotchkiss et al. 2005, Official Google Blog post on February 6, 2009, and Lew 2009). More specifically, in the Enquiro-EyeTools study, 50 percent of participants looked at the first side sponsored link, while only 40 percent looked at the second sponsored link (Hotchkiss et al. 2005). In order to represent this phenomenon, we use ε in our model to capture a decrease in the exposure effect when moving down to the second slot from the first slot. Thus, the exposure effect in the second slot is given by $(1-\varepsilon)E_1$, while that in the first slot remains to be E_1 (for Firm L). In fact, ε is the rate at which the exposure effect decreases and we assume this rate remains the same across firms. By definition, ε is bounded between 0 and 1.

The context effect can be observed in two scenarios: (NY) when only Firm L advertises, and (YY) when both Firm H and Firm L advertise. In the first case, although not shown side by side, both Firm H's link (in the organic result section) and Firm L's

link (in the sponsored result section) are shown together in the search engine result page and thus can be compared. On the other hand, in the latter case, Firm L's sponsored link is more likely to be compared with Firm H's sponsored link rather than Firm H's organic link. Thus the comparison may be greater in the latter case, because the two links are closer to each other. Figure 2 illustrates this difference in the distance between the two brands' links in (NY) and (YY) scenarios. To capture the effect of the physical distance on the magnitude of context effect, we use E_2 to represent the context effect in scenario NY and introduce δ to denote an increase in the context effect due to geographic proximity. Finally, we assume that the improvement in one firm's perceived quality and the loss in the other firm's perceived quality are symmetric. Thus, half of the context effect is attributed to the increase in the quality perception of one brand and the other half is subtracted from the perceived quality of the other brand.
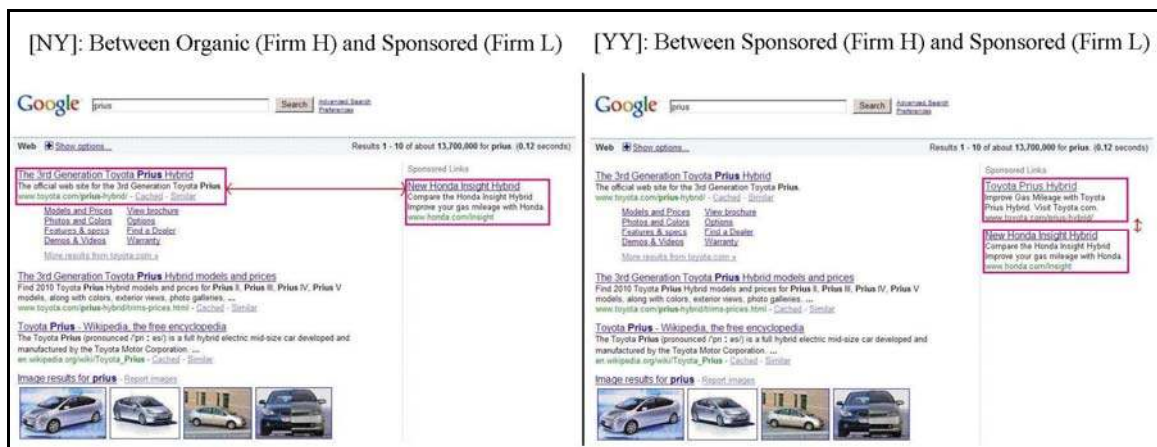
Figure 2: Distance of Two Brands in (NY) and (YY) Scenario

We summarize in Table 1 our discussion of the effect of branded keyword advertising on perceived quality. In particular, if there is no additional information (as in NN scenario), consumers form the quality perception of the products based on prior knowledge. We assume their perception is unbiased, i.e., it is the same as the actual quality level. However, after being exposed to an advertisement, their quality perception is affected by the advertisement as described above. Specifically, when only Firm H advertises (YN Scenario), Firm H's perceived quality increases by $(1+\gamma)E_1$, while Firm L's perceived quality remains the same. When only Firm L advertises (NY Scenario), its quality perception is increased by E_1 due to the exposure effect. However, at the same time, it is contrasted or assimilated with Firm H and thus experiences an additional loss (when contrasted) or gain (when assimilated) in quality perception, represented by $-\frac{1}{2}E_2$. The same amount of effect of the opposite sign is added to Firm H's quality perception by context effect. Finally, when both firms advertise (YY Scenario), two subcases arise: (1) Firm H takes the first slot while Firm L takes the second slot (YY1) or (2) Firm H takes the second slot while Firm L takes the first slot (YY2). In either case, both firms enjoy an increase in perceived quality by exposure and an additional increase or decrease by the context effect. The difference between the two subcases is the degree of exposure effect. In the first case, Firm L has a smaller increase than if it were in the first slot, while in the second case, Firm H has a smaller increase than if it were in the

first slot. In the table, we define $q_{(\cdot)}$ as the quality of each product, $\tilde{q}_{(\cdot)}$ as the perceived quality, and Δq or $\Delta\tilde{q}$ as the corresponding differences of perceived qualities of two products. Later in the analysis, the difference in the perceived quality plays a central role and thus, we also derive the perceived quality difference in the Table.

Table 1: Perceived Quality by Advertising Scenario

Scenario	$\tilde{q}_H$	$\tilde{q}_L$	$\Delta\tilde{q}$
NN	q_H	q_L	Δq
YN	$q_H + (1+\gamma)E_1$	q_L	$\Delta q + (1+\gamma)E_1$
NY	$q_H + \frac{1}{2}E_2$	$q_L + E_1 - \frac{1}{2}E_2$	$\Delta q - E_1 + E_2$
YY1	$q_H + (1+\gamma)E_1 + \frac{1}{2}(1+\delta)E_2$	$q_L + (1-\epsilon)E_1 - \frac{1}{2}(1+\delta)E_2$	$\Delta q + (\gamma+\epsilon)E_1 + (1+\delta)E_2$
YY2	$q_H + (1+\gamma-\epsilon)E_1 + \frac{1}{2}(1+\delta)E_2$	$q_L + E_1 - \frac{1}{2}(1+\delta)E_2$	$\Delta q + (\gamma-\epsilon)E_1 + (1+\delta)E_2$

3.1.2 Bidding Game

For a branded keyword, in addition to the two product market competitors, there can be other advertisers, including retailers and price comparison sites. However, to focus on the role of branded keyword search advertising in the product market competition, we limit the number of these other advertisers to one. We denote this last advertiser by Firm X. Thus, while the search engine can provide multiple advertising slots that can be taken by various kinds of advertisers, we describe the bidding game that has only three potential participants competing for three advertising links available in the search engine result page.

As in practice, the search engine uses an auction to allocate the advertising slots to advertisers, where it ranks advertisers by the product of relevance r_i and bid amount

b_i. In this auction, advertiser i at slot j collects $s_j r_i$ clicks with s_j $(j=1,2)$ being the

slot-specific clickthrough rate, and pays $\dfrac{r_{(j+1)} b_{(j+1)}}{r_i}$ per click, where $(j+1)$ refers to the

advertiser at slot $j+1$. Note that the payment depends on the next slot winner's bid

amount, because the search engine uses so-called generalized second-price auction rule.

By definition, Firm H has the highest relevance to the keyword because it is the

brand owner. On the other hand, Firm X is assumed to have the lowest relevance as well

as the lowest valuation for a click. Thus Firm X always takes the last slot. The role of this

last advertiser is to ensure that both Firm H and Firm L pay the same price per click

when only one of them advertises or when they lose to each other. More specifically, the

total cost of advertising to firm i when advertising alone is given by

$$C_{i0} \equiv s_1 r_i \frac{r_X b_X}{r_i} = s_1 r_X b_X \ (i = H, L), \text{ and when losing to the other firm, by}$$

$$C_{i2} \equiv s_2 r_i \frac{r_X b_X}{r_i} = s_2 r_X b_X \ (i = H, L). \text{ Because we are interested in the strategic interaction}$$

between Firm H and Firm L, we treat b_X as an exogenous variable and assume that

$b_X < b_X^0$, where b_X^0 is defined in the Appendix as a function of the parameters of our

model. This condition guarantees the existence of the parameter space where Firm H

and Firm L purchase the keyword with Firm X's presence in equilibrium. Note also that

the model does not require Firm X. The existence of Firm X with an exogenous bid

amount is mathematically equivalent to an exogenous minimum bid to the position auction.

Finally, when both firms choose to advertise and firm i wins the first slot, it exerts the cost $C_{i1} \equiv s_1 r_i \dfrac{r_{i'} b_{i'}}{r_i} = s_1 r_{i'} b_{i'}$, where i' refers to the other firm and $b_{i'}$ is determined from the Symmetric Nash Equilibrium of the bidding game (Varian 2007). The Symmetric Nash Equilibrium states that in equilibrium every advertiser has no incentive to switch its position with any other advertiser. In particular, if we let π_{ij} denote Firm i's profit when taking the jth slot and suppose Firm i takes the first slot and Firm i' takes the second slot in equilibrium, the Symmetric Nash Equilibrium requires:

$$\pi_{i1} - s_1 r_i p_{i1} \geq \pi_{i2} - s_2 r_i \tilde{p}_{i2} \tag{1}$$

$$\pi_{i'2} - s_2 r_{i'} p_{i'2} \geq \pi_{i'1} - s_1 r_{i'} \tilde{p}_{i'1}, \tag{2}$$

where p_{ij} refers to the per-click equilibrium price Firm i pays in slot j and $\tilde{p}_{ij}$ refers to the per-click price Firm i pays if it switches its position with the firm in slot j. Note that the deviation from the Symmetric Nash Equilibrium is as if two firms switch their positions. Thus, when a firm deviates to an adjacent higher slot, its payment depends on its own bid, because in deviation, the firms originally located in that higher slot now makes the same bid as the focal firm used to bid. For example, the price firm i' pays

when it deviates to slot 1, that is, $\tilde{p}_{i'1}$, is given by $\dfrac{r_i b_{i'}}{r_{i'}}$. This is in sharp contrast with the

deviation in a Nash Equilibrium, where the deviation requires only the focal firm

revising its own bid and thus, the price when deviating to the higher slot is dependent

on the bid of the firm originally located in that higher slot. For further clarification on

the difference between the Nash Equilibrium and the Symmetric Nash Equilibrium,

refer to definitions 1 and 2 in Varian (2007). The above discussion leads the conditions in

(1) and (2) to the following conditions:

$$\pi_{i1} - s_1 r_i b_{i'} \geq \pi_{i2} - s_2 r_X b_X \tag{3}$$

$$\pi_{i'2} - s_2 r_X b_X \geq \pi_{i'1} - s_1 r_i b_{i'}. \tag{4}$$

Now the Symmetric Nash Equilibrium requires that the bidding game is played

under complete information. As we will see later, the profits considered here come from

the product market competition, where each firm observes each other's price and

demand. In addition, each firm can experiment with differen bids in order to uncover

the competitor's relevance. Hence the complete information setting naturally follows. In

fact, the complete information assumption is commonly found in the search advertising

literature (For example, see Varian 2007, Edelman et al. 2007, Katona and Sarvary 2008,

Wilbur and Zhu 2009).

3.1.3 Advertising Budget

We also consider the possibility that advertisers may limit their advertising budgets. Because in search advertising, advertising cost is continuously incurred, search engines allow advertisers to stipulate their budgets and advertisers may choose to do so in order to have some control over their spending. We assume firm i ($i = H, L$) has a budget K_i and stipulates that amount in a designated time period (See also Wilbur and Zhu 2009 and Desai and Shin 2009). If K_i is greater than the total advertising cost C_i that firm i has to pay without budget stipulation, the budget does not affect the decisions in the advertising stage. However, if it is less than C_i, its advertisement stops appearing after exhausting K_i. In practice, the search engine spreads the budget during the entire period so that at any given time, the probability of consumers seeing firm i's advertisement is $\dfrac{K_i}{C_i}$. This implies that with limited advertising budget, the firm's perceived quality is given by the weighted average of that in the scenario where it advertises and that in the other scenario where it does not advertise, with $\dfrac{K_i}{C_i}$ being the weight given to the advertising scenario.

In our main analysis, we assume that the budget is always larger than C_i. However, in an extension of the model (Section 5), we consider the budget-constrained case, where we specifically assume $C_{i0} < K_i < C_{i1}$ ($i = H, L$). This assumption implies

that each firm has a budget that covers the cost it incurs in the second slot or in the first

slot when advertised alone. However, if there is a competitor to advertise together and if

the firm takes the first slot, the cost of advertising is not covered by their budget. Even

though in principle, K_i can take any value, we chose this range of values to highlight

the strategic interaction of the two firms.

3.2 Order of Events

We have two stages in our model: the advertising stage and the pricing stage. In

the advertising stage, the two firms simultaneously decide whether or not to participate

in the auction for Firm H's branded keyword. At the same time, the participating firms

simultaneously submit their bids in the keyword auction.[2] After firms are allocated to

the advertising slots and thus exposed to consumers, in the pricing stage, they set prices

for their own products in the product market. Note that advertising decisions come

before the pricing decision. This is because prices can be easily altered in the product

market. Moreover, the advertising decision, especially the participation decision, is

usually a long-term decision, as is the case in the advertising media decision. Even

though in the search advertising context, firms can easily change their bid amount in a

relatively short time, we do not separate the bidding decision from the participation

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[2]Although the two advertising decisions- participation and bidding- are currently modeled to be simultaneous made, separating out these two decisions does not alter the analysis, because the knowledge of each other's participation is relevant only in one of the four possible scenarios. Thus, in the next section, we analyze our model as if these two were separate decisions.

decision, because a firm cannot participate in the auction without making a bid. In addition, if a firm implements a search advertising plan over a significant period of time, it is very likely that this firm revises its price based on the effectiveness of the advertising campaign. Our model highlights this incentive.[3]

In every stage, firms have complete information. Thus, we derive a Nash Equilibrium in every subgame and a Subgame Perfect Nash Equilibrium in the stage game. Also note that in the bidding subgame, we consider a more stringent set of Nash Equilibria, a Symmetric Nash Equilibrium, where no firm has an incentive to switch their position with anyone else.

3.3 Product Market Competition

In the product market, two firms compete by simultaneously setting prices for their differentiated products. Firm H offers a high quality product of quality q_H while Firm L offers a low quality product of quality q_L, with $q_H > q_L$. The two products are also horizontally differentiated. To represent horizontal differentiation, we consider a Hotelling line where Firm H sits on one end and Firm L sits on the other end. Along this line, consumers of a unit mass are uniformly distributed. Then a consumer located at point x derives utility from each product at the time of purchase decision as follows:

$$U_H = \theta\tilde{q}_H - p_H - tx \tag{5}$$

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[3]In fact, it can be easily shown that the game order does not alter the main result of our model. The current order, by making the advertising decision affect the pricing decision, amplifies the impact of advertising decision on the profit.

$$U_L = \theta \tilde{q}_L - p_L - t(1-x), \tag{6}$$

where θ represents the willingness to pay for quality, t is the transportation cost, and $\tilde{q}_i$ ($i = H, L$) refers to the perceived quality of each product given in Table 1 for each advertising scenario. While consumers are horizontally heterogeneous, we assume for simplicity, that they are vertically homogeneous, that is, $\theta \equiv 1$. Assuming that the market is fully covered, we derive each firm's demand as follows:

$$D_H = \frac{1}{2} + \frac{\Delta\tilde{q} - p_H + p_L}{2t} \tag{7}$$

$$D_L = \frac{1}{2} - \frac{\Delta\tilde{q} - p_H + p_L}{2t}, \tag{8}$$

where $\Delta\tilde{q}$ denotes the difference in the perceived quality. Then both firms' profits are given as follows:

$$\Pi_H = p_H \left(\frac{1}{2} + \frac{\Delta\tilde{q} - p_H + p_L}{2t}\right) - C_H \tag{9}$$

$$\Pi_L = p_L \left(\frac{1}{2} - \frac{\Delta\tilde{q} - p_H + p_L}{2t}\right) - C_L, \tag{10}$$

where C_i ($i = H, L$) refers to the total advertising cost of firm i, which takes one of the following values: C_{i0} when advertising alone, C_{i1} when winning the first slot over the other firm, C_{i2} when losing the first slot to the other firm, and 0 when not advertising. Finally, Table 2 summarizes the notations used in our model.

Table 2: Summary of Notations

	Parameters describing the Effect of Advertising
$q_i, \Delta q$	Actual quality of Firm i and their difference
$\tilde{q}_i, \Delta\tilde{q}$	Perceived quality of Firm i and their difference
E_1	The exposure effect
E_2	The context effect
q^0	The threshold in Δq between the contrast effect and the assimilation effect
γ	The difference in exposure effect between Firm H and Firm L
ϵ	The difference in exposure effect between the first and the second slot
δ	The difference in context effect between the NY and the YY scenario
	Parameters describing the Search Advertising Market Competition
s_j	Slot-specific click-through rate in slot j
r_i	Relevance of Firm i
b_i	Bid amount of Firm i
C_{i0}	Advertising cost to Firm i when advertising alone
C_{i1}	Advertising cost to Firm i when taking the first slot under the YY scenario
C_{i2}	Advertising cost to Firm i when taking the second slot under the YY scenario
K_i	Stipulated advertising budget for Firm i
	Parameters describing the Product Market Competition
t	Transportation cost
p_i	Price of Product i
D_i	Demand for Product i
Π_i	Profit of Firm i
C_i	Total advertising cost to Firm i

4. Theoretical Analysis

We begin our analysis with the case where firms assign sufficient resources to invest in branded keyword advertising. Then we relax this assumption in the extension of the model in Section 5 by considering the case where both firms stipulate a limited budget. In both cases, using backward induction, we derive the product market equilibrium and based on this, we investigate each firm's bidding decisions and finally their participation decisions in the auction of Firm H's branded keyword.

When advertisers have sufficient resources allocated to branded keyword advertising, they can afford to advertise for the entire period and thus the equilibrium does not change throughout the period. We start our investigation from the pricing subgame in the product market. All proofs can be found in the Appendix.

4.1 Pricing Equilibrium

In the pricing subgame, firms set their prices given the advertising decisions. Thus, the story begins with their joint advertising decisions: whether or not to buy the keyword. As a result of this decision, they will be put in one of the four advertising scenarios, and this advertising scenario determines the exposure effect and the context effect they will get from their advertisements. Then, the perceived quality of their product is revised by these effects. Now the perceived quality in turn affects consumer utility, which give to consumers more or less incentive to buy the product. By this, firms can have either higher or lower quality premium, which can be translated into their

prices. This process is depicted in Figure 3. Recall that $\Delta\tilde{q}$ represents the difference in quality perception of the two products in the advertising subgame, which is jointly determined by both firms' participation decision, and that $C_{(.)}$ denotes the total cost associated with search advertising that is determined by both firms' bidding amount as well as participation decision. Then we derive the equilibrium as follows.
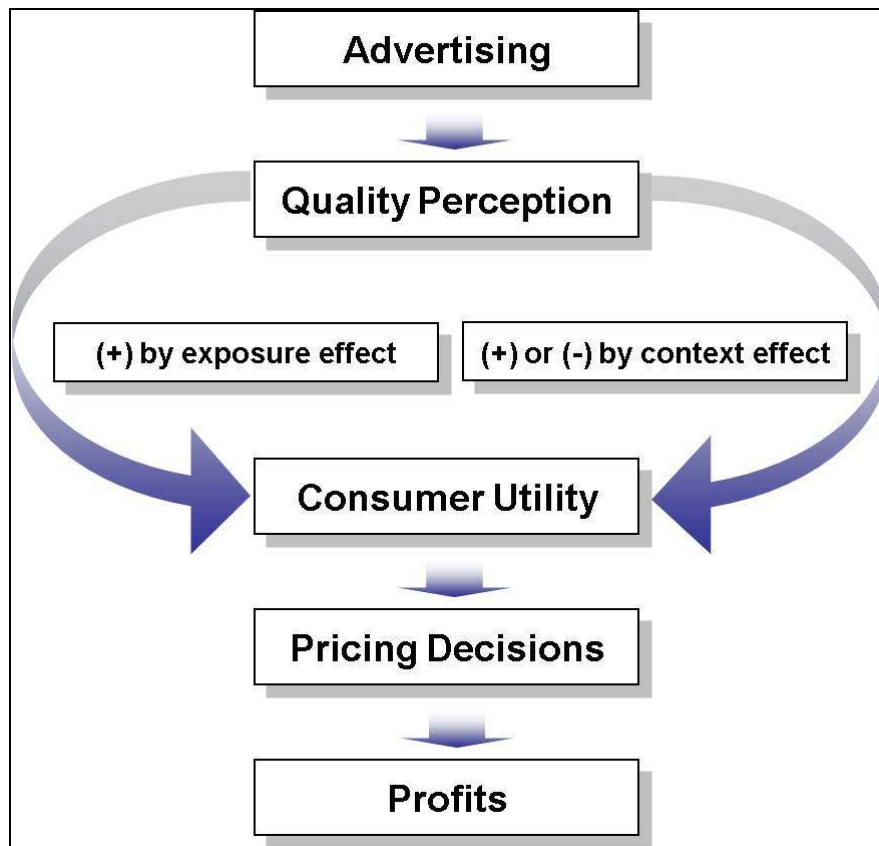

Figure 3: Conceptual Flow of the Model

Lemma 1 *In equilibrium, firms charge* $p_H^* = t + \dfrac{\Delta\widetilde{q}}{3}$ *and* $p_L^* = t - \dfrac{\Delta\widetilde{q}}{3}$ *for their own product;*

make sales of $D_H^* = \dfrac{3t + \Delta\widetilde{q}}{6t}$ *and* $D_L^* = \dfrac{3t - \Delta\widetilde{q}}{6t}$ *; and earn* $\Pi_H^* = \dfrac{(3t + \Delta\widetilde{q})^2}{18t} - C_H$ *and*

$$\Pi_L^* = \dfrac{(3t - \Delta\widetilde{q})^2}{18t} - C_L.$$

The lemma shows that a change in the perceived quality directly affects the equilibrium prices and the equilibrium sales. Thus, firms may have an incentive to advertise in the search engine in order to increase their own product's perceived quality. However, a product's perceived quality is jointly determined by both firms' advertising decisions. Thus, in order to strategically utilize search advertising, firms must understand how the two effects of advertising alters the perceived quality and thus the equilibrium outcome and also, how these changes in the equilibrium outcome are mediated by both firms' advertising decisions. We summarize these implications in the following two propositions by plugging $\Delta\widetilde{q}$ in Table 1 into the equilibrium prices and sales.

Proposition 1 (Exposure Effect)

(1) When only one firm advertises: with larger exposure effect, the advertising firm can charge higher prices and increase its sales, while the other (non-advertising) firm should charge lower prices and get lower sales;

(2) When both firms advertise together: with larger exposure effect, the firm with higher exposure effect can charge higher prices and increase its sales, while the other firm (with lower exposure effect) should charge lower prices and get lower sales.

The proposition shows that the exposure effect basically helps the advertising firm but it favors only one firm with higher sensitivity to the exposure effect when both of them advertise together. This is because the two products are relatively evaluated and thus, any benefit from the exposure effect is canceled out by each other. The following proposition describes the effect of the contrast or the assimilation on the equilibrium outcome.

Proposition 2 (Context Effect) *The context effect has an impact on the equilibrium outcome if and only if Firm L advertises. Suppose Firm L advertises.*

(1) If Firm L's advertisement triggers a contrast, Firm H can charge higher prices and increase its sales, while Firm L should charge lower prices and get lower sales, as the contrast effect increases.

(2) If Firm L's advertisement triggers an assimilation, Firm L can charge higher prices and increase its sales, while Firm H should charge lower prices and get lower sales, as the contrast effect increases.

The proposition first defines the condition for the context effect to be relevant to the equilibrium outcome. Because there always exists the organic link of Firm H (i.e., the brand owner), whenever Firm L has a sponsored link on the search engine result page, it

can be compared with either Firm H's organic link (when Firm H does not have an advertisement) or Firm H's sponsored link (when Firm H has an advertisement). Thus, the relevance of the context effect is dependent on Firm L's advertisement. Now, if Firm L advertises, the proposition shows that as in the case of the exposure effect, the context effect helps one firm while hurting the other. In particular, the contrast effect gives more pricing power to Firm H, while the assimilation effect benefits Firm L in the same way.

Next recall that q^0 is the threshold of the quality difference, determining when the assimilation or the contrast happens. While we are ignorant about what determines this threshold, we can investigate the impact of q^0 on the equilibrium outcome as in the following proposition.

Proposition 3 (Threshold q^0) *As the threshold q^0 becomes larger, Firm L can charge higher prices and increase its sales, while Firm H should charge lower price and face lower sales, whenever Firm L has an advertisement. If Firm L does not advertise, q^0 has no impact on the equilibrium outcome.*

The proposition is easy to understand when we think about what role the threshold q^0 has in determining the context effect. By definition of the context effect, an increase in q^0 results in more occasion of assimilation but less occasion of contrast, given the same quality difference. Thus the impact of q^0 is identical to that of the assimilation effect. As shown in Proposition 2, stronger assimilation helps Firm L while

hurting Firm H, and thus, the threshold q^0 also increases the benefit of Firm L. Finally,

q^0 becomes relevant to the product market competition only when there exists a context

effect, because it affects the competitive horizon only through the context effect.

Therefore, q^0 has any impact on the equilibrium outcome only when Firm L has an

advertisement.

In sum, the above propositions examine how the pricing incentives of both firms

are affected by the two effects of advertising. This pricing equilibrium in turn affects the

firms' incentives to advertise. Thus, in what follows, we investigate how firms consider

the two effects of advertising when making an advertising decision.

Before moving on, we also note from the product market equilibrium that

equilibrium prices and demands are positive, which implies $-3t \leq \Delta \tilde{q}^* \leq 3t$. This

translates into the following inequalities for each of the five scenarios: YN, NY, YY1,

YY2, and NN. Note that we rewrite E_2 using $E_2 \equiv \Delta q - q^0$, because Δq determines the

magnitude and the sign of the context effect.

$$-3t \leq (1+\gamma)E_1 + \Delta q \leq 3t \tag{11}$$

$$-3t \leq -E_1 + 2\Delta q - q^0 \leq 3t \tag{12}$$

$$-3t \leq (\gamma+\varepsilon)E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \leq 3t \tag{13}$$

$$-3t \leq (\gamma-\varepsilon)E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \leq 3t \tag{14}$$

$$-3t \leq \Delta q \leq 3t \tag{15}$$

From this point on, we confine our interest to the case where all inequalities in (11)-(15) are satisfied. This set of inequalities altogether defines the feasible range of E_1 and Δq, the two variables determining the two effects of interest. The other parameters jointly determine the shape of the feasible space. Figure 4 illustrates an example of the parameter space when $t = 50$, $\delta = 0.9$, $\gamma = -0.75$, $\varepsilon = 0.1$, and $q^0 = 50$. Note that we have $E_1 \geq 0$ and $\Delta q \geq 0$ by definition. Although Figure 2 depicts a hexagonal shape, different parameter values may lead to different shapes of parameter space (e.g., pentagon).
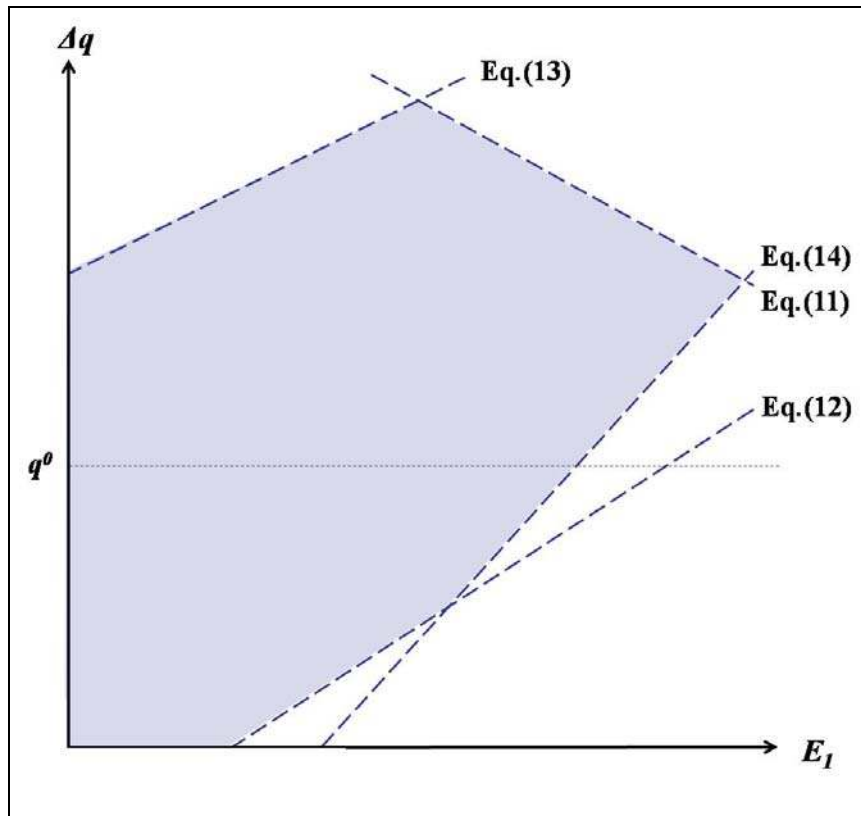
Figure 4: An Example of the Parameter Space

4.2 Bidding Equilibrium

When there is at least one product market competitor that decides to participate, there arises a bidding game among advertisers of Firm H's keyword. Recall that there is an additional advertiser, Firm X, who always participates in the auction. In the bidding subgame, both firms make bids given their joint participation decisions. Thus, we define an equilibrium for each of the three possibilities of participation decision: YN, NY, and YY.

We start with the simplest cases. In scenario YN, among the two product market competitors, only Firm H advertises. Here, by assumption, Firm H always wins the top slot. The payment Firm H makes to the search engine is simply, $C^*_{H0} = s_1 r_X b_X$.[1] Likewise, in scenario NY, only Firm L advertises and thus it always wins the top slot by paying $C^*_{L0} = s_1 r_X b_X$.

Now in scenario YY, both Firm H and Firm L participate in the auction. Here they compete for the top slot. Firm L may want to get better exposure, while Firm H attempts to discourage its competitor from doing so and maximally utilize its own brand equity. The following proposition summarizes the consequence of this competitive interaction.

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[1] Note that b_X is exogenous. In fact, it can be derived from the bidding game between Firm H and Firm X, but in order to focus on more interesting aspect of the game, we leave it as exogenous.

Proposition 4 *When both firms purchase Firm H's branded keyword with sufficient resources,*

in equilibrium, Firm H wins the first slot if and only if $\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0$.

The proposition first defines the case where Firm H successfully deters Firm L's

attempt to fully appropriate its own brand equity by winning the first slot. However, the

lemma also shows that Firm H does not always choose to do so. Instead, it may give up

the first slot and take the second slot when there is a large assimilation effect (i.e., very

small Δq below q^0), or when there is a large exposure effect and Firm H is less sensitive

to the exposure effect than Firm L (i.e., large negative value of γE_1). This is because

under these conditions, the brand owner may not be able to increase its perceived

quality relative to its competitor by spending incremental advertising budget necessary

to get the first slot. Despite its lead in the product market, the brand owner cannot

convert a better impression from the first slot to a higher profit in the product market as

efficiently as its competitor. Thus Firm H's motivation of winning the first slot becomes

weaker than that of Firm L.

We finish the equilibrium derivation in the bidding game by deriving the total

advertising cost based on the equilibrium bids under each equilibrium listing order.

First, the total advertising cost of the second slot winner is always given by $C_{i2}^* = s_2 r_X b_X$,

($i = H, L$), regardless of the equilibrium order. Now the following lemma defines the

advertising cost of the first slot winner.

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Lemma 2 *The payment from Firm H when it takes the first slot is given by,*

$$\frac{2\varepsilon E_1\{3t - \gamma E_1 - (2+\delta)\Delta q - (1+\delta)q^0\}}{9t} + s_2 r_X b_X \le C^*_{H1} \le \frac{2\varepsilon E_1\{3t + \gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0\}}{9t} + s_2 r_X b_X.$$

The payment from Firm L when it takes the first slot is given by,

$$\frac{2\varepsilon E_1\{3t + \gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0\}}{9t} + s_2 r_X b_X \le C^*_{L1} \le \frac{2\varepsilon E_1\{3t - \gamma E_1 - (2+\delta)\Delta q + (1+\delta)q^0\}}{9t} + s_2 r_X b_X.$$

Because the bid amount in a Symmetric Nash Equilibrium is given as an interval, the advertising cost is specified up to an interval. In what follows, we denote the respective equilibrium advertising cost of the first slot winner by C^*_{H1} and C^*_{L1}, for the sake of simplicity. Interestingly, both the upper bound and the lower bound of both C^*_{H1} and C^*_{L1} increase with ε, which suggests that both firms are generally willing to pay more for the first slot, if it generates a better incremental impression. Finally, note that the condition given in proposition 4 guarantees that the upper bound of the costs is greater than the lower bound.

Now we move forward to derive the equilibrium in the participation decision stage.

4.3 Participation Equilibrium

So far we have derived each firm's profit in the product market and the cost associated with search advertising, under each scenario. Now given this information, we investigate the participation decision of the two firms. We start with deriving their contingency plans.

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Proposition 5 (Best Response to Competitor's No Purchase) *When the other firm does not*

buy Firm H's branded keyword,

(1) Firm H buys it if and only if $E_1 > \alpha_1(\Delta q)$ *, and*

(2) Firm L buys it if and only if $E_1 > \alpha_2(\Delta q)$ *,*

where $\alpha_1(\Delta q) \equiv \dfrac{-(3t + \Delta q) + \sqrt{(3t + \Delta q)^2 + 18tC_{H0}^*}}{1 + \gamma}$ *,* $\alpha_2(\Delta q) \equiv -(3t - 2\Delta q + q^0) + \sqrt{(3t - \Delta q)^2 + 18tC_{L0}^*}$ *,*

and C_{H0}^* *and* C_{L0}^* *are the costs defined in the previous section.*

If there is no competitive pressure, each firm will advertise if and only if it gets

sufficient benefit from doing so. Thus, given that the competitor does not purchase the

keyword, each firm makes its own purchase decision solely depending on the benefit it

gets from exposure. More specifically, they decide to purchase the keyword if and only

if they can justify the advertising cost by the exposure benefits from the purchase.

However, note that Firm L's threshold $\alpha_2(\Delta q)$ is an increasing function of the

quality difference Δq, and thus the context effect. To understand why, recall that Firm

H is always displayed in the organic link. Thus, even if Firm L advertises alone, the

effectiveness of its advertising is affected by the interaction between the two links, that

is, the contrast or the assimilation between the organic link of Firm H and the sponsored

link of Firm L. Thus, if Δq is large and thus a contrast occurs, resulting in an additional

loss to Firm L, there must be a higher exposure benefit for Firm L to justify the same

level of cost. On the other hand, if Δq is small and thus there is an assimilation, Firm L

can have a lower threshold because it now gains more with the assimilation effect. Next

we consider the case where the competitor buys the keyword.

Proposition 6 (Best Response to Competitor's Purchase) *When Firm L buys Firm H's*

branded keyword, Firm H also buys it if and only if $\Delta q > \beta_1(E_1)$, *where*

$$\beta_1(E_1) \equiv \beta_{11}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0] + \beta_{12}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 < 0],$$

$$\beta_{11}(E_1) \equiv \frac{-\{2(1+\gamma+\varepsilon)+\delta(\gamma+\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} + \sqrt{\{(2(1+\gamma+\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C_{H1}^*}}{\delta(4+\delta)},$$

$$\beta_{12}(E_1) \equiv \frac{-\{2(1+\gamma-\varepsilon)+\delta(\gamma-\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} + \sqrt{\{(2(1+\gamma-\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C_{H2}^*}}{\delta(4+\delta)},$$

and $I[\cdot]$ *is an indicator function, and* C_{H1}^* *and* C_{H2}^* *are the costs defined in the previous section.*

Proposition 6 shows that all else being constant, if the quality difference between

the two brands is greater than some threshold, Firm H would like to purchase its own

branded keyword contingent upon Firm L's purchase. This is because large quality

difference implies that there is a contrast effect or a weak assimilation effect, under

which Firm H can match Firm L's purchase decision and still benefit itself more than

what it incurs as cost. If there is a strong contrast effect, Firm H can even hurt its

competitor by its contingency plan described above. We finally investigate Firm L's

purchase decision when the brand owner decides to buy its own keyword.

Proposition 7 (Best Response to Competitor's Purchase) *When Firm H buys its own*

branded keyword, Firm L also buys it if and only if $\Delta q < \beta_2(E_1)$, *where*

$$\beta_2(E_1) \equiv \beta_{21}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0] + \beta_{22}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 < 0],$$

$$\beta_{21}(E_1) \equiv \frac{2\{(1-\gamma-2\varepsilon)-\delta(\gamma+\varepsilon)\}E_1 + 6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$-\frac{\sqrt{\{(2+\gamma-\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L2}^*}}{2(3+\delta)(1+\delta)},$$

$$\beta_{22}(E_1) \equiv \frac{2\{(1-\gamma+2\varepsilon)-\delta(\gamma-\varepsilon)\}E_1 + 6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$-\frac{\sqrt{\{(2+\gamma+\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L1}^*}}{2(3+\delta)(1+\delta)},$$

and $I[\cdot]$ is an indicator function, and C_{L1}^* and C_{L2}^* are the costs defined in the previous section.

The proposition shows that in contrast to the previous case, Firm L matches Firm H's purchase decision when the quality difference is small. This is equivalent to the case where there is an assimilation effect or a weak contrast effect. This is because with no or a small additional penalty of being advertised together, Firm L can benefit and thus justify its advertising cost. Thus, Proposition 7 shows that with the help of an assimilation effect, Firm L may profitably attack Firm H's keyword when Firm H is buying.

Given the contingency plans derived above, we now derive the equilibrium participation decisions of both firms. In particular, in the next three propositions, we fully investigate the entire parameter space of E_1 and Δq and characterize the condition for which each scenario is observed in an equilibrium. Here we also note that for some set of parameter values, the YY scenario cannot be observed in equilibrium. However, for the sake of presentation, we consider the parameters that can induce all four

scenarios in equilibrium. We first characterize the condition for an equilibrium corresponding to the NN scenario.

Proposition 8 (Participation Equilibrium: NN) *If the exposure effect is not large, that is,*

$E_1 < \min\{\alpha_1(\Delta q), \alpha_2(\Delta q)\}$, *NN scenario can be observed in equilibrium.*

The proposition is a direct consequence of Proposition 5, which characterizes the condition for each firm's no purchase decision given the competitor's no purchase. Combining the two conditions, we get the condition for neither firm to unilaterally deviate to buy the keyword. The proposition explains why we often observe no sponsored link for some branded keywords: exposure on the search engine may not be impactful enough to be translateed into a significant increase in sales in the product market. This may be true in the case where the keyword is the master brand in its brand hierarchy and thus has little connection to the product market competition. A good example of this is Proctor and Gamble. There is no advertisement under ``Proctor and Gamble'' or ``P and G'' in both Google and Microsoft's Bing.com. However, its subbrands in the detergent category such as Tide and Cheer are currently bought by the company. Another case of the NN equilibrium is when the target market does not use the internet search often and consequently, the exposure to the market through the search engine is not great enough. In all of these cases, the search engine is not an

effective tool for advertising because of a weak exposure effect. Figure 5 graphically

represents the condition for this case in the ($E_1, \Delta q$) space.[2]

From the result of Proposition 8, we can conclude that at least one firm buys the

keyword if $E_1 \geq \min\{\alpha_1(\Delta q), \alpha_2(\Delta q)\}$. Given this, we now proceed to resolve our puzzle

on why we observe different patterns of purchase behavior in each firm. To investigate

the issue, we first determine when the remaining three scenarios can be observed in

equilibrium.

Proposition 9 (Participation Equilibrium: YN, YY, and NY) *Suppose the exposure effect is*

large enough to justify the cost of advertising, that is, $E_1 \geq \alpha_1(\Delta q)$ for Firm H and

$E_1 \geq \alpha_2(\Delta q)$ for Firm L. Then, in equilibrium, the YN scenario is observed if $\Delta q \geq \beta_2(E_1)$,

while the NY scenario is observed if $\Delta q \leq \beta_1(E_1)$. When $\beta_1(E_1) < \Delta q < \beta_2(E_1)$, the YY

scenario is observed.

This proposition provides an integrative framework to understand the

conditions for scenarios YN, NY, and YY. When the exposure in the search advertising is

effective in increasing quality perception, depending on the quality difference between

the two brands, we obtain three different scenarios. If the quality difference is large, the

YN equilibrium is obtained. When it becomes smaller to be around q^0, the YY

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[2]We simplified the shape of each region, for the sake of presentation. While the general shape
remains the same, the details may change depending on the parameter value.

equilibrium can be observed. Finally, if it becomes much smaller than q^0, the NY

equilibrium is obtained. This pattern is illustrated in Figure 5.
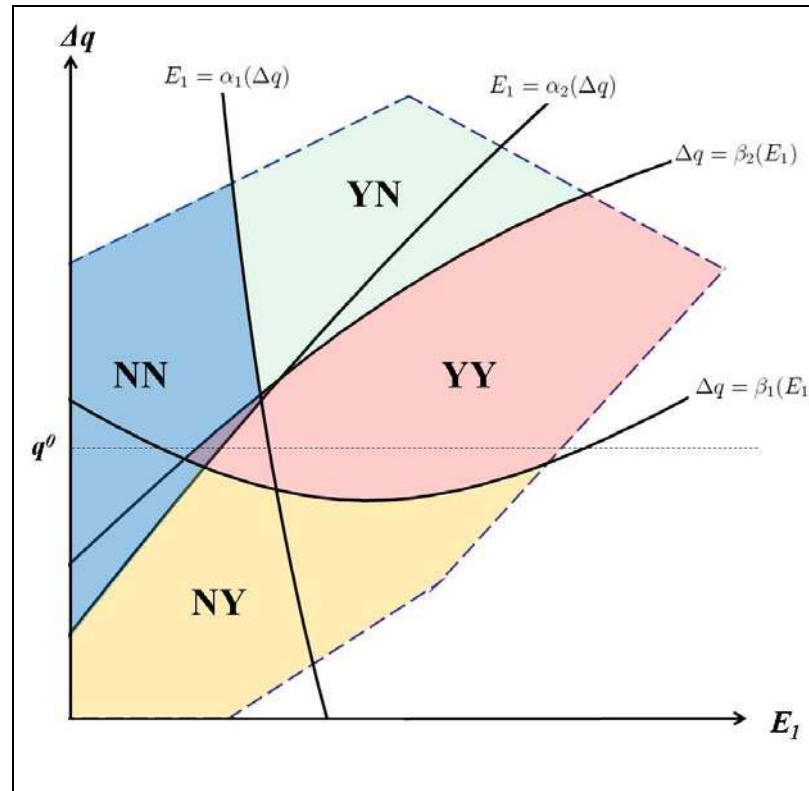
Figure 5: An Illustration of Participation Equilibrium

First consider the YN equilibrium. Here, we only observe Firm H in the

sponsored result. When would it make sense that Firm H advertises alone? This occurs

when the increase in sales due to better exposure can more than offset the cost of

advertising, i.e., when it is worthwhile for Firm H to buy its own keyword. Even though

Firm H already appears in the organic link, it can still increase the benefit of having an

established brand name by additionally advertising in the sponsored link. Next, what

stops Firm L from buying the keyword in the YN equilibrium? It is its fear of being

perceived to be of lower quality than the status quo when listed in parallel with Firm H. If the quality difference is large, there occurs a strong contrast between the two brands. While Firm L may be able to steal part of Firm H's brand value by joining the sponsored list of the keyword, the benefit may be dissipated by the strong contrast effect. Thus, Firm L no longer buys the keyword and thus we observe the YN equilibrium.

The argument is reversed in the NY equilibrium, where we only observe Firm L in the sponsored link. As before, when the exposure effect is strong, Firm H may choose to advertise under its own branded keyword. However, if the exposure effect is so high that even Firm L would like to advertise, Firm H may prefer not to advertise with Firm L, when it expects the assimilation effect to dissipate all of its benefit from exposure. In fact, the condition of the small quality difference implies that there exists a strong assimilation and thus, we obtain the NY equilibrium. Note that even here Firm H cannot completely avoid the assimilation, due to the existence of its organic link. However, it can minimize the negative impact of assimilation by not engaging in search advertising.

Finally, the proposition also suggests that the YY equilibrium is obtained when the quality difference is in the intermediate range. This condition implies that neither contrast effect nor assimilation effect is strong. In line with the above arguments, both firms get involved in advertising only when they find the detrimental effect of being listed together is negligible. Note that in this equilibrium, the strong exposure effect also provides both firms a proper incentive to advertise.

Although Proposition 9 characterized the YY equilibrium when the exposure

effect is strong, the YY equilibrium does not necessarily require a strong exposure effect.

In fact, we can also observe both firms advertising in equilibrium, even when the

exposure effect is small (i.e., where neither would advertise if the other did not

advertise). The following proposition defines such an equilibrium and thus completes

the equilibrium derivation in the entire parameter space of (E_1, Δq).

Proposition 10 (Defensive Purchase) *The YY scenario can be observed in equilibrium if*

$E_1 < \alpha_1(\Delta q)$ *and* $\beta_1(E_1) < \Delta q < \beta_2(E_1)$. *In this equilibrium, Firm H buys its own branded*

keyword only for defensive purposes.

While the proposition derives another case of the YY equilibrium, it also reveals

an underlying incentive for Firm H to buy its own branded keyword.[3] To see this, first

recall from Proposition 5, that the condition $E_1 < \alpha_1(\Delta q)$ implies the exposure effect is

small so that Firm H prefers not to advertise when Firm L does not advertise. However,

when Firm L advertises, Firm H also wants to advertise by Proposition 6, because

$\Delta q > \beta_1(E_1)$. Together, Firm H may not buy the keyword alone but it only buys because

Firm L buys in this equilibrium.

Here we delineate one reason why Firm H buys its own branded keyword: to

defend its own brand from the threat of the competitor. If Firm H lets Firm L buy alone,

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[3]In fact, under the condition of the proposition, we may have multiple equilibria: both YY and
NN scenarios can be obtained in equilibrium for some parameter values.

consumers who conduct their search with Firm H's brand name either because they only

know about Firm H or because they prefer Firm H's product, now can easily associate its

competitor Firm L with Firm H's brand name. This is harmful to Firm H. However, if

Firm H buys the keyword, it can at least minimize the negative impact of Firm L's

purchase. Thus, Firm H's purchase of the keyword is not initiated by its own benefit but

by the threat of Firm L's appropriating its own trademark. This argument resolves one of

our main puzzles on why brand owners buy their own branded keyword while there

does not seem to be much benefit from doing so. They do not choose to advertise but

they are forced to do so by competitive pressure. Region (I) in Figure 6 illustrates the

case where this competitive behavior can arise.

The above proposition shows the case where Firm H buys the keyword to fight

against Firm L but ends up coexisting with it in the search engine result page. However,

in some cases, Firm H may be successful in completely driving out Firm L. The

following proposition characterizes this case.

Proposition 11 (Preclusion) *Even though Firm L find it profitable to buy Firm H's branded*

keyword by itself, it is effectively precluded by Firm H's purchase of its own keyword, when

$E_1 > \alpha_2(\Delta q)$ *and* $\Delta q > \max\{\beta_1(E_1), \beta_2(E_1)\}$.
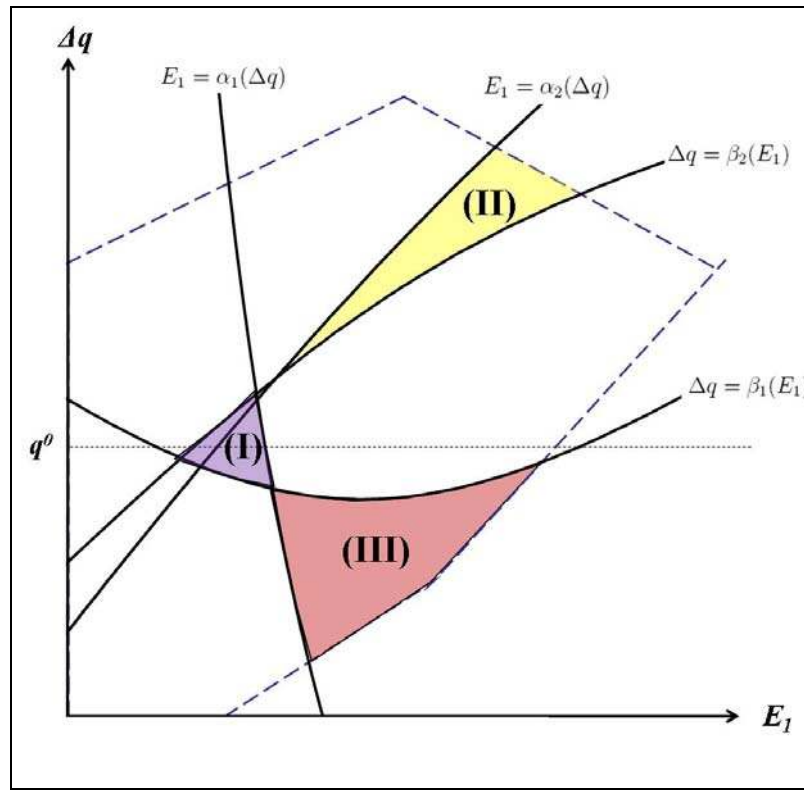
Figure 6: Regions of Strategic Purchase Decision

The proposition shows the case where Firm H successfully defends its intangible asset by driving out its competitor Firm L. To fully appreciate the proposition, consider what happens if Firm H does not buy. Because the exposure effect is large enough, Firm L will buy the keyword without Firm H's purchase. Then Firm H would want to protect its own brand by purchasing the keyword. Unlike in the previous case, now Firm H's threat of buying contingent upon Firm L's buying is credible because the contrast effect is strong (implied by the quality difference being large). Then for the same reason (i.e., strong contrast effect), the competitor gives up purchasing the keyword and thus, Firm

H can effectively preclude its competitor from buying the keyword. This shows an interesting case of disadvantaging the competitor just by purchasing the keyword and in this sense, this is an offensive purchase of the keyword by the brand owner.[4] This case is also in contrast with the comparative advertising in the traditional media, where the well-known firm cannot help letting its competitor utilize its well-known brand in associating it with the competitor's less known brand. However, in the search advertising setting, under some conditions, the well-known firm can deter its competitor's attempt to be listed and thus to be compared with itself.

The proposition potentially explains another puzzle on why some firms forgo the opportunity to advertise themselves under other firm's well-known brand name. It suggests that they do not choose to forgo the opportunity but they are forced to do so by the brand owner's threat of buying the keyword. Region (II) in Figure 6 also represents the condition for this preclusion to occur.

Similarly, Firm L can also preclude Firm H. We investigate this possibility in the following proposition.

Proposition 12 (Preclusion) *Firm H may find it profitable to utilize its own brand name but it is effectively discouraged by Firm L, when $E_1 > \alpha_1(\Delta q)$ and $\Delta q < \min\{\beta_1(E_1), \beta_2(E_1)\}$.*

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[4]In fact, the condition of the proposition includes not only the region where Firm H buys the keyword but also the region where Firm H does not buy the keyword. However, under the condition, L will never advertise. Thus more precisely, Firm L is precluded by Firm H's threat of buying the keyword.

When the exposure effect is large, Firm H may prefer to buy its own keyword to induce better perception about its own product. However, whenever possible, Firm L would not let its competitor do this because Firm H's doing so has a negative consequence to Firm L in the product market. Thus, Firm L tries to discourage Firm H from buying its own keyword by threatening Firm H that Firm L will also buy the keyword. This threat is credible if the assimilation effect is strong because then Firm L is better off by buying than not and Firm H is hurt by Firm L's buying. Thus, if the quality difference is small, Firm H's desire to increase its quality perception is discouraged and we do not observe Firm H in equilibrium in the search engine result page. This case is depicted in Region (III) of Figure 6.

Finally, we examine the profits of both firms when they use branded keyword search advertising. Interestingly, we find a prisoner's dilemma case such as the following.

Proposition 13 (Prisoner's Dilemma) *Both firms engage in search advertising but their profits are lower than when none of them advertises, if*

$\max\{\beta_1(E_1), \beta_3(E_1)\} < \Delta q < \min\{\beta_2(E_1), \beta_4(E_1)\}$, *where* $\beta_1(E_1)$ *and* $\beta_2(E_1)$ *are as defined respectively in Proposition 6 and Proposition 7, and*

$\beta_3(E_1) \equiv \beta_{31}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0] + \beta_{32}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 < 0],$

$\beta_4(E_1) \equiv \beta_{41}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0] + \beta_{42}(E_1)I[\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 < 0],$

$\beta_{31}(E_1) \equiv \dfrac{-(2+\delta)(\gamma+\varepsilon)E_1 + \{3t + (\delta+2)q^0\}(\delta+1) - \sqrt{\{(\gamma+\varepsilon)E_1 + (3t - q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{L2}^*}}{(\delta+1)(\delta+3)},$

$$\beta_{32}(E_1) \equiv \frac{-(2+\delta)(\gamma-\varepsilon)E_1 + \{3t + (\delta+2)q^0\}(\delta+1) - \sqrt{\{(\gamma-\varepsilon)E_1 + (3t-q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{H2}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{41}(E_1) \equiv \frac{-(2+\delta)(\gamma+\varepsilon)E_1 - \{3t - (\delta+2)q^0\}(\delta+1) + \sqrt{\{(\gamma+\varepsilon)E_1 - (3t+q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{H1}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{42}(E_1) \equiv \frac{-(2+\delta)(\gamma-\varepsilon)E_1 - \{3t - (\delta+2)q^0\}(\delta+1) + \sqrt{\{(\gamma-\varepsilon)E_1 - (3t+q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{L1}^*}}{(\delta+1)(\delta+3)},$$

and C_{H1}^*, C_{H2}^*, C_{L1}^*, and C_{L2}^* are the costs defined in the previous section.

To see the intuition of the proposition, we first need to consider each firm's incentive to buy the keyword when both firms engage in advertising. As seen before, Firm H buys the keyword to reduce the negative impact of the competitor free-riding on its own brand by increasing its own exposure. Firm L, on the other hand, tries to catch up with Firm H in terms of the exposure. These exposure benefits that both firms expect to achieve by purchasing the keyword would be canceled out by each other when both firms decide to purchase at the same time. While it is true that the context effect may help one firm get out of this trap, it does not overrule the consequences of the exposure effect unless it is large in either domain (positive or negative). The condition of the proposition (i.e., intermediate range of the quality difference) implies that neither the contrast nor the assimilation is large.

Finally, recall that in our model development, we discussed how search advertising affects the consumer perception by using some additional parameters: γ, ε, q^0, and δ. They can also affect the advertising decision of firms. The following proposition starts with the effect of γ.

Proposition 14 (Effect of γ) *As γ increases, Firm H becomes more likely to buy the keyword when Firm L buys, while Firm L gets less likley to buy when Firm H buys. Thus, with larger γ, the YY scenario is observed for smaller quality difference.*

It is easy to understand the proposition if we recall how γ was defined. It represents the difference in the exposure benefit that both firms get from search advertising. Whenever γ gets larger, the relative exposure benefit of Firm H to that of Firm L also gets larger, given the same level of the exposure effect. Then Firm H has more incentive to match the purchase decision of its competitor Firm L, since with larger γ, Firm H gets more benefit by doing so. However, Firm L has less incentive to match, because in a competitive product market, what is good to Firm H is bad to Firm L. As a result, the thresholds for each firm's buying the keyword, defined in Propositions 5 and 6, now go down. This means that we observe both firms buying the keyword for lower values of quality difference, and thus more towards the region for the assimilation as opposed to the contrast. This is because Firm L needs more benefit from an assimilation to compensate for the loss due to larger γ, while Firm H can endure some loss from an assimilation. The opposite effect can be seen in the analysis of q^0 as in the following proposition.

Proposition 15 (Effect of q^0) *As q^0 increases, Firm L becomes more likely to buy the keyword when Firm H buys, while Firm H gets less likley to buy when Firm L buys. Thus, with larger q^0, the YY scenario is observed for larger quality difference.*

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With higher q^0 value, the same level of quality difference is more likely to lead to the assimilation as opposed to the contrast between the two brands. This change is harmful to Firm H but beneficial to Firm L. Thus, Firm L can be more aggressive in its advertising decision, while Firm H becomes more conservative. In addition, when q^0 takes higher value, the same level of context effect can be achieved by having larger quality difference from the definition: $E_2 \equiv \Delta q - q^0$, the YY scenario can be observed with larger q^0 when the quality difference also becomes greater.

Now we move to investigate the effect of ε. Because ε represents the loss of exposure benefit in the second slot relative to the first slot, we state the following proposition in the context of the YY scenario where both firms buy the keyword.

Proposition 16 (Effect of ε) *As ε increases, the first slot winner becomes more likely to buy the keyword, while the second slot winner gets less likley to buy.*

As the exposure benefit in the first slot relative to the second slot increases, both firms will strive for the first slot. If the first slot winner is already determined, the first slot winner is naturally more inclined to buy the keyword with more benefit of being at the first slot, while the second slot winner is less inclined to buy. Finally, δ has the following impact on the equilibrium outcome.

Proposition 17 (Effect of δ) *As δ increases, Firm L becomes more likely to buy the keyword when Firm L buys if and only if $\beta_2(E_1) \leq q^0$. On the other hand, as δ increases, Firm H becomes more likely to buy the keyword when Firm H buys if and only if $\beta_1(E_1) \geq q^0$.*

Recall that δ was defined to be the additional context effect in the YY scenario, compared to the context effect in the NY scenario. When $\beta_2(E_1) \leq q^0$, the context effect takes a form of an assimilation in the region satisfying $\beta_2(E_1) \leq \Delta q \leq q^0$ and thus, as δ increases, Firm L gets more benefits in the YY scenario compared to the YN scenario. In this region, Firm H gets hurt more in the YY scenario than in the NY scenario with larger δ. Therefore, as δ increases, Firm L has more incentive to buy while Firm H has less incentive, when the other firm buys the keyword. On the other hand, if $\beta_1(E_1) \geq q^0$, the context effect takes a form of a contrast in the region satisfying $q^0 \leq \Delta q \leq \beta_1(E_1)$ and thus, as δ increases, Firm H gets more benefits in the YY scenario compared to the NY scenario. Now in this case, Firm L gets hurt more in the YY scenario than in the YN scenario with larger δ. Thus, as δ increases, Firm H has more incentive to buy while Firm L has less incentive, when the other firm buys the keyword.

Discussion In this section, we have shown that two variables, the exposure effect and the quality difference can explain the keyword purchase pattern in equilibrium. Specifically, firms have an incentive to buy the keyword only when the cost of advertising is justified by the exposure benefit but even in that case, each firm buys only when the detrimental context effect is not present: if the quality difference is large and thus there exists a contrast between the two brands, Firm L refrains from buying the keyword, because the contrast effect hurts Firm L. On the other hand, if the quality difference is small and thus two brands are assimilated, Firm H refuses to buy the

keyword. If the quality difference is in the intermediate range so that neither context effect is harmful to neither firm, both firms buy the keyword at the same time. In addition, we found some interesting cases including the one where Firm H buys its own keyword just for the defensive purpose, and the one where Firm H effectively deters Firm L from buying its own keyword. Our investigation extends the advertising literature to the new media where more than two firms intentionally choose to advertise in one space, together with their competitors. While the research on the comparative advertising has highlighted the challenger's incentive to assimilate itself to the incumbent firm, our model allows for both contrast and assimilation and studies the firm's incentive to strategically make use of the opportunity to be compared with the competitor.

So far, we have seen how firms attempt to gain advantage over the competitor by making a participation decision in the search advertising market, in order to gain a better position in the product market competition. However, firms can also affect the product market outcome by making a proper bidding decision. This effect is especially relevant when firms are financially constrained. This issue will be investigated in more detail in the next section where we relax the assumption of unlimited budgets.

5. Analysis under Budget Constraint

In the previous section, we have assumed that advertisers have unlimited budgets. However, in reality, firms allocate a limited amount of budget to advertising for various reasons. In addition, search engines such as Google let the budget-constrained advertisers stipulate their daily budget to gain more control over their spending. Thus, it is important to consider advertisers with insufficient resources and investigate how their behavior changes due to financial constraints. In this section, we extend the model incorporating this aspect and discuss the consequence of having financial constraint in purchasing brand names as keywords.

As before, we have two stages: advertising and pricing. However, because the pricing equilibrium given the perceived quality $\Delta \tilde{q}$ remains the same, we only consider the advertising decisions. We start by assuming that the advertising budgets of both firms are binding at the first slot under the YY scenario. In particular, we assume

$$C_{i0} < K_i < \min\{\frac{2\varepsilon E_1\{3t + \gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0\}}{9t}, \frac{2\varepsilon E_1\{3t - \gamma E_1 - (2+\delta)\Delta q - (1+\delta)q^0\}}{9t}\} + C_{i2}.$$

This implies that scenarios YY1 (where both firms advertise and Firm H takes the first slot) and YY2 (where both firms advertise and Firm L takes the first slot) cannot last to the end of the designated period. In particular, in scenario YY1, Firm H can take the first slot for only $\dfrac{K_H}{C_{H1}}$ fraction of the time. Then, Firm H exhausts its budget and only Firm L is left to continue advertising until the end of the period. Note that Firm L has sufficient

budget to cover the cost of advertising in the NY scenario. Similarly, consider the YY2

scenario. Now it becomes YN after $\dfrac{K_L}{C_{L1}}$ fraction of the period, because Firm L drops out

due to budget constraint. Accordingly, we revise the perceived quality of each product

in the above two scenarios as follows:

$$\tilde{\tilde{q}}_i^{YY1} \equiv (\frac{K_H}{C_{H1}})\tilde{q}_i^{YY1} + (1 - \frac{K_H}{C_{H1}})\tilde{q}_i^{NY} \tag{16}$$

$$\tilde{\tilde{q}}_i^{YY2} \equiv (\frac{K_L}{C_{L1}})\tilde{q}_i^{YY2} + (1 - \frac{K_L}{C_{L1}})\tilde{q}_i^{YN} \tag{17}$$

where $\tilde{\tilde{q}}_i^S$ and $\tilde{q}_i^S$ respectively denote the perceived quality in scenario S with and

without budget constraint. Note that the expression for $\tilde{q}_i^S$ is given in Table 1. The other

scenarios are not affected by the budget constraint and thus $\tilde{\tilde{q}}_i^S = \tilde{q}_i^S$, ($S = YN, NY$).

With this model, we investigate both firms' bidding incentives and the participation

incentives in the next two subsections.

5.1 Bidding Decision

As in the main analysis, Firm H may choose the first slot or the second slot in

equilibrium. In this subsection, we characterize the bidding equilibrium in each case. We

first observe that unlike in the non-binding budget case, the bidding decision of the

second slot winner now affects its own profit. This creates a particular incentive for the

second slot winner in its bidding decision. The following proposition discusses this

incentive.

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Proposition 18 *(Raising Rival's Cost) In the bidding game, the second slot winner may increase*

its own bid as high as possible in order to raise the cost of the first slot winner. In particular,

(1) when Firm H takes the first slot, Firm L always has an incentive to increase its own bid if

$$\lambda_{11}(E_1) \le \Delta q \le \lambda_{12}(E_1);$$

(2) when Firm H takes the second slot, Firm H always has an incentive to increase its own bid if

$$\lambda_{21}(E_1) \le \Delta q \le \lambda_{22}(E_1); \text{ where}$$

$$\lambda_{11}(E_1) \equiv \frac{(3t + 3q^0 + 2\delta q^0)\delta - \{2 + \delta + 2(\gamma + \varepsilon)(1 + \delta)\}E_1 - \sqrt{\{(2 + 2\gamma + 2\varepsilon + \delta)E_1 + (3t - q^0)\delta\}^2 - 36t(\delta + 2)(C_{L0} - C_{L2})}}{2\delta(2 + \delta)}$$

$$\lambda_{12}(E_1) \equiv \frac{(3t + 3q^0 + 2\delta q^0)\delta - \{2 + \delta + 2(\gamma + \varepsilon)(1 + \delta)\}E_1 + \sqrt{\{(2 + 2\gamma + 2\varepsilon + \delta)E_1 + (3t - q^0)\delta\}^2 - 36t(\delta + 2)(C_{L0} - C_{L2})}}{2\delta(2 + \delta)}$$

$$\lambda_{21}(E_1) \equiv \frac{-(1 + \delta)(3t - 3q^0 - 2\delta q^0) + \{2 + \delta - (1 + \delta)\gamma + (3 + 2\delta)\varepsilon\}E_1}{2(1 + \delta)(2 + \delta)}$$

$$-\frac{\sqrt{\{(2 + \gamma + \delta + \varepsilon + \gamma\delta)E_1 + (1 + \delta)(3t + q^0)\}^2 - 36t(\delta + 1)(\delta + 2)(C_{H0} - C_{H2})}}{2(1 + \delta)(2 + \delta)}$$

$$\lambda_{22}(E_1) \equiv \frac{-(1 + \delta)(3t - 3q^0 - 2\delta q^0) + \{2 + \delta - (1 + \delta)\gamma + (3 + 2\delta)\varepsilon\}E_1}{2(1 + \delta)(2 + \delta)}$$

$$+\frac{\sqrt{\{(2 + \gamma + \delta + \varepsilon + \gamma\delta)E_1 + (1 + \delta)(3t + q^0)\}^2 - 36t(\delta + 1)(\delta + 2)(C_{H0} - C_{H2})}}{2(1 + \delta)(2 + \delta)}.$$

In the previous section, we have seen each firm strategically making a

participation decision in the keyword auction with a foresight on the outcome of the

product market competition. Likewise, firms can also strategically set their bid amount

in order to influence the competitive horizon of the product market. In particular, the

proposition shows that firms may have an incentive to increase their own bid amount

and thus the cost of their budget-constrained rival. Raising the rival's cost can be profitable in this context, because the high cost of advertising may run up the competitor's advertising budget. By exhausting the competitor's budget, the firm is able to constrain the period during which the competitor's advertisement is shown and thus, restrict the competitor's ability to charge higher prices by an increase in perceived quality. This results in a decrease in the competitor's profit[1] but an increase in the firm's own profit.

Note that this strategy can be implemented because the search engine uses the generalized second price auction rule, where the payment of the winner depends on the next winner's bid amount. Thus, only the second slot winner can effectively cause damage to its competitor in the first slot, but not vice versa. There also has been an empirical observation of bid jamming, where the second slot winners try to decrease the gap between their own bid and the first slot winners' bid. (Ganchev et al. 2007).

The proposition also shows when firms may have such an incentive. Our investigation of the condition and the parameter space reveals that the incentive exists in most cases but may not apply to Firm H when it gets large benefit from being compared with the competitor (i.e., with high contrast effect) nor to Firm L when it gets small exposure benefit from advertising. When the contrast effect is large, Firm H may prefer

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[1]Each condition in the proposition also guarantees $\dfrac{\partial \Pi_H^{YY1}}{\partial C_{H1}} < 0$ and $\dfrac{\partial \Pi_L^{YY2}}{\partial C_{L1}} < 0$, respectively.

to be compared and thus stay in the YY scenario rather than exhausting the competitor's

budget and thus being left alone. When the exposure effect is small, the incremental

benefit to Firm L from the YY to the NY scenario is not large that Firm L may not find it

profitable to advertise alone by running up the competitor's budget. Thus, even though

the strategy is generally effective in obtaining some strategic advantage in the product

market competition, we may not observe this behavior all the time.

Recall that with unlimited budget, we specified the solution up to an interval.

One implication of the above proposition is that by considering budget-constrained

advertisers, we can actually pin down their bidding strategies in equilibrium. The

following corollaries in particular, define the strategy of the second slot winner in

equilibrium under the condition specified in Proposition 18.

Corollary 1 *Suppose $\lambda_{11}(E_1) \le \Delta q \le \lambda_{12}(E_1)$. Under budget constraint, Firm H takes the first*

slot in equilibrium if $A(E_1, \Delta q) > 0$, where $A(E_1, \Delta q)$ is defined in the Appendix. In this

equilibrium, the optimal bidding strategy of Firm L is given by $b_L^* = \bar{b}_L$, where $\bar{b}_L$ is the solution

to

$$\{3t + (\frac{K_H}{s_1 r_L \bar{b}_L})\Delta \tilde{q}^{YY1} + (1 - \frac{K_H}{s_1 r_L \bar{b}_L})\Delta \tilde{q}^{NY}\}^2 - 18tK_H$$

$$= \{3t + (\frac{K_L}{s_1 r_L \bar{b}_L})\Delta \tilde{q}^{YY2} + (1 - \frac{K_L}{s_1 r_L \bar{b}_L})\Delta \tilde{q}^{YN}\}^2 - 18t\{(\frac{K_L}{s_1 r_L \bar{b}_L})C_{L2} + (1 - \frac{K_L}{s_1 r_L \bar{b}_L})C_{L0}\}.$$

Corollary 2 *Suppose* $\lambda_{21}(E_1) \leq \Delta q \leq \lambda_{22}(E_1)$. *Under budget constraint, Firm L takes the first slot in equilibrium if* $A(E_1, \Delta q) < 0$. *In this equilibrium, the optimal bidding strategy of Firm H is given by* $b_H^* = \bar{b}_H$, *where* $\bar{b}_H$ *is the solution to*

$$\{3t - (\frac{K_L}{s_1 r_H \bar{b}_H})\Delta \tilde{q}^{YY2} - (1 - \frac{K_L}{s_1 r_H \bar{b}_H})\Delta \tilde{q}^{YN}\}^2 - 18t K_L$$

$$= \{3t - (\frac{K_H}{s_1 r_H \bar{b}_H})\Delta \tilde{q}^{YY1} - (1 - \frac{K_H}{s_1 r_H \bar{b}_H})\Delta \tilde{q}^{NY}\}^2 - 18t\{(\frac{K_H}{s_1 r_H \bar{b}_H})C_{H2} + (1 - \frac{K_H}{s_1 r_H \bar{b}_H})C_{H0}\}.$$

As noted above, because of the budget constraint, the second slot winner is now able to affect the duration of its competitor's advertisement and thus the timing of the transition from one scenario to another. Under the conditions of Proposition 18, because the second slot winner is better off by being advertised alone, it chooses to minimize the duration of the YY scenario. Hence we observe the second slot winner's choosing the maximum bid amount in its range. Note that the first slot winner's incentive compatibility condition defines this maximum bid amount.

From the equilibrium bid amount, we derive the first slot winner's equilibrium costs as $C_{H1}^* = s_1 r_L \bar{b}_L$ and $C_{L1}^* = s_1 r_H \bar{b}_H$. The second slot winner's advertising cost is fixed at $s_2 r_X b_X$ by assumption. Based on this calculation, we now move to see the impact of budget constraint on the participation incentives of both firms in the next subsection.

5.2 Participation Decision

Given this analysis of the bidding equilibrium in the YY scenario with budget constraint, we investigate the impact of the budget constraint on the participation

decision. As we have seen in our main analysis, four different scenarios can be obtained

as an equilibrium. However, given the complexity of the problem, instead of discussing

each equilibrium in detail, we examine how the participation incentive changes due to

the budget constraint. In this analysis, we focus on the firm taking the first slot in the

bidding equilibrium. The following proposition investigates the incentive of this firm.

Proposition 19 *When budgets are binding, the firm taking the first slot in the equilibrium*

without budget constraint becomes less likely to buy the keyword against the purchase of the

other firm if the second slot winner makes a more aggressive bid in the bidding game than

without budget constraint. Even when the second slot winner makes a less aggressive bid, if the

exposure effect is large enough, the first slot winner loses some incentive to buy the keyword. In

particular,

(1) in an equilibrium involving H-L-X order, Firm H is less likely to buy the keyword against the

purchase of Firm L if $C_{H1}^{**} \geq C_{H1}^{*}$. *When* $C_{H1}^{**} < C_{H1}^{*}$, *Firm H is still less likely to buy if*

$$E_1 > \frac{1}{2(1+\gamma+\varepsilon)+\delta}\{-(3t-q^0)\delta + \frac{3\sqrt{t}\{\kappa_1(4+\kappa_1\delta)C_{H1}^{*}-(4+\delta)K_H\}}{\sqrt{2\kappa_1(1-\kappa_1)(\kappa_1 C_{H1}^{*}-K_H)}}\},$$

(2) in an equilibrium involving L-H-X order, Firm L is less likely to buy the keyword against the

purchase of Firm H if $C_{L1}^{**} \geq C_{L1}^{*}$. *When* $C_{L1}^{**} < C_{L1}^{*}$, *Firm L is still less likely to buy if*

$$E_1 > \frac{1}{2+\gamma+\delta+\gamma\delta+\varepsilon}\{(3t-q^0)(1+\delta) + \frac{3\sqrt{t}\{\kappa_2(2+\kappa_2(1+\delta))C_{L1}^{*}-(3+\delta)K_L\}}{\sqrt{2\kappa_2(1-\kappa_2)(\kappa_2 C_{L1}^{*}-K_L)}}\},$$

where $\kappa_1 = \dfrac{C_{H1}^{**}}{K_H}$, $\kappa_2 = \dfrac{C_{L1}^{**}}{K_L}$, *and* C_{i1}^{*} *and* C_{i1}^{**} *respectively represent the advertising cost of*

Firm i *in the first slot without and with budget constraint.*

The proposition discusses the impact of the budget constraint on the advertising incentive of the first slot winners in the original equilibrium (without budget constraint). In particular, it states that this firm loses some incentive to buy the keyword when it is financially constrained than when it is not, under some conditions. First, if the equilibrium bid of the second slot winner is higher than without the budget constraint, the first slot winner's cost becomes higher and thus its incentive to buy the keyword becomes smaller.

Now suppose the first slot winner's advertising cost is smaller than without the budget constraint. Consider a point in the ($E_1, \Delta q$) space that falls in the YY region without budget constraint and satisfies the conditions in the proposition. Here, Firm H earns higher profit in the YY scenario than in the NY scenario. However, due to the budget constraint, the YY scenario cannot sustain till the end of the period but it transitions to the less profitable NY scenario. Then, compared to no budget constraint case, YY becomes less profitable to Firm H and it is possible that Firm H prefers NY to YY at the point under consideration. Same intuition holds true for Firm L. Thus, the budget constraint deprives the firms of the incentive to buy the keyword. However, this is true only when the expousre effect is large enough. This is because the reduction in the profit under budget constraint is greater with larger exposure effect, which cancels

out the gain from the reduction in the advertising cost.On the other hand, when the exposure effect is small, the gain from reduced cost is greater than the loss from budget constraint, and thus the first slot winner may have more incentive to buy the keyword.

One implication of the proposition is that when budgets are binding, the first slot winner becomes less likely to participate in the advertising game and thus, more likely to give in the first slot to the other firm. Thus, we will observe the YY scenario less often but the YN or NY scenario more frequently. The proposition also implies that as a result, it becomes easier for one firm to deter its competitor from buying the keyword, under budget constraint.

So far, we theoretically investigated the incentives of firms with and without budget constraint. In the next section, we move to examine these issues by experimental studies.

6. Empirical Investigation

In this section, we report three empirical studies that investigate the validity of our theoretical model and its predictions. First recall that in our theoretical model, we described the change in perceived quality due to advertisement, using the following two effects: (1) the exposure effect to capture the consumer's tendency to positively revise their quality perception after being exposed to the advertisement, and (2) the context effect to capture the bias of relatively evaluating two objects that are shown together. Although these are well-documented phenomena in the literature, it is important to confirm the existence of these effects in the context of search advertising. Thus, in our empirical investigation, we first show the significance of the estimates of these effects from the experimental data. At the same time, we aim to better understand the search engine users' actual behavior by further analyzing these data. Finally, based on experimental data and some other data sources, we validate the model prediction regarding the firm's keyword purchase decision. More specifically, Proposition 4 states that when the exposure effect is large enough, three different outcomes are obtained in equilibrium depending on the quality difference; YN with large difference, YY with medium difference, and NY with small difference. Here we test these qualitative predictions. The rest of the section summarizes the procedure and the result of the empirical analysis that we conducted to these purposes.

6.1 Study 1

This study serves three purposes: (1) to establish the exposure effect and the context effect in the context of search advertising, (2) to show how the context effect is affected by the quality difference between the brand owner and the competitor, and (3) to show an example of Proposition 4. In particular, we test the following three hypotheses:

$H1$: $E_1 > 0$

$H2$: $E_2 > 0$ if the quality difference is large, but $E_2 < 0$ if the quality difference is small.

$H3$: When E_1 is large enough, in equilibrium, YN is likely to be observed with $\Delta q > q^0$, while NY is likely to be observed with $\Delta q < q^0$.

We start by describing the experimental procedure.

Method We conducted an experiment where each participant was asked to rate the quality of three products on a seven point scale, after being exposed to a search engine result page. The study used a two (Brand owner's advertisement: Shown or Not shown) by three (Competitor's advertisement: High-quality shown, Low-quality shown, or None shown) between-subjects design. Here the "high" quality competitor is still of lower quality than the brand owner. We used the keyword `Sony TV'. Participants who were in the Brand owner's advertisement Shown condition were exposed to a Sony advertisement on the search page, and those in the Not shown condition were not

shown the advertisement. Participants who were in the Competitor's advertisement

High-quality shown condition were exposed to a Panasonic advertisement, those in the

Low-quality shown condition were exposed to a Haier advertisement, and those in the

None shown condition were not exposed to any competitor's advertisement. These

stimuli are presented in Figure 7. All participants gave ratings for Sony, Panasonic, and

Haier. To ensure that Panasonic and Haier indeed are perceived differently in quality,

we tested whether their quality ratings are different using the ratings given in the Brand

owner advertisement Not shown by Competitor's advertisement None shown condition.

As expected, participants gave different ratings for Panasonic and Haier ($\tilde{q}_P = 4.19$,

$\tilde{q}_H = 2.98$ respectively; $t = 6.01$, $p < 0.001$). We also confirm in this cell that Sony

indeed has higher quality than its competitors ($\tilde{q}_S = 4.19$; $t = 5.55$, $p < 0.001$ in

comparison with Panasonic: $t = 14.27$, $p < 0.001$ in comparison with Haier).

Figure 7: Stimuli used in Study 1

Participants We obtained responses from 286 participants who participated in the study through the online survey web site, www.qualtrics.com.[1] They were randomly assigned to one of six experimental conditions.

Model We tested the first two hypotheses directly by estimating each parameter value in out theoretical model using the quality rating response data. In doing so, we slightly change the notations of our theoretical model as follows. First, we denote the baseline quality of each brand by q_i and the exposure effect by E_1^i. i can be any one of S, P, and H, which respectively represent, Sony, Panasonic, and Haier. Competitor i's exposure effect in the YY scenario is denoted by repeating the superscript, i.e., by E_1^{ii}. Note that the effect of ε of the theoretical model is captured by the difference between E_1^i and E_1^{ii}. We also pooled the effect of γ into Sony's exposure effect E_1^S in this notation. The context effect is given by E_2^i in the case where Sony interacts with brand i ($i = P$ or H). In representing the difference in the context effect between the NY and the YY scenarios, we use E_2^i for the context effect in the NY scenario and E_2^{ii} for that in the YY scenario. The latter notation contains the effect of δ of the theoretical model.[2] Table 3 summarizes the discussion on notations by presenting each firm's perceived quality in

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[1]This study was conducted at the end of another (unrelated) study using the same panel.

[2]To see the conversion of notations from Table 1 to Table 2, observe that E_1^S is defined to be equivalent to $(1+\gamma)E_1$, E_1^{PP} and E_1^{HH} are to $(1-\varepsilon)E_1$, E_2^P and E_2^H are to $\dfrac{1}{2}E_2$, and E_2^{PP} and E_2^{HH} are to $\dfrac{1}{2}(1+\delta)E_2$.

each cell of our experimental design. In each cell, the first row corresponds to the perceived quality of Sony, the second row to that of Panasonic, and the third row to that of Haier. In all, we have 12 parameters to be estimated in our empirical model.

Table 3: Empirical Model of the Perceived Quality by Advertising Scenario

		Competitor's Advertisement		
		H Quality Shown	L Quality Shown	None Shown
		$q_S + E_1^S + E_2^{PP}$	$q_S + E_1^S + E_2^{HH}$	$q_S + E_1^S$
	Shown (Y)	$q_P + E_1^{PP} - E_2^{PP}$	q_P	q_P
Owner's		q_H	$q_H + E_1^{HH} - E_2^{HH}$	q_H
Ad		$q_S + E_2^P$	$q_S + E_2^H$	q_S
	Not Shown (N)	$q_P + E_1^P - E_2^P$	q_P	q_P
		q_H	$q_H + E_1^H - E_2^H$	q_H

Estimation We estimate our empirical model by pooling all the responses on the quality ratings of three different brands, thus utilizing 858 observations (286 participants × 3 responses) on the product quality, based on the regression equation that matches the actual response on the left-hand side to the theoretical decomposition of the quality in each cell (as given in Table 3) on the right-hand side. However, because one participant provides three observations, these three observations may be correlated among themselves, while independent from other observations. In this case, running a simple OLS regression would produce inefficient estimates, although unbiased. We therefore, suggest the following model that explicitly takes into account of the participant-specific effect:

$$\tilde{q}_{in} = x_{in}\beta + u_n + \varepsilon_{in} \qquad (18)$$

where $\beta \equiv [q_S \ q_P \ q_H \ E_1^S \ E_1^P \ E_1^{PP} \ E_1^H \ E_1^{HH} \ E_2^P \ E_2^{PP} \ E_2^H \ E_2^{HH}]'$ is the vector of

parameters to be estimated, $\tilde{q}_{in}$ is the quality rating of brand i by participant n, and x_{in}

is the vector of coefficients of parameters in the empirical model in Table 3 for brand i

in the cell that participant n is assigned.[3] Importantly, u_n captures the participant-

specific unobserved heterogeneity, which represents different scales (i.e., levels) used by

each individual. Finally, ε_{in} is the idiosyncratic error term of the regression, which is

assumed to follow normal distribution with mean zero and variance σ_ε^2.

We can think of two different ways of estimating this model. First, we can see the

model as a random effects model, if we treat the invidual effect u_n to be randomly

distributed across participants, with mean zero and variance σ_u^2 and uncorrelated with

the error ε_{in}. Here the sum of the individual random effect and the error, $u_k + \varepsilon_{ik}$,

works as the regression error term and we can explicitly specify the covariance structure

of this error term as follows:

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[3] For example, if participant n is assigned to cell YH, the emprical model for Sony's quality
rating is given by $q_S + E_1^S + E_2^{PP}$. Then, the corresponding x_{in} vector is given by
    [1   0   0   1   0   0   0   0   0   1   0   0   ]
for each corresponding component:
    (q_S q_P q_H E_1^S E_1^P E_1^{PP} E_1^H E_1^{HH} E_2^P E_2^{PP} E_2^H E_2^{HH}).

$$\Omega = \begin{bmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{bmatrix}, \text{ where } \Sigma = \begin{bmatrix} \sigma_\varepsilon^2 + \sigma_u^2 & \sigma_u^2 & \sigma_u^2 \\ \sigma_u^2 & \sigma_\varepsilon^2 + \sigma_u^2 & \sigma_u^2 \\ \sigma_u^2 & \sigma_u^2 & \sigma_\varepsilon^2 + \sigma_u^2 \end{bmatrix}. \tag{19}$$

This model can be estimated by the Feasible Generalized Least Squares (FGLS), where

we first obtain the estimates of σ_ε^2 and σ_u^2 by the use of any consistent estimator of β

(Greene 2003) and then use these estimates in constructing the covariance matrix to

estimate β by the Generalized Least Squares (GLS) method.

The second way is to see the model as the fixed effects model, where each

invidual effect u_n is treated to be a constant. In this case, simply by subtracting the

group mean from both sides of the equation, we can remove this individual-specific

constant and thus, efficiently estimate model parameters by running an OLS on the

following equation:

$$\tilde{q}_{in} - \bar{\tilde{q}}_i = (x_{in} - \bar{x}_i)\beta + \varepsilon_{in} - \bar{\varepsilon}_i. \tag{20}$$

This is called the within-group estimator, or the least squares dummy variable estimator

of the fixed effects model. The downside of this estimator is that we lose one degree of

freedom and thus we estimate one set of parameters up to their difference from each

other. (In our application, one of q_S, q_S, and q_S is normalized while the other two are

estimated as the difference from this normalized variable.)

The two estimation methods thus far discussed were shown to be able to deal

with the problem of the unobserved participant-specific effect. However, given that

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participants gave ratings of Sony first and then Panasonic and Haier, it may be

appropriate to reflect the order of responses on the esimtation. Threfore, we propose the

following estimation method by integrating the ideas of the above two methods. For the

Sony ratings, we use equation (18) as it is. However, for the other two brands, because

they were answered with an anchor on the Sony's quality rating which has been already

given, we use the difference in the quality ratings of these brands from that of Sony and

have the following equation:

$$\tilde{q}_{in} - \tilde{q}_{Sn} = (x_{in} - x_{Sn})\beta + \varepsilon_{in} - \varepsilon_{Sn}, (i = P, H) \qquad (21)$$

Note that in equation (21), the participant-specific effect is already taken out. Table 4

summarizes the actual estimation equations in each cell for each brand with this

estimation. Then, the regression error has the following structure:

$$\Omega = \begin{bmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{bmatrix}, \text{ where } \Sigma = \begin{bmatrix} \sigma_{\varepsilon}^2 + \sigma_u^2 & \sigma_{\varepsilon}^2 & \sigma_{\varepsilon}^2 \\ \sigma_{\varepsilon}^2 & 2\sigma_{\varepsilon}^2 & \sigma_{\varepsilon}^2 \\ \sigma_{\varepsilon}^2 & \sigma_{\varepsilon}^2 & 2\sigma_{\varepsilon}^2 \end{bmatrix}, \qquad (22)$$

and we estimate this model by FGLS. When we explain the estimation results in the next

subsection, we use the results obtained from this estimation method. In the Appendix,

we also present the esimtation results from OLS, the random effects model, and the fixed

effects model (the "within" estimator) for comparison.

$$\tilde{q}_S^{YH} = q_S + E_1^S + E_2^{PP} + \epsilon_S$$

$$\tilde{q}_P^{YH} - \tilde{q}_S^{YH} = q_P - q_S + E_1^{PP} - E_1^S - 2E_2^{PP} + \epsilon_P - \epsilon_S$$

$$\tilde{q}_H^{YH} - \tilde{q}_S^{YH} = q_H - q_S - E_1^S - E_2^{PP} + \epsilon_H - \epsilon_S$$

$$\tilde{q}_S^{NH} = q_S + E_2^P + \epsilon_S$$

$$\tilde{q}_P^{NH} - \tilde{q}_S^{NH} = q_P - q_S + E_1^P - 2E_2^P + \epsilon_P - \epsilon_S$$

$$\tilde{q}_H^{NH} - \tilde{q}_S^{NH} = q_H - q_S - E_2^P + \epsilon_H - \epsilon_S$$

$$\tilde{q}_S^{YL} = q_S + E_1^S + E_2^{HH} + \epsilon_S$$

$$\tilde{q}_P^{YL} - \tilde{q}_S^{YH} = q_P - q_S - E_1^S - E_2^{HH} + \epsilon_P - \epsilon_S$$

$$\tilde{q}_H^{YL} - \tilde{q}_S^{YH} = q_H - q_S + E_1^{HH} - E_1^S - 2E_2^{HH} + \epsilon_H - \epsilon_S$$

$$\tilde{q}_S^{NL} = q_S + E_2^H + \epsilon_S$$

$$\tilde{q}_P^{NL} - \tilde{q}_S^{NH} = q_P - q_S - E_2^H + \epsilon_P - \epsilon_S$$

$$\tilde{q}_H^{NL} - \tilde{q}_S^{NH} = q_H - q_S + E_1^H - 2E_2^H + \epsilon_H - \epsilon_S$$

$$\tilde{q}_S^{YN} = q_S + E_1^S + \epsilon_S$$

$$\tilde{q}_P^{YN} - \tilde{q}_S^{YN} = q_P - q_S - E_1^S + \epsilon_P - \epsilon_S$$

$$\tilde{q}_H^{YN} - \tilde{q}_S^{YN} = q_H - q_S - E_1^S + \epsilon_H - \epsilon_S$$

$$\tilde{q}_S^{NN} = q_S + \epsilon_S$$

$$\tilde{q}_P^{NN} - \tilde{q}_S^{NN} = q_P - q_S + \epsilon_P - \epsilon_S$$

$$\tilde{q}_H^{NN} - \tilde{q}_S^{NN} = q_H - q_S + \epsilon_H - \epsilon_S$$

Finally, one may consider the possibility that the errors are heteroscedastic: because some participants give more extreme answers than others, the within-participant variance of responses may vary across participants, even after taking out individual level effect. In this case, the model remains to be the same but the error term

ε_{in} now has the individual-specific variance: σ_n^2. Thus the covariance strucutre in case

of our last approach becomes,[4]

$$\Omega = \begin{bmatrix} \Sigma_1 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_{286} \end{bmatrix}, \text{ where } \Sigma_n = \begin{bmatrix} \sigma_n^2 + \sigma_u^2 & \sigma_n^2 & \sigma_n^2 \\ \sigma_n^2 & 2\sigma_n^2 & \sigma_n^2 \\ \sigma_n^2 & \sigma_n^2 & 2\sigma_n^2 \end{bmatrix}. \tag{23}$$

However, given the number of observations from one individual (which is three), it is

very hard for σ_n^2 to be consistently estimated, because consistency requires that the

number of observations goes to infinity. Thus, the estimates under heteroscedastic errors

are not reliable. Nontheless, we estimate this model and report the result in the

Appendix.

Results The parameter estimates are reported in Table 5. First, the quality

ratings are positive and are in the order of Sony, Panasonic, and Haier, which is

consistent with our design. We also find the exposure effects of competitor brands to be

positive and (marginally) significant in every case ($p = 0.0036$ for E_1^{PP}; $p = 0.0930$ for

E_1^{HH}; $p = 0.0807$ for E_1^{P}; $p = 0.0818$ for E_1^{H}). This result supports the existence of

---

[4] In case of the random effects model, the covariance structure is given by,

$$\Omega = \begin{bmatrix} \Sigma_1 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_{286} \end{bmatrix}, \text{ where } \Sigma_n = \begin{bmatrix} \sigma_n^2 + \sigma_u^2 & \sigma_u^2 & \sigma_u^2 \\ \sigma_u^2 & \sigma_n^2 + \sigma_u^2 & \sigma_u^2 \\ \sigma_u^2 & \sigma_u^2 & \sigma_n^2 + \sigma_u^2 \end{bmatrix}.$$

exposure effect ($H1$). However, the exposure effect of the brand owner (Sony) is -0.1221

and is not significantly different from zero ($p = 0.2195$). There may not be much

additional gain in terms of perceived quality by advertising to those who already

recognize Sony as a high-quality brand. We can interpret this as the differentiation factor

γ's being close to -1. This result suggests that the brand owner does not always benefit

from exposure of the advertisement.

Table 5: Parameter Estimates

Parameter	Estimates	Standard errors	p values
q_S	5.3432	0.7147	< .0001
q_P	3.3914	1.4124	0.0082
q_H	2.0748	1.4144	0.0712
E_1^S	-0.1221	0.1577	0.2195
E_1^P	0.6897	0.4924	0.0807
E_1^{PP}	1.2251	0.4551	0.0036
E_1^H	0.6994	0.4806	0.0818
E_1^{HH}	0.6314	0.4775	0.0930
E_2^P	0.1195	0.1911	0.2660
E_2^{PP}	0.3005	0.1722	0.0259
E_2^H	0.3349	0.1876	0.0545
E_2^{HH}	0.2991	0.1793	0.0476

The last column represents the p values in a one-tailed test.

Next we examine the context effect. We expect a contrast in a brand that is far

apart in quality from the high-quality brand owner but an assimilation in a brand that is

close to the high-quality brand owner. Our results partially support these expectations.

Specifically, we test the hypotheses: $E_2^P < 0$, $E_2^{PP} < 0$, $E_2^H > 0$, and $E_2^{HH} > 0$. When

testing the hypotheses, we reject the null hypothesis that Haier's context effect is greater

than zero ($p = 0.0545$ for $E_2^H > 0$; $p = 0.0476$ for $E_2^{HH} > 0$). Thus, the contrast effect

for Haier is confirmed. However, the current hypotheses for Panasonic cannot be tested,

since the estimates are positive. Now given that $\hat{E}_2^P$ and $\hat{E}_2^{PP}$ are positive, we test

revised hypotheses: $E_2^P > 0$ and $E_2^{PP} > 0$, in which case the null hypothesis for the latter

is rejected ($p = 0.0259$), while that for the former is not ($p = 0.2660$). Thus, even for

Panasonic, there is a weak contrast effect, rather than an assimilation effect. It may be the

case that Panasonic is not close enough to Sony in the quality dimension. Another

observation of this result is that the contrast effect found when only the competitor

advertises is larger for Haier than for Panasonic (i.e., $\hat{E}_2^H > \hat{E}_2^P$: $F_{(1,841)} = 3.4741$,

$p = 0.0627$). However, the contrast effects when both firms advertise were not

significantly different from each other ($F_{(1,841)} = 0.0030$, $p = 0.9563$). This provides

partial support for our conjecture that the contrast effect is more likely to be obtained

with larger quality difference.

Finally, given the current estimate, we can predict the purchase pattern of both

firms. According to Proposition 4, if the quality difference is large, then YN is likely to

be obtained while NY is likely to be obtained if it is small. Between Sony and Panasonic,

Δq ($\equiv q_S - q_P = 1.9518$) is greater than q^0 ($\equiv \Delta q - E_2^P = 1.8323$) and between Sony

and Haier, Δq ($\equiv q_S - q_H = 3.2684$) is also greater than q^0 ($\equiv \Delta q - E_2^H = 2.9335$).

Thus in both cases, YN is likely to be observed in equilibrium. In Google, from

September 2008 until March 2010, it has been the case that only Sony appears in the

sponsored link under the keyword `Sony TV' and neither Panasonic nor Haier has ever

appeared on that page. Therefore, the qualitative prediction from our model is

consistent with the empirical observation.

In sum, in Study 1, we have confirmed the existence of the exposure effect and

the contrast effect by experimental data and showed that the general specification of the

quality in our theoretical model is well-supported. Moreover, from parameter estimates,

we showed that the keyword purchase pattern predicted by our model is confirmed by

the actual observation. However, in this study, we were not able to show the existence of

the assimilation effect that might occur if there were small quality differences between

the two brands. Thus in the following subsection, we present another study to show the

assimilation effect.

6.2 Study 2

In this study, we replicate the prior study with a different product category in

order to illustrate the assimilation effect that we did not observe there. To this purpose,

we first conducted a pretest where we asked participants to rate the quality of the

hybrid car brands. Based on the result of this pretest, we decided to use the brand

Toyota Prius as the brand owner and Honda Insight as the competitor, because the difference in their quality ratings was relatively small but significant ($\Delta q = 0.86$, $p = 0.0005$).[5]

Method We repeated the same experimental procedure as in the prior study in a two (Brand owner's advertisement: Shown or Not shown) by two (Competitor's advertisement: Shown or Not shown) between-subject design. As noted above, we used the keyword `Prius'. The advertisements on the right-hand side were manipulated while the organic results on the left-hand side remained the same. Figure 8 presents these stimuli. After being exposed to this search engine result page, each participant gave ratings for Prius and Insight in order. A total of 719 participants gave responses through the online survey web site, www.qualtrics.com.

Model and Estimation We used the same model and the same estimation procedure as in Study 1. In particular, we use the same measurement model given in equation (18) which explicitly models the individual level effect, and consider the order of responses in the estimation of this model. While we use the equation (18) in estimating Prius's quality rating, we based our estimation of Insight's rating on its difference from Prius's rating as in equation (21), given that Insight's rating was made

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[5]Note that in Study 1, the estimated quality differences inducing the contrast effect were 1.9518 between Sony and Panasonic, and 3.2684 between Sony and Haier.

with an anchor on the Prius's rating. Then we estimate this model by FGLS with the following covariance structure of the error term:

$$\Omega = \begin{bmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma \end{bmatrix}, \text{ where } \Sigma = \begin{bmatrix} \sigma_\varepsilon^2 + \sigma_u^2 & \sigma_\varepsilon^2 \\ \sigma_\varepsilon^2 & 2\sigma_\varepsilon^2 \end{bmatrix}. \tag{24}$$
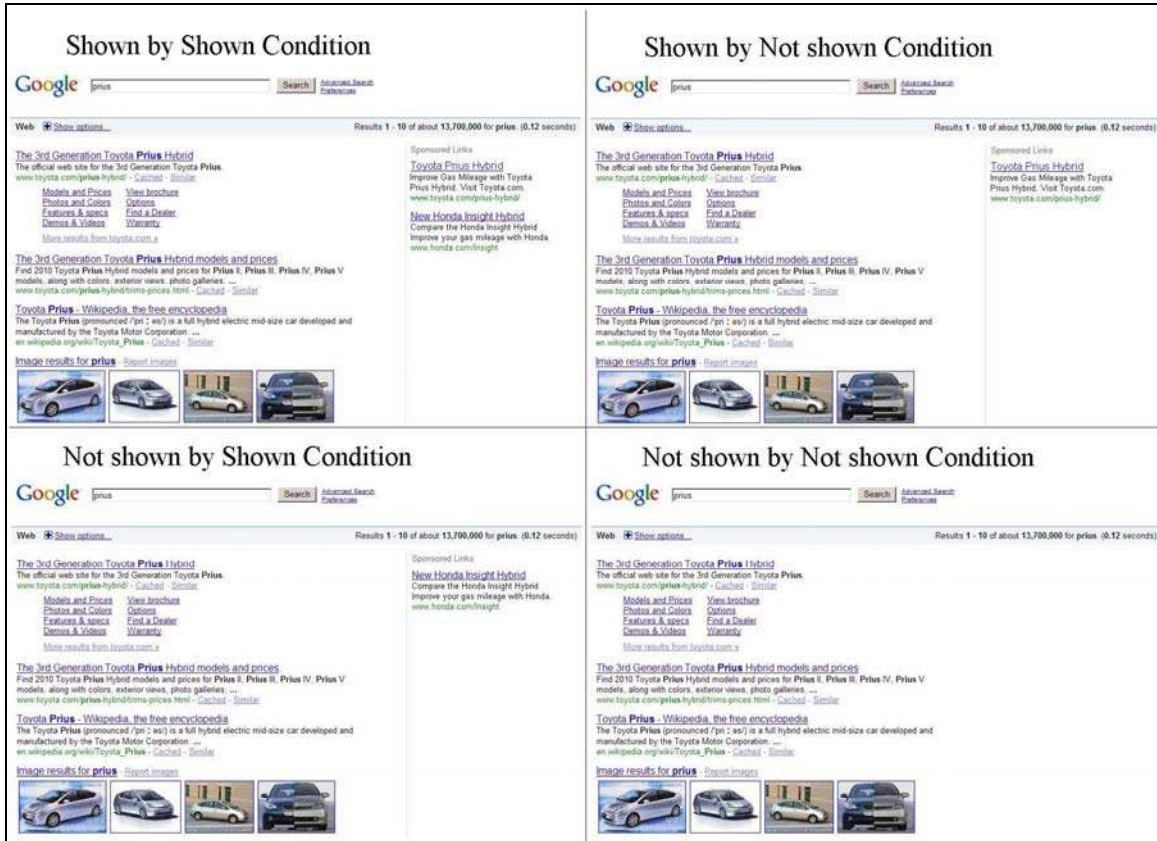

Figure 8: Stimuli used in Study 2

We discuss the estimation result from this approach in the next subsection. In addition, we also estimate the random effects model and the fixed effects model, and the model with heteroscedastic error as in the prior study. We report these estimation results in the Appendix. Finally, the empirical model and the estimating equations are respectively given in Tables 6 and 7. Here, note that P now represents Prius, while I is Insight. In this case, there are seven parameteres that need to be estimated (excluding the variances).

Table 6: Perceived Quality by Advertising Scenario

		Competitor's Advertisement	
		Shown (Y)	Not Shown (N)
Owner's	Shown (Y)	$q_P + E_1^P + E_2^{II}$	$q_P + E_1^P$
		$q_I + E_1^{II} - E_2^{II}$	q_I
Ad	Not Shown (N)	$q_P - E_2^I$	q_P
		$q_I + E_1^I - E_2^I$	q_I

Table 7: Estimating Equations

$$
\begin{aligned}
y_P^{YY} &= q_P + E_1^P + E_2^{II} + \epsilon_P \\
y_I^{YY} - y_P^{YY} &= q_I - q_P + E_1^{PP} - E_1^P - 2E_2^{II} + \epsilon_I - \epsilon_P \\
y_P^{NY} &= q_P + E_2^I + \epsilon_P \\
y_I^{NY} - y_P^{NY} &= q_I - q_P + E_1^I - 2E_2^I + \epsilon_I - \epsilon_P \\
y_P^{YN} &= q_P + E_1^P + \epsilon_P \\
y_I^{YN} - y_P^{YN} &= q_I - q_P - E_1^P + \epsilon_I - \epsilon_P \\
y_P^{NN} &= q_P + \epsilon_P \\
y_I^{NN} - y_P^{NN} &= q_I - q_P + \epsilon_I - \epsilon_P
\end{aligned}
$$

Table 8: Parameter Estimates

Parameter	Estimates	Standard errors	p values
q_P	4.6257	0.6500	$<.0001$
q_I	3.3790	1.2921	0.0045
E_1^P	0.1694	0.1047	0.0528
E_1^I	-0.0538	0.4843	0.4558
E_1^{II}	-0.0538	0.4727	0.4547
E_2^I	0.0507	0.1872	0.3933
E_2^{II}	-0.1936	0.1828	0.1447

The last column represents the p values in a one-tailed test.

Results The parameter estimates are reported in Table 8. The quality ratings are

consistent with the pretest result: Prius has slightly higher ratings than Insight with the

difference 1.2467 .[6] Now the exposure effect of Prius is positive and marginally

significant ($p = 0.0528$), while those of Insight are not different from zero. This suggests

that to those consumers looking at the search result associated with Prius, Insight may

not be as strongly recognized as Prius. Next the context effect in NY cell (i.e., E_2^I) is

positive but small and thus it is not significantly different from zero. However, the

context effect in YY cell (i.e., E_2^{II}) is negative and sizable, although not significant. In

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[6]The quality difference here is larger than the pretest result. This is because the current ratings are
taken after showing the participants the search engine result page associated with the keyword
`Prius', which reflects the fact that consumers are also affected by the organic results. In other
words, q_P contains the exposure effect associated with the organic search information.

fact, the random effets model estimates E_2^{II} to be (marginally) significantly negative.

(See Table 12, column (2), in the Appendix for this result.) This means that there exists a

weak assimilation between Prius and Insight when they are listed in parallel in the

sponsored section of the search result page. Compared with the case where only Insight

has an advertisement, if Prius also has an advertisement, consumers may infer that Prius

has reacted to its competitor's threat because they are direct competitors of similar

quality. Thus both brands can be assimilated in this case.

When we type `Prius' in Google search box, we observe both Prius and Insight

having advertisement there. This has been consistently observed for a significant period

of time (since March 2009). This is in line with our theoretical finding and the estimation

result above, because a weak assimilation effect is predicted to induce YY purchase

pattern (Proposition 4). In sum, in this study, we showed the existence of the weak

assimilation effect and again confirmed the model prediction on the keyword purchase

decision.

6.3 Study 3

In our previous studies, the focus was on showing the exposure effect and the

context effect by examining the search engine users' behavior, although we also showed

how the estimates can be linked to the empirical observations of firm behavior. In this

study, we move our focus more towards testing the model prediction. First recall that

Proposition 4 defined two thresholds in Δq; $\beta_1(E_1)$, below which only the competitor

buys the keyword and above which both firms buy the keyword, and $\beta_2(E_1)$, below

which both firms buy and above which only the brand owner buys the keyword. Using

some empirical observations from Google, we assess how likely these model predictions

can be validated. While the thresholds in Proposition 4 can be exactly specified with

fixed parameter values, it is hard, although not impossible, to obtain the estimates of all

the parameters for a certain pair of brands, or for a certain industry. Thus, instead of

deriving the thresholds, we take a maximum likelihood approach where we choose

these two thresholds that maximize the matching probability between the empirical

observation and the predicted purchase pattern and see how often our model cannot be

disproved.

Data We consider the product category of hybrid sedan and specifically choose

the following four brands: Toyota Prius, Honda Insight, Nissan Altima Hybrid, and

Ford Fusion Hybrid. We denote these brands respectively by P, I, A, and F. We use

two different sources of data in our study: the quality ratings data and the keyword

purchase patterns data. The former were obtained from a survey of 27 daytime MBA

students. In this survey, we asked them to imagine themselves to consider buying a

hybrid sedan and to rate the quality of the four brands. The mean quality ratings

obtained for each brand are as follows: $q_P = 6.19$, $q_I = 5.33$, $q_A = 5.04$, and $q_F = 4.33$.

Then we also collected the keyword purchase pattern for each keyword from Google

search results page. Note that these patterns were consistently observed two months

prior to and after the period the survey was taken, that is, September 2009. Table 9 summarizes the brand owner and the competitors who purchased each keyword.

Table 9: Keyword Purchase Patterns in Hybrid Sedans Category

Keyword	Brand Owner	Competitors
Prius	Toyota Prius	Honda Insight
Insight	Honda Insight	None
Altima Hybird	Nissan Altima Hybrid	Honda Insight, Ford Fusion Hybrid
Fusion Hybrid	Ford Fusion Hybrid	Honda Insight

Results There are 12 possible pairs with these four brands. Recall that our theoretical model dealt only with the case where the brand owner has higher quality than the competitor, although the result can easily be generalized to the other case. Thus, we first confine our interest to the cases where the brand owner has higher quality than the competitor. There are 6 such cases in out data and we investigate the matching probability of these cases by varying two thresholds β_1 and β_2. It turned out that the empirical observation matches the theoretical prediction in 83 percent of the time if the thresholds satisfy $0 < \beta_1 < 0.7$ and $0.86 < \beta_2 < 1$. Table 10 presents the details of this analysis.

Turing our attention to the case where the brand owner has lower quality than the competitor, we investigate the rest 6 cases. First, we can easily see how our theory can be generalized to this setting. Recall that the two brands are contrasted with large

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Δq but assimilated with small Δq. Now that the brand owner has lower quality, the contrast hurts the brand owner while the assimilation helps this firm. Then in equilibrium, with large Δq, only the competitor buys the keyword (i.e., NY scenario), while with small Δq, only the brand owner buys the keyword (i.e., YN scenario). With the intermediate value, both of them buy the keyword (i.e., YY scenario). We investigate how much this theoretical prediction can be supported by the empirical observations. Here we define another two thresholds: β_3 between the YN and the YY scenarios, and β_4 between the YY and the NY scenarios. Then it turned out that the empirical observation matches with the theoretical prediction in 66 percent of the time if the thresholds satisfy $0 < \beta_3 < 0.29$ and $1 < \beta_4 < 1.15$. The details of this analysis is also included in Table 10.

Across these two analyses, we find that with some specific values of the thresholds, the theoretical prediction can be matched with the empirical observation with the probability up to 83 percent and 66 percent respectively. Therefore, we confirm our prediction that we observe in equilibrium, NY, YY, and YN, in order as the quality difference becomes larger, if the brand owner's quality is greater than that of the competitor and that the order is reversed if the brand owner's quality is smaller than

that of the competitor.[7] While this result does not guarantee a perfect match between the

theory and the empirical observation, we can at least confirm that the general purchase

pattern generated by our model also holds true in the real world.

Table 10: Predicted vs. Observed Keyword Purchase Patterns

Brand owner	Competitor	Δq	Predicted	Observed
[when the brand owner has higher quality ($\beta_1 = 0.2$ and $\beta_2 = 0.9$)]				
Prius	Insight	0.86	YY	YY
Prius	Altima Hybrid	1.15	YN	YN
Prius	Fusion Hybrid	1.86	YN	YN
Insight	Altima Hybrid	0.29	YY	YN
Insight	Fusion Hybrid	1	YN	YN
Altima Hybrid	Fusion Hybrid	0.71	YY	YY
[when the brand owner has lower quality ($\beta_3 = 0.2$ and $\beta_4 = 1.1$)]				
Insight	Prius	0.86	YY	YN
Altima Hybrid	Prius	1.15	NY	NY
Altima Hybrid	Insight	0.29	YY	YY
Fusion Hybrid	Prius	1.86	NY	NY
Fusion Hybrid	Insight	1	YY	YY
Fusion Hybrid	Prius	0.71	YY	NY

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[7]Our empirical application includes all the possibilities: YY, YN, and NY. However, NN was not included because we are interested in the prediction of Proposition 4, which is silent about when the NN scenario can happen.

7. Conclusion

In this paper, we aim to understand when and why we observe different patterns of branded keyword purchase behavior. To investigate these issues, we developed a duopoly model where two firms offer horizontally and vertically differentiated products and engage in brand advertising in the search engine. We analyzed our theoretical model with and without budget constraint, investigated the search engine user behavior by experiments, and empirically tested the model prediction.

Our analysis helps us understand the questions we raised in the introduction.

1. When do we observe one pattern over the other? When the exposure effect is not large, neither firm buys the branded keyword because their cost of advertising is not justified. If the exposure effect is above a certain threshold, both firms become interested in buying the keyword and thus, the context effect comes into play. If the contrast effect is strong, only the brand owner is observed in equilibrium, but if the assimilation effect is strong, only the competitor appears in the sponsored links. If neither effect (contrast or assimilation) is large, we observe both firms advertising together. Because the impact of the context effect is asymmetric, the firm that is unfavorably affected by a strong contrast or assimilation effect gives up advertising, but both firms freely choose to advertise if neither effect is strong.

2. Why do brand owners buy their own branded keyword? Why do some firms forgo the opportunity to advertise under a well-known brand's name? When the

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exposure effect is weak and neither contrast nor assimilation effect is large, the brand owner buys its own keyword, but only for a defensive purpose. It may not want to advertise but does so only because its competitor advertises. The competitor's purchase may hurt the brand owner and thus it needs to defend itself by matching the competitor's purchase decision. However, if both the exposure effect and the contrast effect are strong enough, the brand owner may buy the keyword in order to preclude its competitor from doing the same. In fact, the preclusion is possible because the contrast effect is large enough. In this case, the competitor cannot buy because it is precluded, and not because it does not want to buy.

Moving towards the case where firms have limited advertising budgets, we identified an interesting pattern in the bidding equilibrium: when in the second slot, both firms may have an incentive to increase their own bid as high as possible in order to quickly exhaust the other firm's budget. By doing so, each firm can confine the profitability of the other firm. Given this derivation of bidding equilibrium, we investigated how firms revise their advertising strategies. We found that both firms have less incentive to purchase the keyword when they are financially constrained. This in turn, makes it less likely to observe both firms' advertising together. As a result, we may observe the defensive purchase of the brand owner less frequently, while preclusion may become more prevalent.

Finally, our experimental investigation shows that the exposure effect as well as both of the context effects exists. The parameter estimates from the experiment data predict the purchase pattern in equilibrium, which was consistent with the observed pattern in Google. Additionally, the quality data together with the empirical observation from Google search results reveals that our model prediction is reasonably well aligned with the reality.

Our work contributes to the search advertising literature in that this is the first attempt to model the impact of search advertising on brand value. Also, by expanding our model to consider the effects of the advertising on the product market, we offer insight on how firms behave optimally. The conditions derived for each equilibrium and for each particular case (i.e., the defensive purchase and the preclusion) provide a normative guideline on when to buy one's own keyword and when to buy a competitor's keyword. Most importantly, this work contributes to the theoretical advertising literature, in that it uncovered the firm's incentive in a situation where multiple firms simultaneously advertise in the same space, about which the literature has been silent. Finally, our analysis has a policy implication. By considering the contrast effect, we can identify the case where the brand owner is actually better off by a competitor's trademark infringement. This may help resolve some lawsuits against Google.

This paper is not without limitation. First, we considered a duopoly while in reality, in most cases, more than three firms are competing in the market and more importantly, keywords are usually bought by more than three firms. Related, we focused on a single keyword, but in fact, advertisers optimize over a large number of keywords. In this regard, it will be especially interesting to see how the results change when we consider both firm's keywords at the same time. Second, in our analysis, we consider the quality of the product as the important driver of the purchase decision. However, other variable, such as brand awareness can affect the keyword purchase decision. Considering other variables will be an interesting future research. Third, other advertisers that are not product market competitors, such as retailers, were ignored in our model. Its impact may be explicitly investigated. Finally, it will be an interesting future research to investigate how the consumer inference on the competitor's product quality is affected by the brand owner's product quality in various circumstances.

Appendix

Definition of b_X^0 in footnote 2

We define b_X^0 as follows: $b_X^0 \equiv \min\{b_{X1}^0, b_{X2}^0, b_{X3}^0\}$, where

$$b_{X1}^0 \equiv I^* \frac{(1+\gamma)\{3t(3+\delta)-q^0(1+\delta)\}\{3t(5+\gamma+3\delta(1+\gamma)+4\varepsilon)+k(1+\gamma)(1+\delta)\}}{18ts_1r_X(2+\gamma+\delta+\gamma\delta+\varepsilon)^2} + (1-I^*)\frac{(9t-q^0)(1+\gamma)\{3t(9+3\gamma)+q^0(1+\gamma)\}}{18ts_1r_X(3+2\gamma)^2}$$

with

$$I^* = I[\{2(1+\gamma-\varepsilon)+\delta\}\{3t((3-\delta)\gamma+3(1-\varepsilon))-q^0(1+\gamma(1+\delta)+2\delta-\varepsilon)\}\leq 0],$$

$$b_{X2}^0 \equiv I^* \frac{\{3t(7+2\delta+5\gamma+\delta\gamma-\varepsilon)-q^0(1+\gamma+(2+\gamma)\delta-\varepsilon)\}^2-(1+\gamma)^2\{3t(3+\delta)-q^0(1+\delta)\}^2}{18ts_1r_X(2+\gamma+\delta+\gamma\delta+\varepsilon)^2}$$

$$+ (1-I^*)\frac{9t^2(7\gamma^2+30\gamma+27)+(1+\gamma)^2q^0(18t-q^0)\}}{18ts_1r_X(3+2\gamma)^2},$$

$$b_{X3}^0 \equiv \frac{(3t-q^0)\delta\{3t(8+3\delta)+\delta q^0\}}{18ts_2r_X(2+\delta)^2}$$

and,

$$b_{X4}^0 \equiv \frac{q^0(1+\delta)\{6t+q^0(1+\delta)\}}{18ts_2r_X}$$

In fact, $b_X \leq b_{X1}^0$ ensures that Firm H buys the keyword in the presence of Firm X, while

$b_X \leq b_{X2}^0$ ensures that Firm L buys the keyword in the presence of Firm X. Similarly,

$b_X \leq b_{X3}^0$ ensures Firm H's purchase of the keyword in the presence of Firm L and Firm

X, while $b_X \leq b_{X4}^0$ guarantees Firm L's purchase of the keyword in the presence of Firm

H and Firm X. □

Proof of Lemma 1

The profit functions given in equations (9) and (10) are concave in their own prices. Thus, by taking the first derivatives and setting them to be zero, we obtain two equations of prices. Then, simultaneously solving them, we obtain equilibrium prices. Finally, plugging them back into equations (7), (8), (9), and (10), we derive the equilibrium demands and profits. $\square$

Proof of Proposition 1

First, note that by Lemma 1, the equilibrium prices and sales are given by $p_H^* = t + \dfrac{\Delta \tilde{q}}{3}$,

$p_L^* = t - \dfrac{\Delta \tilde{q}}{3}$, $D_H^* = \dfrac{3t + \Delta \tilde{q}}{6t}$, and $D_L^* = \dfrac{3t - \Delta \tilde{q}}{6t}$. From the expressions given Table 1, $\Delta \tilde{q}$

in each scenario is given as follows:

$$\Delta \tilde{q}^{NN} = \Delta q$$

$$\Delta \tilde{q}^{YN} = \Delta q + (1 + \gamma) E_1$$

$$\Delta \tilde{q}^{NY} = \Delta q - E_1 + E_2$$

$$\Delta \tilde{q}^{YY1} = \Delta q + (\gamma + \varepsilon) E_1 + (1 + \delta) E_2$$

$$\Delta \tilde{q}^{YY2} = \Delta q + (\gamma - \varepsilon) E_1 + (1 + \delta) E_2.$$

Then, the first part of the proposition is proved by considering the following two scenarios:

- In Scenario YN: Firm H is the advertising firm and, $\dfrac{\partial p_H^*}{\partial E_1} = \dfrac{1+\gamma}{3} > 0$ and

$\dfrac{\partial D_H^*}{\partial E_1} = \dfrac{1+\gamma}{6t} > 0$, while Firm L is the non-advertising firm, and $\dfrac{\partial p_L^*}{\partial E_1} = -\dfrac{1+\gamma}{3} < 0$ and

$\dfrac{\partial D_L^*}{\partial E_1} = -\dfrac{1+\gamma}{6t} < 0$.

- In Scenario NY: Firm L is the advertising firm and, $\dfrac{\partial p_L^*}{\partial E_1} = \dfrac{1}{3} > 0$ and

$\dfrac{\partial E_L^*}{\partial E_1} = \dfrac{1}{6t} > 0$, while Firm H is the non-advertising firm and, $\dfrac{\partial p_H^*}{\partial E_1} = -\dfrac{1}{3} < 0$ and

$\dfrac{\partial D_H^*}{\partial E_1} = -\dfrac{1}{6t} < 0$.

The second part is proved by considering the following two scenarios:

- In Scenario YY1: $\dfrac{\partial p_H^*}{\partial E_1} = \dfrac{\gamma + \varepsilon}{3} > 0$, $\dfrac{\partial D_H^*}{\partial E_1} = \dfrac{\gamma + \varepsilon}{6t} > 0$, $\dfrac{\partial p_L^*}{\partial E_1} = -\dfrac{\gamma + \varepsilon}{3} < 0$, and

$\dfrac{\partial D_L^*}{\partial E_1} = -\dfrac{\gamma + \varepsilon}{6t} < 0$, if and only if $\gamma + \varepsilon > 0$, where $\gamma + \varepsilon$ denotes the relative size of the

exposure effect of Firm H, compared with that of Firm L.

- In Scenario YY2: $\dfrac{\partial p_H^*}{\partial E_1} = \dfrac{\gamma - \varepsilon}{3} > 0$, $\dfrac{\partial D_H^*}{\partial E_1} = \dfrac{\gamma - \varepsilon}{6t} > 0$, $\dfrac{\partial p_L^*}{\partial E_1} = -\dfrac{\gamma - \varepsilon}{3} < 0$, and

$\dfrac{\partial D_L^*}{\partial E_1} = -\dfrac{\gamma - \varepsilon}{6t} < 0$, if and only if $\gamma - \varepsilon > 0$, where $\gamma - \varepsilon$ denotes the relative size of the

exposure effect of Firm H, compared with that of Firm L. $\square$

Proof of Proposition 2

First note by definition, that there is a contrast if $E_2 > 0$ and an assimilation if $E_2 < 0$.

Thus, $\dfrac{\partial(\cdot)_i^*}{\partial E_2} > 0$ means that the equilibrium quantity (price or sales) of Firm i increases

with a contrast but decreases with an assimilation. As in the previous proof, we consider

the derivatives in the following scenarios.

- In Scenario NY: $\dfrac{\partial p_H^*}{\partial E_2} = \dfrac{1}{3} > 0$, $\dfrac{\partial D_H^*}{\partial E_2} = \dfrac{1}{6t} > 0$, $\dfrac{\partial p_L^*}{\partial E_2} = -\dfrac{1}{3} < 0$, and

$\dfrac{\partial D_L^*}{\partial E_2} = -\dfrac{1}{6t} < 0$. Thus, with a contrast, Firm H's price and sales go up, while they go

down with an assimilation. The reverse is true for Firm L.

- In Scenarios YY1 and YY2: $\dfrac{\partial p_H^*}{\partial E_2} = \dfrac{1+\delta}{3} > 0$, $\dfrac{\partial D_H^*}{\partial E_2} = \dfrac{1+\delta}{6t} > 0$,

$\dfrac{\partial p_L^*}{\partial E_2} = -\dfrac{1+\delta}{3} < 0$, and $\dfrac{\partial D_L^*}{\partial E_2} = -\dfrac{1+\delta}{6t} < 0$. The same conclusion as in Scenario NY can be

reached in this case.

- In Scenarios NN and YN: $\dfrac{\partial p_E^*}{\partial E_2} = 0$. This proves the first line of the

proposition. $\square$

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Proof of Proposition 3

From the definition: $E_2 \equiv \Delta q - q^0$, we have $\dfrac{\partial E_2}{\partial q^0} = -1 < 0$. By noting the chain rule:

$$\frac{\partial f(E_2)}{\partial q^0} = \frac{\partial f(E_2)}{\partial E_2} \cdot \frac{\partial E_2}{\partial q^0} \quad \text{for any function } f(E_2), \text{ the result follows from Proposition 2.} \;\square$$

Proof of Proposition 4

Let π_{ij} denote the profit Firm i earns from the product market when taking the jth slot in the search advertising market. Suppose Firm i takes the first slot and Firm i' takes the second slot in equilibrium. Then by definition of Symmetric Nash Equilibrium, we have:

$$\pi_{i1} - s_1 r_{i'} b_{i'} \geq \pi_{i2} - s_2 r_X b_X \tag{A1}$$

$$\pi_{i'2} - s_2 r_X b_X \geq \pi_{i'1} - s_1 r_{i'} b_{i'}. \tag{A2}$$

Summing two inequalities, we obtain the following condition:

$$\pi_{i1} + \pi_{i'2} \geq \pi_{i2} + \pi_{i'1}. \tag{A3}$$

Rearranging the order, we have:

$$\pi_{i1} - \pi_{i2} \geq \pi_{i'1} - \pi_{i'2}, \tag{A4}$$

which implies that in equilibrium, the firm with higher incremental profit of moving from the second slot to the first slot wins the first slot. Thus, Firm H takes the first slot, if and only if

$$\pi_{H1} - \pi_{H2} \geq \pi_{L1} - \pi_{L2}. \tag{A5}$$

From the product market competition equilibrium, we know,

$$\pi_{H1} = \frac{\{3t + (\gamma + \varepsilon)E_1 + (2+\delta)\Delta q - (1+\delta)q^0\}^2}{18t} \tag{A6}$$

$$\pi_{L2} = \frac{\{3t - (\gamma + \varepsilon)E_1 - (2+\delta)\Delta q + (1+\delta)q^0\}^2}{18t} \tag{A7}$$

$$\pi_{H2} = \frac{\{3t + (\gamma - \varepsilon)E_1 + (2+\delta)\Delta q - (1+\delta)q^0\}^2}{18t} \tag{A8}$$

$$\pi_{L1} = \frac{\{3t - (\gamma - \varepsilon)E_1 - (2+\delta)\Delta q + (1+\delta)q^0\}^2}{18t}. \tag{A9}$$

Note that we replace E_2 by $\Delta q - q^0$ in deriving the above profits. Therefore, equation (5) is equivalent to,

$$8\varepsilon E_1\{\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0\} \geq 0. \tag{A10}$$

Then the result follows. $\square$

Proof of Lemma 2

By inequalities in (A1) and (A2), the second slot winner's equilibrium bid $b_{i'}$ must satisfy

$$\pi_{i'1} - \pi_{i'2} + s_2 r_X b_X \leq s_1 r_i b_{i'} \leq \pi_{i1} - \pi_{i2} + s_2 r_X b_X. \tag{A11}$$

Noting that $C_{i1} \equiv s_1 r_i b_{i'}$, the rest follows from the expressions given in (A6)-(A9). $\square$

Proof of Proposition 5

Let $\Pi_i^{(\cdot\cdot)}$ denote Firm i's profit under scenario $(\cdot\cdot)$. Then when Firm L does not buy,

Firm H buys if and only if $\Pi_H^{YN} \geq \Pi_H^{NN}$, where

$$\Pi_H^{YN} = \frac{\{3t + (1+\gamma)E_1 + \Delta q\}^2}{18t} - C_{H0}^* \tag{A12}$$

$$\Pi_H^{NN} = \frac{\{3t + \Delta q\}^2}{18t}. \tag{A13}$$

This is equivalent to

$$E_1 \leq \frac{-(3t + \Delta q) - \sqrt{(3t + \Delta q)^2 + 18tC_{H0}^*}}{1+\gamma} \, or E_1 \geq \frac{-(3t + \Delta q) + \sqrt{(3t + \Delta q)^2 + 18tC_{H0}^*}}{1+\gamma}. \tag{A14}$$

Recall from equation (11), that the parameter space is confined such that $E_1 \geq -\dfrac{3t + \Delta q}{1+\gamma}$.

Thus, we rewrite the condition in (A14) as,

$$E_1 \geq \alpha_1(E_2) \equiv \frac{-(3t + \Delta q) + \sqrt{(3t + \Delta q)^2 + 18tC_{H0}^*}}{1+\gamma}. \tag{A15}$$

Also, note that $b_X \leq b_{X1}^0$ implies that there exists at least one point in the parameter

space that satisfies (A15).

Now when Firm H does not buy, Firm L buys if and only if $\Pi_L^{NY} \geq \Pi_L^{NN}$, where

$$\Pi_L^{NY} = \frac{\{3t + E_1 - 2\Delta q + q^0\}^2}{18t} - C_{L0}^* \tag{A16}$$

$$\Pi_L^{NN} = \frac{\{3t - \Delta q\}^2}{18t}. \tag{A17}$$

This is equivalent to

$$E_1 \leq -(3t - 2\Delta q + q^0) - \sqrt{(3t - \Delta q)^2 + 18tC_{L0}^*} \quad \text{or} \tag{A18}$$

$$E_1 \geq -(3t - 2\Delta q + q^0) + \sqrt{(3t - \Delta q)^2 + 18tC_{L0}^*}.$$

Recall from equation (12), that the parameter space is confined such that

$E_1 \geq -(3t - 2\Delta q + q^0)$. Thus, we rewrite the condition in (A18) as,

$$E_1 \geq -(3t - 2\Delta q + q^0) + \sqrt{(3t - \Delta q)^2 + 18tC_{L0}^*}. \tag{A19}$$

Finally, note that $b_X \leq b_{X2}^0$ implies that there exists at least one point in the parameter

space that satisfies (A18). $\square$

Proof of Proposition 6

When Firm H buys, Firm L also buys if and only if $\Pi_H^{YY} \geq \Pi_H^{NY}$. Note that by Lemma 2,

Π_H^{YY} is given by Π_H^{YY1} if $\gamma E_1 + (2 + \delta)\Delta q - (1 + \delta)q^0 \geq 0$ and by Π_H^{YY2} otherwise, where

$$\Pi_H^{YY1} = \frac{\{3t + (\gamma + \varepsilon)E_1 + (2 + \delta)\Delta q - (1 + \delta)q^0\}^2}{18t} - C_{H1}^* \tag{A20}$$

$$\Pi_H^{YY2} = \frac{\{3t + (\gamma - \varepsilon)E_1 + (2 + \delta)\Delta q - (1 + \delta)q^0\}^2}{18t} - C_{H2}^*. \tag{A21}$$

Also, Π_H^{NY} is defined by

$$\Pi_H^{NY} = \frac{\{3t - E_1 + 2\Delta q - q^0\}^2}{18t}. \tag{A22}$$

Now when $\gamma E_1 + (2 + \delta)\Delta q - (1 + \delta)q^0 \geq 0$, from $\Pi_H^{YY1} \geq \Pi_H^{NY}$, we have,

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$$\Delta q \geq \frac{-\{2(1+\gamma+\varepsilon)+\delta(\gamma+\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} + \sqrt{\{(2(1+\gamma+\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C^*_{H1}}}{\delta(4+\delta)} \quad (A23)$$
, or

$$\Delta q \leq \frac{-\{2(1+\gamma+\varepsilon)+\delta(\gamma+\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} - \sqrt{\{(2(1+\gamma+\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C^*_{H1}}}{\delta(4+\delta)}. \quad (A24)$$

Equations (12) and (13) cross on the point outside of the parameter space in the south,

which is given by $(\dfrac{(3t+q^0)\delta}{2(1+\gamma+\varepsilon)+\delta}, -\dfrac{(3t-q^0)(1+\gamma+\varepsilon)-\delta q^0}{2(1+\gamma+\varepsilon)+\delta})$. It turns out that this

point is the center of the hyperbola defined by $\Pi_H^{YY1} = \Pi_H^{NY}$ and that equation (12) is the

transverse axis of this hyperbola. Thus, the region described by (A24) is outside of the

parameter space, while that described by (A23) overlaps with the parameter space. (This

latter fact is guaranteed by $b_X \leq b_{X3}^0$.) Then, the above condition becomes, $\Delta q \geq \beta_{11}(E_1)$

where

$$\beta_{11}(E_1) \equiv \frac{-\{2(1+\gamma+\varepsilon)+\delta(\gamma+\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} + \sqrt{\{(2(1+\gamma+\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C^*_{H1}}}{\delta(4+\delta)}. \quad (A25)$$

Finally, when $\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 < 0$, from $\Pi_H^{YY2} \geq \Pi_H^{NY}$, we have,

$$\Delta q \geq \frac{-\{2(1+\gamma-\varepsilon)+\delta(\gamma-\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} + \sqrt{\{(2(1+\gamma-\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C^*_{H1}}}{\delta(4+\delta)}, or \quad (A26)$$

$$\Delta q \leq \frac{-\{2(1+\gamma-\varepsilon)+\delta(\gamma-\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} - \sqrt{\{(2(1+\gamma-\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C^*_{H1}}}{\delta(4+\delta)}. \quad (A27)$$

Similarly, equations (12) and (14) defines a point outside of the parameter space in the

south, which is given by $(\dfrac{(3t+q^0)\delta}{2(1+\gamma-\varepsilon)+\delta}, -\dfrac{(3t-q^0)(1+\gamma-\varepsilon)-\delta q^0}{2(1+\gamma-\varepsilon)+\delta})$ and turns out to be

the center of the hyperbola defined by $\Pi_H^{YY2} = \Pi_H^{NY}$. Again, equation (12) is the

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transverse axis of this hyperbola. Thus, the region described by (A27) is outside of the parameter space, while that described by (A26) overlaps with the parameter space, by $b_X \leq b_{X3}^0$. Thus, the above condition becomes, $\Delta q \geq \beta_{12}(E_1)$ where

$$\beta_{12}(E_1) \equiv \frac{-\{2(1+\gamma-\varepsilon)+\delta(\gamma-\varepsilon)\}E_1 - \delta\{3t - q^0(3+\delta)\} + \sqrt{\{(2(1+\gamma-\varepsilon)+\delta)E_1 - \delta(3t+q^0)\}^2 + 18t\delta(4+\delta)C_{H1}^*}}{\delta(4+\delta)}. \quad (A28)$$

Thus follows the lemma. $\square$

Proof of Proposition 7

When Firm H buys, Firm L also buys if and only if $\Pi_L^{YY} \geq \Pi_L^{YN}$. Note that by proposition 1, Π_L^{YY} is given by Π_L^{YY1} if $\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0$ and by Π_L^{YY2} otherwise, where

$$\Pi_L^{YY1} = \frac{\{3t - (\gamma+\varepsilon)E_1 - (2+\delta)\Delta q + (1+\delta)q^0\}^2}{18t} - C_{L2}^* \quad (A29)$$

$$\Pi_L^{YY2} = \frac{\{3t - (\gamma-\varepsilon)E_1 - (2+\delta)\Delta q + (1+\delta)q^0\}^2}{18t} - C_{L1}^*. \quad (A30)$$

Also, Π_L^{YN} is defined by

$$\Pi_L^{YN} = \frac{\{3t - (1+\gamma)E_1 - \Delta q\}^2}{18t}. \quad (A31)$$

Now when $\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0$, from $\Pi_L^{YY1} \geq \Pi_L^{YN}$, we have,

$$\Delta q \geq \frac{2\{(1-\gamma-2\varepsilon) - \delta(\gamma+\varepsilon)\}E_1 + 6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$+ \frac{\sqrt{\{(2+\gamma-\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L2}^*}}{2(3+\delta)(1+\delta)} \quad (A32)$$

, or

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$$\Delta q \leq \frac{2\{(1-\gamma-2\varepsilon)-\delta(\gamma+\varepsilon)\}E_1 + 6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$-\frac{\sqrt{\{(2+\gamma-\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L2}^*}}{2(3+\delta)(1+\delta)}. \tag{A33}$$

By equations (11) and (13), one vertex in the north of the parameter space is given by

$(\dfrac{(1+\delta)(3t-q^0)}{2+\gamma-\varepsilon+\delta(1+\gamma)}, \dfrac{3t(1-\varepsilon)-q^0(1+\gamma)(1+\delta)}{2+\gamma-\varepsilon+\delta(1+\gamma)})$. It also turns out that this point is the

center of the hyperbola defined by $\Pi_L^{YY1} = \Pi_L^{YN}$ and that equation (11) is the transverse

axis of this hyperbola. Thus, the region described by (A32) is outside of the parameter

space, while that described by (A33) overlaps with the parameter space. (This latter fact

is guaranteed by $b_X \leq b_{X4}^0$.) Therefore, the condition for $\Pi_L^{YY1} \geq \Pi_L^{YN}$ becomes $\Delta q \leq \beta_{21}$,

where

$$\beta_{21}(E_1) \equiv \frac{2\{1-\gamma-2\varepsilon)-\delta(\gamma+\varepsilon)\}E_1 + 6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$-\frac{\sqrt{\{(2+\gamma-\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L2}^*}}{2(3+\delta)(1+\delta)}. \tag{A34}$$

When $\gamma E_1 + (1+\delta)\Delta q - (1+\delta)q^0 < 0$, from $\Pi_L^{YY2} \geq \Pi_L^{YN}$, we have,

$$\Delta q \geq \frac{2\{1-\gamma+2\varepsilon)-\delta(\gamma-\varepsilon)\}E_1 + +6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$+\frac{\sqrt{\{(2+\gamma+\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L2}^*}}{2(3+\delta)(1+\delta)} \tag{A35}$$
, or

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$$\Delta q \leq \frac{2\{1 - \gamma + 2\varepsilon) - \delta(\gamma - \varepsilon)\}E_1 + +6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$-\frac{\sqrt{\{(2+\gamma+\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L2}^*}}{2(3+\delta)(1+\delta)}. \tag{A36}$$

As above, $\Pi_L^{YY2} = \Pi_L^{YN}$ defines a hyperbola with the center

$(\dfrac{(1+\delta)(3t-q^0)}{2+\gamma+\varepsilon+\delta(1+\gamma)}, \dfrac{3t(1+\varepsilon)-q^0(1+\gamma)(1+\delta)}{2+\gamma+\varepsilon+\delta(1+\gamma)})$ (which is outside, in the north of the

parameter space) and equation (11) is again the transverse axis of this hyperbola. Thus,

the region described by (A35) is outside of the parameter space, while that described by

(A36) overlaps with the parameter space, by $b_X \leq b_{X4}^0$. Therefore, the condition for

$\Pi_L^{YY2} \geq \Pi_L^{YN}$ becomes $\Delta q \leq \beta_{22}(E_1)$, where

$$\beta_{22}(E_1) \equiv \frac{2\{1 - \gamma + 2\varepsilon) - \delta(\gamma - \varepsilon)\}E_1 + +6t(1+\delta) + 2q^0(2+\delta)(1+\delta)}{2(3+\delta)(1+\delta)}$$

$$-\frac{\sqrt{\{(2+\gamma+\varepsilon+\delta(\gamma+1))E_1 - (3t-q^0)(1+\delta)\}^2 + 72t(3+\delta)(1+\delta)C_{L2}^*}}{2(3+\delta)(1+\delta)}. \ \square \tag{A37}$$

Proof of Proposition 8

Suppose $E_1 < \min\{\alpha_1(\Delta q), \alpha_2(\Delta q)\}$. Then, by the first part of Lemma 4, Firm H has no

incentive to deviate from the NN scenario. Likewise, by the second part of Lemma 4,

Firm L does not have an incentive to deviate from the NN scenario. Thus, NN scenario

emerges as an equilibrium if $E_1 < \min\{\alpha_1(\Delta q), \alpha_2(\Delta q)\}$. $\square$

Proof of Proposition 9

We prove the proposition in three parts. First, we derive the condition for YN to be observed in equilibrium. By Lemma 4, if $E_1 > \alpha_1(\Delta q)$, Firm H has no incentive to deviate from the YN scenario. In addition, if $\Delta q \geq \beta_2(E_1)$, by Lemma 6, Firm L does not have an incentive to deviate from the YN scenario either. Thus, YN is observed when both $E_1 > \alpha_1(\Delta q)$ and $\Delta q \geq \beta_2(E_1)$ are satisfied.

Next, we prove the NY equilibrium. By Lemma 4, if $E_1 > \alpha_2(\Delta q)$, Firm L has no incentive to deviate from NY scenario. Likewise, if $\Delta q \leq \beta_1(E_1)$, by Lemma 5, Firm H has no incentive to deviate from NY. Thus, we obtain the NY equilibrium when both $E_1 > \alpha_2(\Delta q)$ and $\Delta q \leq \beta_1(E_1)$ hold.

Finally, combining the results from Lemma 5 and Lemma 6, we obtain the YY equilibrium if $\beta_2(E_1) < \Delta q < \beta_1(E_1)$. $\square$

Proof of Proposition 10

First, by proposition 2, YY scenario emerges as an equilibrium as long as $\beta_2(E_1) < \Delta q < \beta_1(E_1)$, even with additional condition on E_1. Now, the defensive strategy is defined in the text, as follows: (1) Firm H does not buy the keyword if Firm L also does not buy the keyword, but (2) Firm H buys the keyword if Firm L buys the keyword. There conditions are equivalent to (1): $\Pi_H^{NN} \geq \Pi_H^{YN}$ and (2): $\Pi_H^{YY} \geq \Pi_H^{NY}$.

Finally, to ensure that Firm L buys the keyword in equilibrium so that Firm H's

defensive strategy is triggered, we need an additional condition (3): $\Pi_L^{YY} \geq \Pi_L^{YN}$. By

Lemma 5 and Lemma 6, the conditions (1) and (3) are translated into

$\beta_2(E_1) < \Delta q < \beta_1(E_1)$, and by Lemma 4, the second condition is equivalent to

$E_1 \leq \alpha_1(\Delta q)$. $\square$

Proof of Proposition 11

The effective deterrence of purchase of Firm L by Firm H is defined as follows in the

text: (1) Firm L wants to buy the keyword without Firm H's buying but (2) it does not

want to buy if Firm H buys. (3) Knowing this, Firm H buys the keyword if Firm L buys.

These conditions are translated as follows:

(1) $\Pi_L^{NY} \geq \Pi_L^{NN}$, which is equivalent to $E_1 > \alpha_2(\Delta q)$ by Lemma 4,

(2) $\Pi_L^{YN} \geq \Pi_L^{YY}$, which is equivalent to $\Delta q > \beta_2(E_1)$ by Lemma 6, and

(3) $\Pi_H^{YY} \geq \Pi_H^{NY}$, which is equivalent to $\Delta q > \beta_1(E_1)$ by Lemma 5.

Therefore, we have $E_1 > \alpha_2(\Delta q)$ and $\Delta q > \max\{\beta_1(E_1), \beta_2(E_1)\}$. $\square$

Proof of Proposition 12

The proof here is similar to that of Proposition 6. The conditions are the same as in

Proposition 6, with Firm H and Firm L switched. Then the conditions are translated as

follows:

(1) $\Pi_H^{YN} \geq \Pi_H^{NN}$, which is equivalent to $E_1 > \alpha_1(\Delta q)$ (by Lemma 4),

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(2) $\Pi_H^{NY} \geq \Pi_H^{YY}$, which is equivalent to $\Delta q < \beta_1(E_1)$ (by Lemma 5), and

(3) $\Pi_L^{YY} \geq \Pi_L^{YN}$, which is equivalent to $\Delta q < \beta_2(E_1)$ (by Lemma 6).

Therefore, we have $E_1 > \alpha_1(\Delta q)$ and $\Delta q < \min\{\beta_1(E_1), \beta_2(E_1)\}$. $\square$

Proof of Proposition 13

The prisoner's dilemma case is obtained when both firms make less profits in YY

scenarios than in NN scenario but both buy the keyword in equilibrium. Thus, the

conditions are given as follows:

(1) $\beta_1(E_1) < \Delta q < \beta_2(E_1)$, which ensures that in equilibrium both firms buy the

keyword, by Proposition 5 and Proposition 6,

(2-1) If $\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 \geq 0$: $\Pi_H^{YY1} \leq \Pi_H^{NN}$ and $\Pi_L^{YY1} \leq \Pi_L^{NN}$, which are

respectively equivalent to $\beta_{31}(E_1) \leq \Delta q \leq \beta_{33}(E_1)$ and $\beta_{43}(E_1) \leq \Delta q \leq \beta_{41}(E_1)$, where

$$\beta_{31}(E_1) \equiv \frac{-(2+\delta)(\gamma+\varepsilon)E_1 + \{3t + (\delta+2)q^0\}(\delta+1) - \sqrt{\{(\gamma+\varepsilon)E_1 + (3t-q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{L2}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{33}(E_1) \equiv \frac{-(2+\delta)(\gamma+\varepsilon)E_1 + \{3t + (\delta+2)q^0\}(\delta+1) + \sqrt{\{(\gamma+\varepsilon)E_1 + (3t-q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{L2}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{43}(E_1) \equiv \frac{-(2+\delta)(\gamma+\varepsilon)E_1 - \{3t - (\delta+2)q^0\}(\delta+1) - \sqrt{\{(\gamma+\varepsilon)E_1 - (3t+q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{H1}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{41}(E_1) \equiv \frac{-(2+\delta)(\gamma+\varepsilon)E_1 - \{3t - (\delta+2)q^0\}(\delta+1) + \sqrt{\{(\gamma+\varepsilon)E_1 - (3t+q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{H1}^*}}{(\delta+1)(\delta+3)}.$$

(2-2) If $\gamma E_1 + (2+\delta)\Delta q - (1+\delta)q^0 < 0$: $\Pi_L^{YY2} \leq \Pi_L^{NN}$ and $\Pi_H^{YY2} \leq \Pi_H^{NN}$, which are

respectively equivalent to $\beta_{32}(E_1) \leq \Delta q \leq \beta_{34}(E_1)$ and $\beta_{44}(E_1) \leq \Delta q \leq \beta_{42}(E_1)$, where

$$\beta_{32}(E_1) \equiv \frac{-(2+\delta)(\gamma-\varepsilon)E_1 + \{3t+(\delta+2)q^0\}(\delta+1) - \sqrt{\{(\gamma-\varepsilon)E_1 + (3t-q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{H2}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{34}(E_1) \equiv \frac{-(2+\delta)(\gamma-\varepsilon)E_1 + \{3t+(\delta+2)q^0\}(\delta+1) + \sqrt{\{(\gamma-\varepsilon)E_1 + (3t-q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{H2}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{44}(E_1) \equiv \frac{-(2+\delta)(\gamma-\varepsilon)E_1 - \{3t-(\delta+2)q^0\}(\delta+1) - \sqrt{\{(\gamma-\varepsilon)E_1 - (3t+q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{L1}^*}}{(\delta+1)(\delta+3)},$$

$$\beta_{42}(E_1) \equiv \frac{-(2+\delta)(\gamma-\varepsilon)E_1 - \{3t-(\delta+2)q^0\}(\delta+1) + \sqrt{\{(\gamma-\varepsilon)E_1 - (3t+q^0)(1+\delta)\}^2 + 18t(\delta+1)(\delta+3)C_{L1}^*}}{(\delta+1)(\delta+3)}.$$

Now, observe $\beta_{43}(E_1) < \beta_{31}(E_1) < \beta_{41}(E_1) < \beta_{33}(E_1)$ and

$\beta_{44}(E_1) < \beta_{32}(E_1) < \beta_{42}(E_1) < \beta_{34}(E_1)$. Then, the condition in (2-1) becomes

$\beta_{31}(E_1) \leq \Delta q \leq \beta_{41}(E_1)$, while that in (2-2) becomes $\beta_{32}(E_1) \leq \Delta q \leq \beta_{42}(E_1)$. By defining

$\beta_3(E_1)$ and $\beta_4(E_1)$ as in the text, both conditions can be altogether written as

$\beta_3(E_1) \leq \Delta q \leq \beta_4(E_1)$.

Therefore, $\max\{\beta_1(E_1), \beta_3(E_1)\} < \Delta q < \min\{\beta_2(E_1), \beta_4(E_1)\}$ defines the prisoner's

dilemma. $\square$

Proof of Proposition 14

First, observe that $-1 < \dfrac{A}{\sqrt{A^2+k}} < 1$ for any A as long as $k > 0$. Now, using this fact,

$$\frac{\partial \beta_{11}(E_1)}{\partial \gamma} = \frac{E_1}{\delta(4+\delta)}\left(\frac{2\{(2+2\gamma+2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}}{\sqrt{4\{(2+2\gamma+2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}^2 + 18t\delta(4+\delta)C_{H1}^*}} - 2 - \delta \right) < \frac{E_1}{\delta(4+\delta)}(2-2-\delta) < 0$$

and

$$\frac{\partial \beta_{12}(E_1)}{\partial \gamma} = \frac{E_1}{\delta(4+\delta)}\left(\frac{2\{(2+2\gamma-2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}}{\sqrt{4\{(2+2\gamma-2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}^2 + 18t\delta(4+\delta)C_{H2}^*}} - 2 - \delta \right) < \frac{E_1}{\delta(4+\delta)}(2-2-\delta) < 0.$$

Therefore, $\dfrac{\partial \beta_1(E_1)}{\partial \gamma} < 0$, which implies that Firm H is more likely to buy the keyword

against the purchase of Firm L, since the condition in Proposition 6 is given by

$\Delta q > \beta_1(E_1)$. Likewise,

$$\frac{\partial \beta_{21}(E_1)}{\partial \gamma} = -\frac{E_1}{(4+\delta)}\left(\frac{2\{(2+\gamma-\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}}{\sqrt{4\{(2+\gamma-\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}^2 + 72t\delta(4+\delta)(1+\delta)C_{L2}^*}} - 1 \right) < 0, \text{ and}$$

$$\frac{\partial \beta_{22}(E_1)}{\partial \gamma} = -\frac{E_1}{(4+\delta)}\left(\frac{2\{(2+\gamma+\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}}{\sqrt{4\{(2+\gamma+\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}^2 + 72t\delta(4+\delta)(1+\delta)C_{L1}^*}} - 1 \right) < 0.$$

Therefore, $\dfrac{\partial \beta_2(E_1)}{\partial \gamma} < 0$, which implies that Firm L is less likely to buy the keyword against

the purchase of Firm H, since the condition in Proposition 7 is given by $\Delta q < \beta_2(E_1)$.

The last part of the proposition follows these results. $\square$

Proof of Proposition 15

Observe,

$$\frac{\partial \beta_{11}(E_1)}{\partial q^0} = \frac{1}{(4+\delta)}\left(\frac{\{(2+2\gamma+2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}}{\sqrt{\{(2+2\gamma+2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}^2 + 18t(4+\delta)C_{H1}^*}} + 3 + \delta \right) > 0 \text{ and}$$

$$\frac{\partial \beta_{11}(E_1)}{\partial q^0} = \frac{1}{(4+\delta)}\left(\frac{\{(2+2\gamma-2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}}{\sqrt{\{(2+2\gamma-2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}^2 + 18t(4+\delta)C_{H2}^*}} + 3 + \delta \right) > 0.$$

Therefore, $\dfrac{\partial \beta_1(E_1)}{\partial q^0} > 0$, which implies that Firm H is less likely to buy the keyword

against the purchase of Firm L. Likewise,

$$\frac{\partial \beta_{21}(E_1)}{\partial q^0} = \frac{E_1}{(3+\delta)}\left(2+\delta - \frac{\{(2+\gamma-\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}}{\sqrt{\{(2+\gamma-\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}^2 + 18t\delta(4+\delta)C_{L2}^*}} \right) > 0, \text{ and}$$

$$\frac{\partial \beta_{22}(E_1)}{\partial q^0} = \frac{E_1}{(3+\delta)}\left(2+\delta + \frac{\{(2+\gamma+\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}}{\sqrt{\{(2+\gamma+\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}^2 + 18t\delta(4+\delta)C^*_{L2}}}\right) > 0$$

Therefore, $\dfrac{\partial \beta_2(E_1)}{\partial q^0} > 0$, which implies that Firm L is more likely to buy the keyword

against the purchase of Firm H. The last part of the proposition follows these results. $\square$

Proof of Proposition 16

When Firm H takes the first slot,

$$\frac{\partial \beta_{11}(E_1)}{\partial \varepsilon} = \frac{1}{\delta(4+\delta)}\left(\frac{2\{(2+2\gamma+2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}}{\sqrt{4\{(2+2\gamma+2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}^2 + 18t(4+\delta)C^*_{H1}}} - 2-\delta\right) < 0 \text{ : it is more likely to}$$

buy;

when Firm H takes the second slot,

$$\frac{\partial \beta_{12}(E_1)}{\partial \varepsilon} = \frac{1}{\delta(4+\delta)}\left(\frac{2\{(2+2\gamma-2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}}{\sqrt{4\{(2+2\gamma-2\varepsilon+\delta)E_1 - (3t+q^0)\delta\}^2 + 18t(4+\delta)C^*_{H1}}} + 2+\delta\right) > 0 \text{ : it is less likely to}$$

buy;

when Firm L takes the second slot,

$$\frac{\partial \beta_{21}(E_1)}{\partial \varepsilon} = \frac{E_1}{(3+\delta)(1+\delta)}\left(\frac{\{(2+\gamma-\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}}{\sqrt{\{(2+\gamma-\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}^2 + 18t\delta(3+\delta)(1+\delta)C^*_{L2}}} - 2-\delta\right) < 0 \text{ : it is}$$

less likely to buy;

when Firm L takes the first slot,

$$\frac{\partial \beta_{22}(E_1)}{\partial \varepsilon} = \frac{E_1}{(3+\delta)(1+\delta)}\left(\frac{\{(2+\gamma+\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}}{\sqrt{\{(2+\gamma+\varepsilon+\delta+\gamma\delta)E_1 - (3t-q^0)(1+\delta)\}^2 + 18t\delta(3+\delta)(1+\delta)C^*_{L2}}} + 2+\delta\right) > 0 \text{ : it is}$$

more likely to buy.

In sum, the first slot winner becomes more likely to buy and the second slot winner becomes less likely to buy

□

Proof of Proposition 17

First observe that $\dfrac{\partial \beta_{11}(E_1)}{\partial \delta} > 0 \Leftrightarrow E_1^* < E_1 < E_1^{**}$ where

$$E_1^* \equiv \frac{-(3t+q^0)(1+\gamma+\varepsilon) - \sqrt{(3t+q^0)^2(1+\gamma+\varepsilon)^2 + 18tC_{H1}^*(-1+\gamma^2+2\gamma\varepsilon+\varepsilon^2)}}{-1+\gamma^2+2\gamma\varepsilon+\varepsilon^2} \quad \text{and}$$

$$E_1^{**} \equiv \frac{-(3t+q^0)(1+\gamma+\varepsilon) + \sqrt{(3t+q^0)^2(1+\gamma+\varepsilon)^2 + 18tC_{H1}^*(-1+\gamma^2+2\gamma\varepsilon+\varepsilon^2)}}{-1+\gamma^2+2\gamma\varepsilon+\varepsilon^2}.$$

It also turned out that $\beta_{11}(E_1) < q^0 \Leftrightarrow E_1^* < E_1 < E_1^{**}$, which implies that $\dfrac{\partial \beta_{11}(E_1)}{\partial \delta} > 0$ is equivalent to $\beta_{11}(E_1) < q^0$.

Now, observe that $\dfrac{\partial \beta_{12}(E_1)}{\partial \delta} > 0 \Leftrightarrow E_1^{***} < E_1 < E_1^{****}$ where

$$E_1^{***} \equiv \frac{-(3t+q^0)(1+\gamma-\varepsilon) - \sqrt{(3t+q^0)^2(1+\gamma-\varepsilon)^2 + 18tC_{H2}^*(-1+\gamma^2-2\gamma\varepsilon+\varepsilon^2)}}{-1+\gamma^2-2\gamma\varepsilon+\varepsilon^2} \quad \text{and}$$

$$E_1^{****} \equiv \frac{-(3t+q^0)(1+\gamma-\varepsilon) + \sqrt{(3t+q^0)^2(1+\gamma-\varepsilon)^2 + 18tC_{H2}^*(-1+\gamma^2-2\gamma\varepsilon+\varepsilon^2)}}{-1+\gamma^2-2\gamma\varepsilon+\varepsilon^2}.$$

It also turned out that $\beta_{12}(E_1) < q^0 \Leftrightarrow E_1^{***} < E_1 < E_1^{****}$, which implies that $\dfrac{\partial \beta_{12}(E_1)}{\partial \delta} > 0$ is equivalent to $\beta_{12}(E_1) < q^0$.

For Firm L, observe that $\dfrac{\partial \beta_{21}(E_1)}{\partial \delta} < 0 \Leftrightarrow E_1^o < E_1 < E_1^{oo}$ where

$$E_1^o \equiv \frac{(3t-q^0)(1-\varepsilon) - \sqrt{(3t-q^0)^2(1-\varepsilon)^2 - 18tC_{L2}^*(1-\varepsilon)(1+2\gamma+\varepsilon)}}{(1-\varepsilon)(1+2\gamma+\varepsilon)} \quad \text{and}$$

$$E_1^{oo} \equiv \frac{(3t-q^0)(1-\varepsilon) + \sqrt{(3t-q^0)^2(1-\varepsilon)^2 - 18tC_{L2}^*(1-\varepsilon)(1+2\gamma+\varepsilon)}}{(1-\varepsilon)(1+2\gamma+\varepsilon)}.$$

It also turned out that $\beta_{21}(E_1) > q^0 \Leftrightarrow E_1^o < E_1 < E_1^{oo}$, which implies that $\frac{\partial \beta_{11}(E_1)}{\partial \delta} < 0$ is

equivalent to $\beta_{21}(E_1) > q^0$.

Now, observe that $\frac{\partial \beta_{21}(E_1)}{\partial \delta} < 0 \Leftrightarrow E_1^{ooo} < E_1 < E_1^{oooo}$ where

$$E_1^{ooo} \equiv \frac{(3t-q^0)(1+\varepsilon) - \sqrt{(3t-q^0)^2(1+\varepsilon)^2 - 18tC_{L2}^*(1+\varepsilon)(1+2\gamma-\varepsilon)}}{(1+\varepsilon)(1+2\gamma-\varepsilon)} \quad \text{and}$$

$$E_1^{oooo} \equiv \frac{(3t-q^0)(1+\varepsilon) + \sqrt{(3t-q^0)^2(1+\varepsilon)^2 - 18tC_{L2}^*(1+\varepsilon)(1+2\gamma-\varepsilon)}}{(1+\varepsilon)(1+2\gamma-\varepsilon)}.$$

It also turned out that $\beta_{21}(E_1) > q^0 \Leftrightarrow E_1^{ooo} < E_1 < E_1^{oooo}$, which implies that $\frac{\partial \beta_{11}(E_1)}{\partial \delta} < 0$ is

equivalent to $\beta_{21}(E_1) > q^0$. $\square$

Proof of Proposition 18

Let i denote the second slot winner and b_i its bid amount. Then the advertising cost of

the competitor i' is given by $C_{i'1} = s_1 r_i b_i$ and thus, we show the incentive of advertiser i

by investigating the sign of $\frac{\partial \Pi_i^{YY1}}{\partial C_{i'1}}$. Now the equilibrium profit of Firm L in scenario

YY1 is given by,

$$\Pi_L^{YY1} = \frac{\{3t - (\frac{K_H}{C_{H1}})\Delta\tilde{q}^{YY1} - (1 - \frac{K_H}{C_{H1}})\Delta\tilde{q}^{NY}\}^2}{18t} - \{(\frac{K_H}{C_{H1}})C_{L2} + (1 - \frac{K_H}{C_{H1}})C_{L0}\} \qquad (A38)$$

where $\Delta\tilde{q}^S$ defines the difference in the quality perception between the two brands

under scenario S without budget constraint.

Taking the first derivative of this profit with respect to Firm H's cost C_{H1}, we have

$$\frac{\partial\Pi_L^{YY1}}{\partial C_{H1}} = (\frac{K_H}{C_{H1}^2})[\frac{(\Delta\tilde{q}^{YY1} - \Delta\tilde{q}^{NY})\{3t - (\frac{K_H}{C_{H1}})\Delta\tilde{q}^{YY1} - (1 - \frac{K_H}{C_{H1}})\Delta\tilde{q}^{NY}\}}{9t} - (C_{L0} - C_{L2})]. \qquad (A39)$$

Then $\dfrac{\partial\Pi_L^{YY1}}{\partial C_{H1}} > 0$ is equivalent to

$$\{(3t - \Delta q^{NY})(\Delta q^{YY1} - \Delta q^{NY}) - 9t(C_{L0} - C_{L2})\}C_{H1} > (\Delta q^{YY1} - \Delta q^{NY})^2 K_H. \qquad (A40)$$

Since by assumption $C_{H1} \geq K_H$ (i.e., the budget is constrained), this inequality holds for

any value of K_H if

$$(3t - \Delta q^{NY})(\Delta q^{YY1} - \Delta q^{NY}) - 9t(C_{L0} - C_{L2}) > (\Delta q^{YY1} - \Delta q^{NY})^2. \qquad (A41)$$

This last inequality is equivalent to $\lambda_{11}(H_1) \leq \Delta q \leq \lambda_{12}(E_1)$, where

$$\lambda_{11}(E_1) \equiv \frac{(3t + 3q^0 + 2\delta q^0)\delta - \{2 + \delta + 2(\gamma + \varepsilon)(1 + \delta)\}E_1 - \sqrt{\{(2 + 2\gamma + 2\varepsilon + \delta)E_1 + (3t - q^0)\delta\}^2 - 36t(\delta + 2)(C_{L0} - C_{L2})}}{2\delta(2 + \delta)}$$

$$\lambda_{12}(E_1) \equiv \frac{(3t + 3q^0 + 2\delta q^0)\delta - \{2 + \delta + 2(\gamma + \varepsilon)(1 + \delta)\}E_1 + \sqrt{\{(2 + 2\gamma + 2\varepsilon + \delta)E_1 + (3t - q^0)\delta\}^2 - 36t(\delta + 2)(C_{L0} - C_{L2})}}{2\delta(2 + \delta)}.$$

Therefore, if $\lambda_{11}(E_1) \leq \Delta q \leq \lambda_{12}(E_1)$, then $\dfrac{\partial\Pi_L^{YY1}}{\partial C_{H1}} > 0$ regardless of the value of K_H, and

thus, Firm L increases its bid as high as possible.

Likewise, from the equilibrium profit for Firm E in scenario YY2,

$$\Pi_H^{YY2} = \frac{\{3t + (\frac{K_L}{C_{L1}})\Delta\tilde{q}^{YY2} - (1 - \frac{K_L}{C_{L1}})\Delta\tilde{q}^{YN}\}^2}{18t} - \{(\frac{K_L}{C_{L1}})C_{E2} + (1 - \frac{K_L}{C_{L1}})C_{H0}\}, \qquad (A42)$$

we get,

$$\frac{\partial\Pi_H^{YY2}}{\partial C_{L1}} = (\frac{K_L}{C_{L1}^2})[\frac{(\Delta\tilde{q}^{YY2} - \Delta\tilde{q}^{YN})\{3t - (\frac{K_L}{C_{L1}})\Delta\tilde{q}^{YY2} - (1 - \frac{K_L}{C_{L1}})\Delta\tilde{q}^{YN}\}}{9t} - (C_{H0} - C_{H2})]. \qquad (A43)$$

Then $\dfrac{\partial\Pi_H^{YY2}}{\partial C_{L1}} > 0$ is equivalent to

$$\{(3t - \Delta q^{YN})(\Delta q^{YY2} - \Delta q^{YN}) - 9t(C_{L0} - C_{L2})\}C_{L1} > (\Delta q^{YY2} - \Delta q^{YN})^2 K_L. \qquad (A44)$$

Since by assumption $C_{L1} \geq K_L$ (i.e., the budget is constrained), this inequality holds for

any value of K_L if

$$(3t + \Delta q^{YN})(\Delta q^{YN} - \Delta q^{YY2}) - 9t(C_{L0} - C_{L2}) > (\Delta q^{YN} - \Delta q^{YY2})^2. \qquad (A45)$$

This last inequality is equivalent to $\lambda_{21}(E_1) \leq \Delta q \leq \lambda_{22}(E_1)$ where

$$\lambda_{21}(E_1) \equiv \frac{-(1+\delta)(3t - 3q^0 - 2\delta q^0) + \{2 + \delta - (1+\delta)\gamma + (3 + 2\delta)\varepsilon\}E_1}{2(1+\delta)(2+\delta)}$$

$$- \frac{\sqrt{\{(2 + \gamma + \delta + \varepsilon + \gamma\delta)E_1 + (1+\delta)(3t + q^0)\}^2 - 36t(\delta + 1)(\delta + 2)(C_{H0} - C_{H2})}}{2(1+\delta)(2+\delta)}$$

$$\lambda_{22}(E_1) \equiv \frac{-(1+\delta)(3t - 3q^0 - 2\delta q^0) + \{2 + \delta - (1+\delta)\gamma + (3 + 2\delta)\varepsilon\}E_1}{2(1+\delta)(2+\delta)}$$

$$+\frac{\sqrt{\{(2+\gamma+\delta+\varepsilon+\gamma\delta)E_1+(1+\delta)(3t+q^0)\}^2-36t(\delta+1)(\delta+2)(C_{H0}-C_{H2})}}{2(1+\delta)(2+\delta)}.$$

Therefore, if $\lambda_{21}(E_1)\le\Delta q\le\lambda_{22}(E_1)$, then $\dfrac{\partial\Pi_E^{YY2}}{\partial C_{U1}}>0$ regardless of the value of K_U, and

thus, Firm H increases its bid as high as possible. $\square$

Proof of Corollary 1

If $\lambda_{11}(E_1)\le\Delta q\le\lambda_{12}(E_1)$, by Proposition 18, $\dfrac{\partial\Pi_L^{YY1}}{\partial C_{H1}}>0$ and thus, $\dfrac{\partial\Pi_L^{YY1}}{\partial b_L}>0$. Therefore,

the optimal bid of Firm L is the upper bound of its range, which is given as a function of

the upper bound of C_{H1}. Now, the feasible range of C_{H1} is derived from the incentive

compatibility conditions of both firms, which are given as follows: in an equilibrium

involving the H-L-X listing order,

$$\frac{\{3t+(\frac{K_H}{C_H})\Delta\tilde{q}^{YY1}+(1-\frac{K_H}{C_H})\Delta\tilde{q}^{NY}\}^2}{18t}-K_H\ge\frac{\{3t+(\frac{K_L}{C_H})\Delta\tilde{q}^{YY2}+(1-\frac{K_L}{C_H})\Delta\tilde{q}^{YN}\}^2}{18t}-\{(\frac{K_L}{C_H})C_2+(1-\frac{K_L}{C_H})C_0\} \tag{A46}$$

$$\frac{\{3t-(\frac{K_H}{C_H})\Delta\tilde{q}^{YY1}-(1-\frac{K_H}{C_H})\Delta\tilde{q}^{NY}\}^2}{18t}-\{(\frac{K_H}{C_H})C_2+(1-\frac{K_H}{C_H})C_0\}\ge\frac{\{3t-(\frac{K_L}{C_H})\Delta\tilde{q}^{YY2}-(1-\frac{K_L}{C_H})\Delta\tilde{q}^{YN}\}^2}{18t}-K_L. \tag{A47}$$

Note that since $C_{H0}=C_{H2}$ and $C_{L0}=C_{L2}$, we define $C_0\equiv C_{H0}=C_{H2}$ and

$C_2\equiv C_{L0}=C_{L2}$ and use C_0 and C_2 this point on. Noting that $C_H\ge K_H$, the solutions to

(46) and (47) are respectively given by $C_H\le\rho$ and $C_H\ge\phi$, where ρ is the solution to

the equation:

$$\frac{\{3t+(\frac{K_H}{\rho})\Delta\tilde{q}^{YY1}+(1-\frac{K_H}{\rho})\Delta\tilde{q}^{NY}\}^2}{18t}-K_H=\frac{\{3t+(\frac{K_L}{\rho})\Delta\tilde{q}^{YY2}+(1-\frac{K_L}{\rho})\Delta\tilde{q}^{YN}\}^2}{18t}-\{(\frac{K_L}{\rho})C_2+(1-\frac{K_L}{\rho})C_0\}, \quad (A48)$$

and ϕ is the solution to the equation:

$$\frac{\{3t-(\frac{K_H}{\phi})\Delta\tilde{q}^{YY1}-(1-\frac{K_H}{\phi})\Delta\tilde{q}^{NY}\}^2}{18t}-\{(\frac{K_H}{\phi})C_2+(1-\frac{K_H}{\phi})C_0\}=\frac{\{3t-(\frac{K_L}{\phi})\Delta\tilde{q}^{YY2}-(1-\frac{K_L}{\phi})\Delta\tilde{q}^{YN}\}^2}{18t}-K_L. \quad (A49)$$

Here, note that both (A48) and (A49) have two solutions and ρ and ϕ stand for the

larger solution to each of the equation.

Now on comparing ϕ and ρ, we find that $\phi < \rho$ if and only if $A(E_1,\Delta q) > 0$, where

$$A(E_1,\Delta q) \equiv -\frac{1}{3\Delta q^2+(q^0)^2+2E_1(q^0-6t)+18C_0t-6q^0t-18K_Ht+\Delta q(-6E_1-4q^0+6t)}\{2\Delta qE_1K_H-E_1^2K_H-E_1q^0K_H$$

$$-\Delta q^2K_L+E_1^2K_L+\Delta qq^0K_L+E_1q^0K_L+3E_1K_Ht-9C_0K_Lt+9C_2K_Lt-3\Delta qK_Lt+3E_1K_Lt$$

$$+3q^0K_Lt+2\Delta qE_1K_H\varepsilon-E_1^2K_H\varepsilon-E_1q^0K_H\varepsilon+\Delta qE_1K_L\varepsilon+E_1^2K_L\varepsilon+3E_1K_Ht\varepsilon+3E_1K_Lt\varepsilon+\frac{\sqrt{A_1}}{2}\}$$

$$-\frac{1}{3\Delta q^2+(q^0)^2-18C_0t+6q^0t+18K_Lt-2\Delta q(3E_1+2q^0+3t)+2E_1(q^0+6t)}\{-2\Delta qE_1K_H+E_1^2K_H+E_1q^0K_H$$

$$+\Delta q^2K_L-E_1^2K_L-\Delta qq^0K_L-E_1q^0K_L-9C_0K_Ht+9C_2K_Ht+3E_1K_Ht-3\Delta qK_Lt+3E_1K_Lt$$

$$+3q^0K_Lt-2\Delta qE_1K_H\varepsilon+E_1^2K_H\varepsilon+E_1q^0K_H\varepsilon-\Delta qE_1K_L\varepsilon-E_1^2K_L\varepsilon+3E_1K_Ht\varepsilon+3E_1K_Lt\varepsilon+\frac{\sqrt{A_2}}{2}\},$$

and A_1 and A_2 above are defined as,

$$A_1 \equiv -4(3\Delta q^2+(q^0)^2+2E_1(q^0-6t)+18C_0t-6q^0t-18K_Ht+\Delta q(-6E_1-4q^0+6t))$$

$$\times(-(\Delta q-q^0)^2K_L^2+2E_1(\Delta q-q^0)K_L^2(1+\varepsilon)+E_1^2(K_H^2-K_L^2)(1+\varepsilon)^2)$$

$$+4(\Delta q^2K_L+3(3C_0-3C_2-q^0)K_Lt+E_1^2(K_H-K_L)(1+\varepsilon)$$

$$+ E_1(-3(K_H + K_L)t(1+\varepsilon) + q^0(K_H - K_L + K_H\varepsilon)) - \Delta q(K_L(q^0 - 3t) + E_1(K_L\varepsilon + 2K_H(1+\varepsilon))))^2$$

$$A_2 \equiv -4(3\Delta q^2 + (q^0)^2 - 18C_0t + 6q^0t + 18K_Lt - 2\Delta q(3E_1 + 2q^0 + 3t) + 2E_1(q^0 + 6t))$$

$$\times (-(\Delta q - q^0)^2 K_L^2 + 2E_1(\Delta q - q^0)K_L^2(1+\varepsilon) + E_1^2(K_H^2 - K_L^2)(1+\varepsilon)^2)$$

$$+ 4(\Delta q^2 K_L + 3(-3C_0K_H + 3C_2K_H + q^0K_L)t + E_1^2(K_H - K_L)(1+\varepsilon)$$

$$+ E_1(3(K_H + K_L)t(1+\varepsilon) + q^0(K_H - K_L + K_H\varepsilon)) - \Delta q(K_L(q^0 + 3t) + E_1(K_L\varepsilon + 2K_H(1+\varepsilon))))^2.$$

Thus, the feasible range of C_{H1} is given by $\phi \leq C_{H1} \leq \rho$ if $A > 0$. Otherwise, no C_{H1}

satisfies both (A46) and (A47) and thus, there is no equilibrium in the bidding game.

Therefore, the optimal bid is given by $\dfrac{\rho}{s_1 r_L}$ with ρ defined as in (A48), whenever

$A(E_1, \Delta q) > 0$. $\square$

Proof of Corollary 2

When $\lambda_{21}(E_1) \leq \Delta q \leq \lambda_{22}(E_1)$, by Proposition 18, $\dfrac{\partial \Pi_H^{YY2}}{\partial C_{L1}} > 0$ and thus, $\dfrac{\partial \Pi_H^{YY2}}{\partial b_H} > 0$.

Therefore, the optimal bid of Firm H is the upper bound of its range, which is given as a

function of the upper bound of C_{L1}. Now, the feasible range of C_{L1} is derived from the

incentive compatibility conditions of both firms, which are given as follows: in an

equilibrium involving the L-H-X listing order,

$$\frac{\{3t + (\frac{K_H}{C_L})\Delta\tilde{q}^{YY1} + (1 - \frac{K_H}{C_L})\Delta\tilde{q}^{NY}\}^2}{18t} - K_H \leq \frac{\{3t + (\frac{K_L}{C_L})\Delta\tilde{q}^{YY2} + (1 - \frac{K_L}{C_L})\Delta\tilde{q}^{YN}\}^2}{18t} - \{(\frac{K_L}{C_L})C_2 + (1 - \frac{K_L}{C_L})C_0\} \quad \text{(A50)}$$

$$\frac{\{3t - (\frac{K_H}{C_L})\Delta\tilde{q}^{YY1} - (1-\frac{K_H}{C_L})\Delta\tilde{q}^{NY}\}^2}{18t} - \{(\frac{K_H}{C_L})C_2 + (1-\frac{K_H}{C_L})C_0\} \le \frac{\{3t - (\frac{K_L}{C_L})\Delta\tilde{q}^{YY2} - (1-\frac{K_L}{C_L})\Delta\tilde{q}^{YN}\}^2}{18t} - K_L. \tag{A51}$$

Noting that $C_L \ge K_L$, the solutions to (A50) and (A51) are respectively given by $C_L \ge \rho$

and $C_L \le \phi$, where ρ and ϕ are respectively defined in (A48) and (A49). Then, $\phi > \rho$ if

and only if $A(E_1, \Delta q) < 0$, where $A(E_1, \Delta q)$ is defined in the proof of Corollary 1.

Thus, the feasible range of C_{L1} is given by $\rho \le C_{L1} \le \phi$ if $A < 0$. Otherwise, no C_{L1}

satisfies both (A50) and (A51) and thus, there is no equilibrium in the bidding game.

Therefore, the optimal bid is given by $\dfrac{\phi}{s_1 r_H}$ with ϕ defined as in (A49), whenever

$A(E_1, \Delta q) < 0$. $\square$

Proof of Proposition 19

We first prove the case for Firm H. Under the budget constraint, $\Pi_H^{YY1} \ge \Pi_H^{NY}$ is

equivalent to the following:

$$\frac{\{3t + \kappa_1\Delta\tilde{q}^{YY1} + (1-\kappa_1)\Delta\tilde{q}^{NY}\}^2}{18t} - K_H \ge \frac{\{3t + \Delta\tilde{q}^{NY}\}^2}{18t} \tag{A52}$$

where $\kappa_1 = \dfrac{K_H}{C_{H1}^{**}}$ and C_{i1}^{**} is the equilibrium advertising cost of Firm i in slot 1 under

budget constraint. This is equivalent to $\Delta q > \beta_3(E_1)$, where

$$\beta_3(E_1) \equiv \frac{-\{2\kappa_1(1+\gamma+\varepsilon)+\delta(\gamma+\varepsilon)\kappa_1^2 - \delta\kappa_1(1-\kappa_1)\}E_1 - \kappa_1\delta\{3t - q^0(3+\kappa_1\delta)\}}{\kappa_1\delta(4+\kappa_1\delta)}$$

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$$+\frac{\sqrt{\kappa_1\{(2(1+\gamma+\varepsilon)+\delta)E_1-\delta(3t+q^0)\}^2+18t\kappa_1\delta(4+\kappa_1\delta)K_H}}{\kappa_1\delta(4+\kappa_1\delta)}. \tag{A53}$$

Recall from Proposition 6 that $\beta_{11}(E_1)$ is the threshold of Δq between the scenarios: YY1

and NY, without any budget constraint. Within that expression, C_{H1}^* stands for the

equilibrium advertising cost of Firm H in slot 1 without budget constraint. Now, on

comparing $\beta_{11}(E_1)$ and $\beta_3(E_1) > 0$, we have;

- if $\kappa_1 \leq \dfrac{K_H}{C_{H1}^*}(\Leftrightarrow C_{H1}^{**} \geq C_{H1}^*)$: $\beta_{11}(E_1) < \beta_3(E_1)$

- if $\kappa_1 > \dfrac{K_H}{C_{H1}^*}(\Leftrightarrow C_{H1}^{**} < C_{H1}^*)$: $\beta_{11}(E_1) < \beta_3(E_1)$ if and only if

$$E_1 > \frac{1}{2(1+\gamma+\varepsilon)+\delta}\{-(3t-q^0)\delta+\frac{3\sqrt{t}\{\kappa_1(4+\kappa_1\delta)C_{H1}^*-(4+\delta)K_H\}}{\sqrt{2(1-\kappa_1)\kappa_1(\kappa_1 C_{H1}^*-K_H)}}\}.$$

We now prove the case for Firm L. Under the budget constraint, $\Pi_L^{YY2} \geq \Pi_L^{YN}$ is

equivalent to the following:

$$\frac{\{3t-\kappa_2\Delta\tilde{q}^{YY2}-(1-\kappa_2)\Delta\tilde{q}^{YN}\}^2}{18t}-K_L \geq \frac{\{3t-\Delta\tilde{q}^{YN}\}^2}{18t} \tag{A54}$$

where $\kappa_2 = \dfrac{K_L}{C_{L1}^{**}}$. This is equivalent to $\Delta q > \beta_4(E_1)$, where

$$\beta_4(E_1) \equiv \frac{\kappa_2\{\kappa_2(1+\delta+\varepsilon+\delta\varepsilon)+\varepsilon-\gamma-\delta-\gamma\delta\}E_1+\kappa_2\{3t-q^0(1+\kappa_2+\kappa_2\delta)\}}{\kappa_2(2+\kappa_2(1+\delta))(1+\delta)}$$

$$-\frac{\sqrt{\kappa_2^2\{(2+\gamma+\varepsilon+\delta(\gamma+1))E_1-(3t-q^0)(1+\delta)\}^2+18t\kappa_2(1+\delta)(2+\kappa_2(1+\delta))K_L}}{\kappa_2(2+\kappa_2(1+\delta))(1+\delta)}. \tag{A55}$$

Recall from Proposition 7 that $\beta_{22}(E_1)$ is the threshold of Δq between the scenarios: YY2 and YN, without any budget constraint. Within that expression, C_{L1}^* stands for the equilibrium advertising cost of Firm L in slot 1 without budget constraint. Now, on comparing $\beta_{22}(E_1)$ and $\beta_4(E_1) > 0$, we have;

- if $\kappa_2 \leq \dfrac{K_L}{C_{L1}^*}$ ($\Leftrightarrow C_{L1}^{**} \geq C_{L1}^*$): $\beta_{22}(E_1) > \beta_4(E_1)$

- if $\kappa_2 > \dfrac{K_L}{C_{L1}^*}$ ($\Leftrightarrow C_{L1}^{**} < C_{L1}^*$): $\beta_{22}(E_1) < \beta_4(E_1)$ if and only if

$$E_1 > \frac{1}{2+\gamma+\delta+\gamma\delta+\varepsilon}\{(3t-q^0)(1+\delta)+\frac{3\sqrt{t}\{\kappa_2(2+\kappa_2(1+\delta))C_{L1}^*-(3+\delta)K_L\}}{\sqrt{2\kappa_2(1-\kappa_2)(\kappa_2C_{L1}^*-K_L)}}\}.$$

$\square$

Study 1: Estimation Results from Alternative Estimation Methods

In the text, we reported the estimation result based on the model taking account of the response order. Here we report the estimation results by other estimation methods. In particular, we summarize in the following table, results from (1) OLS, (2) GLS in the random effects model, (3) GLS in the random effects model with heteroscedasticity, and (4) within-group estimator in the fixed effects model, (5) GLS in the model of response order (which is the model used in the text), and (6) GLS in the model of response order with heteroscedasticity. Note that in the fixed effects model, the estimates for q_S and q_P are in fact the estimate for the difference from q_H, which was normalized to be zero.

Parameter	(1)	(2)	(3)	(4)	(5)	(6)
q_S	5.4712	5.3556	2.1521	2.8595	5.3432	5.5557
	(0.4035)	(0.4817)	(2.9377)	(0.2346)	(0.7147)	(0.2690)
q_P	3.7085	3.7023	3.4826	1.3469	3.3914	3.8847
	(0.3511)	(0.4458)	(2.9327)	(0.1676)	(1.4124)	(0.4917)
q_H	2.4621	2.4347	2.1504	.	2.0748	2.6055
	(0.3525)	(0.4473)	(2.9316)		(1.4144)	(0.4890)
E_1^S	-0.2830	-0.1324	-0.3915	0.0140	-0.1221	-0.1718
	(0.2947)	(0.2683)	(0.2695)	(0.2925)	(0.1577)	(0.1222)
E_1^P	0.4845	0.6662	0.7542	0.8168	0.6897	0.7595
	(0.3958)	(0.4060)	(0.3064)	(0.5508)	(0.4924)	(0.2181)
E_1^{PP}	1.5655	1.4025	0.9511	1.0249	1.2251	1.1090
	(0.3586)	(0.3694)	(0.2444)	(0.5066)	(0.4551)	(0.1800)
E_1^H	-0.0448	0.3516	0.8059	1.1170	0.6694	0.7381
	(0.3871)	(0.3968)	(0.3439)	(0.5393)	(0.4806)	(0.2254)
E_1^{HH}	0.8964	0.7167	0.7644	0.4943	0.6314	0.6905
	(0.3719)	(0.3848)	(0.3077)	(0.5295)	(0.4775)	(0.2251)
E_2^P	-0.0344	0.1111	0.0479	0.2568	0.1195	0.1016
	(0.3112)	(0.2899)	(0.2486)	(0.3451)	(0.1911)	(0.1310)
E_2^{PP}	0.4958	0.3936	0.3207	0.2020	0.3349	0.3568
	(0.2780)	(0.2594)	(0.2032)	(0.3107)	(0.1722)	(0.1095)
E_2^H	-0.0161	0.2262	0.3741	0.6396	0.3005	0.3837
	(0.3058)	(0.2849)	(0.2659)	(0.3390)	(0.1876)	(0.1410)
E_2^{HH}	0.4438	0.3227	0.4356	0.1691	0.2991	0.3642
	(0.2871)	(0.2684)	(0.2463)	(0.3231)	(0.1793)	(0.1272)

(The standard errors are reported in the parenthesis.)

Study 2: Estimation Results from Alternative Estimation Methods

We also report the estimation results from alternative estimation methods in Study 2.

We omit here the within-group estimator, because it cannot estimate E_2^I and E_2^{II}. In the

following table, we summarize results from (1) OLS, (2) GLS in the random effects model, (3) GLS in the random effects model with heteroscedasticity, and (4) GLS in the model of response order, and (5) GLS in the model of response order with heteroscedasticity. Note that since we only have two observations in a group (participant), the estimates for models with heteroscedastic error are not reliable.

Table 12: Parameter Estimates by Different Estimation Methods in Study 2

Parameter	(1)	(2)	(3)	(4)	(5)
q_P	4.0136	3.9993	5.8840	4.6257	4.9278
	(0.4056)	(0.4849)	(97.4106)	(0.6500)	(0.0981)
q_I	3.1553	3.1565	4.9584	3.3790	4.0204
	(0.3987)	(0.4804)	(97.4050)	(1.2921)	(0.1009)
E_1^P	0.1554	0.1859	0.0711	0.1694	0.0898
	(0.1605)	(0.1416)	(1.1157)	(0.1047)	(0.0271)
E_1^I	-0.0637	-0.0484	-0.5068	-0.0538	-0.1203
	(0.2099)	(0.2456)	(1.1147)	(0.4843)	(0.3352)
E_1^{II}	-0.0392	-0.0543	-0.5176	-0.0538	0.0825
	(0.2052)	(0.2399)	(2.1381)	(0.4727)	(0.3342)
E_2^I	0.0418	0.0572	-0.2162	0.0507	-0.0142
	(0.1591)	(0.1546)	(1.0805)	(0.1872)	(0.1676)
E_2^{II}	-0.1819	-0.1970	-0.4022	-0.1936	-0.1027
	(0.1556)	(0.1511)	(1.5127)	(0.1828)	(0.1640)

(The standard errors are reported in the parenthesis.)

References

Agarwal, A., K. Hosanagar, and M. D. Smith (2008), ``Location, Location, Location: An Analysis of Profitability of Position in Online Advertising Markets,'' Working paper, University of Pennsylvania.

Anderson, R.E. (1973), ``Consumer Dissatisfaction: The Effect of Disconfirmed Expectancy on Perceived Product Performance,'' *Journal of Marketing Research*, 10(1), 38-44.

Athey, S., and G. Ellison (2008), ``Position Auctions with Consumer Search,'' Working paper, Harvard University.

Baker, W.E. (1993), ``The Relevance Accessibility Model of Advertising Effectiveness,'' in A. A. Mitchell (Ed.), *Advertising Exposure, Memory, and Choice,* New Jersey, Lawrence Erlbaum Associates, Inc.

Baker, W.E. and R.J. Lutz (2000), ``An Empirical Test of an Updated Relevance-Accessibility Model of Advertising Effectiveness,'' *Journal of Advertising*, 29(1), 1-14.

Balachander, S. and K. Kannan (2008), ``A Theoretical and Empirical Analysis of Alternate Auction Policies for Search Advertisements,'' Working paper, Purdue University.

Batra, R., J.G. Myers, and D.A. Aaker (1996), *Advertising Management,* Prentice Hall, Upper Saddle River, NJ.

Boulding, B., A. Kalra, and R. Staelin (1999), ``The Quality Double Whammy,'' *Marketing Science*, 18(4), 463-484.

Chen, Y. and C. He (2006), ``Paid Placement: Advertising and Search on the Internet,'' Working paper, University of Colorado and Boulder.

Chen, Y., Y. V. Joshi, J. S. Raju, and Z. J. Zhang (2009), ``A Theory of Combative Adveritising,'' *Marketing Science*, 28(1), 1-19.

Dellarocas, C. and S. Viswanathan (2008), ``The Holy Grail of Advertising? Allocative Efficiency and Revenue Implications of ``Pay-Per-Action'' Advertising in Environments with Quality Uncertainty,'' Working paper, University of Maryland.

Desai, P., and W. Shin (2006), ``Allocation Mechanisms in Search Advertising: A Two-Sided Market Approach,'' Working paper, Duke University.

Desai, P., and W. Shin (2009), ``Advertiser-Specific Minimum Bids and Advertising Budgets in Keyword Search Auctions,'' Working paper, Duke University.

Diamond, P.A. (1971), ``A Model of Price Adjustment,'' *Journal of Economic Theory*, 3, 156-168.

Edelman, B., M. Ostrovsky, and M. Schwarz (2007), ``Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords,'' *American Economic Review*, 97(1), 242-258.

Enquiro Research and Google (2007), *The Brand Lift of Search: Enquiro Online Survey and Eye Tracking Study*, July 2007.

Feng, J., and J. Xie (2009), ``Performance-Based Pricing: Advertising as Signal of Product Quality,'' Working paper, University of Florida.

Ganchev, K., A. Kulesza, J. Tan, G. Ryan, Q. Liu, and M. Kearns (2007), ``Empirical Price Modeling for Sponsored Search,'' *Proceedings of the 3rd International Workshop On Internet And Network Economics*, San Diego, CA, Springer.

Ghose, A. and S. Yang (2009), ``An Empirical Analysis of Search Engine Advertising: Sponsored Search in Electronic Markets,'' *Management Science*, Forthcoming.

Google, ``Eye-Tracking Studies: More than Meets the Eye'', *Official Google Blog*, February 6, 2009, [Available at: http://googleblog.blogspot.com/2009/02/eye-tracking-studies-more-than-meets.html (Accessed March 19,2010)].

Google and Media Screen (2008), *The Brand Value of Search: Consumer Packaged Goods Industry Study*, July 2008.

Gorn, R.J., and C. B. Weinberg (1984), ``The Impact of Comparative Advertising on Perception and Attitude: Some Positive Findings,'' *Journal of Consumer Research*, 11(2), 719-727.

Greene, W. (2003), *Econometric Analysis*, New Jersey, Prentice Hall.

Grewal, D., R. Krishnan, J. Baker, and N. Borin (1998), ``The Effects of Store Name, Brand Name, and Price Discounts on Consumers' Evaluations and Purchase Intentions,'' *Journal of Retailing*, 74(3), 331-352.

Helft, M. (2009) ``Companies Object to Google Policy on Trademarks,'' *The New York Times*, May 14, 2009.

Hovland, C.I., O.J.Harvey, and M. Sherif (1957), ``Assimilation and Contrast Effects in Reactions to Communication and Attitude Change,'' *Journal of Abnormal and Social Psychology*, 55, 244-252.

Interactive Advertising Bureau (2008), *IAB Internet Adverrising Revenue Report, 2007 Full-Year Result*, May 2008, PriceWaterhouseCoopers.

Interactive Advertising Bureau (2009), *IAB Internet Adverrising Revenue Report, 2008 Full-Year Result*, May 2009, PriceWaterhouseCoopers.

Interactive Advertising Bureau (2010), *IAB Internet Adverrising Revenue Report, 2009 Second Quarter and Fist Six Months Result*, October 2010, PriceWaterhouseCoopers.

Iyer, G., D. Soberman, and J. M. Villas-Boas (2005), ``The Targeting of Adveritising,'' *Marketing Science*, 24(3), 461-476.

Janiszewski, C. (1993), ``Preattentive Mere Exposure Effects,'' *Journal of Consumer Research*, 20(3), 376-392.

Jerath, K., L. Ma, Y-H. Park, and K. Srinivasan (2009), ``A ``Position Paradox'' in Sponsored Search Auctions,'' Working paper, Carnegie Mellon University.

Katona, Z., and M. Sarvary (2010), ``The Race for Sponsored Links: Bidding Patterns for Search Advertising,'' *Marketing Science*, 29(2), 199-215.

Lahaie (2006), ``An Analysis of Alternative Slot Auction Designs for Sponsored Search,'' In Proceeding. Second Workshop on Sponsored Search Auctions at ACM Conference on Electronic Commerce (EC'06), June 11, Ann Arbor, MI.

Ledgerwood, A., and S. Chaiken (2007), ``Priming Us and Them: Automatic Assimilation and Contrast in Group Attitudes,'' *Journal of Personality and Social Psychology*, 93(6), 940-956.

Lee, K. (2009), ``New Research: Paid Search Builds Branding,'' *ClickZ*, April 17, 2009 [Available at: http://www.clickz.com/3633431 (Accessed March 19,2010)].

Lew, G. (2009), ``The Role of Eye Tracking in User Experience Research,'' *HFES Webinar Serires*, User Centric, Inc., December 11, 2009 [Available at:

http://www.usercentric.com/sites/usercentric.com/files/HFES-ET-webinar2009.pdf (Accessed March 19,2010)].

Lichtenstein D.R., and W.O. Bearden (1989), ``Contextual Influences on Perceptions of Merchant-Supplied Reference Prices,'' *Journal of Consumer Research*, 16, 55-66.

Liu, D., J. Chen, and A. B. Whinston (2008) ``Ex-Ante Information and the Design of Keyword Auctions,'' *Information Systems Research*, Forthcoming.

Mussweiler, T. (2001), ``Focus of Comparisons as a Determinant of Assimilation Versus Contrast in Social Comparison,'' *Personality and Social Psychology Bulletin*, 27(1), 38-47.

Mussweiler, T. (2003), ``Comparison Processes in Social Judgment: Mechanisms and Consequences,'' *Psychological Review*, 110(3), 472-489.

Nedungadi, P., A. A. Mitchell, and I.E. Berger (1993), ``A Framework for Understanding the Effects of Advertising Exposure on Choice,'' in A. A. Mitchell (Ed.), *Advertising Exposure, Memory, and Choice,* New Jersey, Lawrence Erlbaum Associates, Inc.

Obermiller, C. (1985), ``Varieties of Mere Exposure: The Effects of Processing Style and Repetition on Affective Response,'' *Journal of Consumer Research*, 12(1), 17-30.

Orey, M. (2009) ``Google's Rival Keyword Sales Go Wide,'' *Businessweek,* May 11, 2009.

Olshavsky, R.W., and J.A. Miller (1972), ``Consumer Expectations, Product Performance, and Perceived Quality,'' *Journal of Marketing Research*, 9(1), 19-21.

Online Publishers Association (2009), *The Silent Click: Building Brands Online*, June 2009.

Rutz, O. and R. Bucklin (2007a) ``From Generic to Branded: A Model of Spillover Dynamics in Paid Search Advertising,'' Working paper, Yale University.

Rutz, O. and R. Bucklin (2007b) ``A Model of Individual Keyword Performance in Paid Search Advertising,'' Working paper, Yale University.

Salop, S.C., and D.T., Scheffman (1983) ``Raising Rivals' Costs,'' *AEA Papers and Preceedings*, 73(2), 267-271.

Salop, S.C., and D.T., Scheffman (1987) ``Cost-Raising Strategies,'' *Journal of Industrial Economics*, 36(1), 19-34.

Sherif, M, D. Taub, and C.I. Hovland (1958), ``Assimilation and Contrast Effects of Anchoring Stimuli on Judgments,'' *Journal of Experimental Psychology*, 55(2), 150-155.

Stewart, K.J., and R.A. Malaga (2009), ``Contrast and Assimilation Effects on Consumers' Trust in Internet Companies,'' *International Journal of Electronic Commerce*, 13(3), 71-93.

Swinyard, W. R. (1981), ``The Interaction Between Comparative Advertising and Copy Claims Variation,'' *Journal of Marketing Research*, 18(2)), 175-186.

Tellis, G. J. (1988), ``Advertising Exposure, Loyalty, and Brand Purchase: A Two-stage Model of Choice,'' *Journal of Marketing Research*, 25(2), 134-144.

Wilbur, K., and Y. Zhu (2008) ``Click Fraud,'' *Marketing Science*, 28(2), 293-308.

Winter, F.W. (1973) ``A Laboratory Experiment of Individual Attitude Response to Advertising Exposure,'' *Journal of Marketing Research*, 10(2), 130-140.

Varian, H. (2007) ``Position Auctions,'' *International Journal of Industrial Organization*, 25, 1163-1178.

Xu, L., J. Chen, and A.B. Whinston (2009) ``To Place Better or Price Cheaper? Bidding and Pricing under Keyword Advertising,'' Working paper, University of Texas, Austin.

Yang, S. and A. Ghose (2010) ``Analyzing the Relationship Between Organic and Sponsored Search Advertising: Positive, Negative, or Zero Interdependence?,'' *Marketing Science*, Forthcoming.

Yao, S. and C. Mela (2009) ``A Dynamic Model of Sponsored Search Advertising,'' Working paper, Duke University.

Zajonc, R.B. (1968) ``Attitudinal Effects of Mere Exposure,'' *Journal of Personality and Social Psychology*, 9, 1-27.

Biography

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