The Comparison of Two Vertical Outsourcing Structures under Push and Pull Contracts

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In a three-tier supply chain comprising an original equipment manufacturer (OEM), a contract manufacturer (CM) and a supplier, there exist two typical outsourcing structures: control and delegation. Under the control structure, the OEM contracts with the CM and the supplier respectively. Under the delegation structure, the OEM contracts with the CM only and the CM subcontracts with the supplier. We compare the two outsourcing structures under a push contract (whereby orders are placed before demand is realized) and a pull contract (whereby orders are placed after demand is realized). For all combinations of outsourcing structures and contracts, we derive the corresponding equilibrium wholesale prices, order quantities and capacities. We find that the equilibrium production quantity is higher under control than under delegation for the push contract whereas the reverse holds for the pull contract. Both the OEM and the CM prefer control over delegation under the push contract. However, under the pull contract, the OEM prefers control over delegation whereas the CM and the supplier prefer delegation over control. We also show that for a given outsourcing structure, the OEM prefers the pull contract over the push contract. In extending our settings to a general two-wholesale-price (TWP) contract, we find that when wholesale prices are endogenized decision variables, the TWP contract under our setting degenerates to either a push or a pull contract.

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1 Introduction

Nowadays, there are unprecedented opportunities for original equipment manufacturers (OEMs) to outsource all of their assembly functions to contract manufacturers (CMs). By doing so, OEMs can enjoy the benefits of reduced labor costs, freed-up capital and improved worker productivity. Facilitating these gains are the CMs' special strengths, which may include location in a low-wage area, economies of scale, and exposure to the engineering and development processes of the products handled for other OEMs (Arruñda and Vázquez 2006).

The relocation of manufacturing processes to low-cost destinations has driven China, for instance, to become the "world's factory." By 2007, China accounted for 13.2% of all the manufacturing in the world and is set to overtake the USA as the number one destination for manufacturing (Jayaraman 2009). In today's global economy, the CM networks in such countries serve as an important manufacturing base for numerous goods, ranging from garments, toys, mobile handsets, and computers to household appliances and even musical instruments. For instance, more than 90% of Chinese home electronics companies are engaged in the CM business (Yang and Wu 2008). In another example, the Chinese microwave manufacturer Galanz produces microwave ovens for more than 250 international brands, holding a market share of more than 40% of all microwaves sold worldwide (Yang and Wu 2008). In an AMR research report based on an extensive survey of more than 700 brand-owners/OEMs and CMs located primarily in North America, about 65% of the respondents frequently used CMs in mainland China and Taiwan (Swanton et al. 2005). There is also a growing trend for the OEMs located in China to outsource their assembly function to CMs such as Flextronics, Foxconn, Compal and Wistron. For example, Lenovo signed an agreement with Flextronics to manufacture its commercial desktop, server and workstation products (Ligan et al. 2009). Acer named Compal and Wistron as its two biggest contract manufacturers of laptops (Schofield 2010). Huawei outsourced the production of its broadband products to Foxconn (Shen 2007).

However, outsourcing activities enlarge the distance between the supply chain parties and lengthen lead time. This gives rise to greater risks in production planning and capacity decisions for CMs and suppliers, as these decisions need to be made well before demand is observed. It is thus necessary to explore (1) how outsourcing structures and contracting arrangements affect the inventory/capacity risks in a supply chain and (2) which outsourcing arrangement can make the OEM better off and under what conditions. We term this type of questions as "*who should order?*"

Consider a serial three-tier supply chain comprising an OEM, a CM and a supplier. Compared with a two-tier supply chain, this multi-tier supply chain provides one more layer of flexibility to the OEM by allowing it not only to decide how to share its inventory/capacity risk with the upstream parties but also to choose the way in which it outsources manufacturing: The OEM can either outsource just the product manufacturing function to the CM and control the procurement of components from the supplier, or it can outsource both the product manufacturing and component procurement functions to the CM. We call these two outsourcing structures *control* and *delegation*, represented by C and D respectively; see Figure 1. In the automobile industry, General Motors (GM), Ford, Chrysler and some European automobile companies have increasingly delegated component procurement responsibility to manufacturers over the past two decades (Kayiş et al. 2009). In contrast, Motorola, which delegated the component purchasing to its CMs in the 1990s, resorted to a control structure after 2003 (Jorgensen 2004, Smock 2004). Clearly, outsourcing structures change the ownership of inventories in a supply chain.

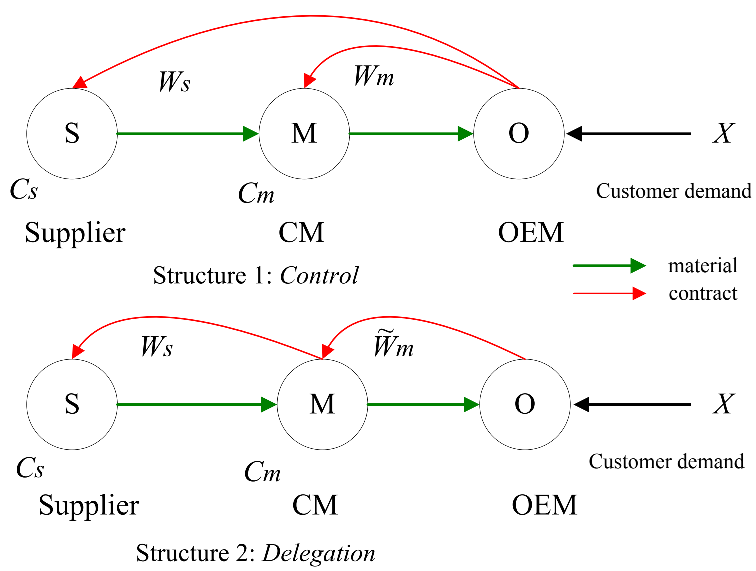

Figure 1: Control and Delegation

The sharing of inventory/capacity risk is also affected by the timing of orders, a type of question called "*when to order*?". In practice, some downstream OEMs ease the uncertainty of their upstream CMs and suppliers by adopting the *push* contract; that is, they place orders with the upstream CMs and suppliers before the selling season and hence bear all inventory risk. Take the fireworks industry as an example. It usually takes about four to six weeks to ship fireworks from China, where most fireworks are made, to the US (Quint and Shorten 2005). However, about 95% of US fireworks sales occur between May 15th and July 4th, which is a very short selling season. Given this, firework retailers in US have to purchase before the selling season and hence the push contract is adopted (Prasad et al. 2011). In contrast to the push contract, there exists another type of contract, the *pull* contract, under which OEMs place orders during the selling season and the upstream CMs and suppliers have to bear all inventory risk. In industrial practice, the vendor managed inventory (VMI) agreement is a typical pull contract, in which the suppliers commit capacities/resources for the OEMs and make capital investment without receiving payment until after the resources are used (Li and Scheller-Wolf 2011). For example, Flextronics provides a VMI service to Lenovo, a world leading Chinese personal computer provider (Ligan et al. 2009). Another example is CEPA, which makes sheet metal components for OEMs such as ABB, Alfa Laval and Hasselblad; it holds all inventory in its warehouses and delivers the components to OEMs only when they receive firm customer orders (Jukka et al. 2007). Other than these two extreme risk-allocation schemes, a third intermediate two-wholesale-price (TWP) contract is also observed in practice, under which the inventory/capacity risk is partially shared between downstream OEMs and their upstream supply chain parties as the OEMs both prebook and make at-once orders and the prebooking price is lower than that for the at-once order. For example, in the LCD industry, OEM manufacturers such as Innolux Display, TPV Technology and LG have signed advance commitment contracts with their LCD panel supplier–Taiwan Chunghwa Picture Tubes (Hayes 2007).

According to the foregoing discussion, we have four supply chain scenarios that are the combinations of two contracts (when to order) and two vertical outsourcing structures (who should order): *Push+Control, Push+Delegation, Pull+Control,* and *Pull+Delegation*. For each scenario, we analyze the game among the three supply chain parties to obtain the equilibrium ordering, wholesale price and capacity decisions. These results answer our questions about *how much should be ordered* and *at what price should orders be placed*. Then, by comparing the performance of each party across the four scenarios, we can answer questions 1 and 2, about who should order and when orders should be placed. At the end of this paper, we also consider the combination of the pull and push contracts and study a unified TWP contract under which both pre-orders and at-once orders are allowed, each corresponding to a wholesale price.

By comparing the OEM's profits from delegation and control, we can answer question 1. Under the push contract where the ordering happens before demand realization, the CM and the supplier set up their capacity the same as what the OEM orders and the OEM bears all of the inventory risk. The decision on whether to adopt control or delegation depends solely on which structure generates a lower total procurement price for the OEM. Note that the pricing sequence is different under these two structures. Under control, we assume that the CM and the supplier simultaneously offer a wholesale price to the OEM. However, under delegation, the pricing game is sequential: the supplier first decides a wholesale price for the component and then the CM decides a wholesale price for the finished product. We find that the supplier charges a higher wholesale price under delegation than under control, which eventually drives up the total procurement price for the OEM and makes the OEM prefer control over delegation. This implies that the middleman role of the CM under delegation actually exacerbates the *double marginalization* effect. Interestingly, we find that the CM also prefers control over delegation. This is somewhat surprising as one might believe that the CM, as the middleman, may profit from procuring components for the OEM. The explanation is that a higher procurement price for the OEM under delegation reduces its incentives to place large orders, which in turn hurts the CM.

Under the pull contract, the OEM's ordering happens after demand realization. The CM and the supplier have to bear their inventory risks as their capacities are set up before demand realization. In such a case, a lower procurement price for the OEM need not mean that it is better off because that can reduce the CM's and the supplier's incentives for capacity building. Indeed, we find that if the wholesale price paid by the OEM to the CM under delegation falls in a moderate range, then delegating the procurement function to the CM is more beneficial to the OEM. That means a too high or a too low wholesale price will reduce the benefit of delegation. This is intuitive. On the one hand, a high wholesale price under delegation hurts the OEM's profit margin and reduces its incentive to adopt the delegation structure. On the other hand, a low wholesale price hurts the CM and reduces its incentive to build up a large capacity, which eventually hurts the OEM. Different from a two-tier supply chain where capacity is usually decided by one party, the whole supply chain capacity in our three-tier supply chain is decided by the capacities of two parties, the CM and the supplier. The equilibrium wholesale prices, regardless of what gaming sequences are adopted, must balance the two parties' capacity setting-up incentives as no party has the incentive to build up more capacity than the other. This balance condition yields a conclusion that the equilibrium wholesale prices of the CM and the supplier are always *proportional* to their respective production costs. This simple yet intuitive markup pricing rule is broadly used in outsourcing practice. Based on this finding, we further demonstrate that the CM's and the supplier's preferences for outsourcing structures are *aligned*. Specifically, we find that the OEM always prefers control over delegation whereas the CM and the supplier always prefer delegation over control.

Another comparison along the order timing dimension allows us to answer question 2. For a given outsourcing structure, we show that both the production quantity of the supply chain and the profit of the OEM are higher under the pull contract than under the push contract. We note that similar results are obtained in Cachon (2004) within a two-tier supply chain setting. Here, we demonstrate that pull is still preferable to the OEM in a three-tier supply chain setting. Readers, however, should note that our conclusion and that of Cachon (2004) are obtained under the assumption that the pricing sequences under pull and push contracts are different. Under the push contract, the CM and the supplier decide their wholesale prices, while under the pull contract the OEM decides the wholesale price. Such a pricing sequence, according to Cachon (2004), is a natural setting.

We finally consider a TWP contract under which the supply chain parties are allowed to contract over more dimensions of variables. The interesting, perhaps surprising, analytical conclusion is that when wholesale prices are endogenized decision variables, the TWP contract actually reduces to either the push or the pull contract or a combination of them (the OEM adopts the pull contract with the CM whereas the CM adopts the push contract with the supplier). A comparison of the OEM's profits under reduced-form contracts shows that the OEM still prefers control.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model setting and preliminaries. Sections 4 and 5 study and compare the performance of the outsourcing structures, control and delegation under the push and pull contracts, respectively, by considering both the exogenous and endogenized wholesale price scenarios. Section 6 provides comparison results between push and pull contracts. In section 7 we extend the discussion to the general TWP contract. We provide concluding remarks in section 8. All the proofs are relegated to the online appendix.

2 Literature Review

Our work is closely related to the literature on quantity commitment and advance purchase in supply chain management. Push and pull contracts were first studied in Cachon (2004). Later Dong and Zhu (2007) consider an unified TWP contract in a two-tier supply chain under which the buyer places both early and late orders. These early orders are quantity committed by the buyer and are given price discounts. Note that TWP with a null early order is reduced to a pull contract and TWP with only an early order and no at-once orders during the selling season is a push contract. Other than these two studies, Cachon and Lariviere (2001) characterize a contract composed of firm commitments and options, to convey demand information. Lariviere and Porteus (2001) study a price-only contract whereby the retailer buys before the random demand is realized. Ferguson (2003) and Ferguson et al. (2005) focus on the manufacturer's commitment time decision (i.e., before or after demand realization). They illustrate the effect of the power structure of the supply chain and demand uncertainty. Özer et al. (2005) consider earlier commitment in a push system when the market is still unknown to the retailer and show that the entire supply chain can achieve Pareto optimization. Özer and Wei (2006) study an upstream firm with dominating power and show that advance purchasing can enable the downstream firm to reveal its private forecast information. Netessine and Rudi (2006) combine the traditional (push) and drop-shipping (pull) channel into a dual-strategy (advance-purchase discount) supply chain. They find that a drop-shipping supply chain can result in higher profits. Taylor (2006) investigates the circumstances under which a manufacturer would prefer to sell early or late, and assumes that the demand is retail-price dependent. Selling early and selling late are similar to push and pull contracts, respectively. These circumstances involve whether information is symmetrical and whether the retailer exerts sales effort. Bernstein et al. (2006) show that the pull-type VMI can coordinate the supply chain by considering two simple pricing schemes. Chen (2007) proposes a push-type purchasing mechanism whereby the buyer offers a quantity-payment contract and the supplier bids an up-front, lump-sum fee. Recently, Li and Scheller-Wolf (2011) consider a situation in which n suppliers with their private cost information bid for a buyer's order via an English auction. They then compare the buyer's performance under push and pull contracts. In addition to the aforementioned literature on push, pull and advance-purchase contracts, many other papers discuss wholesale price contracts. See the reviews by Cachon (2003) and Lariviere (1998) for a more detailed discussion.

Our work is not a simple extension of Cachon (2004) and Dong and Zhu (2007) from a two-tier supply chain into a three-tier supply chain. Our contribution also relies on addressing a new managerial problem "who should take the role of procurement?", which cannot be addressed in a two-tier supply chain setting.

Our work is also closely related to the research on the decentralized capacity decisions in multiple-tier supply chains. Bernstein and DeCroix (2004) investigate a modular assembly system in which the final assembler oursources some of the assembly tasks to subassemblers, and the subassembler buys the components from suppliers. They then discuss the optimal capacity decision for this system and characterize the equilibrium price and capacity choices. Gerchak and Wang (2004) study both a revenue-sharing contract and a wholesale-price contract in an assemble-to-order (ATO) system in which n suppliers' components are assembled to be a final product by the assembler. Bernstein et al. (2007) consider the equilibrium price and capacity decisions in an assembly system where multiple-type products share a common component. Chen et al. (2010a) study three pricing power structures in an ATO system with one assembler and two suppliers. The assembler can push or pull the two suppliers or can push one supplier but pull the other. Although these four papers investigate an ATO system with multiple suppliers and one assembler, which is quite different from our serial chain setting, there exist some similarities between those models and ours if we treat the CM and the supplier in our model as two complementary suppliers to the OEM. When the control outsourcing structure is adopted, similar to Gerchak and Wang (2004), we show that the CM and the supplier will provide the same amount of components to the OEM and there exists a unique optimal wholesale pricing equilibrium. When the CM is delegated with the purchasing function, then the model setting is similar to that of Bernstein and DeCroix (2004), except that their assembly price is determined first followed by the component wholesale prices whereas we consider the reversed sequence. Under our setting, we show that there exists a unique pricing equilibrium, whereas under their setting, multiple pricing equilibria can exist.

There exist extensive studies on comparing delegation and control structures in the literature of economics. Baron and Besanko (1992) consider the setting where the CM and supplier have private cost information. They show that delegation can not perform better than control because of loss-of-control cost. Mookherjee and Tsumagari (2004) show that if the collusion among agents exists, delegation may result in strictly more profits for the manufacturer than control. Cai and Cont (2004) study the optimal design issue of a delegation contract with a consideration of moral hazard and adverse selection problems. Severinov (2008) compares the three organizational forms, centralization, control and delegation, and finds that the principal's preference over organization structure depends on the degree of complementarity/substitutability between the inputs in the final use. Mookherjee and Tsumagari (2009) propose a one-principal-multi-agent model in which communication among the players is costly. They show that delegation can sometimes perform better than control and the value of delegation mainly depends on the principal's ability of verifying the messages exchanged among agents. See Mookherjee (2006) for a comprehensive review on comparison between delegation and control under asymmetric cost information.

Studies on comparing delegation and control structures in multiple-tier supply chains begin in recent years. Guo et al. (2010) study the impact of information distortion induced by different outsourcing structures. They show that with a long-term contract delegation performs better than control even with information distortion. Kayiş et al. (2012) consider delegation and control in a three-tier supply chain under the Newsvendor setting. They compare the optimal menu contract with the price-only contract and find that either delegation or control may be preferable, depending on the manufacturer's prior information on the suppliers' costs. Chen et al. (2010b) consider a situation in which a manufacturer either decides how to allocate its capacity among multiple retailers or delegates the decision to its distributor. Chen et al. (2012) study a three-tier supply chain in which the CM is a competitor of the OEM in the end-market. By comparing buy-sell and turnkey outsourcing structures, they identify the conditions for one to be preferable to the other.

3 Model Setting and Preliminaries

We use subscripts o, m and s to label the OEM, the CM and the supplier, and superscripts C and D to denote control and delegation outsourcing structures, respectively. The market price for the end product is exogenously given and denoted by p. One unit of the end product the CM produces requires one unit of the supplier's component. Assume that the CM and the supplier incur a cost of c_m and c_s for building one unit of their capacities, respectively. We also assume that the related fixed costs are sunk. To guarantee a positive profit margin, $p > c_m + c_s$ is assumed. The demand distribution and capacity installing costs are all common knowledge (see Plarmbeck and Taylor (2007) and Nagarajan and Bassok (2008) for a discussion on this assumption).

Consider that a long lead-time is required for production and there exist two ordering opportunities: an early order before production and a late order just before or during the selling season. Denote the pre-selling period as period 1 and the selling season as period 2. Assume that the wholesale prices are determined in period 1 before the orders and production take place. Under the control outsourcing structure, denote the wholesale price of player i in period t as w^C_{it}, $i = m, s$, $t = 1, 2$; and under the delegation outsourcing structure, denote the wholesale price of the supplier as w^D_{st}, $t = 1, 2$, and that of the CM as $\tilde{w}^D_{mt}, t = 1, 2$. Note that $\tilde{w}^D_{mt} \geq c_m + w^D_{st}, t = 1, 2$ is required because the wholesale price of the CM in this situation needs to cover both its manufacturing cost and its component procurement cost. To avoid the trivial case, we focus on the wholesale price region $\left\{w^C_{m1}, w^C_{m2}, w^C_{s1}, w^C_{s2}, w^D_{s1}, w^D_{s2}\right\} \in [c_m, p] \times [c_m, p] \times [c_s, p] \times [c_s, p] \times [c_s, p] \times [c_s, p]$. We also assume that $p - w^C_{mt} - w^C_{st} > 0$ and $p - \tilde{w}^D_{mt} > 0$, $t = 1, 2$.

Customer demand for the end product is random and denoted by a random variable X with a probability density function (pdf) f and a cumulative distribution function (cdf) F. Define $\bar{F}(x) = 1 - F(x)$. We further assume that the demand distribution has an increasing generalized failure rate (IGFR). Many common distributions have this property, including uniform, normal, logistic, extreme value, chi-square, chi, exponential, Laplace, Weibull ($r \geq 1$), gamma ($b \geq 1$), and beta ($\alpha \geq 1, \beta \geq 1$). This assumption has been widely used in the operations management literature: see Lariviere and Porteus (2001) and the references therein for further information. To facilitate our analysis in the following sections, let $D(Q) = E[min(X, Q)]$ denote the expected demand that can be satisfied by production quantity Q. Also, define

$$g(x) = \frac{xf(x)}{\bar{F}(x)};\ j(x) = \frac{D(x)}{\bar{F}(x)};\ h(x) = \frac{f(x)}{\bar{F}(x)}.$$

It can be shown that for the IGFR distribution, $g(x)$ increases in x ($g(x)' > 0$) and $j(x)h(x)$ increases in x ($(j(x)h(x))' > 0$) (Cachon 2004).

4 Push Contract

Under a push contract, there is no at-once order. The downstream supply chain party bears all of the inventory risk and orders before the demand is realized. Therefore, the upstream supply chain party just builds capacity for what is committed. Similar to Cachon (2004) and

Gerchak and Wang (2004), we assume that the CM and the supplier decide their respective wholesale prices.

4.1 Push and control

Under push and control, the game sequence is defined as follows.

1. In period 1, the CM and the supplier decide their unit wholesale prices w_{m1}^C and w_{s1}^C, respectively.

2. The OEM then announces its prebooking quantity Q to the CM and the supplier. (It is never in the best interests of the OEM to prebook different quantities to the CM and the supplier as the components of the CM and the supplier are complements.)

3. The CM and the supplier then build their capacities according to the OEM's prebook order.

Our model setting can be regarded as a special case of the one-manufacturer-multiple-supplier ATO system of Gerchak and Wang (2004) if we consider the CM as another supplier. Hence, most of the following results can be directly derived from theirs. However, we still provide the details on the intermediate steps derived in our setting.

In period 2, demand is realized and all revenues and costs are incurred. As a result, the profit functions of the three parties are, respectively,

$$\Pi_o = pD(Q) - (w_{m1}^C + w_{s1}^C)Q, \quad \Pi_m = (w_{m1}^C - c_m)Q, \quad \text{and} \quad \Pi_s = (w_{s1}^C - c_s)Q.$$

Therefore, the decision problem for the OEM is a Newsvendor-type problem. The following conclusion can be easily obtained from the above Newsvendor-type problem.

Proposition 1. *Under push and control, the OEM's optimal prebook*

$$Q^C = \bar{F}^{-1}\left(\frac{w_{m1}^C + w_{s1}^C}{p}\right). \tag{1}$$

Note that $Q^C = \bar{F}^{-1}\left(\frac{w_{m1}^C+w_{s1}^C}{p}\right)$ is also the supply chain capacity (the minimum of the capacities of the CM and the supplier).

Next, similar to Lariviere and Porteus (2001), from equation (1) we can derive the following one-to-one relationships:

$$w_{m1}^C(Q^C) = p\bar{F}(Q^C) - w_{s1}^C; \tag{2}$$

$$w_{s1}^C(Q^C) = p\bar{F}(Q^C) - w_{m1}^C. \tag{3}$$

Then substituting (2) and (3) respectively into the CM's and the supplier's profit functions yields

$$\begin{aligned}
\text{Max } \Pi_m(Q^C) &= \left(w_{m1}^C(Q^C) - c_m\right) Q^C = \left(p\bar{F}(Q^C) - w_{s1}^C - c_m\right) Q^C; \\
\text{Max } \Pi_s(Q^C) &= \left(w_{s1}^C(Q^C) - c_s\right) Q^C = \left(p\bar{F}(Q^C) - w_{m1}^C - c_s\right) Q^C.
\end{aligned}$$

It can be shown that $\Pi_m(Q^C)$ and $\Pi_s(Q^C)$ are quasi-concave in Q^C when the demand distribution has IGFR (Lariviere and Porteus 2001), and the optimal system capacity (equivalently, the OEM's ordering quantity) will simultaneously satisfy the following two first-order conditions (FOCs), which can be directly obtained from equation (23) of Gerchak and Wang (2004):

$$\frac{\partial \Pi_m(Q^C)}{\partial Q^C} = p\bar{F}(Q^C)(1 - g(Q^C)) - w_{s1}^C - c_m = 0; \tag{4}$$

$$\frac{\partial \Pi_s(Q^C)}{\partial Q^C} = p\bar{F}(Q^C)(1 - g(Q^C)) - w_{m1}^C - c_s = 0. \tag{5}$$

As $\Pi_m(Q^C)$ and $\Pi_s(Q^C)$ are continuous and quasi-concave in Q^C, and the strategy space for the wholesale prices $w_{i1}^C, i = m, s$ is a nonempty, compact, and convex set, from Theorem 1.2 of Fudenberg and Tirole (1991) and Proposition 7 of Gerchak and Wang (2004), we have the following proposition.

Proposition 2. *Under push and control,*

(1) there exists a unique equilibrium ordering/production quantity Q^{CS} that satisfies

$$\bar{F}(Q^{CS}) - 2Q^{CS}f(Q^{CS}) = \frac{c_m + c_s}{p}. \tag{6}$$

(2) there exists a unique wholesale pricing Nash equilibrium $(w_{m1}^{CS}, w_{s1}^{CS})$ that satisfies $w_{m1}^{CS} = p\bar{F}(Q^{CS})(1 - g(Q^{CS})) - c_s$, and $w_{s1}^{CS} = p\bar{F}(Q^{CS})(1 - g(Q^{CS})) - c_m$. And $w_{m1}^{CS} + c_s = w_{s1}^{CS} + c_m$.

In our three-tier supply chain, the system optimal production quantity denoted by Q^* can be shown to satisfy

$$\bar{F}(Q^*) = \frac{c_m + c_s}{p}.$$

A comparison of this with equation (6) in Proposition 2 shows that the system production quantity under push and control Q^{CS} is less than Q^*, where the additive term $2Q^{CS}f(Q^{CS})$

measures system capacity loss due to decentralization. As $p\bar{F}(Q^{CS}) = w_{s1}^{CS} + w_{m1}^{CS}$, equation (6) can be re-written as

$$\underbrace{p - (w_{s1}^{CS} + w_{m1}^{CS})}_{(1)} + 2pQ^{CS} f(Q^{CS}) = \underbrace{p - (c_m + c_s)}_{(2)},$$

where item (1) is the OEM's profit margin in the decentralized setting while item (2) is that in a centralized setting. Hence, $2Q^{CS} f(Q^{CS})$ also measures the degree of double marginalization in this decentralized supply chain. Note that in Cachon (2004), for a two-tier supply chain under the push contract, the following relationship holds

$$\underbrace{p - w_1^{CS}}_{(1)} + pQ^{CS} f(Q^{CS}) = \underbrace{p - c}_{(2)},$$

where c is the supplier's cost. Our results here show that when the supply chain expands from two to three tiers, the degree of double marginalization further increases.

Proposition 2 also shows that in the pricing equilibrium, the CM and the supplier will set their wholesale prices in such a way that they receive the same profit margin from selling one unit of the component/end-product, that is, $w_{m1}^{CS} - c_m = w_{s1}^{CS} - c_s$.

4.2 Push and delegation

Under push and delegation, the game sequence is as follows.

1. In period 1, the supplier decides its wholesale price w_{s1}^{D} first and after that the CM decides its wholesale price $\tilde{w}_{m1}^{D}$.

2. Then the OEM announces its prebooking quantity Q to the CM. The CM then announces the OEM's prebooking quantity Q to the supplier. (It is never in the best interests of the CM to prebook a different quantity than Q to the supplier because of complementarity between the CM's and the supplier's products.)

3. The CM and the supplier build their capacities according to their prebooked orders.

In period 2, demand is realized and all revenues and costs are incurred. Similarly, we can write the profit functions of the supply chain parties as

$$\Pi_o = pD(Q) - \tilde{w}_{m1}^{D} Q, \quad \Pi_m = (\tilde{w}_{m1}^{D} - w_{s1}^{D} - c_m)Q, \text{ and } \Pi_s = (w_{s1}^{D} - c_s)Q.$$

Again, the OEM's optimization problem is a Newsvendor-type problem. Then we have the following proposition.

Proposition 3. *Under push and delegation, the optimal prebooking quantity of the OEM and the CM is* $Q^D = \bar{F}^{-1}\left(\frac{\tilde{w}^D_{m1}}{p}\right)$.

Note that $Q^D = \bar{F}^{-1}\left(\frac{\tilde{w}^D_{m1}}{p}\right)$ is also the supply chain capacity, based on which we derive the following one-to-one relationship:

$$\tilde{w}^D_{m1}(Q^D) = p\bar{F}(Q^D).$$

Then the CM's profit function can be rewritten as

$$\text{Max } \Pi_m(Q^D) = (\tilde{w}^D_{m1}(Q^D) - w^D_{s1} - c_m)Q^D = \left(p\bar{F}(Q^D) - w^D_{s1} - c_m\right)Q^D.$$

It can be shown that $\Pi_m(Q^D)$ is quasi-concave in Q^D, and the optimal Q^D satisfies the following FOC:

$$p\bar{F}(Q^D)(1 - g(Q^D)) - c_m = w^D_{s1}. \tag{7}$$

Substituting (7) into the supplier's profit function yields

$$\text{Max } \Pi_s(Q^D) = (w^D_{s1} - c_s)Q^D = \left(p\bar{F}(Q^D)(1 - g(Q^D)) - c_m - c_s\right)Q^D.$$

Taking the first-order derivative of $\Pi_s(Q^D)$ with respect to Q^D results in

$$\frac{\partial \Pi_s(Q^D)}{\partial Q^D} = p\bar{F}(Q^D)(1 - g(Q^D)) - c_m - c_s + pQ^D \frac{\partial[\bar{F}(Q^D)(1 - g(Q^D))]}{\partial Q^D}.$$

Unfortunately, it is difficult to verify whether the profit function $\Pi_s(Q^D)$ is unimodal in Q^D. However, it can be shown that $\frac{\partial \Pi_s(Q^D)}{\partial Q^D}|_{Q^D=0} = p - c_m - c_s > 0$ and $\frac{\partial \Pi_s(Q^D)}{\partial Q^D}|_{Q^D=\infty} < 0$. As $\Pi_s(Q^D)$ is a continuous function, there exists at least a Q^{DS} that satisfies

$$\bar{F}(Q^{DS})\underbrace{(1 - g(Q^{DS}))}_{(1)} + \underbrace{Q^{DS}\frac{\partial(\bar{F}(Q^{DS})(1 - g(Q^{DS})))}{\partial Q^{DS}}}_{(2)} = \frac{c_m + c_s}{p}, \tag{8}$$

and the corresponding equilibrium wholesale prices $(\tilde{w}^{DS}_{m1}, w^{DS}_{s1})$ are

$$\tilde{w}^{DS}_{m1} = p\bar{F}(Q^{DS}); \quad w^{DS}_{s1} = p\bar{F}(Q^{DS})(1 - g(Q^{DS})) - c_m.$$

Equation (8) shows that under the delegation structure the degree of double marginalization takes a more complex format than under control structure: a multiplicative term (1) and an additive term (2). Fortunately, the unimodality of $\Pi_s(Q^D)$ is not required when we conduct a system performance comparison among different scenarios. We provide such a comparison result in the following section.

4.3 Control vs. delegation

First, we assume that the wholesale prices are exogenous, then the difference between the OEM's profits under the two outsourcing structures can be written as

$$\Pi_o^D - \Pi_o^C = [pD(Q^D) - \tilde{w}_{m1}^D Q^D] - [pD(Q^C) - \tilde{w}_{m1}^D Q^C] + [(w_{m1}^C + w_{s1}^C) - \tilde{w}_{m1}^D]Q^C. \qquad (9)$$

Proposition 4. *Given the exogenous wholesale prices, under the push contract, if* $\tilde{w}_{m1}^D < (w_{m1}^C + w_{s1}^C)$*, then* $Q^D > Q^C$ *and* $\Pi_o^D > \Pi_o^C$*; if* $\tilde{w}_{m1}^D = (w_{m1}^C + w_{s1}^C)$*, then* $Q^D = Q^C$ *and* $\Pi_o^D = \Pi_o^C$*; otherwise,* $Q^D < Q^C$ *and* $\Pi_o^D < \Pi_o^C$*.*

Proposition 4 shows that if $\tilde{w}_{m1}^D < (w_{m1}^C + w_{s1}^C)$, delegating the component procurement function to the CM is more beneficial to the OEM; if $\tilde{w}_{m1}^D = (w_{m1}^C + w_{s1}^C)$, then the OEM is indifferent about control and delegation; otherwise, the OEM will keep this function in-house. The reason is that the condition $\tilde{w}_{m1}^D < (w_{m1}^C + w_{s1}^C)$ implies not only that the OEM can obtain a lower unit wholesale price and achieve cost saving by delegating the procurement function to the CM, but also that the OEM is willing to bear more inventory risk as $Q^D \geq Q^C$. This cost saving and higher system capacity lead to a higher expected profit for the OEM under delegation than that under control.

Next, we compare the performance of the supply chain parties under the two outsourcing structures with the endogenized wholesale prices when the push contract is adopted. To facilitate our analysis, we further assume that the demand distribution has an increasing failure rate (IFR), that is, $(f(x)/\bar{F}(x))' > 0$, which implies that it is also IGFR.

In the foregoing analysis, we show that the optimal ordering quantity under control (Q^{CS}) satisfies equation (6) and that under delegation (Q^{DS}) satisfies equation (8). Comparing these two equations leads to the following lemma.

Lemma 1. *For the IFR demand distribution, under the push contract the equilibrium ordering quantity is higher under control than under delegation, that is,* $Q^{CS} > Q^{DS}$*.*

Based on Lemma 1, we obtain the following corollary.

Corollary 1. *For the IFR demand distribution, under the push contract the supplier charges a higher wholesale price under delegation than under control, that is,* $w_{s1}^{CS} < w_{s1}^{DS}$*.*

Corollary 1 shows that the supplier can obtain a higher wholesale price under delegation than under control. This might be related to our assumption on pricing sequence: the supplier first decides the wholesale price with the CM and then the CM decides the wholesale price with the OEM.

Next, we compare the performance of the supply chain parties under the two structures. Although the OEM's profit function may not be unimodal under delegation, we can show that its maximal profit is increasing in the production quantity Q. As $Q^{CS} > Q^{DS}$, the OEM always prefers control, regardless of the value of Q^{DS}. We are able to obtain the following analytical result by comparing the two structures.

Proposition 5. *Under the push contract, both the OEM and the CM prefer control outsourcing structure over delegation. However, the supplier prefers delegation over control.*

Note that the total procurement cost for the OEM is $p\bar{F}(Q^{CS})$ under control and $p\bar{F}(Q^{DS})$ under delegation. The inequality $Q^{CS} > Q^{DS}$ thus implies that control structure can achieve both a lower unit procurement cost and a higher system capacity for the OEM. Thus, the OEM prefers control over delegation. The difference on the procurement cost under two outsourcing structures is caused by the difference on their respective contract sequences. From the CM's and the supplier's viewpoints, the OEM's response function can be regarded as a demand function of their wholesale prices. The pricing game between the CM and the supplier can then be regarded as a simultaneous game under control and a sequential-move game under delegation. The supplier, as the price leader in the sequential-move game, can always obtain a profit at least as large as the one in the simultaneous game because it can always choose the best price response point on the CM's best response curve, whereas the Nash equilibrium of the simultaneous game is at the intersection of their best response functions (see Vives, 2001). Therefore, the supplier prefers delegation over control. One may believe that the supplier, in order to obtain a higher profit, shall charge a lower price to induce a larger order from the OEM. Nevertheless, doing that under delegation will provide more room for the follower—the CM—to overcharge the OEM, resulting in a less elastic demand function with respect to the supplier's wholesale price. Consequently, the supplier shall charge a higher wholesale price under delegation than under control. As to the CM, under the optimal wholesale prices its profit increases in the equilibrium production quantity. Thus, the CM also prefers control over delegation.

5 Pull Contract

Under a pull contract, the CM and the supplier need to invest in their capacities Q_m and Q_s in advance and there is no prebooking from the OEM. Thus, both the CM and the supplier bear their own capacity risks. We study the pull contract with the wholesale prices as endogenized decision variables. Further, we assume that the downstream supply chain parties decide the unit wholesale prices they pay for the component/end-product of the upstream parties, and then the upstream parties build their capacities. This assumption is consistent with that of Cachon (2004), Bernstein and DeCroix (2004) and Bernstein et al. (2007).

5.1 Pull and control

Under pull and control, the game sequence is defined as follows.

1. First, in period 1, the OEM decides the unit wholesale prices w_{m2}^C and w_{s2}^C.
2. Next, given w_{m2}^C and w_{s2}^C, the CM and the supplier build their capacities Q_m and Q_s in anticipation of the OEM's at-once order.
3. In period 2, the market demand is observed. The OEM sends the at-once orders to the CM and the supplier to satisfy the observed demand.

Again, if we consider the CM as a supplier, our model setting here becomes another special case of one-manufacturer-multiple-supplier ATO system studied by Gerchak and Wang (2004). Some results can be directly derived from theirs but we still provide the details on the intermediate steps to aid reading.

We need to solve this game by backward induction. First, in period 2, the OEM makes the at-once order $x \wedge Q_m \wedge Q_s$, where x is the realized demand and $a \wedge b = min(a, b)$. The order quantity $x \wedge Q_m \wedge Q_s$ actually represents the effective demand that the whole supply chain can satisfy using the available capacities of the CM and the supplier.

Next, in period 1, anticipating the OEM's at-once order, the CM and the supplier decide how much capacity to build up to maximize their respective expected profits:

$$\text{Max } \Pi_m(Q_m|Q_s) = w_{m2}^C D(Q_m \wedge Q_s) - c_m Q_m, \text{ and } \text{Max } \Pi_s(Q_s|Q_m) = w_{s2}^C D(Q_m \wedge Q_s) - c_s Q_s.$$

For now we assume that the capacity game between the CM and the supplier is simultaneous. We first derive the best response function of the CM given the supplier's capacity decision

Q_s. As the CM's and the supplier's products are complements, it is never optimal for the CM to build capacity $Q_m > Q_s$. We can show that given the supplier's capacity Q_s, the best response function of the CM is to build

$$Q_m^*(Q_s) = min(Q_m^C, Q_s),$$

where $Q_m^C = \bar{F}^{-1}\left(\frac{c_m}{w_{m2}^C}\right)$ and is the CM's optimal newsvendor capacity decision by assuming the supplier's capacity Q_s is ample (much larger than Q_m) [1]. It represents the maximum amount of capacity that the CM has the incentive to build under control. Similarly, the best response function of the supplier is

$$Q_s^*(Q_m) = min(Q_s^C, Q_m),$$

where $Q_s^C = \bar{F}^{-1}\left(\frac{c_s}{w_{s2}^C}\right)$ and also represents the maximum amount of capacity that the supplier has the incentive to build under control. Solving these two best response functions simultaneously yields the equilibrium capacities of the CM and the supplier under pull and control as

$$Q^C = Q_m^C \wedge Q_s^C.$$

Consequently, the supply chain capacity is also $Q_m^C \wedge Q_s^C$.

Proposition 6. *Under pull and control, the equilibrium capacities of the CM and the supplier are* $Q^C = Q_m^C \wedge Q_s^C = \bar{F}^{-1}\left(\frac{c_m}{w_{m2}^C}\right) \wedge \bar{F}^{-1}\left(\frac{c_s}{w_{s2}^C}\right)$.

Remark 1. *Analogous to the foregoing analysis, we can show that the capacity equilibrium remains the same, that is,* $Q^C = Q_m^C \wedge Q_s^C$ *under pull and control if the capacity game between the CM and the supplier is a sequential one in which the CM/supplier decides its capacity first and then the supplier/CM decides its own capacity.*

Next, anticipating the capacity decisions of the CM and the supplier, the OEM makes its wholesale pricing decisions to

$$\underset{w_{m2}^C, w_{s2}^C}{\text{Max}} \ \Pi_o = (p - w_{m2}^C - w_{s2}^C) D(Q_m^C \wedge Q_s^C). \tag{10}$$

Optimizing the above profit function (10) leads to the following lemma, which is similar to Proposition 3 of Gerchak and Wang (2004).

[1] Note that when the supplier's capacity is ample, the CM's expected profit function becomes $\Pi_m(Q_m) = w_{m2}^C D(Q_m) - c_m Q_m$. This corresponds to equation (5) of Gerchak and Wang (2004).

Lemma 2. *Under pull and control, the OEM will set $c_m/w_{m2}^C = c_s/w_{s2}^C$. That is, $Q_m^C = Q_s^C = Q^C$.*

Lemma 2 shows that it is in the best interests of the OEM to offer the wholesale prices to the CM and the supplier in such a way that they have the same capacity building incentives. In addition, based on Lemma 2, we obtain the following one-to-one relationship between the wholesale prices and the system capacity: $w_{m2}^C = c_m/\bar{F}(Q^C)$ and $w_{s2}^C = c_s/\bar{F}(Q^C)$. Then the OEM's decision problem can be rewritten as

$$\underset{Q^C}{\text{Max}}\ \Pi_o = \left(p - \frac{c_m + c_s}{\bar{F}(Q^C)}\right) D(Q^C).$$

Proposition 7. *Under pull and control,*

(1) the OEM's profit function is concave in Q^C, and the optimal system capacity Q^{CL}, which is also the equilibrium production quantity of the CM and the supplier, satisfies

$$\frac{\bar{F}(Q^{CL})}{1 + j(Q^{CL})h(Q^{CL})} = \frac{c_m + c_s}{p}. \tag{11}$$

(2) the optimal wholesale prices satisfy $w_{m2}^{CL} = c_m/\bar{F}(Q^{CL})$ and $w_{s2}^{CL} = c_s/\bar{F}(Q^{CL})$.

In §4.1, we show that under push and control, an additive term $2Q^{CS}f(Q^{CS})$ measures the double marginalization effect of the decentralized serial outsourcing supply chain. Here in Proposition 7, a multiplicative term $1 + j(Q^{CL})h(Q^{CL})$ in equation (11) measures the double marginalization effect under pull and control. Furthermore, we show that the OEM will set the equilibrium wholesale prices of the CM and the supplier *proportional* to their respective production costs. Thus, the CM and the supplier have the same capacity building incentives. This simple markup pricing rule is actually broadly used in outsourcing practice.

5.2 Pull and delegation

Under pull and delegation, the game sequence is defined as follows.

1. In period 1, the CM decides the component price w_{s2}^D first and after that, the OEM decides $\tilde{w}_{m2}^D$.

2. Next, given $\tilde{w}_{m2}^D$ and w_{s2}^D in period 2, the CM and the supplier build their capacities Q_m and Q_s in anticipation of the OEM's at-once order.

3. In period 2, the market demand is observed. The OEM places an at-once order with the CM and then the CM places an at-once order with the supplier.

Similarly we solve this game backwards. Again the OEM and the CM make the at-once order $x \wedge Q_m \wedge Q_s$ in period 2. In period 1, the CM and the supplier simultaneously make their respective capacity decisions by maximizing their expected profit functions:

Max $\Pi_m(Q_m|Q_s) = (\tilde{w}_{m2}^D - w_{s2}^D)D(Q_m \wedge Q_s) - c_m Q_m$, and Max $\Pi_s(Q_s|Q_m) = w_{s2}^D D(Q_m \wedge Q_s) - c_s Q_s$.

Let

$$Q_m^D \equiv \bar{F}^{-1}\left(\frac{c_m}{\tilde{w}_{m2}^D - w_{s2}^D}\right), \quad \text{and} \quad Q_s^D \equiv \bar{F}^{-1}\left(\frac{c_s}{w_{s2}^D}\right),$$

then Q_m^D (Q_s^D) is the optimal capacity that the CM (supplier) will invest in under pull and delegation assuming that the supplier (CM) has ample capacity. It represents the maximum amount of capacity that the CM (supplier) has the incentive to build. Consequently, the equilibrium capacities of the CM and the supplier under pull and delegation are

$$Q^D = Q_m^D \wedge Q_s^D,$$

which is also the corresponding supply chain capacity.

Proposition 8. *Under pull and delegation, the equilibrium capacities of the CM and the supplier are* $Q^D = Q_m^D \wedge Q_s^D = \bar{F}^{-1}\left(\frac{c_m}{\tilde{w}_{m2}^D - w_{s2}^D}\right) \wedge \bar{F}^{-1}\left(\frac{c_s}{w_{s2}^D}\right)$.

Remark 2. *We can also show that the capacity equilibrium remains the same, that is,* $Q^D = Q_m^D \wedge Q_s^D$ *under pull and delegation if the capacity game between the CM and the supplier is a sequential one in which the CM/supplier decides its capacity first and then the supplier/CM decides its own capacity.*

Based on Proposition 8, the wholesale prices and the capacities of the supply chain parties have the following relationship:

$$\tilde{w}_{m2}^D = \frac{c_m}{\bar{F}(Q_m^D)} + w_{s2}^D; \quad w_{s2}^D = \frac{c_s}{\bar{F}(Q_s^D)}.$$

Next, given w_{s2}^D and anticipating the capacity decisions of the CM and the supplier, the OEM decides its wholesale price $\tilde{w}_{m2}^D$ to

$$\underset{\tilde{w}_{m2}^D}{\text{Max}} \; \Pi_o(\tilde{w}_{m2}^D | w_{s2}^D) = (p - \tilde{w}_{m2}^D) D(Q_m^D \wedge Q_s^D),$$

which can be rewritten as

$$\operatorname*{Max}_{Q_m^D}\ \Pi_o(Q_m^D|Q_s^D) = \left(p - \frac{c_m}{\bar{F}(Q_m^D)} - \frac{c_s}{\bar{F}(Q_s^D)}\right) D(Q_m^D \wedge Q_s^D).$$

Lemma 3. *$\Pi_o(Q_m^D|Q_s^D)$ is unimodal in Q_m^D and the optimal $Q_m^{D*}(Q_s^D) = \min(Q_s^D, Q_m^D)$ where Q_m^D satisfies the following FOC:*

$$\frac{\bar{F}(Q_m^D)}{1 + j(Q_m^D)h(Q_m^D)} = \frac{c_m}{p - \frac{c_s}{\bar{F}(Q_s^D)}}. \tag{12}$$

Q_m^D decreases in Q_s^D.

Given the best response function of the OEM $Q_m^{D*}(Q_s^D) = Q_m^D \wedge Q_s^D$, the CM will decide its wholesale price to

$$\operatorname*{Max}_{w_{s2}^D}\ \Pi_m(w_{s2}^D) = (\tilde{w}_{m2}^D - w_{s2}^D)D(Q_m^{D*}(Q_s^D) \wedge Q_s^D) - c_m(Q_m^{D*}(Q_s^D) \wedge Q_s^D),$$

which can be re-written as

$$\operatorname*{Max}_{Q_s^D}\ \Pi_m(Q_s^D) = \frac{c_m}{\bar{F}(Q_m^D \wedge Q_s^D)} D(Q_m^D \wedge Q_s^D) - c_m(Q_m^D \wedge Q_s^D), \tag{13}$$

where Q_m^D satisfies equation (12). Optimizing (13) leads to the following proposition.

Proposition 9. *Under pull and delegation,*

(1) the equilibrium capacities of the CM and the supplier are $Q_s^{DL} = Q_m^{DL} = Q^{DL}$, where Q^{DL} satisfies

$$\bar{F}(Q^{DL}) = \frac{(1 + j(Q^{DL})h(Q^{DL}))c_m + c_s}{p} \tag{14}$$

(2) the optimal wholesale prices $(\tilde{w}_{m2}^{DL}, w_{s2}^{DL})$ satisfy

$$\tilde{w}_{m2}^{DL} = \frac{c_m + c_s}{\bar{F}(Q^{DL})}; \quad w_{s2}^{DL} = \frac{c_s}{\bar{F}(Q^{DL})}.$$

Proposition 9 shows that the optimal wholesale prices satisfy the following relationship: $w_{s2}^{DL}/\tilde{w}_{m2}^{DL} = c_s/(c_m + c_s)$. Similar to that under pull and control, here the optimal wholesale prices of the CM and the supplier are proportional to the unit capacity building cost of their respective products. Equation (14) also shows that under pull and delegation, a multiplicative form $1 + j(Q^{CL})h(Q^{CL})$ over the CM's unit capacity cost c_m measures the double marginalization effect. A comparison of equations (11) and (14) shows that the double marginalization effect is stronger under control than under delegation. That is, delegation helps to mitigate the double marginalization effect when a pull contract is adopted, a result in contrast to that under the push contract.

5.3 Control vs. delegation

5.3.1 Exogenous wholesale prices

In this subsection, we compare two vertical outsourcing structures under the pull contract with exogenous wholesale prices. First, we compare the system capacity under the two outsourcing structures and obtain the following corollary.

Corollary 2. *Under the pull contract , if* $\tilde{w}_{m2}^D - w_{s2}^D \leq (>) w_{m2}^C$ *and* $w_{s2}^D \leq (>) w_{s2}^C$*, then* $Q^D \leq (>) Q^C$*.*

Corollary 2 shows that if the net wholesale price paid to the CM/supplier under delegation is lower/higher than that under control, then the CM/supplier will build less/more capacity under delegation than under control. As a result, the system capacity will be lower under delegation.

Next, we compare the performance of the OEM under two outsourcing structures. Define the relative gain of the OEM by switching from control to delegation as

$$\gamma = \frac{\Pi_o^D - \Pi_o^C}{\Pi_o^C} = \frac{(p - \tilde{w}_{m2}^D) D(Q_m^D \wedge Q_s^D)}{(p - w_{m2}^C - w_{s2}^C) D(Q_m^C \wedge Q_s^C)} - 1.$$

Then we have the following conclusion on the unimodality of γ function.

Lemma 4. γ *is quasi-concave in* $\tilde{w}_{m2}^D$*.*

The quasi-concavity of γ implies that it crosses 0 at most twice. Therefore, compared to $w_{m2}^C + w_{s2}^C$, the total wholesale price the OEM pays under control, if the wholesale price paid to the CM under delegation, $\tilde{w}_{m2}^D$, falls into a moderate range, then delegation is more beneficial to the OEM. However, if the wholesale price paid to the CM under delegation is either too high or too low, then control is more beneficial to the OEM. The possible driving force behind this is the tradeoff between the cost saving of the unit wholesale price and the potential loss of high demand. Under delegation, when $\tilde{w}_{m2}^D$ is too high, then the OEM has a small profit margin and when the realized demand is small, it may hurt the OEM's profits. Similarly, when $\tilde{w}_{m2}^D$ is too low, the CM is not willing to build a large capacity and the OEM will lose sales when realized demand is high. This may explain why the OEM prefers control over delegation when $\tilde{w}_{m2}^D$ is either too high or too low.

Assume that the customer demand follows a *truncated* normal distribution with a mean μ and a standard deviation σ. Then the coefficient of variation (CV) is $CV = \sigma/\mu$. Let $p = 20$,

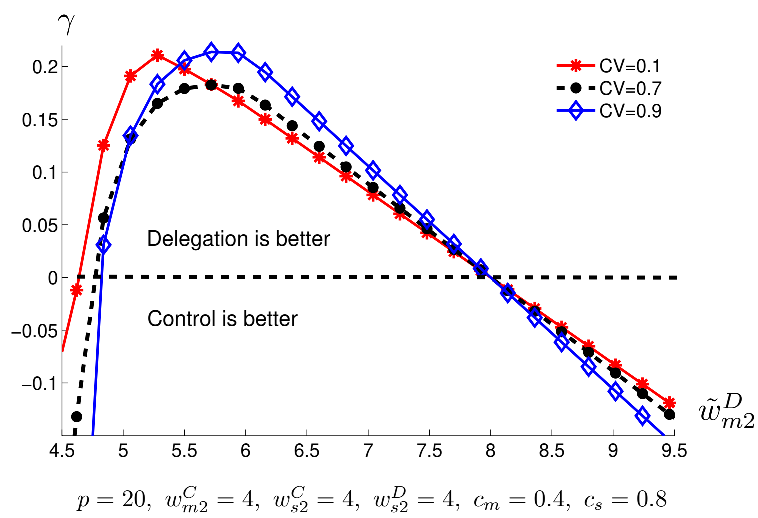

Figure 2: Impact of $\tilde{w}_{m2}^D$ and CV on γ

$w_{m2}^C = 4$, $w_{s2}^C = 4$, $w_{s2}^D = 4$, $c_m = 0.4$ and $c_s = 0.8$, by varying $\tilde{w}_{m2}^D$ and CV, we numerically examine how customer demand and the wholesale price paid to the CM under delegation affect γ, a measurement of the OEM's preference over the two outsourcing structures under the pull contract (see Figure 2). We can observe from Figure 2 that delegation is more likely to be preferred by the OEM if the customer demand has a small CV. That is, it is better for the OEM to control the procurement function instead of delegating it to the CM when facing high demand uncertainty. Figure 2 also shows that delegation is preferred by the OEM when $\tilde{w}_{m2}^D$ is in a moderate range.

5.3.2 Endogenized wholesale prices

In this subsection, we compare two vertical outsourcing structures with endogenously determined wholesale prices under the pull contract. We first compare equilibrium production capacities Q^{CL} and Q^{DL}, and obtain the following lemma.

Lemma 5. *The equilibrium production quantity under control is smaller than under delegation, $Q^{CL} < Q^{DL}$.*

Lemma 5 shows that the system capacity is higher under delegation than under control.

Next, we compare the performance of the supply chain parties under control and delegation, and obtain Proposition 10.

Proposition 10. *Under the pull contract, the OEM prefers control over delegation whereas the CM and the supplier prefer delegation over control.*

The result in Proposition 10 is similar to that of Bernstein and DeCroix (2004) in which the assembler's (in our context, the OEM's) expected profit is lower under delegation than under control.

6 Which Contract to Adopt: Push or Pull?

When an outsourcing structure is given, which contract, pull or push, should be adopted from the point view of the OEM? We examine this question here.

We first compare the equilibrium production quantities under the two contracts and obtain the following comparison results.

Proposition 11. *The equilibrium production quantity is higher under the pull contract than under the push contract, i.e.,* $Q^{CL} > Q^{CS}$ *and* $Q^{DL} > Q^{DS}$.

One potential reason for a higher capacity/production quantity under the pull contract than under the push contract is that the OEM bears all of the supply chain risk —demand risk and inventory risk— under the push contract whereas under the pull contract, the supply chain risk is shared among the supply chain parties: the CM and the supplier bear the inventory risk and the OEM faces the demand uncertainty, especially when the demand exceeds the system capacity.

Next, we compare the OEM's profits under the two contracts and obtain the following proposition.

Proposition 12. *The OEM prefers the pull contract to the push contract, i.e.,* $\Pi_o^{CS} < \Pi_o^{CL}$ *and* $\Pi_o^{DS} < \Pi_o^{DL}$.

Interestingly, we obtain the same results, regardless of the outsourcing structure in a multi-tier supply chain, as those from Cachon's (2004) two-tier supply chain setting in that both the retailer's profit and the system production quantity are higher under the pull contract than under the push contract. One main driving force behind this probably is that the wholesale prices are decided by the downstream buyer (the OEM) under the pull contract whereas they are decided by the upstream suppliers under the push contract; see Li and Scheller-Wolf (2011).

7 Two-Wholesale-Price Contract

Under the TWP contract, there exist two ordering opportunities for the OEM: in periods 1 and 2. Thus, besides the committed capacities for the prebooking placed in period 1, the CM and the supplier may both build extra capacity to satisfy potential at-once orders in period 2.

The sequence of events under TWP and control is as follows.

1. In period 1, the OEM decides the at-once wholesale prices (w_{m2}^C, w_{s2}^C) first and then the CM and the supplier decide their prebooking wholesale prices (w_{m1}^C, w_{s1}^C).

2. Given the unit wholesale price pairs (w_{m1}, w_{s1}) and (w_{m2}, w_{s2}), the OEM decides the prebooking q_{m1} and q_{s1}, its quantity commitments to the CM and the supplier, respectively.

3. The CM and the supplier then simultaneously decide how much extra capacity to install, q_{m2} and q_{s2}, respectively.

4. In period 2, demand is observed. The OEM may make at-once orders based on the available capacity and satisfy as much demand as possible.

Under this scenario, the game between the OEM and the CM/supplier follows a Stackelberg setting, whereas the capacity game between the CM and the supplier is simultaneous. The customer demand that can be satisfied by the supply chain is $D((q_{m1} + q_{m2}) \wedge (q_{s1} + q_{s2}))$. We also solve this game sequence backwards.

First, given the committed prebookings q_{m1} and q_{s1} from the OEM, the CM and the supplier decide on their additional capacities that will maximize their expected profits.

$$\textbf{CM}: \underset{q_{m2}}{\text{Max}} \quad \Pi_m^C = w_{m1}q_{m1} + w_{m2}[D((q_{m1}+q_{m2}) \wedge (q_{s1}+q_{s2})) - D(q_{m1})] - c_m(q_{m1}+q_{m2}), \quad (15)$$

$$\textbf{Supplier}: \underset{q_{s2}}{\text{Max}} \quad \Pi_s^C = w_{s1}q_{s1} + w_{s2}[D((q_{m1}+q_{m2}) \wedge (q_{s1}+q_{s2})) - D(q_{s1})] - c_s(q_{s1}+q_{s2}). \quad (16)$$

It can be shown that the objective function in (15) is concave in q_{m2} and, given the supplier's additional capacity q_{s2}, the best response function of the CM is

$$q_{m2}^C(q_{s2}) = min(Q_m^C, q_{s1} + q_{s2}) - q_{m1},$$

where Q_m^C is defined in §5.1. Similarly, the objective function in (16) is concave in q_{s2} and given the CM's additional capacity q_{m2}, the best response function of the supplier is

$$q_{s2}^C(q_{m2}) = min(Q_s^C, q_{m1} + q_{m2}) - q_{s1},$$

where Q_s^C is also defined in §5.1. Then, the equilibrium extra capacities that the CM and the supplier build are

$$q_{m2}^C = (Q_m^C \wedge Q_s^C - q_{m1})^+, \quad \text{and} \quad q_{s2}^C = (Q_m^C \wedge Q_s^C - q_{s1})^+, \tag{17}$$

where $x^+ = max(x, 0)$. From the above expression, we find that when the OEM's advance quantity commitment is more than $Q_m^C \wedge Q_s^C$, the capacity that the CM and the supplier have the incentive to build under the pull contract, then the CM and the supplier will produce just that amount and there will be no capacity available for the at-once order. However, when the OEM's prebook amount is small, the CM and the supplier will build their total capacity to $Q_m^C \wedge Q_s^C$. Thus, the supply chain capacity is $\max(Q_m^C \wedge Q_s^C, q_{m1} \wedge q_{s1})$.

Anticipating the CM's and supplier's capacity decisions, the OEM will decide on its prebooking quantities to maximize its expected profit.

$$\begin{aligned} \textbf{OEM:} \quad \underset{q_{m1}, q_{s1}}{\text{Max}}\ \Pi_o^C &= pD((q_{m1} + q_{m2}^C) \wedge (q_{s1} + q_{s2}^C)) \\ &\quad - w_{m1} q_{m1} - w_{m2}[D((q_{m1} + q_{m2}^C) \wedge (q_{s1} + q_{s2}^C)) - D(q_{m1})] \\ &\quad - w_{s1} q_{s1} - w_{s2}[D((q_{m1} + q_{m2}^C) \wedge (q_{s1} + q_{s2}^C)) - D(q_{s1})]. \end{aligned} \tag{18}$$

Based on the OEM's optimal prebooking quantities and the CM's and the supplier's capacity building decisions, we solve the pricing game among the three parties and obtain the following proposition.

Proposition 13. *Under control, the endogenized TWP contract degenerates to a pull contract.*

Interestingly, under the control outsourcing structure, if the wholesale prices are endogenized variables, then the general TWP contract will degenerate to the extreme pull contract. This is rather surprising: One may believe that the upstream parties, the CM and the supplier, are willing to offer price discounts for those pre-orders. Our analytical results show that they actually are better off by not offering price discounts.

Remark 3. *Our result is not in conflict with that of Dong and Zhu (2007). According to part (3) of Lemma 4 of Dong and Zhu (2004), the supplier's profit function is shown to be unimodal in w_1 in the push regime, increasing in the PAB regime and flat in the pull regime. Therefore, if the wholesale price is determined by the supplier, only push or pull regimes appear. Here, we further demonstrate that the supplier's local maximum in the pull regime is larger than that in the push regime and it does not have the incentive to offer price discounts for pre-orders. However, readers should note that this conclusion is based on the assumption that the CM and the supplier have full price-determining power.*

The sequence of events under TWP and delegation is as follows.

1. In period 1, the OEM and the CM first sequentially decide their respective at-once wholesale prices $(\tilde{w}_{m2}, w_{s2})$, and then the supplier and the CM sequentially decide their prebooking wholesale prices $(\tilde{w}_{m1}, w_{s1})$.

2. Given the unit wholesale price pairs $(\tilde{w}_{m1}, w_{s1})$ and $(\tilde{w}_{m2}, w_{s2})$, the OEM decides the prebooking quantity to be committed to the CM, q_{m1}. Then the CM decides the prebooking quantity committed to the supplier, q_{s1}.

3. Next, the CM and the supplier decide how much extra capacity they both want to build, q_{m2} and q_{s2}, respectively.

4. In period 2, demand is observed. The OEM and the CM can make at-once orders to satisfy as much demand as possible.

Note that when the CM is delegated the procurement function, to satisfy the OEM's prebook and taking the complementarity between the CM's and the supplier's products into consideration, the CM's prebook q_{s1} should be no less than q_{m1}, i.e., $q_{s1} \geq q_{m1}$. This also means that the total capacity the CM builds, $q_{m1} + q_{m2}$ is no less than q_{s1}.

Similar to the situation under TWP and control, we can show that given the OEM's and CM's prebook q_{m1} and q_{s1}, under TWP and delegation the equilibrium extra capacities that the CM and the supplier will build are

$$q_{m2}^D = (max(q_{s1}, Q_m^D \wedge Q_s^D) - q_{m1})^+, \quad \text{and} \quad q_{s2}^D = (Q_m^D \wedge Q_s^D - q_{s1})^+, \tag{19}$$

where Q_m^D and Q_s^D are defined in §5.2. Thus, the system capacity is $max(q_{s1}, Q_m^D \wedge Q_s^D)$.

Anticipating the equilibrium (q^D_{m2}, q^D_{s2}), the CM decides on prebooking q_{s1} to maximize its expected profit:

$$\textbf{CM:} \quad \underset{q_{s1}\geq q_{m1}}{\text{Max}}\ \Pi^D_m \;=\; \tilde{w}_{m1}q_{m1} + \tilde{w}_{m2}[D((q_{m1}+q^D_{m2}) \wedge (q_{s1}+q^D_{s2})) - D(q_{m1})] - c_m(q_{m1}+q^D_{m2}) \\ -w_{s1}q_{s1} - w_{s2}[D((q_{m1}+q^D_{m2}) \wedge (q_{s1}+q^D_{s2})) - D(q_{s1})]. \quad (20)$$

Denote the optimal prebooking as q^D_{s1}. Then anticipating the CM's and the supplier's capacity and ordering decisions, the OEM decides its prebooking amount by solving the following problem.

$$\underset{q_{m1}\geq 0}{\text{Max}}\ \Pi^D_o = pD((q_{m1}+q^D_{m2})\wedge(q^D_{s1}+q^D_{s2})) - \tilde{w}_{m1}q_{m1} - \tilde{w}_{m2}[D((q_{m1}+q^D_{m2})\wedge(q^D_{s1}+q^D_{s2})) - D(q_{m1})].$$

Based on the OEM's and the CM's optimal prebooking quantities and the CM's and the supplier's capacity building decisions, we then solve the pricing game among the three parties. The following proposition summarizes our findings.

Proposition 14. *Under delegation, the endogenized TWP contract will degenerate to one of the following two contracts: a pull contract or a combination OEM-Pull-the-CM-but-CM-Push-the-Supplier contract.*

Similar to the situation under TWP and control, when one supply chain party has full price-determining power, the endogenized wholesale pricing decisions again leads to a degenerate TWP. However, here we observe a combination of push and pull contracts where the OEM makes the at-once order but the CM makes the quantity commitment to the supplier. Thus, all of the component inventory risk is borne by the CM instead of the OEM. From the proof of Proposition 14, we find that the *OEM-Pull-the-CM-but-CM-Push-the-Supplier* contract is more likely to be preferred by the OEM when $\tilde{w}^D_{m2}$ is in a moderate range. The reason behind this is that a moderate at-once wholesale price will lead to more *balanced* capacity building incentives between the CM and the supplier, and therefore the CM faces less inventory risk when it prebooks with the supplier.

Comparison of the reduced forms immediately generates the following conclusion.

Proposition 15. *Under the TWP, when the wholesale prices are decision variables, only the pull contract appears and the OEM prefers the control outsourcing structure.*

Therefore, although the supply chain parties have more options on contracting, the final contract is still pull, under which the OEM prefers the control outsourcing structure.

8 Concluding Remarks

In this paper, we have studied two inventory/capacity risk allocation mechanisms, push and pull contracts, in a multi-tier supply chain composed of an OEM, a CM and a supplier by allowing the OEM to choose between two outsourcing structures, control and delegation. For each combination of the risk allocation contracts and the outsourcing structures, we have derived the corresponding optimal equilibrium ordering/capacity and pricing decisions.

When the wholesale prices are exogenously give, we showed that under the push contract, the OEM prefers delegation over control as long as it can achieve a cost-saving advantage of the total procurement price by delegating the component procurement function to the CM. For the pull contract, we showed that the OEM may prefer control over delegation when the wholesale price it pays to the CM under delegation is either too high or too low. Only when the wholesale price is in a moderate range and demand for the final product is stable can delegation be more preferable.

Table 1 lists the results on the preference over the two outsourcing structures for different supply chain parties when the wholesale prices are endogenized decision variables. An important observation is that the preference over the outsourcing structure can never be aligned among all of the supply chain parties but it can happen between two parties. In particular, under a push contract, both the OEM and the CM prefer control whereas under a pull contract, both the CM and the supplier prefer delegation.

Table 1: Main Comparison Results between Control and Delegation Structures

	Capacity	OEM's preference	CM's preference	Supplier's preference
Push	$Q^{CS} > Q^{DS}$	Control	Control	Delegation
Pull	$Q^{CL} < Q^{DL}$	Control	Delegation	Delegation

The main message delivered by our study is that control, under endogenized wholesale prices, allows the OEM to achieve a lower procurement cost than delegation. The paper, however, does not consider the associated administrative cost associated with the direct control structure. According to McIvor et al. (1998), control can result in an increase in the administration cost for the OEM. If such a cost is very large, delegation can still be preferred by the OEM. Indeed, the tradeoff between the procurement cost saving and administrative cost plays a critical role in choosing the outsourcing structures for many OEMs. According to McIvor et al. (1998) and Kayis et al. (2009), the American automobile companies choose delegation mainly due to administration cost reduction factor: Delegation helps the OEM to reduce the managerial effort in procurement, and therefore results in fewer employees in the procurement department. As a result, GM, Ford, Chrysler and some European automobile manufacturers have increased their use of subcontracting (McIvor et al. 1998). It is worth noting that the information technology such as e-procurement and B2B allows the OEM to reduce the managerial cost and manage the supplier relationship in a more cost-efficient way. Consequently, many OEMs have established centralized purchase departments and switched from delegation back into control. For example, Motorola has stated that its strategic choice of moving back to control is mainly due to procurement cost and supplier relationship issues (Jorgensen 2004, Smock 2004).

We also studied the OEM's contract preference under a given outsourcing structure and found that the OEM always prefers the pull contract to the push contract, which is consistent with the two-tier supply chain studied by Cachon (2004).

Finally, we investigated the TWP contract. Analytically, we demonstrated that when the wholesale prices are decision variables, then the TWP contract will degenerate to either pull or push contract under a Stackelberg pricing setting.

In this paper, we have assumed that there is no cost- and demand-information asymmetry. If such an asymmetry exists, the conclusions obtained in our paper on the delegation structure shall be taken with caution. The CM, as the middle party, can selfishly distort the procurement price to achieve its own benefit. It would be interesting to explore the moral hazard and optimal mechanism design issue in such situations.

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Online Appendices

"The Comparison of Two Vertical Outsourcing Structures under Push and Pull Contracts"

Appendix A Proofs

Proof of Proposition 2: As Q^{CS} satisfies both (4) and (5), therefore,

$$p\bar{F}(Q^{CS})(1-g(Q^{CS})) = w_{s1}^{CS} + c_m; \quad (21)$$

$$p\bar{F}(Q^{CS})(1-g(Q^{CS})) = w_{m1}^{CS} + c_s. \quad (22)$$

Thus, part 2 is proved.

From (21) and (22), we have

$$2p\bar{F}(Q^{CS}) - 2Q^{CS}f(Q^{CS}) = w_{s1}^{CS} + w_{m1}^{CS} + c_m + c_s.$$

Note that $p\bar{F}(Q^{CS}) = w_{s1}^{CS} + w_{m1}^{CS}$. Thus part 1 is proved.

Proof of Proposition 4: In equation (9), the first part $[pD(Q^D) - \tilde{w}_{m1}^D Q^D] - [pD(Q^C) - \tilde{w}_{m1}^D Q^C]$ is non-negative because $Q^D = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}^D}{p}\right)$ is the optimal solution of the function $pD(Q) - \tilde{w}_{m1}^D Q$, but the sign of the second part, $[(w_{m1}^C + w_{s1}^C) - \tilde{w}_{m1}^D]Q^C$, depends on whether $\tilde{w}_{m1}^D$ is smaller than $w_{m1}^C + w_{s1}^C$. If $\tilde{w}_{m1}^D \le (w_{m1}^C + w_{s1}^C)$, then the second part is also non-negative, so $\Pi_o^D \ge \Pi_o^C$.

If $\tilde{w}_{m1}^D > w_{m1}^C + w_{s1}^C$, then $Q^D < Q^C$. Let us consider the following problem:

$$\frac{\Pi_o^C - \Pi_o^D}{Q^C} = \underbrace{\tilde{w}_{m1}^D - (w_{m1}^C + w_{s1}^C)} - \underbrace{\frac{[pD(Q^D) - \tilde{w}_{m1}^D Q^D] - [pD(Q^C) - \tilde{w}_{m1}^D Q^C]}{Q^C}}.$$

Let $y = \tilde{w}_{m1}^D - (w_{m1}^C + w_{s1}^C) > 0$ and $z = \frac{[pD(Q^D) - \tilde{w}_{m1}^D Q^D] - [pD(Q^C) - \tilde{w}_{m1}^D Q^C]}{Q^C}$. Then, fixing $w_{m1}^C + w_{s1}^C$, both y and z are increasing in $\tilde{w}_{m1}^D$. Moreover, we have

$$\frac{\partial z}{\partial \tilde{w}_{m1}^D} = \frac{Q^C - Q^D}{Q^C} = 1 - \frac{Q^D}{Q^C} < 1.$$

Therefore, the increasing speed of z is always slower than y. Note that $y = z = 0$ if $\tilde{w}_{m1}^D = w_{m1}^C + w_{s1}^C$. we thus have $y > z$ if $\tilde{w}_{m1}^D > w_{m1}^C + w_{s1}^C$. Therefore, $\Pi_o^C > \Pi_o^D$ if $\tilde{w}_{m1}^D > w_{m1}^C + w_{s1}^C$.

Proof of Lemma 1: First, we define

$$\begin{aligned} A(x) &= p\bar{F}(x)(1-g(x)) - c_m - c_s + px\frac{\partial(\bar{F}(x)(1-g(x)))}{\partial x} \\ &= p\bar{F}(x) - pxf(x) - c_m - c_s - px(2f(x) + xf(x)') \\ C(x) &= p\bar{F}(x) - 2pxf(x) - c_m - c_s. \end{aligned}$$

Note that when the demand distribution has an IFR property,

$$\left(\frac{f(x)}{\bar{F}(x)}\right)' = \frac{f'(x)\bar{F}(x) + f^2(x)}{[\bar{F}(x)]^2} > 0,$$

which implies $f'(x) > -\frac{f^2(x)}{\bar{F}(x)}$.

Next, we consider the domain of x where $g(x) \le 1$ (otherwise, the objective function for the supplier is negative under push and delegation). We can show that $A(0) = C(0) = p - c_m - c_s$, and

$$\begin{aligned} A(x) - C(x) &= -pxf(x) - px^2 f'(x) \\ &< -pxf(x) + p\frac{x^2 f^2(x)}{\bar{F}(x)} \\ &= -pxf(x)(1-g(x)) \\ &< 0. \end{aligned}$$

Since $A(Q^{DS}) = 0$ and $C(Q^{CS}) = 0$, it follows that $Q^{DS} < Q^{CS}$.

Proof of Corollary 1: w_{s1}^{CS} and w_{s1}^{DS} satisfy equations (21) and (7), respectively. It can be verified that the function $P(x) = p\bar{F}(x)(1-g(x)) - c_m$ decreases in x. Thus, $w_{s1}^{CS} < w_{s1}^{DS}$ as $Q^{DS} < Q^{CS}$.

Proof of Proposition 5: First, under the push contract the OEM's profit functions under control and delegation can be written as

$$\begin{aligned} \Pi_o^{CS}(Q^{CS}) &= pD(Q^{CS}) - (w_{m1}^{CS} + w_{s1}^{CS})Q^{CS} \\ &= p(D(Q^{CS}) - \bar{F}(Q^{CS})Q^{CS}); \\ \Pi_o^{DS}(Q^{DS}) &= pD(Q^{DS}) - \tilde{w}_{m1}^{DS}Q^{DS} \\ &= p(D(Q^{DS}) - \bar{F}(Q^{DS})Q^{DS}). \end{aligned}$$

We can show that $D(x) - \bar{F}(x)x$ is increasing in x as

$$\frac{d(D(x) - \bar{F}(x)x)}{dx} = xf(x) > 0.$$

Recall that $Q^{DS} < Q^{CS}$, and therefore $\Pi_o^{CS}(Q^{CS}) > \Pi_o^{DS}(Q^{DS})$.

Next, the CM's profit functions under control and delegation can be written as

$$\begin{aligned}
\Pi_m^{CS}(Q^{CS}) &= \left(w_{m1}^{CS}(Q^{CS}) - c_m\right) Q^{CS} \\
&= \left(p\bar{F}(Q^{CS}) - w_{s1}^{CS} - c_m\right) Q^{CS} \\
&= p(Q^{CS})^2 f(Q^{CS}); \\
\Pi_m^{DS}(Q^{DS}) &= (\tilde{w}_{m1}^{DS}(Q^{DS}) - w_{s1}^{DS} - c_m) Q^{DS} \\
&= \left(p\bar{F}(Q^{DS}) - w_{s1}^{DS} - c_m\right) Q^{DS} \\
&= p(Q^{DS})^2 f(Q^{DS}).
\end{aligned}$$

It can be verified that $px^2 f(x)$ is increasing in x as $[\bar{F}(x)(1-g(x))]' < 0$. Recall that $Q^{DS} < Q^{CS}$, and therefore $\Pi_m^{CS}(Q^{CS}) > \Pi_m^{DS}(Q^{DS})$.

Finally, we consider the supplier's profit. Suppose the supplier charges the same wholesale price under delegation as the equilibrium one under control. It then follows from the CM's best response functions (4) and (7) that the CM will also charge the same wholesale price under control and delegation. The OEM's quantity decision is also the same under the two structures. Hence, the supplier, as the first mover in the pricing game under delegation, must obtain at least the same profit as the one in the simultaneous game (control).

Proof of Remark 1 As under pull and control, the best response function of the CM is $Q_m^*(Q_s) = min(Q_m^C, Q_s)$. If the CM makes its capacity decision first, we then substitute $Q_m^*(Q_s)$ into the supplier's profit function and have $\Pi_s(Q_s) = w_{s2}^C D(Q_m^C \wedge Q_s) - c_s Q_s$.

If $Q_m^C \wedge Q_s = Q_m^C$, then the supplier's profit function changes to $\Pi_s(Q_s) = w_{s2}^C D(Q_m^C) - c_s Q_s$, a decreasing function of Q_s. Thus, the supplier will set $Q_s = Q_m^C$. If $Q_m^C \wedge Q_s = Q_s$, then the supplier's profit function changes to $\Pi_s(Q_s) = w_{s2}^C D(Q_s) - c_s Q_s$, a concave function of Q_s. Thus, the supplier will set $Q_s = Q_s^C \wedge Q_m^C$, where $Q_s^C = \bar{F}^{-1}\left(\frac{c_s}{w_{s2}^C}\right)$ is the unique solution of $\frac{\partial \Pi_s}{\partial Q_s} = 0$. Consequently, the system capacity is again $Q^C = Q_m^C \wedge Q_s^C$. Similarly, we can show that the same results hold if the supplier decides its capacity first.

Proof of Corollary 2: If $\frac{c_m}{w_{m2}^C} \leq (>) \frac{c_m}{\tilde{w}_{m2}^D - w_{s2}^D}$, or equivalently, $\tilde{w}_{m2}^D - w_{s2}^D \leq (>) w_{m2}^C$, then $Q_m^C \geq Q_m^D$. If $\frac{c_s}{w_{s2}^C} \leq \frac{c_s}{w_{s2}^D}$, or equivalently, $w_{s2}^D \leq w_{s2}^C$, then $Q_s^C \geq Q_s^D$. Consequently, we prove Corollary 2.

Proof of Lemma 2: Similar to Gerchak and Wang (2004), we have the following. If

$Q_m^C \leq Q_s^C$, then (10) changes to

$$\underset{w_{m2}^C, w_{s2}^C}{\text{Max}} \ \Pi_o = (p - w_{m2}^C - w_{s2}^C) D(\bar{F}^{-1}\left(c_m/w_{m2}^C\right)).$$

Then the OEM can improve its profit by reducing w_{s2}^C to an extent so that $c_m/w_{m2}^C = c_s/w_{s2}^C$ without changing the system capacity but improving profit margin $p - w_{m2}^C - w_{s2}^C$. This result also holds if $Q_m^C > Q_s^C$.

Proof of Proposition 7: The FOC of Π_o with respect to Q^C is

$$\begin{aligned} \frac{\partial \Pi_o}{\partial Q^C} &= -\frac{D(Q^C) f(Q^C)}{[\bar{F}(Q)]^2}(c_m + c_s) + p\bar{F}(Q) - (c_m + c_s) \\ &= p\bar{F}(Q) - (c_m + c_s)[1 + j(Q^C)h(Q^C)], \end{aligned} \tag{23}$$

where $j(x) = \frac{D(x)}{\bar{F}(x)}$ and $h(x) = \frac{f(x)}{\bar{F}(x)}$. If the IGFR property holds, then Π_o is concave in Q^C as $[j(Q^C)h(Q^C)]' > 0$ (Cachon 2004). Thus, we have Proposition 7. This proof is similar to that of Gerchak and Wang (2004).

Proof of Lemma 3: If $Q_m^D \leq Q_s^D$, then $\Pi_o = \left(p - \frac{c_m}{\bar{F}(Q_m^D)} - \frac{c_s}{\bar{F}(Q_s^D)}\right) D(Q_m^D)$. Taking the first- and second- order derivatives with respect to Q_m^D yields

$$\begin{aligned} \frac{\partial \Pi_o}{\partial Q_m^D} &= -\frac{f(Q_m^D) D(Q_m^D)}{[\bar{F}(Q_m^D)]^2} c_m + (p - \frac{c_s}{\bar{F}(Q_s^D)})\bar{F}(Q_m^D) - c_m, \\ &= (p - \frac{c_s}{\bar{F}(Q_s^D)})\bar{F}(Q_m^D) - c_m(1 + j(Q_m^D)h(Q_m^D)) \\ \frac{\partial^2 \Pi_o}{\partial (Q_m^D)^2} &= -(p - \frac{c_s}{\bar{F}(Q_s^D)}) f(Q_m^D) - c_m (j(Q_m^D)h(Q_m^D))'. \end{aligned}$$

Since $[j(Q_m^D)h(Q_m^D)]' > 0$ (Cachon 2004), Π_o is concave in Q_m^D when $Q_m^D \leq Q_s^D$ and thus we derive the FOC (12).

If $Q_m^D \geq Q_s^D$, then $\Pi_o = (p - \frac{c_m}{\bar{F}(Q_m^D)} - \frac{c_s}{\bar{F}(Q_s^D)}) D(Q_s^D)$, which is decreasing in Q_m^D. Thus, $Q_m^{D*} = Q_s^D$.

Next, taking the first-order derivative of (12) with respect to Q_s^D yields

$$-(p - \frac{c_s}{\bar{F}(Q_s^D)}) f(Q_m^D) \frac{\partial Q_m^D}{\partial Q_s^D} - \frac{c_s f(Q_s^D)}{(\bar{F}(Q_s^D))^2}\bar{F}(Q_m^D) - c_m (j(Q_m^D)h(Q_m^D))' \frac{\partial Q_m^D}{\partial Q_s^D} = 0,$$

from which we obtain

$$\frac{\partial Q_m^D}{\partial Q_s^D} = \frac{-\frac{c_s f(Q_s^D)}{(\bar{F}(Q_s^D))^2}\bar{F}(Q_m^D)}{c_m (j(Q_m^D)h(Q_m^D))' + (p - \frac{c_s}{\bar{F}(Q_s^D)}) f(Q_m^D)} < 0,$$

as $(j(Q_m^D)h(Q_m^D))' > 0$. Hence, Q_m^D decreases in Q_s^D.

Proof of Proposition 9: First we investigate the CM's profit function (13). If $Q_s^D \geq Q_m^D$, then (13) changes to $\text{Max}_{Q_s^D}\ \Pi_m(Q_s^D) = \frac{c_m}{\bar{F}(Q_m^D)} D(Q_m^D) - c_m Q_m^D$. As

$$\frac{\partial \Pi_m(Q_s^D)}{\partial Q_s^D} = \frac{f(Q_m^D)D(Q_m^D)}{(\bar{F}(Q_m^D))^2} \frac{\partial Q_m^D}{\partial Q_s^D} c_m < 0,$$

because $\frac{\partial Q_m^D}{\partial Q_s^D} < 0$. Thus, $\Pi_m(Q_s^D)$ decreases in Q_s^D if $Q_s^D \geq Q_m^D$, and the optimal Q_s^D should be $Q_s^D = Q_m^D$.

Next, if $Q_s^D < Q_m^D$, then $\text{Max}_{Q_s^D}\ \Pi_m(Q_s^D) = \frac{c_m}{\bar{F}(Q_s^D)} D(Q_s^D) - c_m Q_s^D$. Thus

$$\frac{\partial \Pi_m(Q_s^D)}{\partial Q_s^D} = \frac{f(Q_s^D)D(Q_s^D)}{(\bar{F}(Q_s^D))^2} c_m > 0.$$

Hence, $\Pi_m(Q_s^D)$ is increasing in Q_s^D if $Q_s^D < Q_m^D$, and hence, the optimal Q_s^D should be $Q_s^D = Q_m^D$.

Thus, in the capacity equilibrium, $Q_s^{DL} = Q_m^{DL} = Q^{DL}$. Substituting this relationship into (12), we obtain

$$p\bar{F}(Q^{DL}) - j(Q^{DL})h(Q^{DL})c_m - (c_m + c_s) = 0.$$

Thus, $Q_m^{DL} = \bar{F}^{-1}\left(\frac{c_m}{\tilde{w}_{m2}^{DL} - w_{s2}^{DL}}\right) = Q_s^{DL} = \bar{F}^{-1}\left(\frac{c_s}{w_{s2}^{DL}}\right)$. Proposition 9 is proved.

Proof of Lemma 4: Note that $D(Q_m^D \wedge Q_s^D) = E(Q_m^D \wedge \min(Q_s^D, X))$. $\min(Q_s^D, X)$ is independent of $\tilde{w}_{m2}^D$ while Q_m^D increases in $\tilde{w}_{m2}^D$. Therefore, if $\tilde{w}_{m2}^D$ is small enough, then we have $E(Q_m^D \wedge \min(Q_s^D, X) = Q_m^D$. Consequently, Π_o^D is quasi-concave in $\tilde{w}_{m2}^D$ (Lariviere and Porteus 2001). If $\tilde{w}_{m2}^D$ is large enough, then we have $E(Q_m^D \wedge \min(Q_s^D, X) = E(Q_s^D \wedge X)$, and Π_o^D decreases in $\tilde{w}_{m2}^D$. Hence, γ is quasi-concave in $\tilde{w}_{m2}^D$.

Proof of Lemma 5: First, we define

$$H(x) = p\bar{F}(x) - (c_m + c_s)[1 + j(x)h(x)];\ \ J(x) = p\bar{F}(x) - j(x)h(x)c_m - (c_m + c_s).$$

Note that both $H(x)$ and $J(x)$ are decreasing functions of x for the IGFR distributions. Moreover, $H(0) = J(0) = p - c_m - c_s > 0$, and $H(x) < J(x)$ for $x > 0$. Also note that $H(Q^{CL}) = 0$ and $J(Q^{DL}) = 0$. Thus, $Q^{CL} < Q^{DL}$, see Figure 3 for the illustration.

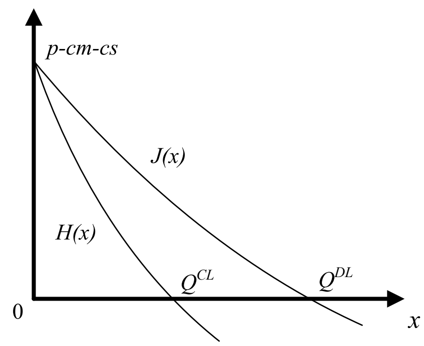

Figure 3: Comparison between Q^{CL} and Q^{DL}

Proof of Proposition 10: The profits of the OEM under the control and delegation are, respectively,

$$\Pi_o^C = \left(p - \frac{c_m + c_s}{\bar{F}(Q^{CL})}\right) D(Q^{CL}); \quad \Pi_o^D = \left(p - \frac{c_m + c_s}{\bar{F}(Q^{DL})}\right) D(Q^{DL}).$$

As Q^{CL} maximizes $\left(p - \frac{c_m+c_s}{\bar{F}(x)}\right) D(x)$, $\Pi_o^C \geq \Pi_o^D$. That is, the OEM prefers control.

Next, the CM's profit functions under the two structures are

$$\Pi_m^C = c_m \left(\frac{D(Q^{CL})}{\bar{F}(Q^{CL})} - Q^{CL}\right); \quad \Pi_m^D = c_m \left(\frac{D(Q^{DL})}{\bar{F}(Q^{DL})} - Q^{DL}\right).$$

It can be verified that the function $D(x)/\bar{F}(x) - x$ is increasing in x. As $Q^{CL} < Q^{DL}$, the CM prefers delegation over control. Similarly, we can show that the supplier prefers delegation over control.

We give a lemma to be used in some proofs.

Lemma 6. *For an IGFR distribution,*

$$(1) \quad \frac{1}{1 + j(x)h(x)} \geq 1 - g(x) \geq 0; \quad (2) \quad \frac{x\bar{F}(x)}{D(x)} \geq 1 - g(x) \geq 0.$$

Proof of Lemma 6: First, from Lariviere and Porteus (2001) we have $1 - g(x) \geq 0$. Next, to prove $\frac{1}{1+j(x)h(x)} \geq 1-g(x)$ is equivalent to proving that

$$\begin{aligned}
& \frac{1}{1+j(x)h(x)} \geq 1-g(x) \\
\Rightarrow\ & \frac{\bar{F}(x)}{1+\frac{f(x)D(x)}{(\bar{F}(x))^2}} \geq \bar{F}(x)-xf(x) \\
\Rightarrow\ & (\bar{F}(x))^3 \geq ((\bar{F}(x))^2+f(x)D(x))(\bar{F}(x)-xf(x)) \\
\Rightarrow\ & x(\bar{F}(x))^2+xf(x)D(x) \geq \bar{F}(x)D(x) \\
\Rightarrow\ & x\bar{F}(x)+g(x)D(x)-D(x) \geq 0.
\end{aligned}$$

Let $M(x)=x\bar{F}(x)+g(x)D(x)-D(x)$. As $M(0)=0$ and

$$\begin{aligned}
\frac{\partial M(x)}{\partial x} &= \bar{F}(x)-xf(x)+\bar{F}(x)g(x)+D(x)g(x)'-\bar{F}(x) \\
&= D(x)g(x)' \\
&\geq 0,
\end{aligned}$$

where the last inequality is due to the IGFR property. Therefore, $M(x)$ is increasing in x and $M(x)\geq 0$. Thus, we prove Lemma 6.

Proof of Proposition 11:

First consider the control structure. Under the push contract, Proposition 2 shows that

$$\bar{F}(Q^{CS})-2Q^{CS}f(Q^{CS})=\frac{c_m+c_s}{p}. \tag{24}$$

Under the pull contract, Proposition 7 shows that

$$\frac{\bar{F}(Q^{CL})}{1+j(Q^{CL})h(Q^{CL})}=\frac{c_m+c_s}{p}. \tag{25}$$

Applying Lemma 6, from (25), we have

$$\bar{F}(Q^{CL})-Q^{CL}f(Q^{CL}) \leq \frac{\bar{F}(Q^{CL})}{1+j(Q^{CL})h(Q^{CL})}=\frac{c_m+c_s}{p}.$$

From (24), we have

$$\frac{c_m+c_s}{p}=\bar{F}(Q^{CS})-2Q^{CS}f(Q^{CS})<\bar{F}(Q^{CS})-Q^{CS}f(Q^{CS}).$$

Thus, $\bar{F}(Q^{CL})-Q^{CL}f(Q^{CL})<\bar{F}(Q^{CS})-Q^{CS}f(Q^{CS})$. As $N(x)=\bar{F}(x)-xf(x)=\bar{F}(x)(1-g(x))$ decreases in x (Lariviere and Porteus 2001), $Q^{CL}>Q^{CS}$ is proved.

Second, we consider the delegation structure. Under the push contract, §5.2 shows that Q^{DS} satisfies

$$p\bar{F}(Q^{DS})(1-g(Q^{DS}))-c_m-c_s+pQ^{DS}\frac{\partial[\bar{F}(Q^{DS})(1-g(Q^{DS}))]}{\partial Q^{DS}}=0.$$

Under the pull contract, Proposition 9 shows that Q^{DL} satisfies

$$p\bar{F}(Q^{DL})-j(Q^{DL})h(Q^{DL})c_m-(c_m+c_s)=0.$$

Next, define

$$\begin{aligned}A(x) &= p\bar{F}(x)(1-g(x))-c_m-c_s+px\frac{\partial[\bar{F}(x)(1-g(x))]}{\partial x}\\ &= p\bar{F}(x)-pxf(x)-c_m-c_s-px[2f(x)+xf(x)']\\ B(x) &= p\bar{F}(x)-j(x)h(x)c_m-c_m-c_s.\end{aligned}$$

Then,

$$A(x)-B(x) = -pxf(x)-px[2f(x)+xf(x)']+j(x)h(x)c_m. \tag{26}$$

From Lemma 6, we have

$$\frac{1}{1+j(x)h(x)}\geq 1-g(x)\geq 0 \;\Rightarrow\; j(x)h(x)\leq\frac{xf(x)}{\bar{F}(x)-xf(x)}.$$

Thus, (26) can be rewritten as

$$\begin{aligned}A(x)-B(x) &\leq -pxf(x)-px(2f(x)+xf(x)')+\frac{xf(x)}{\bar{F}(x)-xf(x)}c_m\\ &= -xf(x)\left(\frac{p(\bar{F}(x)-xf(x))-c_m}{\bar{F}(x)-xf(x)}\right)-px(2f(x)+xf(x)')\\ &= -xf(x)\left(\frac{p\bar{F}(x)(1-g(x))-c_m}{\bar{F}(x)(1-g(x))}\right)-px(2f(x)+xf(x)').\end{aligned}$$

Recall that $w_{s1}^{DS}=p\bar{F}(Q^{DS})(1-g(Q^{DS}))-c_m>0$. Hence, $A(Q^{DS})-B(Q^{DS})<0$. As $A(Q^{DS})=0$, $B(Q^{DS})>0$. Note that $B(x)$ is decreasing in x, and $B(Q^{DL})=0$. Thus, $Q^{DL}>Q^{DS}$.

Proof of Proposition 12: For the control structure, we have the following. Under the push contract, we have

$$\Pi_o^{CS}=pD(Q^{CS})-(w_{m1}^{CS}+w_{s1}^{CS})Q^{CS}=pD(Q^{CS})-p\bar{F}(Q^{CS})Q^{CS}. \tag{27}$$

Under the pull contract, from (25), we have

$$\begin{aligned}\Pi_o^{CL} &= (p - w_{m1}^{CL} - w_{s1}^{CL})D(Q^{CL}) \\ &= \left(p - \frac{c_m + c_s}{\bar{F}(Q^{CL})}\right) D(Q^{CL}) \\ &= pD(Q^{CL}) \left(1 - \frac{1}{1 + j(Q^{CL})h(Q^{CL}))}\right). \end{aligned} \tag{28}$$

Next, Lemma 6 shows that $\frac{x\bar{F}(x)}{D(x)} \geq 1 - g(x) \geq 0$. We rearrange it and obtain

$$\frac{x\bar{F}(x)}{D(x)} \geq 1 - g(x) \geq 0 \quad \Rightarrow \quad x + xj(x)h(x) - \frac{D(x)}{\bar{F}(x)} \geq 0 \Rightarrow \frac{1}{1 + j(x)h(x)} \leq \frac{x\bar{F}(x)}{D(x)}.$$

Substituting this inequality into (28) results in

$$\Pi_o^{CL} \geq pD(Q^{CL}) \left(1 - \frac{\bar{F}(Q^{CL})Q^{CL}}{D(Q^{CL})}\right) = pD(Q^{CL}) - p\bar{F}(Q^{CL})Q^{CL}.$$

Define $K(x) = pD(x) - px\bar{F}(x)$. Then, $K(x)' = pxf(x) > 0$. Hence $K(x)$ is increasing in x. As $Q^{CL} > Q^{CS}$, $\Pi_o^{CL} \geq K(Q^{CL}) > K(Q^{CS}) = pD(Q^{CS}) - p\bar{F}(Q^{CS})Q^{CS} = \Pi_o^{CS}$.

For the delegation structure, the OEM's profits under the two contracts are, respectively,

$$\begin{aligned}\Pi_o^{DS} &= pD(Q^{DS}) - \tilde{w}_{m2}^{DS}Q^{DS} = p\left[D(Q^{DS}) - \bar{F}(Q^{DS})Q^{DS}\right] \\ \Pi_o^{DL} &= (p - \tilde{w}_{m2}^{DL})D(Q^{DL}) = pD(Q^{DL}) - (c_m + c_s)\frac{D(Q^{DL})}{\bar{F}(Q^{DL})}.\end{aligned}$$

Next, define

$$\Pi_o^S(x) = p[D(x) - x\bar{F}(x)]; \quad \Pi_o^L(x) = pD(x) - (c_m + c_s)\frac{D(x)}{\bar{F}(x)}.$$

Then using the Lemma 4 of Cachon (2004), we know that

(1) there exists an unique Q^P at which $\Pi_o^S(Q^P) = \Pi_o^L(Q^P)$; and

(2) $Q^P > Q^{DL} > Q^{DS}$.

See Figure 4 for illustration. From Lemma 4 of Cachon (2004), we have $\Pi_o^{DS} < \Pi_o^{DL}$.

Proof of Proposition 13

First assume that $Q_m^C \leq Q_s^C$. Again taking into consideration the complementarity between the CM and the supplier's products, and their incentives to set up no more than $Q_m^C \leq Q_s^C$ units of capacities, there exist only three prebook options for the OEM: (1) $\max(q_{m1}, q_{s1}) \leq Q_m^C$; (2) $\max(q_{s1}, Q_m^C) \leq q_{m1} \leq Q_s^C$; and (3) $Q_s^C \leq q_{m1}$.

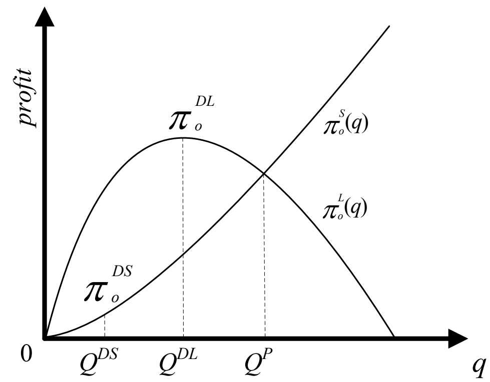

Figure 4: Comparison between Π_o^{DS} and Π_o^{DL}

Under option 1, $\max(q_{m1}, q_{s1}) \leq Q_m^C$. Thus, the OEM's profit function is

$$\Pi_o^C = pD(Q_m^C) - w_{m1}q_{m1} - w_{m2}(D(Q_m^C) - D(q_{m1})) - w_{s1}q_{s1} - w_{s2}(D(Q_m^C) - D(q_{s1})).$$

It can be shown that the optimal prebook quantities are

$$q_{m1}^C = \bar{F}^{-1}\left(\frac{w_{m1}}{w_{m2}}\right) \wedge Q_m^C, \quad \text{and} \quad q_{s1}^C = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right) \wedge Q_m^C$$

Under option 2, $(q_{m1} + q_{m2}^C) \wedge (q_{s1} + q_{s2}^C) = q_{m1}$. Then, the OEM pushes the CM, and the CM will produce only the prebooked quantity. Thus, $q_{m2} = 0$. Therefore, if the OEM decides to prebook $q_{s1} < q_{m1}$, then

$$\Pi_o^C = pD(q_{m1}) - w_{m1}q_{m1} - w_{s1}q_{s1} - w_{s2}(D(q_{m1}) - D(q_{s1})).$$

Thus, the optimal prebook quantities are

$$q_{s1}^C = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right), q_{m1}^C = max\left(\bar{F}^{-1}\left(\frac{w_{m1}}{p - w_{s2}}\right) \wedge Q_s^C, Q_m^C\right), \text{ and } q_{s1}^C < q_{m1}^C.$$

If the OEM decides to prebook $q_{s1} = q_{m1}$, then we can show that

$$q_{m1}^C = q_{s1}^C = max\left(Q_m^C, \bar{F}^{-1}\left(\frac{w_{m1} + w_{s1}}{p}\right) \wedge Q_s^C\right)$$

Finally, we consider $q_{m1} \geq Q_s^C$. Then, naturally, the OEM pushes both the CM and the supplier and $q_{m1} = q_{s1} = q$: $\Pi_o^C = pD(q) - (w_{m1} + w_{s1})q$. We can show that

$$q_{m1}^C = q_{s1}^C = max\left(\bar{F}^{-1}\left(\frac{w_{m1} + w_{s1}}{p}\right), Q_s^C\right)$$

If $Q_m^C > Q_s^C$, then we can derive the similar results, thus we can summarize the findings in Proposition 16.

Define $\Omega = (q_{m1}, q_{s1}) \in [0, \infty) \times [0, \infty)$, and the subsets $\Omega_1 = [0, Q_m^C \wedge Q_s^C) \times (\cdot)$, $\Omega_2 = [Q_m^C \wedge Q_s^C, \max(Q_m^C, Q_s^C)) \times (\cdot)$ and $\Omega_3 = [\max(Q_m^C, Q_s^C), \infty) \times (\cdot) \in \Omega - \Omega_1 - \Omega_2$.

Proposition 16. *Given the wholesale prices, the OEM's expected profit under control and TWP is locally concave on the above three sets, respectively, but not globally concave in (q_{m1}, q_{s1}). The optimal prebooking (q_{m1}^C, q_{s1}^C) takes one of the following forms.*

1. *The local optimum on $[0, Q_m^C \wedge Q_s^C) \times (\cdot)$ is $q_{m1}^C = \bar{F}^{-1}\left(\frac{w_{m1}}{w_{m2}}\right) \wedge Q_m^C \wedge Q_s^C$, and $q_{s1}^C = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right) \wedge Q_m^C \wedge Q_s^C$, where the OEM prebooks and also makes at-once orders after demand realization.*

2. *The local optimum on $[Q_m^C \wedge Q_s^C, \max(Q_m^C, Q_s^C)) \times (\cdot)$ depends on the outcome of $Q_m^C \wedge Q_s^C$.*

   (a) *if $Q_m^C \wedge Q_s^C = Q_m^C$, the OEM prebooks to the CM no less than what it prebooks to the supplier where*

   $$q_{s1}^C = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right),\ q_{m1}^C = max\left(\bar{F}^{-1}\left(\frac{w_{m1}}{p - w_{s2}}\right) \wedge Q_s^C, Q_m^C\right),\ \text{and } q_{m1}^C > q_{s1}^C,$$

   or $q_{m1}^C = q_{s1}^C = \max\left(Q_m^C, \bar{F}^{-1}\left(\frac{w_{m1}+w_{s1}}{p}\right) \wedge Q_s^C\right)$.

   (b) *if $Q_m^C \wedge Q_s^C = Q_s^C$, the OEM prebooks to the supplier no less than what it prebooks to the CM where*

   $$q_{m1}^C = \bar{F}^{-1}\left(\frac{w_{m1}}{w_{m2}}\right),\ q_{s1}^C = max\left(Q_s^C, \bar{F}^{-1}\left(\frac{w_{s1}}{p - w_{m2}}\right) \wedge Q_m^C\right),\ \text{and } q_{m1}^C < q_{s1}^C,$$

   or $q_{m1}^C = q_{s1}^C = \max\left(Q_s^C, \bar{F}^{-1}\left(\frac{w_{m1}+w_{s1}}{p}\right) \wedge Q_m^C\right)$.

3. *The local optima on $[\max(Q_m^C, Q_s^C), \infty) \times (\cdot)$ is*

   $$q_{m1}^C = q_{s1}^C = max\left(\bar{F}^{-1}\left(\frac{w_{m1} + w_{s1}}{p}\right), Q_m^C, Q_s^C\right),$$

   where the OEM pushes the CM and the supplier to produce more than their maximum capacity building incentives.

Next, we solve the pricing game of the supply chain parties for each of the above scenarios in Proposition 16.

First assume that $Q_m^C \le Q_s^C$. Then, under scenario 1 of Proposition 16, the system capacity is Q_m^C and we have

$$q_{m1} = \bar{F}^{-1}\left(\frac{w_{m1}}{w_{m2}}\right), \quad \text{and} \quad q_{s1} = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right) \wedge Q_m^C$$

Hence, there is a one-to-one relationship between w_{m1}^C and q_{m1}: $w_{m1}^C(q_{m1}) = w_{m2}^C \bar{F}(q_{m1})$. Thus, the CM's profit function is

$$\begin{aligned} \text{Max } \Pi_m(q_{m1}|w_{m2}^C) &= w_{m1}^C q_{m1} + w_{m2}^C(D(Q_m^C) - D(q_{m1})) - c_m Q_m^C \\ &= w_{m2}^C \bar{F}(q_{m1}) q_{m1} + w_{m2}^C(D(Q_m^C) - D(q_{m1})) - c_m Q_m^C. \end{aligned}$$

It can be shown that $\Pi_m(q_{m1}|w_{m2}^C)$ is decreasing in q_{m1} as

$$\frac{\partial \Pi_m(q_{m1}|w_{m2}^C)}{\partial q_{m1}} = -w_{m2}^C f(q_{m1}) q_{m1} + w_{m2}^C \bar{F}(q_{m1}) - w_{m2}^C \bar{F}(q_{m1}) = -w_{m2}^C f(q_{m1}) q_{m1} < 0.$$

As a result, the CM sets $q_{m1} = 0$. Thus $w_{m1}^C = w_{m2}^C$.

The supplier's profit function is

$$\text{Max } \Pi_s(w_{s1}^C|w_{s2}^C) = w_{s1}^C q_{s1} + w_{s2}^C(D(Q_m^C) - D(q_{s1})) - c_s Q_m^C.$$

We take its first-order condition with respect to w_{s1}^C and have

$$\frac{\partial \Pi_s(w_{s1}^C|w_{s2}^C)}{\partial w_{s1}^C} = q_{s1} + (w_{s1}^C - w_{s2}^C \bar{F}(q_{s1})) \frac{\partial q_{s1}}{\partial w_{s1}^C} = q_{s1} > 0.$$

Thus, the supplier sets $w_{s1}^C = w_{s2}^C$, and $q_{s1} = 0$. Hence, in scenario 1, the TWP contract degenerates to a pull contract which was studied in §5.

Under scenario 2, the system capacity is q_{m1} and $q_{m2} = 0$. The CM's profit function is

$$\text{Max } \Pi_m(w_{m1}^C) = (w_{m1}^C - c_m) q_{m1}.$$

According to Proposition 16, when the OEM decides to prebook $q_{s1} < q_{m1}$, then

$$q_{s1} = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right), q_{m1} = max\left(\bar{F}^{-1}\left(\frac{w_{m1}}{p - w_{s2}}\right) \wedge Q_s^C, Q_m^C\right).$$

If $q_{m1} = Q_m^C$, then scenario 2 is reduced to scenario 1. If $q_{m1} = Q_s^C$, then $\Pi_m(w_{m1}^C)$ is increasing in w_{m1}^C. If $q_{m1} = \bar{F}^{-1}\left(\frac{w_{m1}^C}{p - w_{s2}^C}\right)$, then there is a one-to-one relationship between w_{m1}^C and q_{m1}: $w_{m1}^C = (p - w_{s2}^C)\bar{F}(q_{m1})$. Substituting it into the CM's profit function yields

$$\text{Max } \Pi_m(q_{m1}) = ((p - w_{s2}^C)\bar{F}(q_{m1}) - c_m) q_{m1},$$

which is concave in q_{m1} (Lariviere and Porteus 2001).

$$\begin{aligned}\frac{\partial \Pi_m(q_{m1})}{\partial q_{m1}} &= -(p-w_{s2}^C)f(q_{m1})q_{m1} + (p-w_{s2}^C)\bar{F}(q_{m1}) - c_m \\ &= (p-w_{s2}^C)\bar{F}(q_{m1})(1-g(q_{m1})) - c_m.\end{aligned}$$

As a result, the optimal q_{m1} satisfies

$$(p-w_{s2}^C)\bar{F}(q_{m1})(1-g(q_{m1})) - c_m = 0.$$

The corresponding wholesale price satisfies

$$w_{m1}^C = (p-w_{s2}^C)\bar{F}(q_{m1}) = (p-w_{s2}^C)f(q_{m1})q_{m1} + c_m.$$

For the supplier, there is also a one-to-one relationship between w_{s1}^C and q_{s1}, that is, $w_{s1}^C = w_{s2}^C\bar{F}(q_{s1})$. Its profit function is thus

$$\begin{aligned}\text{Max } \Pi_s(q_{s1}|w_{s2}^C) &= w_{s1}^C q_{s1} + w_{s2}^C(D(q_{m1}) - D(q_{s1})) - c_m q_{m1} \\ &= w_{s2}^C\bar{F}(q_{s1})q_{s1} + w_{s2}^C(D(q_{m1}) - D(q_{s1})) - c_m q_{m1}.\end{aligned}$$

One can verify that $\Pi_s(q_{s1}|w_{s2}^C)$ is decreasing in q_{s1} as

$$\frac{\partial \Pi_s(q_{s1}|w_{s2}^C)}{\partial q_{s1}} = -w_{s2}^C f(q_{s1})q_{s1} < 0.$$

As a result, the supplier sets $q_{s1} = 0$, which implies that $w_{s1}^C = w_{s2}^C$.

The OEM's profit function can be written as

$$\begin{aligned}\Pi_o(w_{s2}^C) &= pD(q_{m1}) - w_{m1}^C q_{m1} - w_{s2}^C D(q_{m1}) \\ &= (p-w_{s2}^C)D(q_{m1}) - (p-w_{s2}^C)\bar{F}(q_{m1})q_{m1} \\ &= (p-w_{s2}^C)(D(q_{m1}) - \bar{F}(q_{m1})q_{m1}).\end{aligned}$$

The first-order condition of $\Pi_o(w_{s2}^C)$ with respect to w_{s2}^C is

$$\frac{\partial \Pi_o(w_{s2}^C)}{\partial w_{s2}^C} = -\underbrace{(D(q_{m1}) - \bar{F}(q_{m1})q_{m1})}_{(1)} + (p-w_{s2}^C)f(q_{m1})q_{m1}\frac{\partial q_{m1}}{\partial w_{s2}^C}.$$

Item (1) is positive as $D(x) - \bar{F}(x)x$ increases in x and $(D(x) - \bar{F}(x)x)|_{x=0} = 0$. Recall that q_{m1} satisfies

$$(p-w_{s2}^C)\bar{F}(q_{m1})(1-g(q_{m1})) - c_m = 0.$$

Taking the first derivative with respective to w_{s2}^C yields

$$-\bar{F}(q_{m1})(1-g(q_{m1})) + (p - w_{s2}^C)[\bar{F}(q_{m1})(1-g(q_{m1}))]' \frac{\partial q_{m1}}{\partial w_{s2}^C} = 0,$$

which indicates that

$$\frac{\partial q_{m1}}{\partial w_{s2}^C} = \frac{\bar{F}(q_{m1})(1-g(q_{m1}))}{(p - w_{s2}^C)[\bar{F}(q_{m1})(1-g(q_{m1}))]'} < 0.$$

The last inequality is due to $[\bar{F}(x)(1-g(x))]' < 0$ for the IGFR distribution (Lariviere and Porteus 2001). Therefore, $\frac{\partial \Pi_o(w_{s2}^C)}{\partial w_{s2}^C} < 0$. Hence, the OEM shall set w_{s2}^C as low as possible. Note that here we assume that $Q_m^C \le Q_s^C$ and recall that Q_s^C increases in w_{s2}^C. Thus, the OEM will set w_{s2}^C so that $Q_m^C = Q_s^C$ and scenario 2 no longer exists.

Under scenario 3, there is no at-once order and the TWP contract is reduced to the push contract, which was studied in §4.

When $Q_m^C > Q_s^C$, we can again derive the similar results for the above three scenarios. Then based on the discussion §6, we can shown that the pull contract will be adopted by the OEM.

Proof of Proposition 14

Here we discuss the gaming decisions of the supply chain parties under two cases.

Case 1: $Q_m^D \le Q_s^D$

When $Q_m^D \le Q_s^D$, if $q_{m1} < Q_m^D$, then the CM has no incentives to prebook $q_{s1} > Q_m^D$. and they will produce up to Q_m^D. So

$$\Pi_m^D = \tilde{w}_{m1} q_{m1} + \tilde{w}_{m2}(D(Q_m^D) - D(q_{m1})) - w_{s1} q_{s1} - w_{s2}(D(Q_m^D) - D(q_{s1})) - c_m Q_m^D.$$

The CM will prebook to the supplier

$$q_{s1}^D = Q_m^D \wedge \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right).$$

The corresponding OEM's profit function is

$$\Pi_o^D = pD(Q_m^D) - \tilde{w}_{m1} q_{m1} - \tilde{w}_{m2}(D(Q_m^D) - D(q_{m1})).$$

It can be shown that the optimal prebooking to the CM $q_{m1}^D = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D$.

If $q_{m1} \ge Q_m^D$, then the CM will prebook $q_{s1} = q_{m1}$. And $\Pi_o^D = pD(q) - \tilde{w}_{m1} q$. Then, $q_{s1}^D = q_{m1}^D = max\left(\bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{p}\right), Q_m^D\right)$. We summarize the findings in the following proposition.

Proposition 17. *Under TWP and delegation, suppose $Q_m^D \le Q_s^D$. Then the OEM and the CM's optimal prebookings take one of the following:*

(1). $q_{m1}^D = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D$; $q_{s1}^D = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right) \wedge Q_m^D$.

(2). $q_{s1}^D = q_{m1}^D = max\left(\bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{p}\right), Q_m^D\right)$.

When $K_m^D \le K_s^D$, the CM has less capacity building incentives than the supplier. From Proposition 17, we know there exist two prebooking equilibria. In the first equilibrium (a), the downstream party, the OEM (CM), shares the inventory risk with the upstream party, the CM (supplier), by prebooking no more than K_m^D, the amount that the CM and the supplier have incentives to build up, and there exists a second ordering opportunity in period 2. This is a *partial commitment* strategy. In the second prebook equilibrium (b), the OEM bears all of the inventory risk and pushes the CM to produce more than K_m^D, which is the maximum capacity the CM is willing to produce, and consequently, the CM prebooks to the supplier what it receives from the OEM. There is no capacity available for period 2, so this is a *push* strategy. Whether the OEM will choose a partial commitment or a push strategy will be derived by comparing its expected profits under the two strategies. Next, based on Proposition 17, we solve the pricing game for the case $Q_m^D \le Q_s^D$.

Under scenario 1, the system capacity is Q_m^D. The CM's profit function can be written as

$$\Pi_m = \tilde{w}_{m1}^D q_{m1} + \tilde{w}_{m2}^D\left(D(Q_m^D) - D(q_{m1})\right) - w_{s1}^D q_{s1} - w_{s2}^D\left(D(Q_m^D) - D(q_{s1})\right) - c_m Q_m^D.$$

According to Proposition 17, q_{s1} is independent of $\tilde{w}_{m1}^D$. If $q_{m1} = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}^D}{\tilde{w}_{m2}^D}\right) \wedge Q_m^D = Q_m^D$, then the CM prebooks $q_{s1} = q_{m1} = Q_m^D$. Thus, scenario 1 is reduced to scenario 2. If $q_{m1} = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}^D}{\tilde{w}_{m2}^D}\right)$, then there exists a one-to-one relationship between $\tilde{w}_{m1}^D$ and q_{m1}: $\tilde{w}_{m1}^D = \tilde{w}_{m2}^D\bar{F}(q_{m1})$. Substituting it into the CM's profit function yields

$$\text{Max } \Pi_m(q_{m1}) = \tilde{w}_{m2}^D\bar{F}(q_{m1})q_{m1} + \tilde{w}_{m2}^D\left(D(Q_m^D) - D(q_{m1})\right) - w_{s1}^D q_{s1} - w_{s2}^D\left(D(Q_m^D) - D(q_{s1})\right) - c_m Q_m^D.$$

Taking the first-order condition of $\Pi_m(q_{m1})$ with respect to q_{m1} shows that Π_m is decreasing in q_{m1}, which implies that Π_m is increasing in $\tilde{w}_{m1}^D$. As a result, the CM will set $\tilde{w}_{m1}^D = \tilde{w}_{m2}^D$. Consequently, $q_{m1} = 0$.

Next, we study the supplier's pricing behavior. If $q_{s1} = Q_m^D$, then scenario 1 is reduced to scenario 2. Further, if $q_{s1} = \bar{F}^{-1}\left(\frac{w_{s1}^D}{w_{s2}^D}\right)$, then there is a one-to-one relationship between w_{s1}^D and q_{s1}: $w_{s1}^D = w_{s2}^D\bar{F}(q_{s1})$. Substituting it into the supplier's profit function yields

$$\text{Max } \Pi_s(q_{s1}) = w_{s2}^D\bar{F}(s1) + w_{s2}^D(D(Q_m^D) - D(q_{s1})) - c_s Q_m^D,$$

which is decreasing in q_{s1}. Thus, the supplier will set $q_{s1} = 0$, which implies that $w_{s1}^D = w_{s2}^D$. Consequently, in scenario 1, the TWP contract is reduced to a pull contract, which was studied in §5.

In scenario 2, there is no at-once order and the TWP contract degenerates to a push contract, which was studied in §4.

Case 2: $Q_m^D \le Q_s^D$

When $Q_m^D > Q_s^D$, if the prebooking to the CM $q_{m1} < Q_s^D$, then the CM needs to decide: should it push the supplier and prebook $q_{s1} > Q_s^D$, or not? If the CM decides not, then the supplier and the CM produce up to Q_s^D, and the CM's profit function is

$$\Pi_m^D = \tilde{w}_{m1} q_{m1} + \tilde{w}_{m2}\left(D(Q_s^D) - D(q_{m1})\right) - w_{s1} q_{s1} - w_{s2}\left(D(Q_s^D) - D(q_{s1})\right) - c_m Q_s^D.$$

It can be shown that $q_{s1}^D = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right) \wedge Q_s^D$. If the CM decides to prebook $q_{s1} > Q_s^D$, then both the supplier and the CM will produce up to q_{s1}, but note that it is never optimal for the CM to prebook $q_{s1} > Q_m^D$. Under this scenario, the CM's profit function is

$$\Pi_m^D = \tilde{w}_{m1} q_{m1} + \tilde{w}_{m2}(D(q_{s1}) - D(q_{m1})) - c_m q_{s1} - w_{s1} q_{s1}.$$

Thus, the optimal prebooking $q_{s1}^D = max\left(Q_s^D, \bar{F}^{-1}\left(\frac{c_m + w_{s1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D\right)$. The CM will compare these above two decisions and choose the one that maximizes its own expected profit.

As to the OEM, knowing that the system capacity now is $\max(q_{s1}, Q_s^D)$, it needs to decide q_{m1} to maximize its own profit:

$$pD(\max(q_{s1}, Q_s^D)) - \tilde{w}_{m1} q_{m1} - \tilde{w}_{m2}(D(\max(q_{s1}, Q_s^D)) - D(q_{m1})).$$

It can be shown that the optimal prebooking quantity $q_{m1}^D = \left(\bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{\tilde{w}_{m2}}\right) \wedge Q_s^D\right)$.

Next, if the prebooking to the CM $Q_s^D < q_{m1} < Q_m^D$, then the CM has to prebook $q_{m1} \le q_{s1} \le Q_m^D$, and the system capacity is again q_{s1}. Thus, similarly, the CM is going to prebook $q_{s1}^D = max\left(q_{m1}, \bar{F}^{-1}\left(\frac{c_m + w_{s1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D\right)$. The corresponding OEM's profit function becomes $pD(q_{s1}) - \tilde{w}_{m1} q_{m1} - \tilde{w}_{m2}(D(q_{s1}) - D(q_{m1}))$. Plugging $q_{s1}^D = max\left(q_{m1}, \bar{F}^{-1}\left(\frac{c_m + w_{s1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D\right)$ into the above profit function, we can derive that the optimal prebooking quantity is either $q_{m1}^D = q_{s1}^D = max\left(Q_s^D, \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{p}\right) \wedge Q_m^D\right)$ or

$$q_{m1}^D = max\left(Q_s^D, \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D\right), q_{s1}^D = \bar{F}^{-1}\left(\frac{c_m + w_{s1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D, \quad \text{and} \quad q_{m1}^D < q_{s1}^D.$$

Finally, if the OEM prebooks $q_{m1} > Q_m^D$, then the CM will prebook $q_{s1}^D = q_{m1}$. Then the OEM's profit function is $\Pi_o^D = pD(q) - \tilde{w}_{m1}q$. The optimal prebookings are $q_{m1}^D = q_{s1}^D = max\left(\bar{F}^{-1}\left(\frac{w_{m1}+w_{s1}}{p}\right), Q_m^D\right)$.

Proposition 18. *Under delegation and TWP, suppose that* $Q_m^D > Q_s^D$*. Then the OEM and the CM's equilibrium prebookings take one of the following formats:*

(1). $q_{m1}^D = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{\tilde{w}_{m2}}\right) \wedge Q_s^D$ *and* $q_{s1}^D = \bar{F}^{-1}\left(\frac{w_{s1}}{w_{s2}}\right) \wedge Q_s^D$*;*

(2). $q_{m1}^D = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{\tilde{w}_{m2}}\right) \wedge Q_s^D$ *and* $q_{s1}^D = max\left(Q_s^D, \bar{F}^{-1}\left(\frac{c_m+w_{s1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D\right)$*.*

(3). $q_{m1}^D = max\left(Q_s^D, \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D\right)$*, and* $q_{s1}^D = max\left(q_{m1}^D, \bar{F}^{-1}\left(\frac{c_m+w_{s1}}{\tilde{w}_{m2}}\right) \wedge Q_m^D\right)$*;*

(4). $q_{m1}^D = q_{s1}^D = max\left(\bar{F}^{-1}\left(\frac{\tilde{w}_{m1}}{p}\right), Q_m^D, Q_s^D\right)$*.*

Proposition 18 shows that when $K_m^D > K_s^D$, that is, the CM has higher capacity building incentives than the supplier, there exist more prebooking equilibria than in Proposition 17. This means that when the CM has higher capacity building incentives than the supplier, TWP under delegation offers more flexibility to allocate the inventory/capacity risk among the three supply chain parties. In the first equilibrium, both the OEM and the CM share the inventory/capacity risk with their upstream party by prebooking no more than K_s^D, the capacity both the CM and the supplier are willing to install, which is again a *partial commitment* strategy. In the second equilibrium, the OEM still partially commits to the CM by prebooking less than K_s^D, but the CM is now pushing the supplier to produce more than K_s^D, the capacity that the supplier is willing to install. This is the so-called *OEM partial commit but CM push supplier* strategy. In the third and the fourth equilibria, the OEM and the CM both prebook more than K_s^D, which is named the *both push supplier* strategy. In the last equilibrium, both the OEM and the CM prebook more than K_m^D, the maximal capacity that the CM is willing to install, which is a *push* strategy. Next, based on Proposition 18, we solve the pricing game for the case $Q_m^D > Q_s^D$.

Under scenario 1, the system capacity is Q_s^D and the CM's profit function can be written as

$$\text{Max } \Pi_m(\tilde{w}_{m1}^D) = \tilde{w}_{m1}^D q_{m1} + \tilde{w}_{m2}^D\left(D(Q_s^D) - D(q_{m1})\right) - w_{s1}^D q_{s1} - w_{s2}^D\left(D(Q_s^D) - D(q_{s1})\right) - c_m Q_s^D.$$

If $q_{m1} = Q_s^D$, then Π_m is increasing in $\tilde{w}_{m1}^D$. And if $q_{m1} = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}^D}{\tilde{w}_{m2}^D}\right)$, there is a one-to-one relationship between $\tilde{w}_{m1}$ and q_{m1}: $\tilde{w}_{m1} = \tilde{w}_{m2}\bar{F}(q_{m1})$. Substituting it into the CM's profit function and taking the first-order derivative, we can show that Π_m is increasing in $\tilde{w}_{m1}^D$. As a result, the CM will set $\tilde{w}_{m1}^D = \tilde{w}_{m2}^D$, which implies that $q_{m1} = 0$.

Next we study the supplier's optimal price decisions. The supplier's profit function can be written as

$$\text{Max } \Pi_s(w_{s1}^D) = w_{s1}^D q_{s1} + w_{s2}^D \left(D(Q_s^D) - D(q_{s1})\right) - c_s Q_s^D.$$

Using the one-to-one relationship $w_{s1}^D = w_{s2}^D \bar{F}(q_{s1})$, we can show that Π_s is decreasing in q_{s1}. As a result, $q_{s1} = 0$ and the supplier will set $w_{s1}^D = w_{s2}^D$. Then under scenario 1, the TWP contract is reduced to a pull contract, which was studied in §5.

Under scenario 2, the system capacity is q_{s1}, which is independent of $\tilde{w}_{m1}^D$. The CM's profit function can be written as

$$\text{Max } \Pi_m(\tilde{w}_{m1}^D) = \tilde{w}_{m1}^D q_{m1} + \tilde{w}_{m2}^D \left(D(q_{s1}) - D(q_{m1})\right) - w_{s1}^D q_{s1} - c_m q_{s1}.$$

Analogous to the discussion in scenario 1, we can show that the CM will set $\tilde{w}_{m1}^D = \tilde{w}_{m2}^D$. Thus, $q_{m1} = 0$.

The supplier's profit function is $\Pi_s = (w_{s1}^D - c_s) q_{s1}$. If $q_{s1} = Q_s^D$, then scenario 2 is reduced to scenario 1. If $q_{s1} = Q_m^D$, then Π_s is increasing in w_{s1}^D. However, if $q_{s1} = \bar{F}^{-1}\left(\frac{c_m + w_{s1}^D}{\tilde{w}_{m2}^D}\right)$, then there exists a one-to-one relationship between w_{s1}^D and q_{s1}:

$$w_{s1}^D = \tilde{w}_{m2}^D \bar{F}(q_{s1}) - c_m.$$

Substituting it into Π_s yields

$$\text{Max } \Pi_s(q_{s1}) = (\tilde{w}_{m2}^D \bar{F}(q_{s1}) - c_m - c_s) q_{s1}.$$

The first-order condition of $\Pi_s(q_{s1})$ with respect to q_{s1} is

$$\frac{\partial \Pi_s}{\partial q_{s1}} = \tilde{w}_{m2}^D \bar{F}(q_{s1})(1 - g(q_{s1})) - c_m - c_s,$$

which decreases in q_{s1} (Lariviere and Porteus 2001).Hence, $\Pi_s(q_{s1})$ is quasi-concave and the optimal q_{s1} satisfies

$$\tilde{w}_{m2}^D \bar{F}(q_{s1})(1 - g(q_{s1})) - c_m - c_s = 0. \tag{29}$$

In equation (29), taking the derivative with respect to $\tilde{w}_{m2}^D$ yields

$$\bar{F}(q_{s1})(1 - g(q_{s1})) + \tilde{w}_{m2}^D [\bar{F}(q_{s1})(1 - g(q_{s1}))]' \frac{\partial q_{s1}}{\partial \tilde{w}_{m2}^D} = 0.$$

This implies that
$$\frac{\partial q_{s1}}{\partial \tilde{w}_{m2}^D} = -\frac{\bar{F}(q_{s1})(1-g(q_{s1}))}{\tilde{w}_{m2}^D[\bar{F}(q_{s1})(1-g(q_{s1}))]'} > 0.$$
Hence, a higher $\tilde{w}_{m2}^D$ motivates the CM to prebook more from the supplier. The optimal prebook wholesale price offered by the supplier is

$$w_{s1}^D = \tilde{w}_{m2}^D \bar{F}(q_{s1}) - c_m = \tilde{w}_{m2}^D f(q_{s1}) q_{s1} + c_s.$$

As in scenario 2, $q_{s2} = 0$, so the decision on w_{s2}^D is irrelevant. The profit function of the OEM can be written as

$$\Pi_o(\tilde{w}_{m2}^D) = (p - \tilde{w}_{m2}^D) D(q_{s1}).$$

Recall that $\tilde{w}_{m2}^D \bar{F}(q_{s1})(1-g(q_{s1})) - c_m - c_s = 0$. Hence,
$$\tilde{w}_{m2}^D = \frac{c_m + c_s}{\bar{F}(q_{s1})(1-g(q_{s1}))}.$$
Therefore, the OEM's profit function can be rewritten as
$$\text{Max } \Pi_o(q_{s1}) = \left(p - \frac{c_m + c_s}{\bar{F}(q_{s1})(1-g(q_{s1}))}\right) D(q_{s1}).$$
Unfortunately, it is hard to verify whether $\Pi_o(q_{s1})$ is concave in q_{s1}. However, in scenario 2, we can see that the TWP contract is reduced to a combination of *Pull-the-CM* but *Push-the-Supplier.*

Under scenario 3, the system capacity is q_{s1}. If $q_{m1} = Q_s^D$, then the optimization in scenario 3 will degenerate to scenario 2. If $q_{m1} = \bar{F}^{-1}\left(\frac{\tilde{w}_{m1}^D}{\tilde{w}_{m2}^D}\right) \wedge Q_m^D$, then scenario 3 is again reduced to scenario 2.

Under scenario 4, the TWP contract is reduced to a push contract.

Based on the above discussions, we conclude that under delegation the TWP contract degenerates to the following three forms: the pull contract, the push contract and the *OEM-Pull-the-CM-but-CM-Push-the-Supplier* contract. Then we compare the OEM's profits under the above three forms. In §6 we found that the pull contract is preferred by the OEM. Thus we just need to compare the pull contract with the *OEM-Pull-the-CM-but-CM-Push-the-Supplier* contract. For the former, the OEM's profit function is
$$\Pi_o(Q^{DL}) = \left(p - \frac{c_m + c_s}{\bar{F}(Q^{DL})}\right) D(Q^{DL}).$$

For the latter, the OEM's profit function is

$$\Pi_o(q_{s1}) = \left(p - \frac{c_m + c_s}{\bar{F}(q_{s1})(1 - g(q_{s1}))}\right) D(q_{s1}) < \left(p - \frac{c_m + c_s}{\bar{F}(q_{s1})}\right) D(q_{s1}).$$

It can be verified that $\left(p - \frac{c_m+c_s}{\bar{F}(x)}\right) D(x)$ is concave in x. Note that the optimal solutions of $\left(p - \frac{c_m+c_s}{\bar{F}(x)}\right) D(x)$ is Q^{CL} and $Q^{CL} < Q^{DL}$. Thus, if q_{s1} is in a moderate range, it is more likely that the *OEM-Pull-the-CM-but-CM-Push-the-Supplier* contract will be preferred, which requires a moderate $\tilde{w}_{m2}$ (neither too high nor too low).

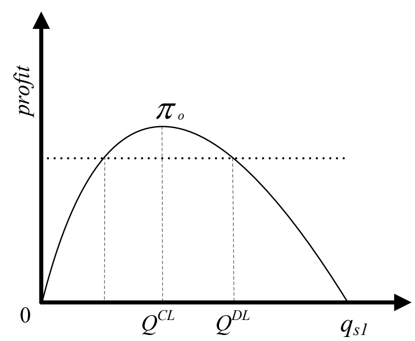

Figure 5: The OEM's Optimal Strategy under Delegation and TWP

Proof of Proposition 15

As shown in §7.1, under the control outsourcing structure the OEM prefers the pull contract over the push contract (Proposition 12), while in §7.2 under the the delegation outsourcing structure, the OEM can prefer both pull and *OEM-Pull-the-CM-but-CM-Push-the-Supplier* contracts. Comparing the OEM's profit under pull and control with that under pull and delegation shows that pull and control leads to a higher profit for the OEM (Proposition 10). Next we compare the OEM's profit under the *OEM-Pull-the-CM-but-CM-Push-the-Supplier* contract and delegation with that under pull and control. As under the *OEM-Pull-the-CM-but-CM-Push-the-Supplier* contract and delegation,

$$\Pi_o(q_{s1}) = \left(p - \frac{c_m + c_s}{\bar{F}(q_{s1})(1 - g(q_{s1}))}\right) D(q_{s1}) < \left(p - \frac{c_m + c_s}{\bar{F}(q_{s1})}\right) D(q_{s1}).$$

Under pull and control,

$$\Pi_o(Q^{CL} = Q_m^{CL} = Q_s^{CL}) = \left(p - \frac{c_m + c_s}{\bar{F}(Q^{CL})}\right) D(Q^{CL}).$$

Note that $\left(p - \frac{c_m + c_s}{\bar{F}(x)}\right) D(x)$ is concave in x and Q^{CL} is its optimal solution. Therefore, $\Pi_o(q_{s1}) < \Pi_o(Q^{CL} = Q_m^{CL} = Q_s^{CL})$. Thus, when wholesale prices are endogenized decision variables, the TWP contract will reduce to a pull contract and the OEM prefers the control outsourcing structure.