If the maximum velocity at every point is W, and if at any latitude the area of the land is a third that of the sea, the mean rate of dissipation of energy would be, from equation (26),

$$\tfrac{3}{4}\pi\rho W^2R^2\sqrt{(\nu\Omega)}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}\cos\tfrac{1}{2}\lambda\cos\lambda\,d\lambda = \pi\rho W^2R^2\sqrt{(2\nu\Omega)}$$

where R is the radius of the earth. Hence from equation (33)

$$W^2 = \frac{1{\cdot}6\times10^{13}}{\pi\rho R^2\sqrt{(2\nu\Omega)}},$$

or

$$W = 2 \text{ feet per second} = 1{\cdot}2 \text{ knots.}$$

My thanks are due to Sir J. Larmor for the interest he has taken in the work, and to Dr. Proudman for help and advice on many points which have arisen.

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## *The Complete Photo-electric Emission from the Alloy of Sodium and Potassium.**

By William Wilson, Ph.D.

(Communicated by Prof. O. W. Richardson, F.R.S. Received February 17, 1917.)

The experimental work described in the present paper suggested itself to the writer in connection with an earlier investigation† on the law governing the temperature variation of the complete photo-electric emission from a hot body, *i.e.* the photo-electric emission from a body in equilibrium with the full (black body) radiation corresponding to its temperature. By making use of hypotheses contained in the quantum theory, the writer obtained the following expression for the current per unit area

$$C = AT(1+2kT/\phi+2k^2T^2/\phi^2)e^{-\phi/kT},$$

where $\phi$ is the work done in removing an electron from the hot body, and is equal to $h\nu$, $\nu$ being the lowest frequency of the radiation capable of producing a photo-electric emission, and $h$ being Planck's constant. The quantity $k$ is the "gas constant" reckoned for one molecule, and A is a quantity independent of T, and characteristic of the substance. As the expression inside the brackets in the above formula does not differ appreciably from unity, the latter is substantially of the same type as Richardson's equation

$$C = AT^{\lambda}e^{-\phi/kT}, \qquad (1)$$

* The expenses of this research were partly defrayed by the aid of a grant from the Government Grant Committee of the Royal Society, to whom my thanks are due.

† W. Wilson, 'Annalen der Physik,' vol. 42, p. 1154 (1913).

for the thermionic emission. Richardson* has also shown that it follows, from thermodynamic considerations, that this law governs the complete photo-electric emission. There is reason to believe that the thermionic emission is not wholly photo-electric in origin,† but it is clear that some portion of it is the complete auto-photo-electric emission of the substance concerned, and that the law governing its temperature variation should be the same as that for the whole thermionic emission. We are thus led to expect that, when a body is exposed to an external source of full radiation, the same law will govern the variation of its complete photo-electric emission with the temperature of the source of the radiation. This expectation has been confirmed by experiments on the alloy of sodium and potassium, the description of which constitutes the subject of the present paper.

*Description of the Apparatus and Experimental Arrangements.*

The part of the apparatus containing the alloy is illustrated by figs. 1 and 2. It consisted of a glass vessel completely sealed up and without taps, many previous attempts to prepare the alloy, and introduce it in satisfactory condition, having failed on account of the great difficulty of getting sufficiently good stop-cocks. The sodium and potassium, in atomic proportions, were introduced into the portion A (fig. 1). This was afterwards sealed on to the rest of the apparatus, which was kept inverted during the preparation of the alloy, and not as actually shown in the figure. The naphtha, in which the metals were immersed, was run off as far as possible before the apparatus was finally closed. The vessel was then evacuated by a Toepler pump and a charcoal tube immersed in liquid air. The metals were then melted, and the alloy formed, by applying a Bunsen flame to the exterior of the tube. After the apparatus had been evacuated as completely as possible, it was sealed off from the pump, and the alloy run through a plug of glass wool E, and brought into the position B (figs. 1 and 2) by inverting the whole vessel, so that the liquid filled the U-shaped portion. About 3 cm. above the surface of the alloy was a horizontal platinum wire D (figs. 1 and 2), about 9 cm. in length

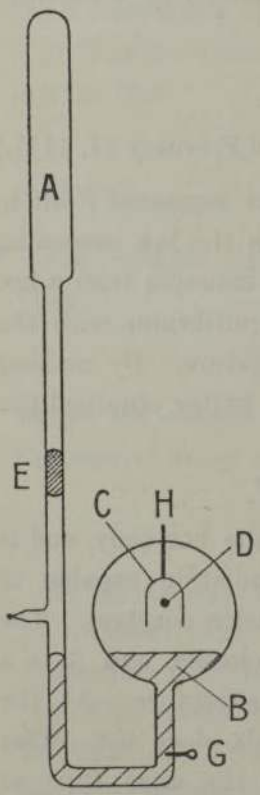

Fig. 1.

* 'Phil. Mag.,' vol. 23, pp. 619 and 620 (1912).

† See a paper by O. W. Richardson, 'Phil. Mag.,' vol. 31, p. 149 (1916).

and 0·2 mm. in diameter. The ends of this wire were fused on, by means of gold, to thicker platinum leads sealed into mercury tubes FF (fig. 2).

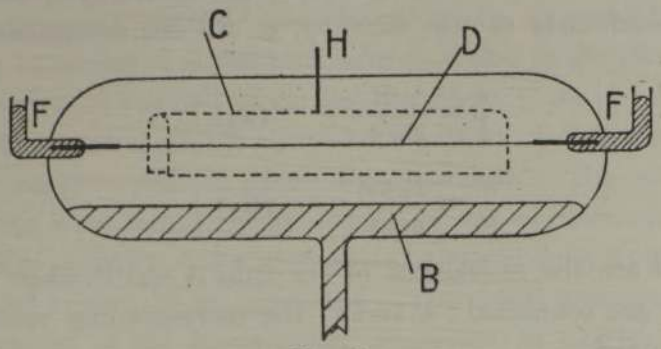


Fig. 2.

Surrounding the wire, but not actually touching it, was a shield of silver foil C (figs. 1 and 2). This was slightly open at the lower side, to allow the radiation from the platinum wire to reach the alloy. This arrangement of silver shield and platinum wire constituted an approximately full radiator.

A battery of 8 or 10 accumulators was used to heat the wire of the radiator, and, at the same time, the connections were so arranged that the wire and its leads constituted one arm of a Wheatstone bridge (see fig. 3), the remaining arms consisting of a resistance R of 2·016 ohms of thick eureka or platinoid wire immersed in paraffin oil, and the two large resistances (of the order of 4000 ohms) contained in a good post-office box. In addition to these four resistances, a fifth one was connected with the bridge. This consisted of a very short piece of the same wire as that of the radiator, and was furnished with mercury tubes and leads exactly similar to those of the radiator wire.

The whole arrangement is shown in fig. 3. By means of the mercury cups, $\alpha$, $\beta$, and $\gamma$, one of the terminals of the bridge galvanometer could be connected in such a way that the short wire and its leads could be included

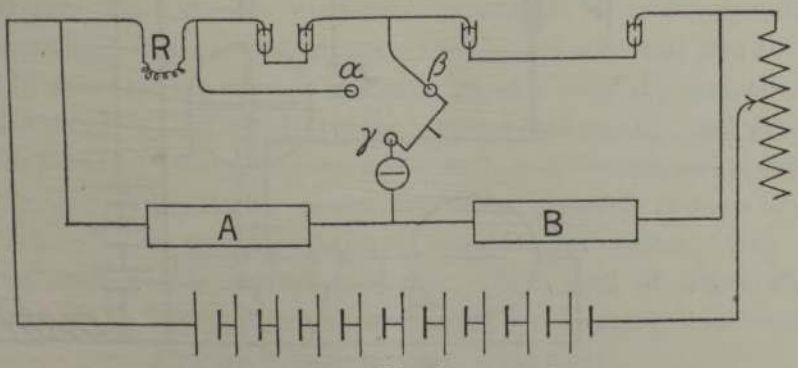


Fig. 3.

in the radiator arm of the bridge, or, when desired, in the arm containing R. By this means the resistance of a uniformly hot portion of the wire of the radiator could be found. If this resistance be denoted by $x$, and that of the leads and colder ends of the wire by $\eta$, we can determine $x$ from the equations

$$\left.\begin{aligned} \frac{a}{b} &= \frac{\mathrm{R}}{2\eta + x}, \qquad (\alpha\gamma) \\ \frac{a'}{b'} &= \frac{\mathrm{R}+\eta}{\eta + x}, \qquad (\beta\gamma) \end{aligned}\right\}, \tag{2}$$

where $a$ and $b$ are the resistances in the arms A and B, when the mercury cups $\alpha$ and $\gamma$ are connected; $a'$ and $b'$, the corresponding resistances when $\beta$ and $\gamma$ are joined.

The bridge galvanometer was of the moving coil type, and had a resistance of, approximately, 100 ohms. The whole arrangement was very sensitive; in fact, such errors as may have entered into the measurements cannot be ascribed to the individual bridge measurements, since a change of 1 in 4000 or 5000 of the arms A and B produced an easily discernible galvanometer deflection.

*Measurement of the Photo-electric Current.*

The methods employed to measure the photo-electric current from the alloy, when subjected to the radiation from the hot wire radiator, are shown diagrammatically in fig. 4. The larger currents were measured by a moving

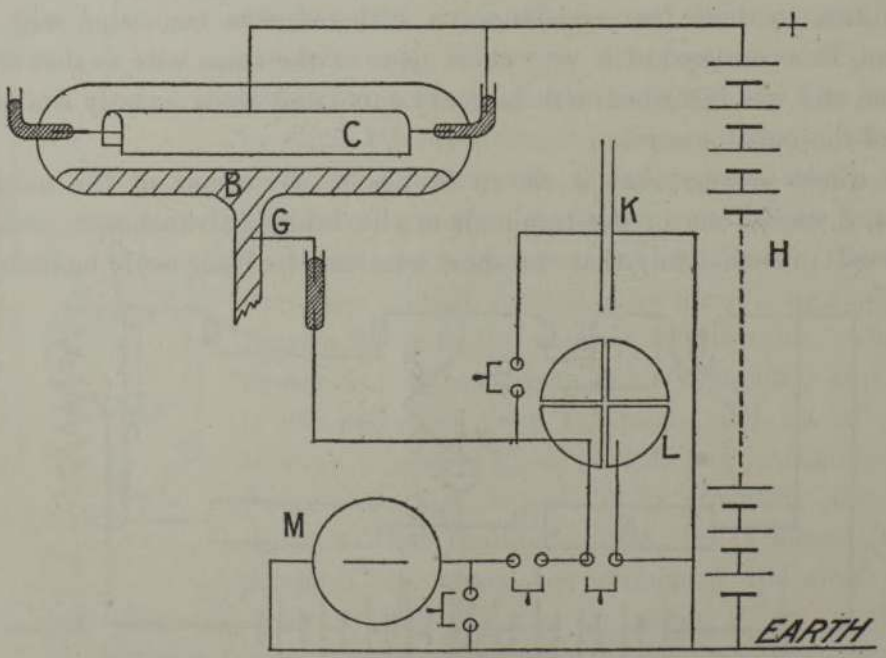


Fig. 4.

coil galvanometer (fig. 4, M), which gave a scale deflection of 1 cm. for $1{\cdot}437 \times 10^{-8}$ ampères, and was provided with a shunt, so that its sensitiveness could be reduced, when desired, to one-eighth of this. One terminal of the galvanometer was "earthed," and the other (fig. 4) could be connected with the alloy by means of a platinum wire G, sealed in the glass.

The radiator wire (or one end of it) and the shield C could be maintained at a fixed positive potential by means of a battery of 240 small dry cells (fig. 4, H), the negative pole of which was "earthed." The smaller photo-electric currents were measured by means of an electrometer L and a condenser K. The capacity of the latter could be varied from 0·001 microfarad to 0·3 microfarad to suit the requirements of individual measurements.

The sensitiveness of the electrometer amounted to nearly 16 cm. scale deflection for 1 volt. The currents were measured by taking the time required by the electrometer and condenser to charge up to the potential represented by a deflection of 10 cm. on the scale. The sensitiveness of the electrometer was determined immediately before and after use. The condenser, electrometer, the part of the apparatus containing the alloy and radiator, and all the connections were enclosed in metal boxes and tubes, which were all connected to the same "earth" as the negative pole of the high-tension battery.

*The Saturation of the Currents.*

A number of measurements were carried out for the purpose of ascertaining what potential would saturate the currents. The form of the apparatus and the comparatively large magnitude of the currents rendered it necessary to use a relatively high potential for this purpose. In fact, the upper limit of the photo-electric currents, which could be satisfactorily measured, was largely determined by the voltage available for saturating them.

The dependence of the photo-electric current on the temperature of the radiator, when an insufficiently high voltage was maintained between the alloy and radiator, is illustrated by fig. 5, which is plotted from actual measurements. The ordinates represent the currents in arbitrary units, and the abscissæ approximate temperatures. The applied potential was 100 volts.

It will be seen that the current increases rapidly with the temperature up to a certain point, beyond which it rises less and less rapidly, and ultimately becomes stationary. This phenomenon is already well known, and is explained by the fact that at the higher temperatures the emission is very large and the applied potential is not sufficiently high for saturation, owing to the mutual repulsion of the emitted electrons. This kind of effect was first observed by J. E. Lilienfeld,* who accounted for it along lines similar to

* 'Phys. Zeitschrift,' vol. 9, p. 193 (1908).

the above. It has also been considered by C. D. Child,* and at greater length, more recently, by I. Langmuir.†

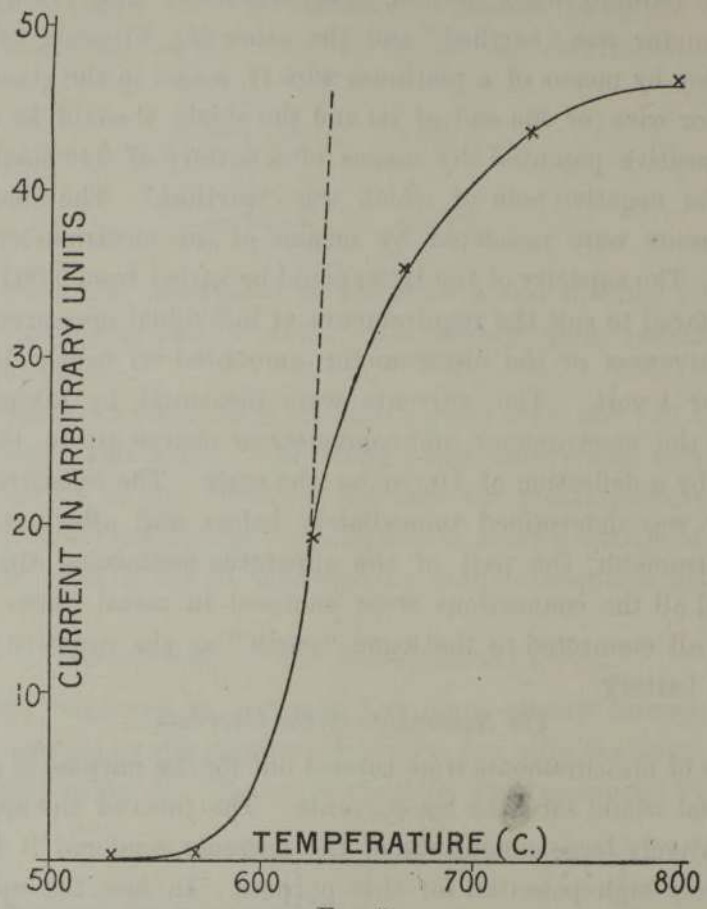


Fig. 5.

In fig. 6 is shown the dependence of the current on the applied potential when the radiator temperature was 573° C. The abscissæ will represent volts when multiplied by 1·46. The current, as will be seen from the figure, is practically saturated with 200 volts. In the actual measurements described below, the applied potential was 350 volts, so that the currents obtained with the temperature stated above, and with all lower temperatures, were undoubtedly saturated. Higher radiator temperatures were employed, but the saturation of the currents was less satisfactory.

The saturation curve corresponding to a radiator temperature of 624° C. is shown in fig. 7. In this case saturation appears to be just reached at the highest potential at my disposal, namely, 350 volts. (The abscissæ in fig. 7 have to be multiplied by 1·46 to give volts.) It is possible that the current

* 'Phys. Rev.,' vol. 32, p. 492 (1911).
† 'Phys. Rev.,' vol. 2, p. 453 (1913).

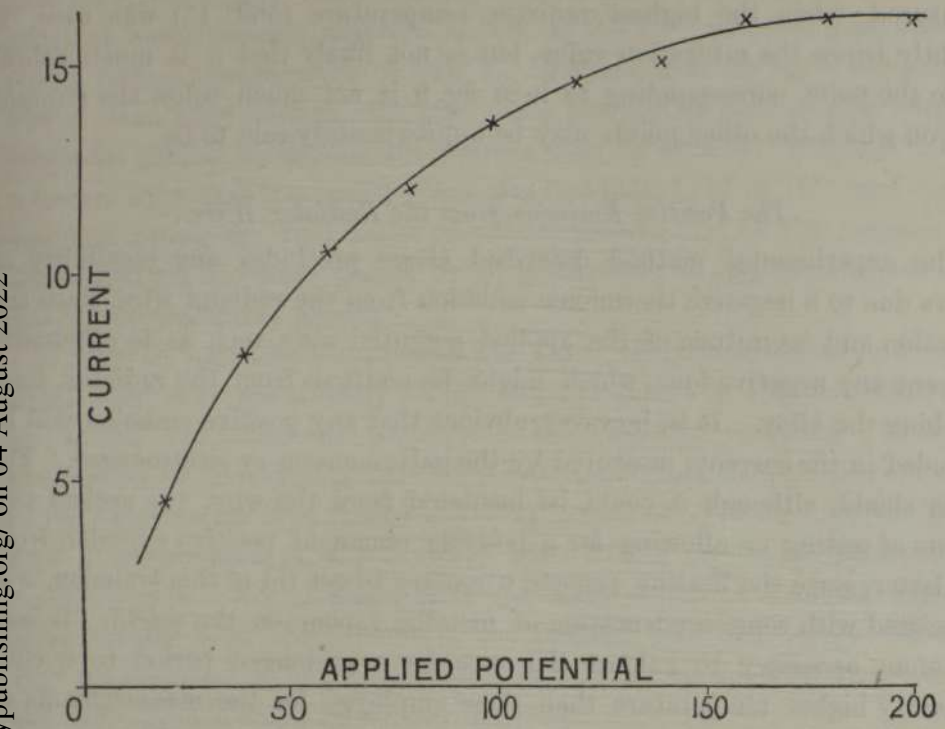


Fig. 6.

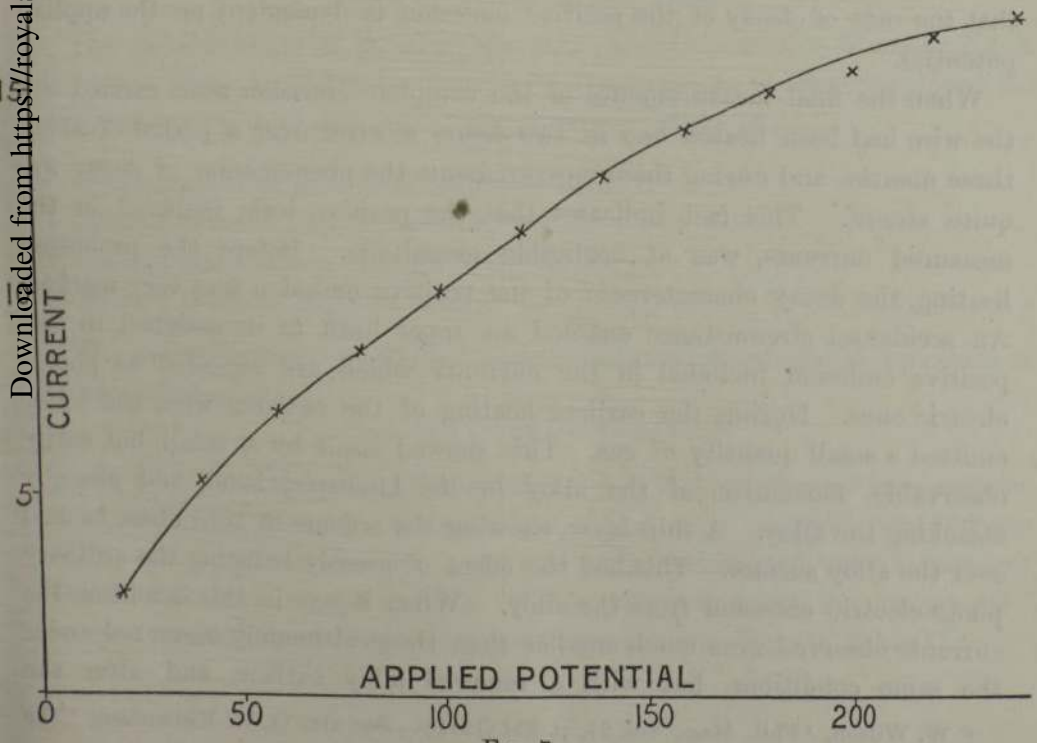


Fig. 7.

measured, when the highest radiator temperature (669° C.) was used, is slightly below the saturation value, but is not likely that it is much below, since the point corresponding to it in fig. 9 is not much below the straight line on which the other points may be approximately said to lie.

*The Positive Emission from the Radiator Wire.*

The experimental method described above precludes any possibility of errors due to a negative thermionic emission from the radiator wire, since the direction and magnitude of the applied potential were such as to effectually prevent any negative ions, which might be emitted from the radiator, from reaching the alloy. It is, however, obvious that any positive emission will be included in the currents measured by the galvanometer or electrometer. The silver shield, although it could be insulated from the wire, was useless as a means of testing or allowing for a possible remanent positive emission from the latter, since the heating process, necessary to get rid of this emission, was associated with some condensation of metallic vapour on the shield. It was therefore necessary to subject the wire for a prolonged period to a considerably higher temperature than those employed for the measurements of the complete photo-electric emission. During this heating a positive potential was applied to the wire, as the writer and other investigators* have observed that the rate of decay of the positive emission is dependent on the applied potential.

When the final measurements of the complete emission were carried out, the wire had been heated one or two hours a week over a period of about three months, and during these measurements the phenomenon of decay was quite absent. This fact indicates that the positive leak, included in the measured currents, was of negligible magnitude. Before the prolonged heating, the decay characteristic of the positive emission was very marked. An accidental circumstance enabled an upper limit to be assigned to any positive emission included in the currents which are regarded as photo-electric ones. During the earliest heating of the radiator wire, the latter emitted a small quantity of gas. This showed itself by a small, but easily observable, movement of the alloy in the U-shaped bend, and also by attacking the alloy. A thin layer, showing the colours of thin films, formed over the alloy surface. This had the effect of greatly reducing the ordinary photo-electric emission from the alloy. When it was in this condition the currents observed were much smaller than those ultimately measured under the same conditions, but with a renewed alloy surface, and after the

* W. Wilson, 'Phil. Mag.,' vol. 21, p. 634 (1911). See also O. W. Richardson, 'The Emission of Electricity from Hot Bodies,' p. 227 *et seq.*

positive emission and decay had been got rid of. In some of the earliest series of measurements, currents as low as $2 \times 10^{-8}$ ampères were observed with a radiator temperature of 669° C.; whereas with the same temperature and a clean alloy surface, and after long heating of the radiator wire, but otherwise similar conditions, the current rose to $48 \times 10^{-8}$ ampères. These numbers show that the positive leak was less than 1/24 of the total current observed during the final series of measurements, and as the radiator wire was heated till the decay ceased to be perceptible, it may be safely assumed that it was practically absent.

It may be mentioned here that the direct thermionic emission from the alloy, which was undoubtedly heated to some extent by the radiation falling on it, did not contribute sensibly to the measured currents. This was established by maintaining the radiator at the highest temperature employed in the experiments for a sufficiently long period to give the alloy at least as high a temperature as any it could have during the actual photo-electric measurements. No emission from the alloy could be detected with the galvanometer after the current in the radiator wire had been switched off. Had the thermionic contribution been only 1/150 of the total emission from the alloy it could easily have been detected.

*Final Measurements and Results.*

The measurements of the complete photo-electric emission from the Na–K alloy are given in Table I. Four fiducial temperatures were employed for determining the temperatures of the radiator. These were:—

Melting point of potassium sulphate, 1070° C.
Melting point of sodium chloride, 801° C.
Melting point of lead chloride, 447° C.
Melting point of naphthalene, 80° C.

The corresponding "platinum" temperatures were calculated by means of Callendar's formula,

$$t - t_p = 1{\cdot}5\{(t/100)^2 - t/100\},$$

where $t$ is the temperature (Centigrade) and $t_p$ the "platinum" temperature. A small crystal of each substance was placed on a radiator wire, and the resistance of the wire (less leads and cold ends) determined when it was just hot enough to melt the crystal. These resistances and the "platinum" temperatures of the corresponding melting points are given in Table II.

Table I.

| Current in radiator wire. | $(\alpha\gamma)$. | | $(\beta\gamma)$. | | Galvanometer deflections. | Electrometer measurements. | | |
|---|---|---|---|---|---|---|---|---|
| | A. | B. | A. | B. | | Deflections. | Time. | Capacity. |
| ampères. | | | | | cm. | cm. | sec. | mf. |
| 2·3 | 4000 | 2417 | 4000 | 1694 | 8·4* | | | |
| 2·25 | 4000 | 2376 | 4000 | 1660 | 5·05* | | | |
| 2·20 | 4000 | 2306 | 4000 | 1626 | 1·90* | | | |
| 2·15 | 7000 | 3957 | 7000 | 2780 | 7·40† | | | |
| 2·10 | 7000 | 3850 | 7000 | 2720 | 2·20† | | | |
| 2·05 | 7000 | 3744 | 7000 | 2660 | 0·70† | | | |
| 2·00 | 7000 | 3650 | 7000 | 2600 | — | 10 | 78 | 0·3 |
| 1·95 | 7000 | 3554 | 7000 | 2536 | — | 10 | 92·5 | 0·1 |
| 1·90 | 7000 | 3480 | 7000 | 2487 | — | 10 | 76 | 0·03 |
| 1·85 | 7000 | 3374 | 7000 | 2417 | — | 10 | 87·2 | 0·01 |
| 1·80 | 7000 | 3294 | 7000 | 2357 | — | 10 | 53 | 0·003 |

* 1 cm. $= 8 \times 1{\cdot}437 \times 10^{-8}$ ampère.

† 1 cm. $= 1{\cdot}437 \times 10^{-8}$ ampère.

NOTE.—An electrometer deflection of 15·55 cm. represents 1·02 volts. A and B are the resistances in ohms of the corresponding bridge arms (fig. 3). $(\alpha\gamma)$, $(\beta\gamma)$ indicate that the corresponding mercury cups were connected.

Table II.

| Substance. | Melting point. | Resistance. |
|---|---|---|
| Potassium sulphate ..... | 914 | 0·9530 |
| Sodium chloride ..... | 717 | 0·7990 |
| Lead chloride ..... | 425 | 0·5461 |
| Naphthalene ..... | 80 | 0·2674 |

These measurements are very consistent with one another, as will be seen from fig. 8. The "platinum" temperatures, corresponding to the measurements given in Table I, were calculated by means of the formula

$$\alpha t_p + \beta - x = 0, \qquad (3)$$

where $t_p$ is the "platinum" temperature, and $x$ the resistance of the wire. For the constants, $\alpha$ and $\beta$, the following values, deduced from the fiducial data in Table II, were employed:—

$$\alpha = 0{\cdot}0008253, \qquad \beta = 0{\cdot}2005.$$

The calculated results are given in Table III. Column 2 contains the resistance of the radiator wire (corresponding to different temperatures) as calculated by equations (2) from the measurements in Table I. In column 3

are the corresponding "platinum" temperatures obtained by equation (3) and the values of $\alpha$ and $\beta$ given above. The Centigrade temperatures computed

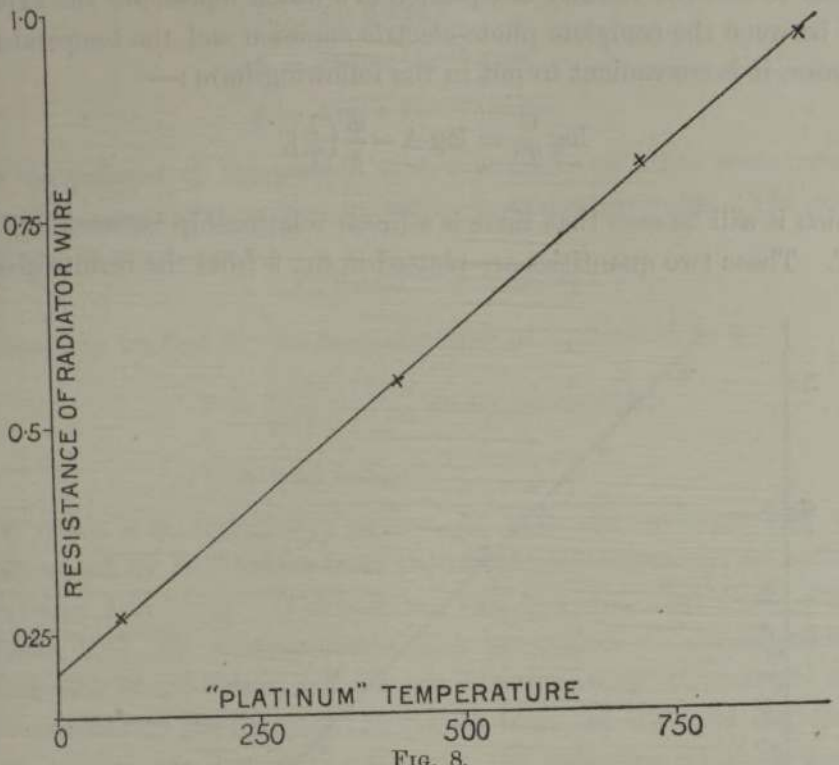


FIG. 8.

by Callendar's formula are in column 4. The remaining four columns contain, respectively, the absolute temperatures, their reciprocals, the common

Table III.

| Heating current in radiator wire. | Resistance of radiator wire, less that of leads and cold ends. | $t_p$. | $t$. | T. | $T^{-1} \times 10^4$. | $\log_{10}$ C. | $\log_{10} C/T^2$. |
|---|---|---|---|---|---|---|---|
| ampères. | ohms. | | | | | | |
| 2·3 | 0·7062 | 612·7 | 669 | 942 | 10·615 | 7·9849 | $\bar{1}$2·0368 |
| 2·25 | 0·6874 | 589·9 | 642 | 915 | 10·929 | 7·7639 | $\bar{1}$3·8411 |
| 2·20 | 0·6749 | 574·8 | 624 | 897 | 11·148 | 7·3394 | $\bar{1}$3·4338 |
| 2·15 | 0·6544 | 549·9 | 594 | 867 | 11·534 | 7·0268 | $\bar{1}$3·1507 |
| 2·10 | 0·6401 | 532·6 | 573 | 846 | 11·820 | 8·5000 | $\bar{1}$4·6452 |
| 2·05 | 0·6258 | 515·3 | 553 | 826 | 12·107 | 8·0026 | $\bar{1}$4·1687 |
| 2·00 | 0·6102 | 496·4 | 531 | 804 | 12·438 | 9·3990 | $\bar{1}$5·5885 |
| 1·95 | 0·5931 | 475·7 | 507 | 780 | 12·821 | $\bar{1}$0·8405 | $\bar{1}$5·0563 |
| 1·90 | 0·5802 | 460·1 | 488·5 | 761·5 | 13·123 | $\bar{1}$0·3848 | $\bar{1}$6·5215 |
| 1·85 | 0·5620 | 438·0 | 463 | 736 | 13·587 | $\bar{1}$1·7696 | $\bar{1}$6·0358 |
| 1·80 | 0·5449 | 417·3 | 440 | 713 | 14·025 | $\bar{1}$1·3166 | $\bar{1}$7·6104 |

logarithms of the photo-electric currents and the common logarithms of the photo-electric currents divided by the squares of the absolute temperatures.

In order to test the validity of equation (1), which represents the expected relation between the complete photo-electric emission and the temperature of the radiator, it is convenient to put in the following form :—

$$\log \frac{C}{T^{\lambda}} = \log A - \frac{\phi}{k}\left(\frac{1}{T}\right), \tag{4}$$

from which it will be seen that there is a linear relationship between $\log C/T^{\lambda}$ and $1/T$. These two quantities are plotted in fig. 9 from the results given in

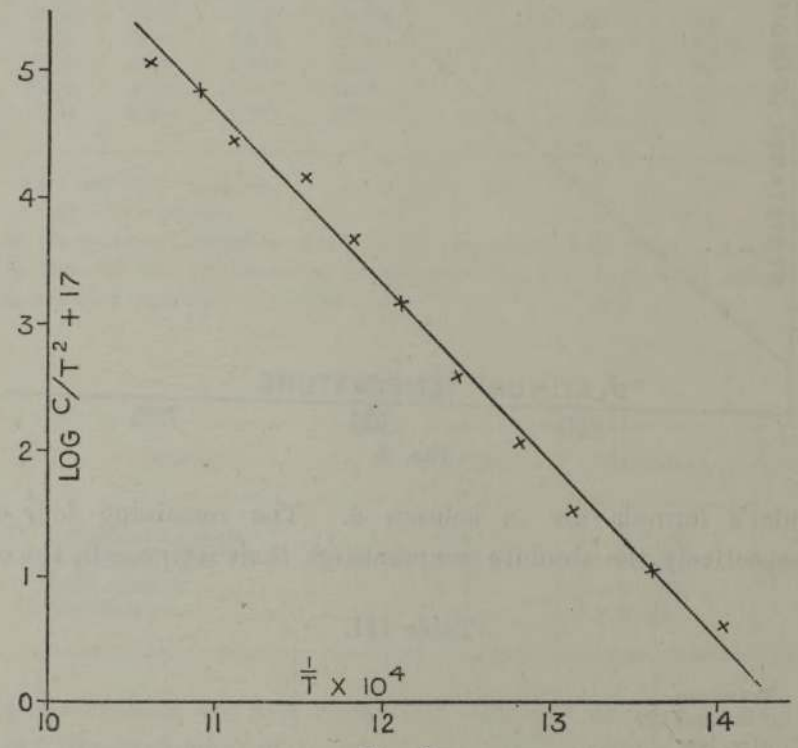


FIG. 9.

Table III ($\lambda$ being taken to be 2). The points, as will be seen, lie very close to a straight line over the whole of the very wide range of the photo-electric currents given in the Table. The logarithm of the largest current exceeds that of the smallest by 4·668 approximately, so that the extreme values of the currents differ by a factor of 46600.

The tangent of the angle, under which the straight line in fig. 9 meets the horizontal (1/T) axis, is to a close approximation 4/3, and therefore, by equation (4)

$$\frac{\phi}{k} = \frac{4 \times 10^4}{3 \times 0{\cdot}4343},$$

where 0·4343 is $\log_{10} e$. The entropy constant $k$ is known to be $1{\cdot}34 \times 10^{-16}$ ergs per degree, so that we get

$$\phi = \frac{4 \times 1{\cdot}34}{3 \times 0{\cdot}4343} \times 10^{-12} \text{ ergs},$$

*i.e.*,
$$\phi = 4{\cdot}02 \times 10^{-12} \text{ ergs},$$

or
$$\phi = 4{\cdot}02 \times 10^{-19} \text{ joules}.$$

For the purpose of comparison with thermionic or other measurements of $\phi$ it will be convenient to give its value in equivalent volts. The charge on an electron is very near

$$1{\cdot}57 \times 10^{-19} \text{ coulombs},$$

and therefore we find for the drop of potential equivalent to $\phi$

$$V = \frac{4{\cdot}02 \times 10^{-19}}{1{\cdot}57 \times 10^{-19}} \text{ joules per coulomb},$$

or
$$V = 2{\cdot}56 \text{ volts}.$$

This result is in satisfactory agreement with the corresponding quantity as determined by Richardson from thermionic measurements on sodium,* his value being 2·65 volts. It should be said, however, that Prof. Richardson considers that his measurements may be subject to considerable errors through lack of saturation and the possible occurrence of chemical effects.

We can deduce the threshold frequency from the value of $\phi$ given above, that is to say, the lowest frequency of the radiation which is capable of causing a photo-electric emission. This is simply effected by the equation

$$\phi = h\nu,$$

which is based on the quantum theory. The quantity $h$ is Planck's constant and may be taken to be

$$6{\cdot}5 \times 10^{-27} \text{ erg} \times \text{sec}.$$

The remaining quantity $\nu$ is the threshold frequency. We have, therefore,

$$\nu = \frac{4{\cdot}02 \times 10^{-12}}{6{\cdot}5 \times 10^{-27}} \text{ sec.}^{-1},$$

or
$$\nu = 6{\cdot}2 \times 10^{-14} \text{ sec.}^{-1}.$$

This corresponds to a wave-length (in air) of

$$\lambda = 485\ \mu\mu$$

approximately. Richardson and Compton† find for sodium

$$\lambda = 587\ \mu\mu \quad \text{or} \quad \nu = 5{\cdot}2 \times 10^{-14} \text{ sec.}^{-1}.$$

* O. W. Richardson, 'Emission of Electrons from Hot Bodies,' p. 69.

† O. W. Richardson and K. T. Compton, 'Phil. Mag.,' vol. 24, p. 585 (1912).

Millikan* finds for the same metal

$$\lambda = 680\,\mu\mu \qquad \text{or} \qquad \nu = 4{\cdot}41 \times 10^{-14}\ \text{sec.}^{-1}.$$

There is no reason to suppose that $\lambda$ is smaller for the Na–K alloy than for sodium. The explanation of the discrepancies in the numbers just given lies in the fact, illustrated by the curves relating to sodium, in a paper by Richardson and Compton,† that $\phi$ (and therefore the threshold frequency) is a function of the state of oxidation of the surface of the metal. The writers just mentioned (*loc. cit.*), although they worked with a good vacuum, noticed a well marked "fatigue." They found, for example, that the photo-electric currents measured $2\frac{1}{2}$ hours after the formation of the fresh sodium surface were considerably smaller than those measured only one hour afterwards. The present writer also noticed the "fatigue" phenomenon, which is, of course, very well known. After the formation of a new alloy surface, the photo-electric current for a given radiator temperature diminished for some time, but soon reached a stationary value, so that the same current could be measured days afterwards, provided the same radiator temperature was employed. Though the alloy surface appeared perfectly clean and bright (and remained so), there is no doubt that, during the time which had to elapse before a series of measurements could be carried out, a very thin film of oxide formed over the surface, so that $\phi$ acquired a somewhat greater value than that corresponding to a perfectly clean surface.

## *Conclusion.*

The law governing the variation of the complete photo-electric emission from the alloy of sodium and potassium, with the temperature of the source of radiation, has been experimentally investigated, and found to be well expressed (as, indeed, theoretical considerations suggest it should be) by Richardson's formula for the temperature variation of the thermionic emission from metals.

I wish to take this opportunity of expressing my thanks to Prof. O. W. Richardson for his valuable advice, and for the interest he has taken in this investigation.

* Millikan, 'Phys. Rev.,' vol. 7, p. 380 (1916).

† O. W. Richardson and K. T. Compton, 'Phil. Mag.,' vol. 26, p. 563 (1913).