The Complexity of Propositional Modal Theories and the Complexity of Consistency of Propositional Modal Theories*

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February 16, 1994

Abstract

This paper is concerned with the computational complexity of the following problems for various modal logics L: (1). The L-deducibility problem: given a finite set of formulas S and a formula A, determine if A is in the modal theory $TH_L(S)$ formed with all theorems of the modal logic L as logical axioms and with all members of S as proper axioms. (2). The L-consistency problem: given a finite set of formulas S, determine if the theory $TH_L(S)$ is consistent.

Table 1 is a comparison of complexity results of these two problems and the corresponding provability and satisfiability problems for modal logics K, T, B, S4, KD45 and S5. The complexity results of the deducibility problem for extensions of K4 are a direct consequence of a modal deduction theorem for K4 (cf. [17, 15]). The NP-completeness of the S4-consistency problem is due to Tiomkin and Kaminski [15].

The main contribution of this paper is that we can show that the deducibility problem and the consistency problem for any modal logic between K and B are EXPTIME-hard; in particular, for K, T and B, both problems are EXPTIME-complete.

Table 1: The complexity of various decision problems for some normal modal logics. Note: all problems are complete for the corresponding complex classes.

Logic(L)	K	T	B	S4	KD45, S5
L-provability	PSPACE	PSPACE	PSPACE	PSPACE	Co-NP
L-satisfiability	PSPACE	PSPACE	PSPACE	PSPACE	NP
L-deducibility	EXPTIME	EXPTIME	EXPTIME	PSPACE	Co-NP
L-consistency	EXPTIME	EXPTIME	EXPTIME	NP	NP

1   Introduction

In the study of logic, there are two kinds of relations that are usually discussed. One is a unary relation consisting of formulas derivable from the logical axioms and inference rules of a logic and is called *the provability relation*; the other is a binary relation, which we call *the deducibility relation* and is concerned with whether a given formula can be derived from the logic with an additional set of formulas given as proper axioms. The provability relation is always a subrelation of the deducibility relation with the given set of proper axioms restricted to empty.

For classical logic, since the deduction theorem holds, if the set of proper axioms is restricted to finite sets, the deducibility relation can be reduced to the provability relation. As a result both relations are inter-reducible and any complexity results about either one apply to the other too.

For modal logic, however, there are at least two deducibility relations that have been defined in the literature [7] depending on whether we admit the rule of necessitation(NEC): "If $S \vdash A$, then $S \vdash \Box A$". For the deducibility relation without the rule NEC, the deduction theorem also holds. Hence its decision problem is reducible to the corresponding provability relation. Consequently, any complexity result about this relation can resort to the better known result of the corresponding provability relation. For this reason, we need only pay our attention to the deducibility relation admitting the rule NEC.

Under the deducibilty relation discussed , we can also define the notion of cosistency. We say that a set of formulas is consistent for a modal logic L (or simply L-consistent) if $\bot$ (falsity) is not derivable from the logic with the set of formulas given as proper axioms.

In this paper we are concerned with the complexity of the deducibility relations and the corresponding consistency relations for some modal logics. We restrict the given set of proper axioms in both relations to finite sets.

In the study of modal logic, traditional logicians seem more interested in the provability relation and its corresponding satisfiability relation; in contrast, the deducibility relation receives less attention. Nevertheless, in the application of modal logics to artificial intelligence, the deducibility relation has been found to be fundamental for nonmonotonic reasoning [12, 15, 14]. In fact, our study of the complexity of these

relations is motivated by Moore's autoepistemic logic (AEL)[13] and McDermott's Non-monotonic modal logic (NML) [12], each suggesting a fixed point semantics and the corresponding fixed point operator being defined in terms of the deducibility relations in some modal logics.

Table 1 is a comparison of the complexities of these two problems and the corresponding provability and satisfiability problems for modal logics K, T, B, S4, KD45 and S5. The complexity results of the deducibility problem for extensions of K4 are a direct consequence of a modal deduction theorem for K4 (cf [17, 15]). The NP-completeness of the S4-consistency problem is due to Tiomkin and Kaminski [15].

The main results of this paper is that we can show that the deducibility problem and the consistency problem for any modal logic between K and B are EXPTIME-hard; in particular, for K, T and B, both problems are EXPTIME-complete.

The rest of this paper is organized as follows. In Section 2 we define the deducibility and the consistency relations and their semantics. Section 3 summarizes all complexity results. Section 4 presents the upper bound result by showing that the deducibility relation for K, T and B is in EXPTIME. Finally, in Section 5 we presents the lower bound result by showing that the consistency relation for any normal logic between K and B is EXPTIME-hard.

2   Preliminaries

All modal logics considered in the paper share a common language whose alphabet Γ includes : (1). a denumerable set PV of propositional variables: $p, q, r \cdots$ and (2). the five symbols: (,), $\neg$, $\wedge$, $\square$. The set of modal formulas MF is defined to be the least set of words over Γ including PV such that if A and B are members of MF, then so are $(A \wedge B)$, $\neg A$ and $\square A$. The other connectives such as $\vee$ (or), $\supset$ (implication), $\diamond$ (possibility), $\perp$ (falsity) as treated as abbreviations in the standard way.

By a *(modal) logic L* we mean any subset of MF that contains all tautologies and is closed under modus ponens (MP) and uniform substitution (US). So the classical propositional logic (in MF) is the smallest modal logic, while the set of all modal formulas is the largest modal logic.

We say that a modal logic L is *normal* if it contains the formula $K : \square(p \supset q) \supset (\square p \supset \square q)$ and is closed under necessitation(NEC): $\square A \in L$ whenever $A \in L$. The smallest normal modal logic is called K.

There are famous normal modal logics that can be defined as smallest normal logics containing parts of the following formulas:

D: $\square p \supset \diamond p$.

T: $\square p \supset p$.

B: $p \supset \square \diamond p$.

4: $\square p \supset \square \square p$.

5: $\square p \supset \diamond \square p$.

As adopted by Chellas[2], we use $Ka_1 \cdots a_n$, where $n \geq 0$ and each $a_i \in \{D, T, B, 4, 5\}$, to denote the smallest normal modal logic containing all logical axioms a_i's. According to such convention, the five famous normal modal logics more usually called D, T, B, $S4$ and $S5$ are in fact equal to KD, KT, KTB, $KT4$ and $KT45$, respectively. In what follows let $\mathcal{M}$ denote the set of logics $\{L \mid L = Ka_1 \cdots a_n, n \geq 0$ and each $a_i \in \{D, T, B, 4, 5\}\}$.

Each member of a modal logic is called a *theorem* of the logic. We use the relational notation $\vdash_L A$ to stand for the fact that $A \in L$. In [11], $\vdash_L$ is called the provability relation for L (or simply the L-provability relation). For any logics L_1, L_2, we say L_2 is an *extension* of L_1 (and L_1 is a *sublogic* of L_2) iff L_1 is a subset of L_2. We say another logic L is between L_1 and L_2 if $L_1 \subseteq L \subseteq L_2$. Therefore, the normal logics D, T, and $K4$ are between K and $S4$; D, T, B and $S4$ are all extensions of K.

Let L be any modal logic, the L-deducibility relation, with some abuse of terminology, also denoted $\vdash_L$, is defined to the the smallest subset of $2^{MF} \times MF$ closed under the following rules:

1. $S \vdash_L A$ whenever $A \in S$ or A is a theorem of L (Axioms).

2. $S \vdash_L B$ if $S \vdash_L A$ and $S \vdash_L A \supset B$ (MP).

3. $S \vdash_L \Box A$ if $S \vdash_L A$ (NEC).

The *L-theory* of S, denoted $TH_L(S)$, is the set $\{A \in MF \mid S \vdash_L A\}$. If $S \vdash_L A$, we say A is deducible from S under L. Each formula in S is called a proper axiom of the theory.

We say that a set of formulas S is *L-consistent* if $\bot$ (falsity) $\notin TH_L(S)$.

2.1   Semantics

The semantics of many modal logics can be defined by *Kripke model*. A (Kripke) model M is a triple $< W, R, h >$ where

- W is a nonempty set (of *worlds*),

- R is a binary relation on W called the *accessibility relation* of M; if $(w, w') \in R$, we say w' is *accessible* from w. The pair (W, R) is called the *frame* of M.

- $h \in W \to 2^{PV}$ is the valuation function, which assigns to each world w in W a subset $h(w)$ of PV with the intention that p is true at world w iff $p \in h(w)$.

Given any Kripke model $M =< W, R, h >$, any world $w \in W$, and any formula $A \in MF$, the truth definition of A at w of M, denoted $M, w \models A$, is defined inductively as follows.

- $M, w \models p$ iff $p \in h(w)$

- $M, w \models \neg A$ iff $M, w \not\models A$

- $M, w \models A \wedge B$ iff $M, w \models A$ and $M, w \models B$.

- $M, w \models \Box A$ iff for every $w' \in W$ accessible from w, $M, w' \models A$.

We say A is M-*satisfiable* if there is a world w in W such that $M, w \models A$ and say A is M-*valid*, denoted $M \models A$, if $M, w \models A$ for every world w in W.

We are particularly interested in Kripke models whose accessibility relations R satisfy parts of the following conditions:

serial(D): for any $w \in W$, there is a w' in W such that wRw'.

reflexive(T); for any $w \in W$, wRw.

symmetric(B): for any $w, w' \in W$, if wRw' then $w'Rw$.

transitive(4): for any $w, w', w'' \in W$, if wRw' and $w'Rw''$, then wRw''.

euclidean(5): for any $w, w', w'' \in W$, if wRw' and wRw'', then $w'Rw''$.

With some abuse of terminology, we use $Kr_1 \cdots r_n$, where ($n \geq 0$) and each $r_i \in \{D, T, 4, 5, B, H\}$, to denote also the class of all kripke models whose accessibility relations satisfy all r_i's. Each member of the class $Kr_1 \cdots r_n$ of models is called a $Kr_1 \cdots r_n$-model. Analogous to modal logics, we also use T, D, B, S4 and S5 to denote the classes of KT, KD, KTB, KT4 and KT45 models, respectively.

Let L be any class of models. We say that a formula A is (locally) *L-satisfiable* if there exists an L-model M and a world w in W such that $M, w \models A$; we say A is *globally L-satisfiable* if there is an L-model in which A is valid. We say that A is *L-valid*, denoted $\models_L A$, if A is valid in every L-model.

By treating every (finite) set of formula as an abbreviation of the conjunction of all its members, we extend the definitions of previously defined notions like satisfiability, validity, etc. to sets of formulas in the obvious way. So, for example, $M, w \models S$ iff $M, w \models A$ for every $A \in S$.

The following well-known proposition establishes the equivalence of the semantical validity relation and the syntactic provability relation for each modal logic given here.

Proposition 1 *[2] Let L be any logic in $\mathcal{M}$. Then for any modal formula A, A is L-provable iff A is L-valid.*

For each normal logic $L \in \mathcal{M}$, Kripke's model can also supply suitable semantics for the L-deducibility relation and the L-consistency relation. Given a set of formula S and a formula A, we say that A is a global L-consequence of S if for all L-models M, if every formula of S is valid in M, then so is A. D. McDermott [12] has shown that for $L \in \{T, S4, S5\}$, the deducibility relation and the consequence relation are identical. The following proposition is a simple generalization of his result.

Proposition 2 *Let L be any logic in $\mathcal{M}$. Then for any set of formula S and for any formula A, $S \vdash_L A$ iff A is a global L-consequence of S.*

The following is a corollary of Proposition 2.

Proposition 3 *Let L be any logic in $\mathcal{M}$. Then for any set of formula S, S is L-consistent iff S is globally L-satisfiable.*

3 Complexity results

We now begin to study the complexity of the deducibility relation and the consistency relation for the logics K, T, B, S4, KD45 and S5. The results, together with the complexity of other related decision problems, are listed in Table 1. The complexity results of the provability and the satisfiability problems for these logics can be found in [11, 8, 3].

For all extensions of K4 we have the following modal deduction theorem.

Proposition 4 *[7, 15, 17] Let L be any extension of $K4$. Then $S \vdash_L A$ iff $\vdash_L (\bigwedge S \wedge \bigwedge \{\Box B \mid B \in S\}) \supset A$.*

According to Proposition 4, for every extension of $K4$, the deducibility relation and the unary provability relation are inter-reducible and hence are in the same complexity class. Since the provability relation for S4 is PSPACE-complete [11] and the provability relation for KD45 and S5 [11, 8] is CoNP-complete, we thus have the following corollary.

Corollary 1 *1. The S4-deducibility relation is PSPACE-complete.*

2. The KD45-deducibility relation and the S5-deducibility relation are CoNP-complete.

That the L-consistency relation for every modal logic L is NP-hard is rather obvious since every Boolean formula(modal formula without modality) is satisfiable for classical propositional logic if and only if it is L-consistent. Hence the consistency relations for KD45 and S5, like their corresponding satisfiability problems, are NP-complete.

We might guess that for S4 the consistency problem has the same complexity as the satisfiability problem. But in fact the guess is incorrect. The following result is due to Tiomkin and Kaminski [15].

Lemma 2 *[15] Let S be any set of modal formulas. Then S is S4-consistent iff S is S5-consistent.*

Corollary 3 *The consistency relation for any of S4, KD45 and S5 is NP-complete.*

Compared with the satisfiability problem, for S4 the consistency relation becomes easier to decide. On the contrary, for all normal logics between K and B, we will show in Section 5 that the consistency relation is EXPTIME-hard; hence for K, T and B the global consistency relation is harder than the satisfiability relation, which has been shown to be PSPACE-complete.

4 The deducibility relations for K, T and B are in EXPTIME

In this section we show that the global deducibility relations (and the global consistency relations) for the three modal logics K, T and B are all in EXPTIME. The result in

fact also holds for many other logics between K and B, such as D, KB, OM, OM^+, OB, OB^+ [16] etc. For simplicity, we choose here only K, T and B as representatives.

To show that the deducibility relation is in EXPTIME, our strategy is to use the semantical notion and show that the global consequence relations for K, T and B can all be reduced to the unsatisfiability problem of propositional dynamic logic with converse (CPDL). Since the latter problem is known to be EXPTIME-complete [9], the deducibility relations thus are in EXPTIME as well. For an understanding of this section, we assume the readers are familiar with propositional dynamic logic with converse. Readers who are unfamiliar with CPDL are referred to Harel[9]

Translations of modal formulas into CPDL formulas

Let L be any of K, T and B. Let a be a fixed atomic program. Define the CPDL programs pr_L as follows.

1. $pr_K = a$,

2. $pr_T = a \cup \top?$,

3. $pr_B = a \cup a^- \cup \top?$.

For each modal formula A, define the CPDL formula

$$Tr_L(A) = A(\Box \leftarrow [pr_L]),$$

where $A(\Box \leftarrow [pr_L])$ is the CPDL formula obtained from A with every occurrence of the $\Box$ connective in A replaced by the modality "$[pr_L]$".

For each pair (S, A) where S is a finite set of modal formulas and A is a modal formula, now define three CPDL formulas

1. $f_K(S, A) = ([a^*]Tr_K(\bigwedge S)) \wedge Tr_K(\neg A)$,

2. $f_T(S, A) = ([a^*]Tr_T(\bigwedge S)) \wedge Tr_T(\neg A)$,

3. $f_B(S, A) = ([(a \cup a^- \cup \top?)^*]Tr_B(\bigwedge S)) \wedge Tr_B(\neg A)$.

Intuitively, the formula $f_B(S, A)$ say that there is a CPDL structure M and there is a world w at which A is false and at every world reachable from which S is true.

Lemma 4 *[3] Let L is any of K, T and B, S a finite set of modal formulas and A a modal formula. Then A is not an L-consequence of S iff $f_L(S, A)$ is satisfiable for CPDL.*

It is easy to see that $f_L(S, A)$ can be obtained from S and A in polynominal time; moreover, since the unsatisfiability problem for CPDL is known to be in EXPTIME [9]. We thus can have the following corollary.

Corollary 5 *For $L \in \{K, T, B\}$, the L-deducibility relation and the L-consistency relation are in EXPTIME.*

5   EXPTIME-hardness of the consistency relations for logics between K and B

We now turn to the derivation of lower bound results.

Theorem 1 *For any modal logic L between $\tilde{K}$ and B, the L-consistency relation is EXPTIME-hard.*

Our method is borrowed and modified from that of Fischer and Ladner[5] for showing that the satisfiability problem of propositional dynamic logic (PDL) is EXPTIME-hard. The method is to reduce any language accepted by an alternating Turing machine operating in polynominal space to the L-consistency relation for any L between K and B. Since APSPACE = EXPTIME [1], the L-consistency relation thus is EXPTIME-hard. Similar methods have been used for showing EXPTIME-hardness of many intensional logics [4, 8, 6].

Let $\mathcal{L}$ be any language accepted by an alternating Turing machine TM which operates in space $s(n)$, where s is a polynominal,

Without loss of generality, assume that $s(n)$ is computable in time polynominal of n and that any computation path of TM on any input never repeats a configuration. See Fischer and Ladner[5] for detail arguments.

Let $TM = (Q, \Gamma, \Sigma, \#, \delta, q_0, U)$ be a one-tape alternating Turing machine, where

- Q is the set of states,

- Γ is the tape alphabet,

- $\Sigma \subset \Gamma$ is the input alphabet,

- $\# \in \Gamma - \Sigma$ is the blank symbol,

- $\delta \subset (Q \times \Gamma) \times (Q \times \Gamma \times \{L, R\})$ is the next move relation,

- $U \subset Q$ is the set of universal states,

- $Q - U$ is the set of existential states, and

- q_0 is the initial state.

Let x be any input string of length n. Assume $s(n) = m$. Since TM never runs outside the first m tape cells, every configuration $x'qy'$ of TM possibly reachable from the initial configuration $q_0 x$ can be represented by the triple (q, i, z), where i is the position (counted from 1) of the tape cell pointed to by the tape head and $z = \#x'y'\#^{(m+1-|x'|-|y'|)}$ is a string of length $m + 2$ storing not only the nonblank part of the configuration but also all tailing blanks up to the $(m + 1)$-th tape cell in the configuration; moreover, a blank is inserted at the front serving as end marker. Now define CF^m to be the set of (representations of) all configurations of TM of length at most m, i.e., $CF^m = \{(q, i, x) \mid q \in Q, 0 \leq i \leq m + 1, |x| \in \#\Gamma^m\#\}$.

We construct a set of modal formulas $S_{TM(x)}$ using propositional variables from $PV_{TM(x)}$, which is given as follows.

1. $PV_{TM(x)}$ consists of the following propositional variables:

- *init*, which is used to identify the initial configuration. *init* has the effect of preventing all predictions about successor configurations from being enforced to the initial configuration.

- $C_{i,a}$ where $0 \leq i \leq m+1$ and $a \in \Gamma$, which is used to represent the tape contents of current configuration so that $C_{i,a}$ is true iff the ith tape cell contains a.

- H_i, where $0 \leq i \leq m+1$, which is used to represent the head position so that H_i is *ture* iff the current tape head points to position i.

- Q_q, where $q \in Q$, which is used to represent the state of the current configuration.

- L_i, where $0 \leq i \leq 2$, which is used to represent the value j mod 3 where j is the length of the computation path from the initial configuration to current configuration. L_i has the effect of preventing all predictions about successor configurations from being enforced to predecessor configurations the current configuration.

2. $S_{TM(x)}$ is the union of the following sets.

- $S_1 = \{ \bigvee_{q \in Q} (Q_q \wedge \bigwedge_{q' \in Q - \{q\}} \neg Q_{q'}) \}$.

  S_1 is the state constraint which says that each configuration has exactly one state.

- $S_2 = \bigcup_{0 \leq i \leq m+1} \{ \bigvee_{a \in \Gamma} (C_{i,a} \wedge \bigwedge_{a' \in \Gamma - \{a\}} \neg C_{i,a'}) \} \cup \{ C_{0,\#}, C_{m+1,\#} \}$

  S_2 says that each tape cell contains a unique symbol in Γ and the initial and the final cells must contain the blank symbol.

- $S_3 = \{ \neg H_0, \neg H_{m+1} \} \cup \{ \bigvee_{1 \leq i \leq m} (H_i \wedge \bigwedge_{1 \leq j \leq m, j \neq i} \neg H_j) \}$

  S_3 says that the tape head must point to exactly one position in $[1, m]$.

- $S_4 = \{ \neg H_i \wedge C_{i,a} \wedge L_j \supset \Box(L_{j+1 \bmod 3} \wedge \neg init \supset C_{i,a}) \mid 0 \leq i \leq m+1, a \in \Gamma, 0 \leq j \leq 2 \}$

  S_4 says that all tape cells not pointed to by tape head remain unchanged in the next configuration. Note that the $L_{j+1 \bmod 3} \wedge \neg init$ appearing inside the box $\Box$ has the effect of preventing the prediction that $C_{i,a}$ must be true from being enforced to the initial configuration, in which *init* must be true, and the current and the predecessor configurations in which $L_{j+1 \bmod 3}$ must be false.

- $S_5 = \{ H_i \wedge C_{i,a} \wedge Q_q \wedge L_j \supset \bigwedge_{(q,a,q',a',R) \in \delta} \Diamond (H_{i+1} \wedge C_{i,a'} \wedge Q_{q'} \wedge L_{j+1 \bmod 3} \wedge \neg init) \wedge$
  $\bigwedge_{(q,a,q',a',L) \in \delta} \Diamond (H_{i-1} \wedge C_{i,a'} \wedge Q_{q'} \wedge L_{j+1 \bmod 3} \wedge \neg init)$
  $\mid 1 \leq i \leq m, a \in \Gamma, q \in U$ and $0 \leq j \leq 2 \}$

  S_4 and S_5 encode the next-move relation for universal states. It describes how the current configuration moves to successor configurations and says that

the current configuration is accepted if all its successors are accepted. Note that the empty conjunction is defined to be ⊤. The $L_{j+1 \bmod 3}$'s appearing at the right hand side of the implication say that if the length modulo 3 of the path from the initial configuration to current configuration is j, then the length modulo 3 of the path from the initial configuration to every successor configuration is $j + 1 \bmod 3$. The "¬$init$"s appearing at the right hand side of the implication indicate that no successor configuration is the initial configuration.

- $S_6 = \{H_i \wedge C_{i,a} \wedge Q_q \wedge L_j \supset \bigvee_{(q,a,q',a',R) \in \delta} \Diamond (H_{i+1} \wedge C_{i,a'} \wedge Q_{q'} \wedge L_{j+1 \bmod 3} \wedge \neg init) \vee \bigvee_{(q,a,q',a',L) \in \delta} \Diamond (H_{i-1} \wedge C_{i,a'} \wedge Q_{q'} \wedge L_{j+1 \bmod 3} \wedge \neg init)$
  $\mid 1 \le i \le m, a \in \Gamma, q \in Q - U$ and $0 \le j \le 2\}$

S_4 and S_6 encode the next-move relation for existential states. Note that the empty disjunction is defined to be ⊥.

- Let $A_{TM(x)}$ denote the encoding of the initial configuration:

$$Q_{q_0} \wedge H_1 \wedge L_0 \wedge C_{0,\#} \wedge \bigwedge_{1 \le i \le n} C_{i,x_i} \wedge \bigwedge_{n+1 \le i \le m+1} C_{i,\#}.$$

Then let $S_0 = \{\Diamond init, init \supset A_{TM(x)}\}$.

S_0 says firstly that there must exist a world at which $init$ is true, and secondly that whenever $init$ is true at a world, it is a world corresponding to the initial configuration. Note that $\Diamond init$ has the side effect of making accessible from each world a world corresponding to the initial configuration.

The following proposition is easy to see.

Proposition 5 $S_{TM(x)}$ can be obtained from (some encoding of) TM and x in polynominal time.

With the above explanation it is not hard to prove the following two lemmas, whose detail proofs can be found in Chen[3].

Lemma 6 [3] If $S_{TM(x)}$ is globally K-satisfiable, then x is accepted by TM.

Lemma 7 [3] If x is accepted by TM then $S_{TM(x)}$ is globally B-satisfiable.

We can now prove the main theorem of this section.

Proof. (of Theorem 1.) Let L be any modal logic between K and B. Let $\mathcal{L}$ be any language in EXPTIME accepted by an alternating TM which operates in polynominal space $s(\cdot)$. Let x be any input string. If x is accepted by TM, then by Lemma 7 and Proposition 2, $S_{TM(x)}$ is globally B-consistent. Hence it is L-consistent. On the other hand, if x is not accepted by TM, by Lemma 6 and Proposition 2, $S_{TM(x)}$ is not K-consistent. Hence it is not L-consistent. As a result, every language $\mathcal{L}$ in EXPTIME is polynominal-time reducible to the global L-consistency relation via the polynominal

time transformation $(TM, x) \rightarrow S_{TM(x)}$. The L-consistency relation thus is EXPTIME-hard. ⋈

Theorem 2 *For $L \in \{K, T, B\}$, the L-consistency relation and the L-deducibility relation are EXPTIME-complete.*

Proof. Direct consequence of Theorem 1 and Corollary 5 and the fact that EXPTIME is closed under complementation. ⋈

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