Mathematical Surveys and Monographs

Volume 89

## The Concentration of Measure Phenomenon

**Michel Ledoux** 



**American Mathematical Society** 

## CONTENTS

~

| INTRODUCTION  | vii  |
|---|--|
| <ol> <li>CONCENTRATION FUNCTIONS<br/>AND INEQUALITIES</li> <li>1.1 First examples</li> <li>1.2 Concentration functions</li> <li>1.3 Deviation inequalities</li> <li>1.4 Observable diameter</li> <li>1.5 Expansion coefficient</li> <li>1.6 Laplace bounds and infimum-convolutions</li> <li>Notes and Remarks</li> </ol> | $ \begin{array}{c} 1\\ 1\\ 3\\ 5\\ 14\\ 15\\ 16\\ 21\\ \end{array} $ |
| <ol> <li>ISOPERIMETRIC AND FUNCTIONAL EXAMPLES</li> <li>Isoperimetric examples</li> <li>Brunn-Minkowski inequalities</li> <li>Semigroup tools</li> <li>Notes and Remarks</li> </ol>   | 23<br>23<br>32<br>38<br>44   |
| <ol> <li>CONCENTRATION AND GEOMETRY</li> <li>Spectrum and concentration</li> <li>Spectral and diameter bounds</li> <li>Lévy families</li> <li>4 Topological applications</li> <li>Euclidean sections of convex bodies</li> <li>Notes and Remarks</li> </ol>   | $47 \\ 47 \\ 53 \\ 55 \\ 57 \\ 60 \\ 65$                             |
| <ul> <li>4. CONCENTRATION IN PRODUCT SPACES</li> <li>4.1 Martingale methods</li> <li>4.2 Convex hull approximation</li> <li>4.3 Control by several points</li> <li>4.4 Convex infimum-convolution</li> <li>4.5 The exponential distribution</li> <li>Notes and Remarks</li> </ul>   | 67<br>67<br>72<br>79<br>82<br>83<br>89                               |
| <ul> <li>5. ENTROPY AND CONCENTRATION</li> <li>5.1 Logarithmic Sobolev inequalities and concentration</li> <li>5.2 Product measures</li> <li>5.3 Modified logarithmic Sobolev inequalities</li> <li>5.4 Discrete settings</li> </ul>  | 91<br>91<br>97<br>101<br>108   |

| 5.5 Covariance identities<br>Notes and Remarks          | $114 \\ 115$ |
|---|--------------|
|   |              |
| 6. TRANSPORTATION COST INEQUALITIES                     | 117          |
| 6.1 Information inequalities and concentration          | 117          |
| 6.2 Quadratic transportation cost inequalities          | 122          |
| 6.3 Transportation for product and non-product measures | 126          |
| Notes and Remarks                                       | 132          |
| 7. SHARP BOUNDS ON GAUSSIAN                             |              |
| AND EMPIRICAL PROCESSES                                 | 133          |
| 7.1 Gaussian processes                                  | 133          |
| 7.2 Bounds on empirical processes                       | 138          |
| 7.3 Sharper bounds via the entropic method              | 142          |
| Notes and Remarks                                       | 149          |
| 8. SELECTED APPLICATIONS                                | 151          |
| 8.1 Concentration of harmonic measures                  | 151          |
| 8.2 Concentration for independent permutations          | 155          |
| 8.3 Subsequences, percolation, assignment               | 159          |
| 8.4 The spin glass free energy                          | 163          |
| 8.5 Concentration of random matrices                    | 167          |
| Notes and Remarks                                       | 170          |
| REFERENCES  | 171          |
| INDEX   | 181          |